Computation of the Free Traverse

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This syllabus explains the computation of the free traverse line. Let's take the traverse line seen on Figure 1 and compute the coordinates of the traverse points 1-4. In this case we are interested in the horizontal coordinates only, therefore only Easting and Northing coordinates are computed.

Please note that one can find an empty computational sheet for the traverse computation on the last page of this document.

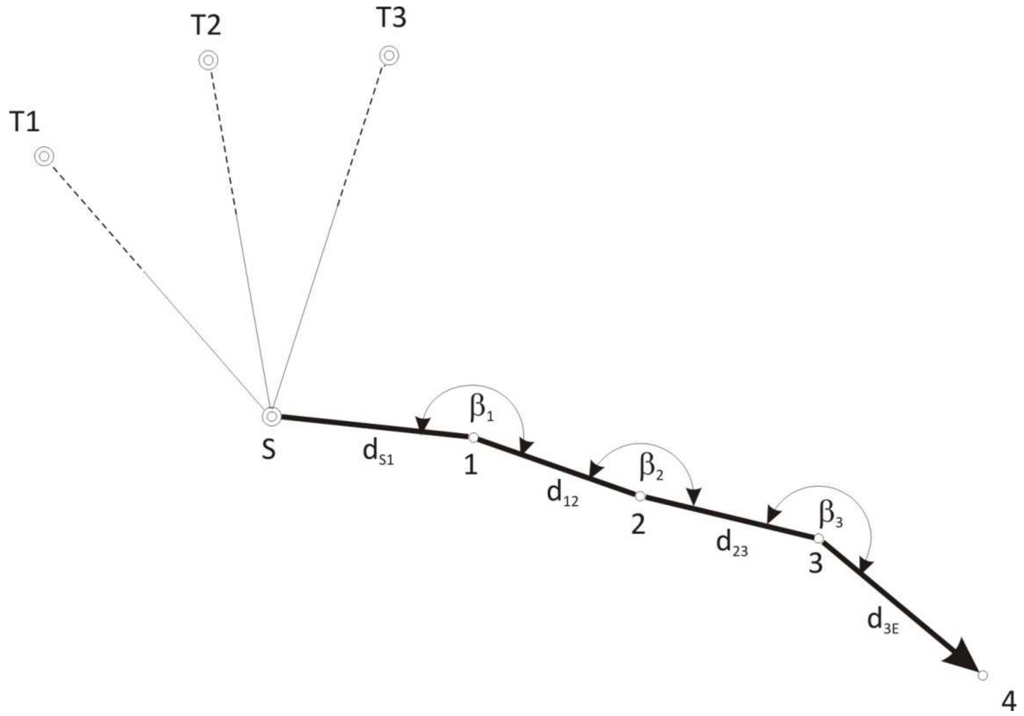

Figure 1. The free traverse

The coordinates found in Table 1 as well as the results of directional and distance observations found in Table 2 can be used for the computations. Please note that stations S and E are the starting and the terminating point of the traverse line, while points T1, T2, T3, T4 and T5 are targets used for the computation of the orientation.

Station	E(asting)	N(orthing)
S	629558.31	184686.23
T1	629413.64	184827.49
T2	629520.96	184917.46
T3	629657.71	184919.04

Table 1. The coordinates of the existing stations (values are in metres)

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Station	Target	Mean Direction			Orientation angle			whole circle bearing /deflection angle			dist
		°	′	″	°	′	″	°	′	″	
S	T1	307	42	54							
	T2	344	13	49							
	1	108	54	51							125.19
	T3	16	31	57							
1	S	25	39	31							125.21
	2	225	30	07							93.73
2	1	325	31	31							93.75
	3	145	40	11							100.85
3	2	241	33	05							100.87
	4	95	02	28							142.65

Table 2. The results of the directional and distance observations.

Computing the Whole Circle Bearing (WCB) of the S-1 ans E-3 directions

Firstly the WCB of the first traverse leg (in this case the S1 section) should be computed. In order to compute the WCB, the mean orientation angle must be computed, since 3-3 orientations have been observed at station S.

The principle of the computation of the orientation angles and the WCBs from the directional observations can be seen on Figure 2. Let’s see the observations at the station S:

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Station	Target	Mean Direction °	ʹ	ʺ	Orientation angle °	ʹ	ʺ	whole circle bearing /deflection angle °	ʹ	ʺ	dist
S	T1	307	42	54							
	T2	344	13	49							
	1	108	54	51							125.19
	T3	16	31	57							

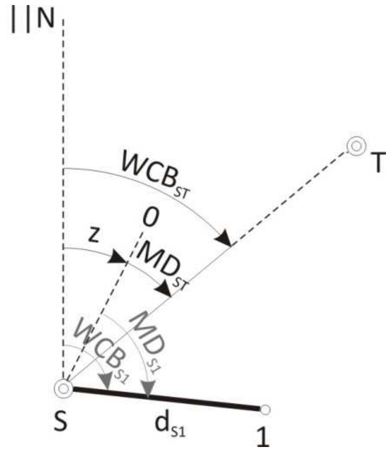

Figure 2. The computation of the orientation angle

In order to compute the WCB_{S1}, first the targets used for the orientation should be identified. Those targets can be used for the computation of the orientation angles, which are listed in the coordinate list (Table 1) and the respective observed mead direction can be found in Table 2. Thus we can identify T1, T2 and T3 as possible targets for the orientation.

The steps of the computation of the WCB_{S1} are the following:

1. Compute the WCB and the horizontal distance from station S to the respective targets used for the orientation (e.g. WCB_{ST}).
2. Subtract the observed mean direction pointing to this target (MD_{ST}). The result is called the orientation angle, and it is denoted by *z*.
3. In case of redundant orientational targets, one can compute the mean orientation angle.
4. The WCBS1 can be computed as the sum of the mean orientation angle (or in case of a single target the orientation angle) and the observed Mead Direction pointing to the first traverse point.

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In order to compute the orientation angles from the observations to T1, T2 and T3 respectively, the respective WCBs and distances must be computed. This can be achieved by the 2nd fundamental task of surveying. Let's take the data of the S-T1 direction:

Coordinates of station (S):	629558.31	184686.23
Coordinates of target (T1):	629413.64	184827.49

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The coordinate differences: ΔE= -144.67 m ΔN= 141.26 m

Please note *that the coordinate differences are always computed as the difference between the target and the stations coordinates!* $\Delta E=E_{Target}-E_{Station}$.

From the coordinate differences, the WCB and the horizontal distances can be computed:

$$\alpha = atan\frac{\Delta E}{\Delta N} = atan\frac{-144.67}{141.26} = -45°41'00''$$

According to the sign of the coordinate differences the appropriate quadrant can be identified, where the target is located. This also defines the appropriate correction between the α angle and the WCB. The decision matrix can be found in Table 3.

Quadrant	ΔE	ΔN	c
I	+	+	0
II	+	-	+180
III	-	-	+180
IV	-	+	+360
WCB=α+c			

Table 3. The decision matrix of the computation of WCB

Since in our case the ΔE is negative and ΔN is positive, therefore the target is located in the 4th quadrant. Thus the WCB is computed as α+360. Therefore the WCB_{S-T1} is:

$$WCB_{S-T1}=314°19'00''$$

On the other hand the horizontal distance between the station and the target can be computed using the Pythagoras' theorem:

$$d_{S-T1} = \sqrt{(\Delta E)^2 + (\Delta N)^2} = 202.197m$$

Using the same approach all the WCBs and distances can be computed for the orientations. Please note that one can identify the targets used for the orientation by comparing the directional observations and the coordinate lists. In case of orientations, both the station and the target must have given coordinates, and the mean directions must be observed as well.

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After computing the whole circle bearings and the distances, the field book should appear like this:

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54				314	19	00	202.197
	T2	344	13	49				350	49	28	234.227
	1	108	54	51							125.19
	T3	16	31	57				23	07	14	253.142

In the next step, the orientation angles should be computed. The orientation angle is the WCB of the index of the horizontal circle (marked with 0 on Fig. 2). Thus it can be computed as the difference between the WCB and the observed mean direction:

$$z_{S-T1} = WCB_{S-T1} - MD_{S-T1} = 6°36'06''.$$

With the same approach all the orientation angles can be computed for all of the targets used for the orientation. Now the fieldbook looks like this:

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51							125.19
	T3	16	31	57	6	35	17	23	07	14	253.142

Since we have three different orientation angles, we should compute the mean value of them. The mean orientation angle is computed as a weighted mean of the individual orientation angles, where the weight is the distance between the station and the target, in kilometers, rounded to one digit (eg. 256.123m will become 0.3 km). Thus:

$$\bar{z} = \frac{z_{S-T1}d_{S-T1} + z_{S-T2}d_{S-T2} + z_{S-T3}d_{S-T3}}{d_{S-T1} + d_{S-T2} + d_{S-T3}} = 6°35'37''$$

Let's write this value to the row of the first traverse point (Target No. 1) as an orientation angle!

Finally the WCB_{S1} should be computed (the WCB of the first traverse leg):

$$WCB_{S1} = \bar{z} + MD_{S1} = 115°30'28''$$

Computation of the Free Traverse

Now the fieldbook should look like this:

Station	Target	Mean Direction °	′	″	Orientation angle °	′	″	whole circle bearing /deflection angle °	′	″	dist
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51	**6**	**35**	**37**	**115**	**30**	**28**	125.19
	T3	16	31	57	6	35	17	23	07	14	253.142

Remarks:

1. *Please note that the orientation angles should be sufficiently close to each other. Usually the maximal difference is less than 1′. When you experience larger differences, please check your computations.*

2. *Another useful check: the mean orientation angle must be between the smallest and the largest orientation angles.*

3. *The mean orientation angles can be computed according to the following method, too:*

Subtract the smallest orientation angle cut to a full degree and minute value from all of the orientation angles (in our case, subtract 6°35′00′′ from all of the orientation angles at the station S). Compute the weighted mean of the residual values:

$$\bar{r} = \frac{66'' \cdot 202 + 39'' \cdot 234 + 17'' \cdot 253}{689} = 37''$$

Finally add the subtracted value to this mean to get the mean orientation angle:

$$\bar{\sigma} = 6°35'00'' + 37'' = 6°35'37''$$

Computing the deflection angles:

The deflection angles are computed as the difference between the forward mean direction and the backward mean direction. Please note that the deflection angles are the angles between the traverse legs on the left side of the traverse line.

Thus the deflection angle at the station 1 is:

$$\beta_1 = (225°30'07'') - (25°39'31'') = 199°50'36''$$

Hint: *It is always a good idea to draw a sketch of the traverse line, because the order of the observations in the field book might differ from the order of the traverse points.*

Computation of the Free Traverse

The completed fieldbook will look like Table 4.

Station	Target	Mean Direction			Orientation angle			whole circle bearing /deflection angle			dist
		°	′	″	°	′	″	°	′	″	
S	T1	307	42	54	6	36	06	314	19	00	202.197
	T2	344	13	49	6	35	39	350	49	28	234.227
	1	108	54	51	**6**	**35**	**37**	**115**	**30**	**28**	125.19
	T3	16	31	57	6	35	17	23	07	14	253.142
1	S	25	39	31							125.21
	2	225	30	07				199	50	36	93.73
2	1	325	31	31							93.75
	3	145	40	11				180	08	40	100.85
3	2	241	33	05							100.87
	4	95	02	28				213	29	23	142.65

Table 4. The completed fieldbook of the traverse observations (including WCBs and deflection angles)

Computing the deflection angles of the first (S)

For the first station, the deflection angle is equal to the WCB between the station and the first traverse point:

$\beta_S = WCB_{S1}$.

Computation of the Free Traverse

Computation of the traverse line

In order to compute the traverse line, the traverse computation form should be used. Let's discuss the notations in this form:

- Point ID: is the ID of the stations found in the traverse line
- distance: the horizontal (grid) distance between the consecutive stations (if it is measured both forwards and backwards, then we have to take the average of them!)
- WCB: the whole circle bearing of the consecutive traverse legs
- β: the computed deflection angle at the stations
- $v\beta$: the corrections of the observed deflection angles (when applicable)
- (ΔE): preliminary coordinate difference in the Easting direction between the consecutive stations (when applicable)
- (ΔN): preliminary coordinate difference in the Northing direction between the consecutive stations (when applicable)
- $v\Delta E$: correction of the preliminary Easting coordinate differences between the consecutive stations (when applicable)
- $v\Delta N$: correction of the preliminary Northing coordinate differences between the consecutive stations (when applicable)
- ΔE: the corrected (final) coordinate difference in the Easting direction between the consecutive stations
- ΔN: the corrected (final) coordinate difference in the Northing direction between the consecutive stations
- E: the Easting coordinate of the station
- N: the Northing coordinate of the station

Let's start to fill the form! Firstly the Station IDs should be written to the consecutive rows. Then the distances (length of the traverse legs) should be filled. Please note that the distances should be written to the row of the station, whose coordinates will be computed using the distance. Thus the first distance comes to the row of station 1 (and not station S!).

Afterwards the deflection angles should be written to the appropriate rows (each station will have a separate deflection angle computed in the fieldbook of the traverse observations.

After these steps the traverse computation form can be seen in Table 5.

Computation of the Free Traverse

Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
S		115	30	28				
1	125.200	199	50	36				
2	93.740	180	08	40				
3	100.860	213	29	23				
4	142.650							

Table 5. Copying the distance observations and deflection angles into the traverse computation form.

Computation of the Free Traverse

In the next step the WCB of the traverse legs should be computed. The WCB of the first traverse leg is computed as the sum of the deflection angle and its correction:

$$WCB_{S-1} = \beta_S$$

Afterwards each whole circle bearing is computed as the sum of the WCB of the previous traverse leg, the respective deflection angle (β) and the respective correction (vβ) minus 180°. Thus:

$$WCB_{1-2} = WCB_{S-1} + \beta_1 - 180°$$

Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
		0	00	00				
S		115	30	28				
		-	--	--				
		115	30	28				
1	125.200	199	50	36				
		-	--	--				
		135	21	04				
2	93.740	180	08	40				
		-	--	--				
		135	29	44				
3	100.860	213	29	23				
		-	--	--				
		168	59	07				
4	142.650	-	--	--				
		-	--	--				

Table 7. The traverse computation form after the computation of the coordinate misclosures.

Computation of the Free Traverse

Computing the coordinate differences

In the next step, the preliminary coordinate differences should be computed. This is done using the first fundamental task of surveying:

$$\Delta E = d \cdot \sin(WCB)$$

$$\Delta N = d \cdot \cos(WCB)$$

Afterwards the coordinates of the station S could be written to the appropriate cells, too.

Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
S		0	00	00				
		115	30	28			629558.310	184686.230
		-	--	--				
1	125.200	115	30	28			112.996	-53.915
		199	50	36				
		-	--	--				
2	93.740	135	21	04			65.877	-66.689
		180	08	40				
		-	--	--				
3	100.860	135	29	44			70.699	-71.933
		213	29	23				
		-	--	--				
4	142.650	168	59	07			27.255	-140.022
		-	--	--				
		-	--	--				

Table 8. The traverse computation form after the computation of the coordinate differences.

Computation of the Free Traverse

Please note that the coordinate differences and the coordinates are computed with an order higher precision than the original coordinates in the coordinate list (Table 1.) Thus the coordinate differences are computed with mm precision instead of cm precision. This is important to minimize the rounding error during the computation.

Finally the coordinates of the stations (1-3) can be computed as the sum of the coordinates of the previous station in the traverse line and the corrected coordinate difference between the stations. For example:

$$E_1 = E_S + \Delta E_{S-1}$$

$$N_1 = N_S + \Delta N_{S-1}$$

The final solution of the traverse line can be seen in Table 9.

Since the coordinates have been computed with mm precision, therefore we must round them to their final values with cm precision, to indicate the accuracy level of the coordinates. Thus the final coordinate solution is given in Table 10.

Remark:

1. *The rounding method in surveying slightly differs from the approach used in mathematics.When the real number is exactly between the two integers, then we always round to the next even number, and not always upwards.*

	Surveying	Mathematics
1.4	1	1
1.5	2	2
1.6	2	2
2.5	**2**	**3**

Various real numbers after rounding to integers using the 'surveying approach' and the normal rounding procedure in Mathematics.

Please note that in the Table 2.5 must be rounded to 2 instead of 3! The reason for this is to minimize the effect of roundings on the mean values of observations.Ssince in case of a large amount of observations, it is likely, that the number of upward and downward roundings are the same, while in mathematics the values are rounded always upwards.

Computation of the Free Traverse

Station ID	Distance	WCB (whole circle bearing) / β (deflection angle) / vβ (correction)			(ΔE) / vΔE	(ΔN) / vΔN	ΔE / **E**	ΔN / **N**
S		0	00	00				
		115	30	28				
		-	--	--			629558.310	184686.230
1	125.200	115	30	28			112.996	-53.915
		199	50	36				
		-	--	--			629671.306	184632.315
2	93.740	135	21	04			65.877	-66.689
		180	08	40				
		-	--	--			629737.183	184565.626
3	100.860	135	29	44			70.699	-71.933
		213	29	23				
		-	--	--			629807.882	184493.693
4	142.650	168	59	07			27.255	-140.022
		-	--	--				
		-	--	--			629835.137	184353.671

Table 9. The traverse computation form after the computation of the coordinate differences.

	E	N
S	629558.31	184686.23
1	629671.31	184632.32
2	629737.18	184565.63
3	629807.88	184493.69
4	629835.14	184353.67

Table 10. The final coordinate solution of the traverse points

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Station ID	Distance	WCB (whole circle bearing)	(ΔE)	(ΔN)	ΔE	ΔN
		β (deflection angle)				
		vβ (correction)	vΔE	vΔN	**E**	**N**

Table 11. An empty computational sheet for free traverse computation