Computation of the Closed Line Traverse

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This syllabus explains the computation of the closed line traverse line according to the Hungarian regulations. Let's take the traverse line seen on Figure 1 and compute the coordinates of the traverse points 1-3. In this case we are interested in the horizontal coordinates only, therefore only Easting and Northing coordinates are computed.

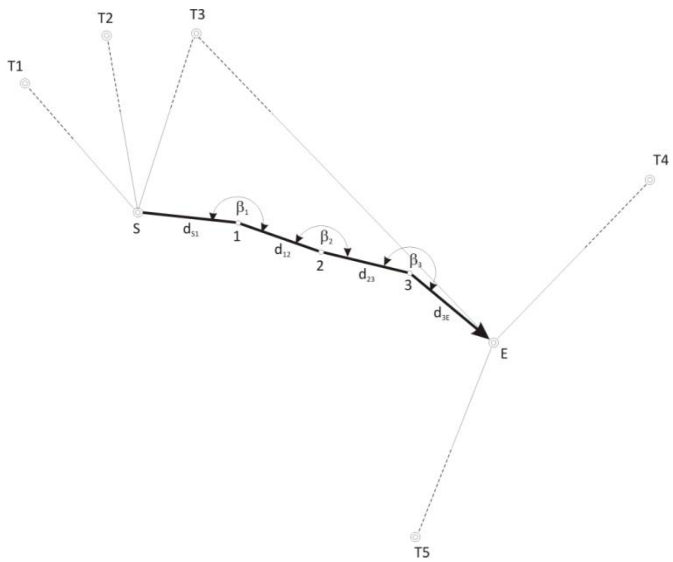

Figure 1. The closed line traverse

The coordinates found in Table 1 as well as the results of directional and distance observations found in Table 2 can be used for the computations. Please note that stations S and E are the starting and the terminating point of the traverse line, while points T1, T2, T3, T4 and T5 are targets used for the computation of the orientation.

Station	E(asting)	N(orthing)
S	629558.31	184686.23
E	629835.08	184353.73
T1	629413.64	184827.49
T2	629520.96	184917.46
T3	629657.71	184919.04
T4	630137.90	184635.48
T5	629746.20	184171.56

Table 1. The coordinates of the existing stations (values are in metres)

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Station	Target	Mean Direction			Orientation angle			whole circle bearing /deflection angle			dist
		°	′	″	°	′	″	°	′	″	
S	T1	307	42	54							
	T2	344	13	49							
	1	108	54	51							125.19
	T3	16	31	57							
1	S	25	39	31							125.21
	2	225	30	07							93.73
2	1	325	31	31							93.75
	3	145	40	11							100.85
3	2	241	33	05							100.87
	E	95	02	28							142.64
E	T3	331	33	54							
	T4	36	02	46							
	T5	194	59	32							
	3	337	58	00							142.66

Table 2. The results of the directional and distance observations.

Computing the Whole Circle Bearing (WCB) of the S-1 ans E-3 directions

Firstly the WCB of the first traverse leg (in this case the S1 section) and the last traverse leg (E3 section) should be computed. In order to compute the WCB, the mean orientation angle must be computed, since 3-3 orientations have been observed at both of the stations (S and E).

The principle of the computation of the orientation angles and the WCBs from the directional observations can be seen on Figure 2. Let’s see the observations at the station S:

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Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54							
	T2	344	13	49							
	1	108	54	51							125.19
	T3	16	31	57							

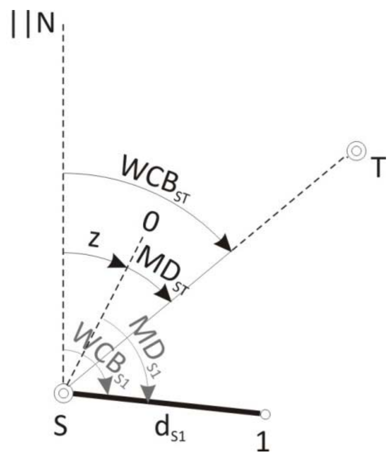

Figure 2. The computation of the orientation angle

In order to compute the orientation angles from the observations to T1, T2 and T3 respectively, the respective WCBs and distances must be computed. This can be achieved by the 2nd fundamental task of surveying. Let's take the data of the S-T1 direction:

Coordinates of station (S): 629558.31 184686.23

Coordinates of target (T1): 629413.64 184827.49

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The coordinate differences: ΔE= -144.67 m ΔN= 141.26 m

Please note *that the coordinate differences are always computed as the difference between the target and the stations coordinates!* $\Delta E = E_{Target} - E_{Station}$.

From the coordinate differences, the WCB and the horizontal distances can be computed:

$$\alpha = atan\frac{\Delta E}{\Delta N} = atan\frac{-144.67}{141.26} = -45°41'00''$$

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According to the sign of the coordinate differences the appropriate quadrant can be identified, where the target is located. This also defines the appropriate correction between the α angle and the WCB. The decision matrix can be found in Table 3.

Quadrant	ΔE	ΔN	c
I	+	+	0
II	+	-	+180
III	-	-	+180
IV	-	+	+360
WCB=α+c			

Table 3. The decision matrix of the computation of WCB

Since in our case the ΔE is negative and ΔN is positive, therefore the target is located in the 4th quadrant. Thus the WCB is computed as α+360. Therefore the WCB_{S-T1} is:

$$WCB_{S-T1}=314°19'00''$$

On the other hand the horizontal distance between the station and the target can be computed using the Pythagoras' theorem:

$$d_{S-T1} = \sqrt{(\Delta E)^2 + (\Delta N)^2} = 202.197m$$

Using the same approach all the WCBs and distances can be computed for the orientations. Please note that one can identify the targets used for the orientation by comparing the directional observations and the coordinate lists. In case of orientations, both the station and the target must have given coordinates, and the mean directions must be observed as well.

After computing the whole circle bearings and the distances, the field book should appear like this:

Station	Target	Mean Direction °	'	''	Orientation angle °	'	''	whole circle bearing /deflection angle °	'	''	dist
S	T1	307	42	54				314	19	00	202.197
	T2	344	13	49				350	49	28	234.227
	1	108	54	51							125.19
	T3	16	31	57				23	07	14	253.142

In the next step, the orientation angles should be computed. The orientation angle is the WCB of the index of the horizontal circle (marked with 0 on Fig. 2). Thus it can be computed as the difference between the WCB and the observed mean direction:

$$z_{S-T1} = WCB_{S-T1} - MD_{S-T1} = 6°36'06''.$$

With the same approach all the orientation angles can be computed. Now the fieldbook looks like this:

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Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51							125.19
	T3	16	31	57	6	35	17	23	07	14	253.142

Since we have three different orientation angles, we should compute the mean value of them. The mean orientation angle is computed as a weighted mean of the individual orientation angles, where the weight is the distance between the station and the target. Thus:

$$\bar{z} = \frac{z_{S-T1}d_{S-T1} + z_{S-T2}d_{S-T2} + z_{S-T3}d_{S-T3}}{d_{S-T1} + d_{S-T2} + d_{S-T3}} = 6°35'37''$$

Let's write this value to the row of target 1 as an orientation angle!

Finally the WCB_{S1} should be computed (the WCB of the first traverse leg):

$$WCB_{S1} = \bar{z} + MD_{S1} = 115°30'28''$$

Now the fieldbook should look like this:

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51	**6**	**35**	**37**	**115**	**30**	**28**	125.19
	T3	16	31	57	6	35	17	23	07	14	253.142

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Let's follow the same approach with the observations at the station E! The results should look like this:

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
E	T3	331	33	54	11	00	55	342	34	49	592.483
	T4	36	02	46	11	01	05	47	03	51	413.622
	T5	194	59	32	11	00	55	206	00	27	202.696
	3	337	58	00	**11**	**00**	**58**	**348**	**58**	**58**	142.66

Remarks:

1. *Please note that the orientation angles should be sufficiently close to each other. Usually the maximal difference is less than 1′. When you experience larger differences, please check your computations.*

2. *Another useful check: the mean orientation angle must be between the smallest and the largest orientation angles.*

3. *The mean orientation angles can be computed according to the following method, too:*

Subtract the smallest orientation angle cut to a full degree and minute value from all of the orientation angles (in our case, subtract 6°35′00′′ from all of the orientation angles at the station S). Compute the weighted mean of the residual values:

$$\bar{r} = \frac{66'' \cdot 0.2 + 39'' \cdot 0.2 + 17'' \cdot 0.3}{0.3} = 37''$$

Finally add the subtracted value to this mean to get the mean orientation angle:

$$\bar{z} = 6°35'00'' + 37'' = 6°35'37''$$

Computing the deflection angles:

The deflection angles are computed as the difference between the forward mean direction and the backward mean direction. Please note that the deflection angles are the angles between the traverse legs on the left side of the traverse line.

Thus the deflection angle at the station 1 is:

$$\beta_1 = (225°30'07'') - (25°39'31'') = 199°50'36''$$

Computation of Closed Line Traverse

Hint: *It is always a good idea to draw a sketch of the traverse line, because the order of the observations in the field book might differ from the order of the traverse points.*

The completed fieldbook will look like Table 4.

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51	**6**	**35**	**37**	**115**	**30**	**28**	125.19
	T3	16	31	57	6	35	17	23	07	14	253.142
1	S	25	39	31							125.21
	2	225	30	07				199	50	36	93.73
2	1	325	31	31							93.75
	3	145	40	11				180	08	40	100.85
3	2	241	33	05							100.87
	E	95	02	28				213	29	23	142.64
E	T3	331	33	54	11	00	55	342	34	49	592.483
	T4	36	02	46	11	01	05	47	03	51	413.622
	T5	194	59	32	11	00	55	206	00	27	202.696
	3	337	58	00	**11**	**00**	**58**	**348**	**58**	**58**	142.66

Table 4. The completed fieldbook of the traverse observations (including WCBs and deflection angles)

Computing the deflection angles of the first (S) and the last station (E)

For the first and the last stations, the deflection angles are computed according to the following:

$\beta_S = WCB_{S1}$ and $\beta_E = 360° - WCB_{E3}$.

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Please note that the deflection angle at the last station is the explementary angle (or conjugate angle) of the WCB of the last traverse leg.

Computation of the traverse line

In order to compute the traverse line, the traverse computation form should be used. Let's discuss the notations in this form:

- Point ID: is the ID of the stations found in the traverse line
- distance: the horizontal (grid) distance between the consecutive stations (if it ismeasured both forwards and backwards, then we have to take the average of them!)
- WCB: the whole circle bearing of the consecutive traverse legs
- β: the computed deflection angle at the stations
- $v\beta$: the corrections of the observed deflection angles (when applicable)
- (ΔE): preliminary coordinate difference in the Easting direction between the consecutive stations
- (ΔN): preliminary coordinate difference in the Northing direction between the consecutive stations
- $v\Delta E$: correction of the preliminary Easting coordinate differences between the consecutive stations
- $v\Delta N$: correction of the preliminary Northing coordinate differences between the consecutive stations
- ΔE: the corrected (final) coordinate difference in the Easting direction between the consecutive stations
- ΔN: the corrected (final) coordinate difference in the Northing direction between the consecutive stations
- E: the Easting coordinate of the station
- N: the Northing coordinate of the station

Let's start to fill the form! Firstly the Station IDs should be written to the consecutive rows. Then the distances (length of the traverse legs) should be filled. Please note that the distances should be written to the row of the station, whose coordinates will be computed using the distance. Thus the first distance comes to the row of station 1 (and not station S!).

Afterwards the deflection angles should be written to the appropriate rows (each station will have a separate deflection angle computed in the fieldbook of the traverse observations.

After these steps the traverse computation form can be seen in Table 5.

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Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
S		115	30	28				
1	125.200	199	50	36				
2	93.740	180	08	40				
3	100.860	213	29	23				
E	142.650	11	01	02				
	Σβ							
	(n-1)180							
	Δβ							

Table 5. Copying the distance observations and deflection angles into the traverse computation form.

The next step is to check the angular criteria. The sum of the deflection angles should be equal to:

$$\sum \beta = (n-1)180°$$

where *n* is the number of the traverse points (in this case *n*=5)

Let's sum up all the deflection angles, and write the sum to the row of Σβ:

$$\sum \beta = 720°00'09''$$

Since the Σβ should be equal to 720°, therefore the angular misclosure (Δβ) can be computed:

$$\Delta\beta = -9''$$

The angular misclosure is the sum of the error in the angular observations. Since the accuracy of the deflection angles is supposed to be the same, therefore each deflection angle should get the same correction rounded to 1 arcsecond.

Let's compute the vβ corrections:

$$v\beta = \frac{\Delta\beta}{n} = -1.8''$$

Hints:

Please note that the sum of the deflection angles should be equal to (n-1)180° (+360°). The extra 360° should be added when the traverse line goes from the east to the west.

Please note that the sum of the corrections must be equal to the angular misclosure as well. Therefore some of the $v\beta$ correction should be rounded upward, and some should be rounded downward. For example when the angular misclosure is 11'' and n=5 then the $v\beta$ correction should be 2.2''. Since this value should be rounded to 1 arcsecond, therefore some $v\beta$ values should be rounded to 2'' and some should be rounded to 3''. Since the sum of the corrections must be equal to the angular misclosure (11'') and n=5 therefore the corrections will be 2'' in four cases and 3'' in a single case.

Please also note that the maximal values of the accepted angular misclosures are prescribed in the Hungarian regulations. Depending on the purpose of the traverse line, the maximal acceptable value of the angular misclosure is ranging between 50'' and 105''. When the computed angular misclosure is larger than the acceptable value, then a part, or even all of the observations must be repeated. When larger angular misclosures are experienced, then one should try to locate the blunder in the deflection angles. It can be done by computing the closed line traverse as a free traverse line from the starting as well as from the terminating station. Comparing the coordinates of the two solutions (two free traverse solutions), the blunder is present, where the coordinates are close to each other in both of the free traverse solutions.

In the next step the WCB of the traverse legs should be computed. The WCB of the first traverse leg is computed as the sum of the deflection angle and its correction:

$$WCB_{S-1} = \beta_S + v\beta_S$$

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Afterwards each whole circle bearing is computed as the sum of the WCB of the previous traverse leg, the respective deflection angle (β) and the respective correction (vβ) minus 180°. Thus:

$$WCB_{1-2} = WCB_{S-1} + \beta_1 + v\beta_1 - 180°$$

Hint: *After the computation of the WCB of the last traverse leg, one should also add the β_E and $v\beta_E$ to this value and subtract 180°from the sum. When all the corrections and the computations are right, then the result should be exactly 0°:*

$$WCB_{3-E} + \beta_E + v\beta_E - 180° = 0°$$

This is a check for the computation of the WCBs.

Computing the preliminary coordinate differences

In the next step, the preliminary coordinate differences should be computed. This is done using the first fundamental task of surveying:

$$(\Delta E) = d \cdot \sin(WCB)$$

$$(\Delta N) = d \cdot \cos(WCB)$$

Afterwards the coordinates of the station S and E could be written to the appropriate cells, too.
In order to check the observed coordinate differences, one should compute the coordinate misclosures. In an ideal case the sum of the observed coordinate differences should be equal to the coordinate difference between station S and E. This is expressed by the coordinate criteria:

$$\sum(\Delta E) = E_E - E_S$$

$$\sum(\Delta N) = N_E - N_S$$

In order to compute the coordinate misclosures in the Easting and Northing directions, the observed coordinate differences should be summed up and the true coordinate differences should be computed between the starting and the terminating station:
The sum of the observed coordinate differences are:

$$\sum(\Delta E) = +276.836m$$

$$\sum(\Delta N) = -332.554m$$

The true coordinate differences between the starting and the terminating station are:

$$\Delta E_{S-E} = +276.770m$$

$$\Delta N_{S-E} = -332.500m$$

The coordinate misclosures are the differences between the true coordinate differences and the observed coordinate differences:

$\Delta\Delta E = \Delta E_{S-E} - \sum(\Delta E) = -0.066m$ $\Delta\Delta N = \Delta N_{S-E} - \sum(\Delta N) = +0.054m$

Computation of Closed Line Traverse

Using the coordinate misclosures in the Easting and Northing directions, the linear misclosure can be computed:

$$\Delta L = \sqrt{\Delta\Delta E^2 + \Delta\Delta N^2} = 0.085m$$

Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
		0	00	00				
S		115	30	28				
				-2			629558.310	184686.230
		115	30	26	112.997	-53.914		
1	125.200	199	50	36				
				-2				
		135	21	00	65.878	-66.688		
2	93.740	180	08	40				
				-2				
		135	29	38	70.701	-71.931		
3	100.860	213	29	23				
				-2				
		168	58	59	27.260	-140.021		
E	142.650	11	01	02				
				-1			629835.080	184353.730
	Σβ	720	00	09	276.836	-332.554	276.770	-332.500
	(n-1)180	720	00	00	-0.066	0.054		
	Δβ			-9’’				
					ΔL	0.085		

Table 6. The traverse computation form after the computation of the coordinate misclosures.

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Since the coordinate misclosure is mainly caused by the error in the distance observations (or a scale error due to the error in the coordinates of the reference points), therefore these misclosures should be distributed as corrections to each coordinate difference proportionally with the respective distances. Thus the vΔE and vΔN can be computed using the following formulae:

$$v\Delta E = \frac{\Delta\Delta E}{\sum d_i} d_i$$

$$v\Delta N = \frac{\Delta\Delta N}{\sum d_i} d_i$$

After the computation of the coordinate corrections, the corrected coordinate differences (ΔE and ΔN) can be computed:

$$\Delta E_i = (\Delta E)_i + v\Delta E_i$$

$$\Delta N_i = (\Delta N)_i + v\Delta N_i$$

Finally the coordinates of the stations (1-3) can be computed as the sum of the coordinates of the previous station in the traverse line and the corrected coordinate difference between the stations. For example:

$$E_1 = E_S + \Delta E_{S-1}$$

$$N_1 = N_S + \Delta N_{S-1}$$

Hints:

Although the coordinates of the existing stations are given with the precision of 1 cm, the computation of the coordinate differences should be made with the precision of 1mm. In this way the rounding error is limited to the mm level!

As a check, the coordinate of the last but one station (3) and the last corrected coordinate difference should be summed up. This sum must be equal to the coordinate of the terminating station.

Please note that the maximal accepted value of the linear misclosure (ΔL) is prescribed in the Hungarian regulations. This value is between 12 and 37 cm depending on the purpose and order of the traverse line.

The final solution of the traverse line can be seen in Table 7.

Computation of Closed Line Traverse

Station ID	Distance	WCB (whole circle bearing)			(ΔE)	(ΔN)	ΔE	ΔN
		β (deflection angle)						
		vβ (correction)			vΔE	vΔN	**E**	**N**
S		0	00	00				
		115	30	28				
				-2			629558.310	184686.230
1	125.200	115	30	26	112.997	-53.914	+112.979	-53.900
		199	50	36				
				-2	-0.018	+0.014(-1)	629671.289	184632.330
2	93.740	135	21	00	65.878	-66.688	+65.865	-66.677
		180	08	40				
				-2	-0.013	+0.011	629737.154	184565.653
3	100.860	135	29	38	70.701	-71.931	+70.686	-71.919
		213	29	23				
				-2	-0.015(-1)	+0.012	629807.840	184493.734
E	142.650	168	58	59	27.260	-140.021	+27.240	-140.004
		11	01	02				
				-1	-0.020	+0.017	629835.080	184353.730
	Σβ	720	00	09	276.836	-332.554	276.770	-332.500
	(n-1)180	720	00	00	-0.066	+0.054		
	Δβ			-9’’				
					ΔL	0.085		

Table 7. The final look of the traverse computation form