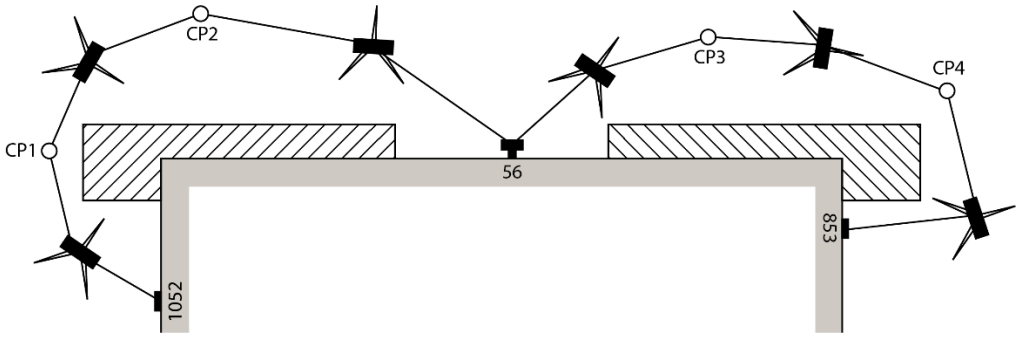
4th PRACTICAL – LINE LEVELING

The goal of this exercise is to determine the unknown elevation of point no. 56 by carrying out a line leveling between two points with known elevations (benchmarks, no. 1052 and no. 853) and including the point no. 56 in the line. A figure of the leveling line's layout:



The field books after taking the forward and backward measurements is as follows (only the numbers in black are measured, the numbers in blue are calculated values):

Forward direction					
Point ID	**Distance**	**Backsight**	**Foresight**	**Rise/Fall**	**Reduced level [m]**
1052	16	1025			110.562
CP1	16		0985	+40	
CP1	12	1432			
CP2	12		1003	+429	
CP2	11	1577			
56	11		1405	+172	
56	15	0988			
CP3	15		1474	-486	
CP3	20	1550			
CP4	20		1635	-85	
CP4	31	1225			
853	31		1047	+178	110.804
Σ	210	7797	7549	+248	+242

Backward direction					
Point ID	**Distance**	**Backsight**	**Foresight**	**Rise/Fall**	**Reduced level [m]**
853	32	1042			110.804
CP5	32		1228	-186	
CP5	21	1630			
CP6	21		1555	75	
CP6	17	1481			
56	17		980	501	
56	13	1401			
CP7	13		1579	-178	
CP7	9	892			
CP8	9		1431	-539	
CP8	15	1024			
1052	15		947	77	110.562
Σ	214	7470	7720	-250	-242

Please note that the elevation of the starting and the endpoint are already filled in, as these are known benchmarks. The Rise/Fall values between two points are calculated by taking the backsight reading (BS) to the first point and subtracting from it the foresight reading (FS) to the second point. For example, the rise value in the 3rd row of the first table is calculated by taking the BS to 1052 and subtracting the FS to CP1 (change point no. 1). This gives us the elevation difference between the two points.

Also note that the distance in each row corresponds to the distance between the point denoted with the point ID in the same row and the point denoted with the point ID in the previous row.

As the leveling line was measured between two benchmarks, we can calculate the true height difference (Δh_{true}) between the two endpoints, using the reduced levels (RL).

$$\Delta h_{\text{true}} = RL_{853} - RL_{1052} = 110.804 - 110.562 = +242 \text{ mm}$$

By calculating the measured elevation difference and comparing it to the true elevation difference, we can calculate the misclosure of the line (Δ). For computing the measured elevation difference, we should create another table:

Point ID	**Distance**	**Forward**	**Backward**	**Average**	**Correction**	**Corrected elevation difference**	**Reduced level [m]**
1052							110.562
	78	+641	-640	+640	-2	+638	
56							**111.200**
	132	-393	+390	-392	-4	-396	
853							110.804
Σ	210			+248	-6	+242	+242 mm

In the table above we filled in the summed distance between 1052-56 and 56-852 from the "forward" field book because they are nearly equal to the summed distances from the "backward" field book. The next three columns contain the summed rise/fall values between the corresponding points (between 1052-56 and 56-953) and the average of the two values. Please note that when calculating the average of the forward and backward elevation differences, we have to change the sign of the backward elevation difference because of the direction difference.

By adding up the average elevation differences, we get that the measured elevation difference between the two endpoints (Δh_{meas}) is equal to +248 mm. To calculate the misclosure, we always take the true elevation difference and subtract from it the measured elevation difference:

$$\Delta = \Delta h_{\text{true}} - \Delta h_{\text{meas}} = +242 - 248 = -6 \text{ mm}$$

We somehow have to distribute the misclosure to the measured elevation differences. For this, we use the distances between the points as weights. We divide the misclosure by the sum of the distances and multiply with the corresponding distance.

$$\Delta_i = \frac{\Delta}{\sum d} \cdot d_i$$

The correction for the measured elevation diff. between 1052-56 and 56-853:

$$\Delta_{1052-56} = -\frac{6}{210} \cdot 78 = -2.22 \approx -2 \text{ mm}$$

$$\Delta_{56-853} = -\frac{6}{210} \cdot 132 = -3.77 \approx -4 \text{ mm}$$

We compute the reduced level of point no. 56 by taking the reduced level of the starting point (1052) and adding the corrected elevation difference between 1052-56 ($\Delta h^c_{1052-56}$). Alternatively, we can take the reduced level of the endpoint and subtract from it the elevation difference between 56-853 (Δh^c_{56-853}). We need subtraction in this case as we are going "backwards" from the endpoint, whereas in the first we are going "forward" from the starting point.

$$RL_{56} = RL_{1052} + \Delta h^c_{1052-56} = 110.562 + 0.638 = 111.200 \text{ m}$$

$$RL_{56} = RL_{853} - \Delta h^c_{56-853} = 110.804 - (-0.396) = 111.200 \text{ m}$$

Please note that the elevation differences in the table are in millimeters, while the elevations are in meters. This means that we have to convert the differences into meters when we add them to the elevations.