The goal of the exercise is to determine the reduced levels of the detail points (numbered 101, 102, 103), which denote characteristic points of certain objects (sewer drains, manholes, buildings, side of pavement etc.). In order to accomplish this, we create a levelling line between two points with known elevations (benchmarks) and guide the line towards the detail points (marked with a black dot on the figure). In a station close to the detail points, in addition to taking the backsight and foresight readings, we put the leveling staff to the detail points as well and take readings on the staff. To distinguish these readings from normal backsight-foresight readings in the leveling line, we call these intersight readings and write them in a different column of the field book.

The layout of the leveling line in our example:


The field book after finishing the measurements:

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC <br> $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 |  |  |  |  |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 |  |  |  |  |
| CP2 | 16 | 1767 |  |  |  |  |  |  |
| 101 | - |  | 1616 |  |  |  |  |  |
| 102 | - |  | 1479 |  |  |  |  |  |
| CP3 | 16 |  |  | 1518 |  |  |  |  |
| CP3 | 10 | 1780 |  |  |  |  |  |  |
| 103 | - |  | 1306 |  |  |  |  |  |
| CP4 | 10 |  |  | 1624 |  |  |  |  |
| CP4 | 13 | 1036 |  |  |  |  |  |  |
| CP5 | 13 |  |  | 1235 |  |  |  |  |
| CP5 | 15 | 1332 |  |  |  |  |  |  |
| 853 | 15 |  |  | 0908 |  |  |  |  |

The first step of the calculation is to compute the leveling line. In this step, we disregard the intersight readings and use only the backsight and foresight readings. This is similar to the leveling line computation in the $4^{\text {th }}$ practical, except now we didn't carry out a backward measurement, we only have a forward one.

First, we calculate the rise and fall values between the starting point, the change points and the end point, and sum up the distances, the backsight and foresight readings. We also compute the true elevation difference between the endpoint and the starting point, using their given reduced levels. In the following table, these are written in red.

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 | +274 |  |  |  |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 | +153 |  |  |  |
| CP2 | 16 | 1767 |  |  |  |  |  |  |
| 101 | - |  | 1616 |  |  |  |  |  |
| 102 | - |  | 1479 |  |  |  |  |  |
| CP3 | 16 |  |  | 1518 | +249 |  |  |  |
| CP3 | 10 | 1780 |  |  |  |  |  |  |
| 103 | - |  | 1306 |  |  |  |  |  |
| CP4 | 10 |  |  | 1624 | +156 |  |  |  |
| CP4 | 13 | 1036 |  |  |  |  |  |  |
| CP5 | 13 |  |  | 1235 | -199 |  |  |  |
| CP5 | 15 | 1332 |  |  |  |  |  |  |
| 853 | 15 |  |  | 0908 | +424 |  |  |  |
| $\Sigma$ | 180 | 9002 |  | 7945 | $\Sigma \Delta h_{\text {meas }}=1057$ |  |  |  |

Remember, that if we subtract the sum of the foresight readings from the sum of the backsight readings, we have to get the sum of the rise/fall values. In our case, $7670-5086=960$, which means, we calculated the rise/fall values correctly.

The next step is to calculate the misclosure of the line from the true elevation difference and the measured elevation difference (the sum of the rise/fall values):

$$
\Delta=\Delta h_{\text {true }}-\Delta h_{\text {meas }}=1067-1057=+10 \mathrm{~mm}
$$

We have to distribute the misclosure of the line as correction to each of the calculated rise/fall values. We use the distances between the points as weights. As an example, the correction of the rise value between point no. 1052 and CP 1 ; and between CP 3 and CP 4 are calculated as the following:

$$
\begin{aligned}
& \Delta_{1052-C P 1}=\frac{\Delta}{\sum d_{i}} \cdot d_{1052-C P 1}=\frac{+10}{180} \cdot 30=+1.66 \approx+2 \mathrm{~mm} \\
& \Delta_{C P 3-C P 4}=\frac{\Delta}{\sum d_{i}} \cdot d_{C P 3-C P 4}=\frac{+10}{180} \cdot 20=+1.11 \approx+1 \mathrm{~mm}
\end{aligned}
$$

We have to check that the sum of the distributed corrections equals the total misclosure. If not, than instead of rounding up, we need to round down some of the correction values (or vice versa). The next table shows the distributed correction values:

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 | +274 | +2 |  |  |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 | +153 | +2 |  |  |
| CP2 | 16 | 1767 |  |  |  |  |  |  |
| 101 | - |  | 1616 |  |  |  |  |  |
| 102 | - |  | 1479 |  |  |  |  |  |
| CP3 | 16 |  |  | 1518 | +249 | +2 |  |  |
| CP3 | 10 | 1780 |  |  |  |  |  |  |
| 103 | - |  | 1306 |  |  |  |  |  |
| CP4 | 10 |  |  | 1624 | +156 | +1 |  |  |
| CP4 | 13 | 1036 |  |  |  |  |  |  |
| CP5 | 13 |  |  | 1235 | -199 | +1 |  |  |
| CP5 | 15 | 1332 |  |  |  |  |  |  |
| 853 | 15 |  |  | 0908 | +424 | +2 |  | 111.629 |
| $\Sigma$ | 180 | 9002 |  | 7945 | $\Sigma \Delta h_{\text {meas }}=1057$ |  |  | $\Sigma \Delta h_{\text {true }}=1067$ |
|  |  |  |  |  | $\Delta=+10 \mathrm{~mm}$ |  |  |  |

Now that the computation of the leveling line is completed, we can determine the reduced level of the detail points. For this, let's take a look at the following figure of the third station (between points CP2 and CP3):


At the third station, in addition to taking a backsight $(B S)$ and a foresight $(F S)$ reading to point CP2 and CP3 respectively, we took intersight readings (IS) to point no. 101 and 102. If we knew the reduced level $(R L)$ of CP 2 , than we could determine the height of collimation (abbreviated HoC, the reduced level of the horizontal plane created by the level) by adding the backsight reading to the reduced level of CP2. After that, in order to compute the reduced level of the detail point 101, we only need to subtract the intersight reading to 101 from the HoC . To calculate the reduced level of detail point 102 , we have to subtract the intersight reading to 102 from the same HoC.
(Please note, that when taking readings on the detail points, we usually cannot keep equal instrumentstaff distances, so we always have to work with an instrument that has no collimation error, or we know the value of the collimation and correct our measurements afterwards.)

So first, we need to compute the elevations of change points. The elevation of CP1 can be calculated by taking the given elevation of the starting point (no. 1052) and adding the corrected rise/fall value $\left(\Delta h^{c}\right)$ between 1052 and CP1:

$$
R L_{C P 1}=R L_{1052}+\Delta h_{1052-C P 1}^{c}=110.562+(\underbrace{0.274}_{\text {rise/fall value }}+\underbrace{0.002}_{\text {correction }})=110.838 \mathrm{~m}
$$

To calculate the reduced level of CP 2 and all the subsequent intermediate points, we have to take the calculated elevation of the previous point and add the corrected rise/fall value to it. The next table shows the calculated reduced levels:

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 | +274 | +2 |  | 110.838 |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 | +153 | +2 |  | 110.993 |
| CP2 | 16 | 1767 |  |  |  |  |  |  |
| 101 | - |  | 1616 |  |  |  |  |  |
| 102 | - |  | 1479 |  |  |  |  |  |
| CP3 | 16 |  |  | 1518 | +249 | +2 |  | 111.244 |
| CP3 | 10 | 1780 |  |  |  |  |  |  |
| 103 | - |  | 1306 |  |  |  |  |  |
| CP4 | 10 |  |  | 1624 | +156 | +1 |  | 111.401 |
| CP4 | 13 | 1036 |  |  |  |  |  | 111.203 |
| CP5 | 13 |  |  | 1235 | -199 | +1 |  |  |
| CP5 | 15 | 1332 |  |  |  |  |  | 111.629 |
| 853 | 15 |  |  | 0908 | +424 | +2 |  | $\sum \Delta h_{\text {true }}=1067$ |
| $\Sigma$ | 180 | 9002 |  | 7945 | $\Sigma \Delta h_{\text {meas }}=1057$ |  |  |  |

From the field book, we can see that two of the detail points were measured between points CP2 and CP3 and one was measured between CP3 and CP4. This means that we have to calculate two HoC values, one between CP2-CP3 and one between CP3-CP4. We compute the HoC between CP2-CP3 by taking the calculated elevation of CP 2 and adding the backsight $(B S)$ reading on CP 2 to it:

$$
H o C_{C P 2-C P 3}=R L_{C P 2}+B S_{C P 2}=110.993+1.767=112.760 \mathrm{~m}
$$

We calculate the HoC between CP3-CP4 similarly:

$$
H o C_{C P 3-C P 4}=R L_{C P 3}+B S_{C P 3}=111.245+1.780=113.024 \mathrm{~m}
$$

Next, we fill in the HoC values into the correct cells:

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 | +274 | +2 |  | 110.838 |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 | +153 | +2 |  | 110.993 |
| CP2 | 16 | 1767 |  |  |  |  | 112.760 |  |
| 101 | - |  | 1616 |  |  |  |  |  |
| 102 | - |  | 1479 |  |  |  |  |  |
| CP3 | 16 |  |  | 1518 | +249 | +2 |  | 111.244 |
| CP3 | 10 | 1780 |  |  |  |  | 113.024 |  |
| 103 | - |  | 1306 |  |  |  |  |  |
| CP4 | 10 |  |  | 1624 | +156 | +1 |  | 111.401 |
| CP4 | 13 | 1036 |  |  |  |  |  |  |
| CP5 | 13 |  |  | 1235 | -199 | +1 |  | 111.203 |
| CP5 | 15 | 1332 |  |  |  |  |  |  |
| 853 | 15 |  |  | 0908 | +424 | +2 |  | 111.629 |
| $\Sigma$ | 180 | 9002 |  | 7945 | $\sum \Delta h_{\text {meas }}=1057$ |  |  | $\sum \Delta h_{\text {true }}=1067$ |
|  |  |  |  |  | $\Delta=+10 \mathrm{~mm}$ |  |  |  |

Finally, we need to take the intersight readings (IS), and subtract them from the corresponding HoC value. That is, for detail points no. 101 and 102, we use the HoC at CP 2 (as these points were measured from station between CP2 and CP3); and for detail point no. 103, we use the HoC at CP3 (103 was measured from the station between CP3 and CP4). For example:

$$
R L_{101}=H o C_{C P 2-C P 3}-I S_{101}=112.761-1.616=111.145 \mathrm{~m}
$$

The following table shows the completed field book after the calculations:

| Point ID | Dist. | Backsight | Intersight | Foresight | Rise/Fall | Correction | HoC $[\mathbf{m}]$ | RL <br> $[\mathbf{m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1052 | 15 | 1551 |  |  |  |  |  | 110.562 |
| CP1 | 15 |  |  | 1277 | +274 | +2 |  | 110.838 |
| CP1 | 21 | 1536 |  |  |  |  |  |  |
| CP2 | 21 |  |  | 1383 | +153 | +2 |  | 110.993 |
| CP2 | 16 | 1767 |  |  |  |  | 112.760 |  |
| 101 | - |  | 1616 |  |  |  |  | $\mathbf{1 1 1 . 1 4 4}$ |
| 102 | - |  | 1479 |  |  |  |  | $\mathbf{1 1 1 . 2 8 1}$ |
| CP3 | 16 |  |  | 1518 | +249 | +2 |  | 111.244 |
| CP3 | 10 | 1780 |  |  |  |  | 113.024 |  |
| 103 | - |  | 1306 |  |  |  |  | $\mathbf{1 1 1 . 7 1 8}$ |
| CP4 | 10 |  |  | 1624 | +156 | +1 |  | 111.401 |
| CP4 | 13 | 1036 |  |  |  |  |  |  |
| CP5 | 13 |  |  | 1235 | -199 | +1 |  | 111.203 |
| CP5 | 15 | 1332 |  |  |  |  |  |  |
| 853 | 15 |  |  | 0908 | +424 | +2 |  | 111.629 |
| $\Sigma$ | 180 | 9002 |  | 7945 | $\Sigma \Delta h_{\text {meas }}=1057$ |  |  | $\sum \Delta h_{\text {true }}=1067$ |
|  |  |  |  |  | $\Delta=+10 \mathrm{~mm}$ |  |  |  |

