## Practical 2: Surveying and geodesy. Coordinate systems. Orthogonal and polar coordinates. Conversion between orthogonal and polar coordinates

## Accessories to be used:

1 scientific calculator for each student

## Contents:

Refreshing elementary coordinate geometry calculations
Coordinate systems used in surveying

## Coordinate systems used in matematics

Position of points on the plain are defined by coordinates. We need a coordinate reference system for this. Two types of coordinate systems are used generally:

- Cartesian (orthogonal) coordinate system
- polar coordinate system


## Cartesian coordinate system:

To define the coordinate system first the origin have to be set up, the intersection point of the two perpendicular coordinate axises. The horizontal axis is called $\mathbf{x}$ axis. The positive rotation is counter clockwise. The other, $\mathbf{y}$ axis is got rotating $\mathbf{x}$ axis by $+90^{\circ}$. The axises divide the plan into four quadrants.



## Polar coordinate system:

A point is also chosen here as the pole (origin) and a ray from this point is taken as the polar axis, which is usually horizontal in mathematics. A point is defined on the plain by the counterclockwise angle from the polar axis and the distance from the pole.


If the origin of a Cartesian coordinate system and the x axis are identical to the pole and polar axis of a polar coordinate system, exchange between x , y coordinates and $\mathrm{r}, \Theta$ can be done.


| $x=r \cdot \cos (\theta)$ |
| :--- |
| $y=r \cdot \sin (\theta)$ |
| $r=\sqrt{x^{2}+y^{2}}$ |


$\alpha=\arctan \left|\frac{y}{x}\right|$ | Quadrant | $\mathbf{x}$ | $\mathbf{y}$ | $\Theta$ |
| :---: | :---: | :---: | :---: |
| I | + | + | $\theta=\alpha$ |
| II | - | + | $\theta=180^{\circ}-\alpha$ |
| III | - | - | $\theta=180^{\circ}+\alpha$ |
| IV | + | - | $\theta=360^{\circ}-\alpha$ |

## Polar to Cartesian

## Exercise 1:

$$
\begin{array}{lr}
r_{A}=\sqrt{2} & \theta=45 \\
x_{A}=1 & y_{A}=1
\end{array}
$$

## Exercise 2:

$$
\begin{array}{ll}
r_{A}=10.45 & \theta=122-52-43 \\
x_{A}=-5.67 & y_{A}=8.78
\end{array}
$$

## Cartesian to polar

## Exercise 3:

$$
\begin{array}{ll}
x_{A}=1 & y_{A}=1 \\
r_{A}=\sqrt{2} & \alpha=\theta=45^{\circ}
\end{array}
$$

## Exercise 4:

$$
\begin{array}{ll}
x_{A}=-1 & y_{A}=1 \\
r_{A}=\sqrt{2} & \alpha=45^{\circ}
\end{array} \quad \theta=180^{\circ}-\alpha=135^{\circ}
$$

## Exercise 5:

$$
\begin{array}{ll}
x_{A}=-1 & y_{A}=-1 \\
r_{A}=\sqrt{2} & \alpha=45^{\circ} \quad \theta=180^{\circ}+\alpha=225^{\circ}
\end{array}
$$

## Exercise 6:

$$
\begin{array}{ll}
x_{A}=1 & y_{A}=-1 \\
r_{A}=\sqrt{2} & \alpha=45^{\circ} \quad \theta=360^{\circ}-\alpha=315^{\circ}
\end{array}
$$

## Exercise 7:

$$
\begin{array}{lll}
x_{A}=-5.67 & y_{A}=8.78 \\
r_{A}=10.45 & \alpha=57-08-46 & \theta=122-51-14
\end{array}
$$

## Using scientific calculator

## Polar to Cartesian:

| $\operatorname{Rect}($ | $\mathrm{r}_{\mathrm{A}}$ | , | $\theta_{\mathrm{A}}$ | $=$ | $\mathrm{x}_{\mathrm{A}}$ | RCL | F | $\mathrm{y}_{\mathrm{A}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Cartesian to polar

| $\operatorname{Pol}($ | $\mathrm{x}_{\mathrm{A}}$ | , | $\mathrm{y}_{\mathrm{A}}$ | $=$ | $\mathrm{r}_{\mathrm{A}}$ | RCL | F | $\theta_{\mathrm{A}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Distance of two points



## Equation of a line

## Coordinate systems used in surveying

There are differences between mathematical and surveyor's coordinate systems. The x axis is often called Easting (E) and the y axis is often called Northing (N). The polar axis is going to North and the positive angles are measured clockwise and called bearing.

## Cartesian coordinates Polar coordinates



Bearing

## Bearings



Whole circle bearing

| Quadrant in which bearing lies | Conversion relation |
| :---: | :---: |
| NE | $\alpha=\theta$ |
| SE | $\alpha=180^{\circ}-\theta$ |
| SW | $\alpha=\theta-180^{\circ}$ |
| NW | $\alpha=360^{\circ}-\theta$ |



Fig. 13.4

Orthogonal and polar coordinates
Conversion between orthogonal and polar coordinates
Calculation of coordinates from a known point and relative polar coordinates.

$$
\begin{aligned}
& E_{B}=E_{A}+d_{A B} \cdot \sin \left(W C B_{A B}\right) \\
& N_{B}=N_{A}+d_{A B} \cdot \cos \left(W C B_{A B}\right)
\end{aligned}
$$

