Practical 3: Introduction to levelling, using the surveyors’ level

Equipment needed per group:

1 tilting level + tripod

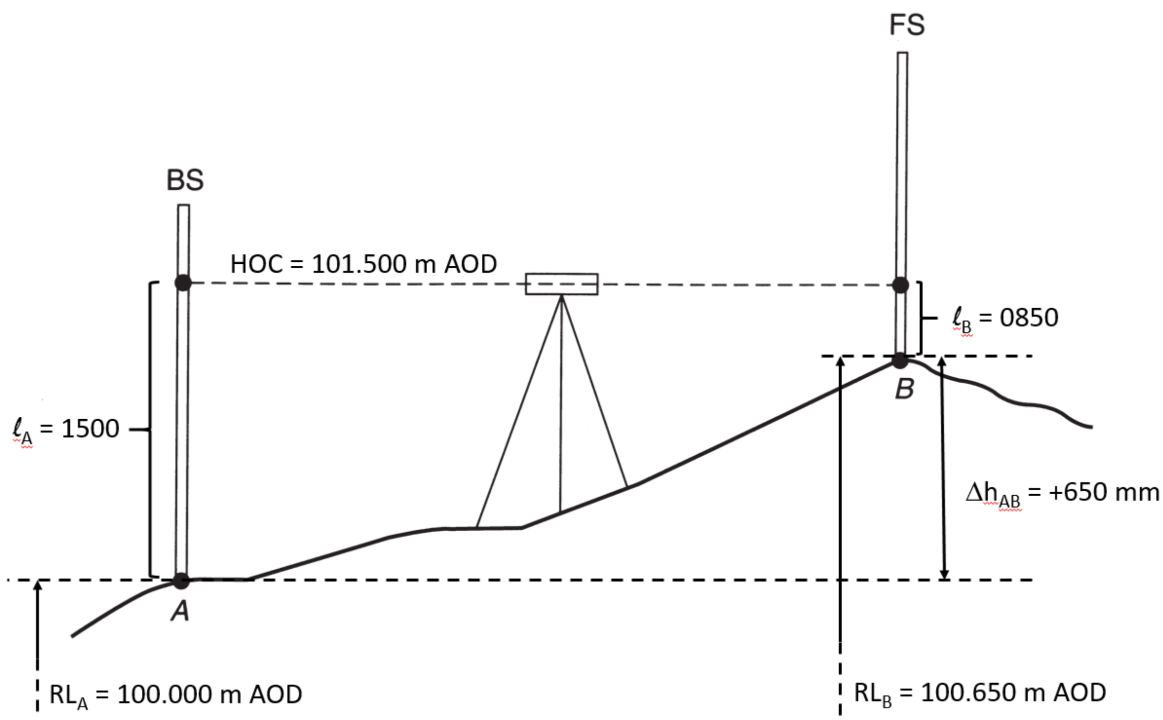
1 automatic level (surveyors’ level) + tripod

2 levelling staves

2 change plates (levelling plates)

Levelling in practice, determining the height difference between two points

The levelling instrument (called the level) is set up between two vertical levelling staves (A and B) at an equal distance from each staff and correctly levelled. Levelling the instrument means, that we use the circular and the levelling bubbles (or only the circular bubble in case of an automatic level) to make the line of sight of the instrument horizontal and therefore create a horizontal plane at the height of the instrument. If the direction of the levelling is from A to B, then point A is called the backsight point (BS) and B is called the foresight point (FS).



Let the reduced level (RL, the height of the point above a reference surface) of point A be 100.000 m AOD (above ordnance datum – a reference surface).

We start with taking a reading on the levelling staff on the BS point (ℓ_A) using the level. The readings are made in mm units and we always note down four digits. Let this reading be 1500 (meaning 1500 mm or 1.500 m). Then, we take a reading on the levelling staff on the FS point (ℓ_B). Let this reading be 0850 (meaning 850 mm or 0.850 m).

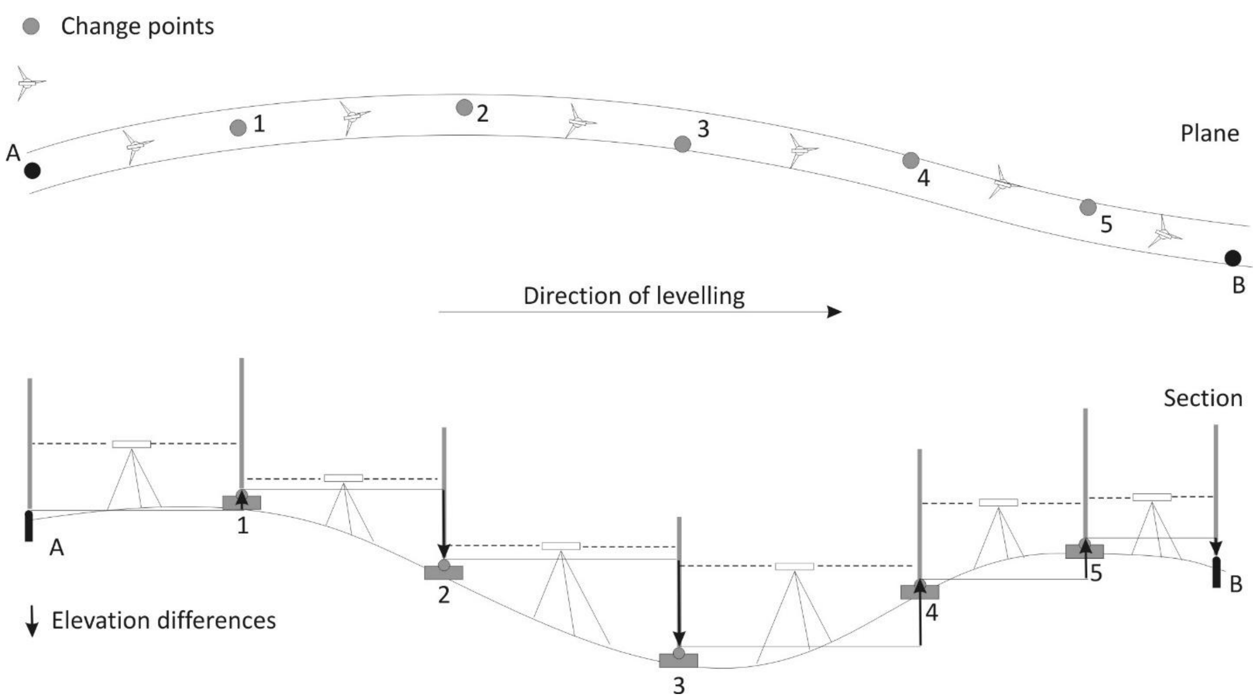
Adding the BS reading to the RL of point A gives us the height of collimation (HOC) or instrument height, which is the height of the reference plane set out by the instrument AOD. In this particular case, the HOC is 100.000 m + 1.500 m = 101.500 m AOD.

The height difference between the backsight point and the foresight point (points A and B respectively in this case) is always given by taking the BS reading and subtracting from it the FS reading: Δh_{AB} = 1500 – 0850 = +650 mm. If the difference is positive, it is called a rise, otherwise it's a fall. (We can visualize this calculation by imagining that we are first at point A, then we use the BS reading and add it to the RL of the point to rise up to the HOC. When we are the level of the HOC, we can use the FS reading to go down to the level of point B.)

Adding the rise/fall value to the RL of point A gives us the RL of point B: 100.000 m AOD + 0.650 m = 100.650 m AOD. We may also compute this as taking the HOC and subtracting the FS reading from it: 101.500 m AOD – 0.850 m = 100.650 m AOD.

Line levelling, determining the height difference between distant points

As the instrument-staff distance is limited due to the size of the scale on the levelling staves (if the staff is too far away from the instrument, the accuracy of the reading deteriorates rapidly), in many cases, we cannot measure the height difference between two points using a single pair of BS and FS readings. We have to create a levelling line and divide it into smaller segments, called stations (see below). In one station, the instrument-staff distance (maximum of ~20-25 m) is small enough to make an accurate reading on the levelling staff.



Source: Sz. Rózsa – Surveying I. lecture notes

Apart from the starting and the end point of the line, all other points are called change points. As the position (and therefore the RL) of these points can be arbitrary, we are not necessarily interested in determining their reduced levels, we only use them to be able to connect to the end point of the line. The starting point of the line is always a benchmark (with known RL) and the end points of the line always has some sort of point marking. Because the change points are not marked, whenever we have to put the levelling staff on a change point, we always use a so-called change plate, which is a metal plate with a rounded top. This ensures, that there is only one position where the levelling staff can be held vertically.

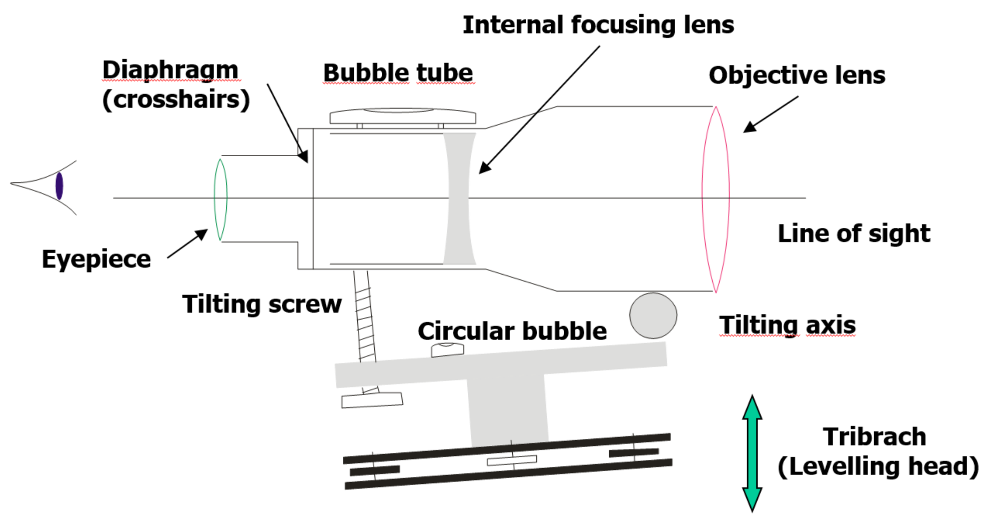
In the first station, we determine the height difference between the starting point (BS) and the first change point (FS). This means, that using the RL of the starting benchmark and the computed height difference (from the readings) between the benchmark and the first change point, we can now compute the RL of the first change point. In the next step, we pick up the instrument and the BS staff, determine the position of the next change point and put the BS staff on it, then put the instrument between the two staves. It is imperative, that the staff which is on the first change point (the FS point in the first station) remain in place! As the change plate has a rounded top, we do not have to pick up the staff, we can simply turn it around while holding it on the change plate. Picking up the staff is bad practice and can result in errors in the levelling, especially if it is not put back precisely in the position it was beforehand.

In all the subsequent stations, we continue the same procedure, until we reach the end point of the line, where we close the line. The line levelling is always carried out in a forward and a backward sense as well in order to have a check for our measurements. If the height difference calculated between A and B from the forward levelling correspond to the height difference calculated from the backward levelling, we can, in greater confidence, say that the measurements are correct. There is usually some discrepancy between the two values due to the uncompensated error sources (random errors) and we usually have an established tolerance for this discrepancy, which depends on the length of the levelling line or the number of stations in the levelling line.

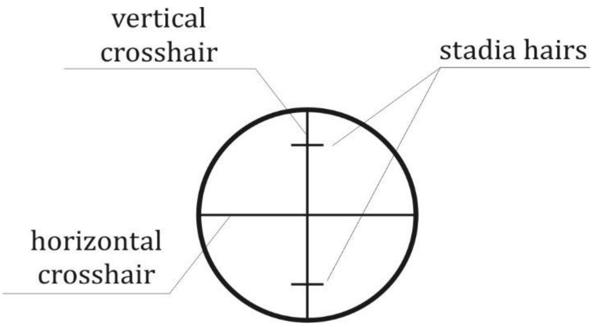
Levelling instruments (levels)

Tilting levels (or dumpy levels)

The tilting level consists of two main sections, the tribrach (or the levelling head) and the alidade, which is located above the tribrach. The tribrach connects the instrument to the tripod and it contains the foot screws that are used to approximately level the alidade. Mounted on the alidade, we have a surveyor's scope, which is fixed to the tribrach on one side with a tilting axis and on the other side with a levelling screw.



Source: Sz. Rózsa – Surveying I. lecture notes



The view inside the scope of the level

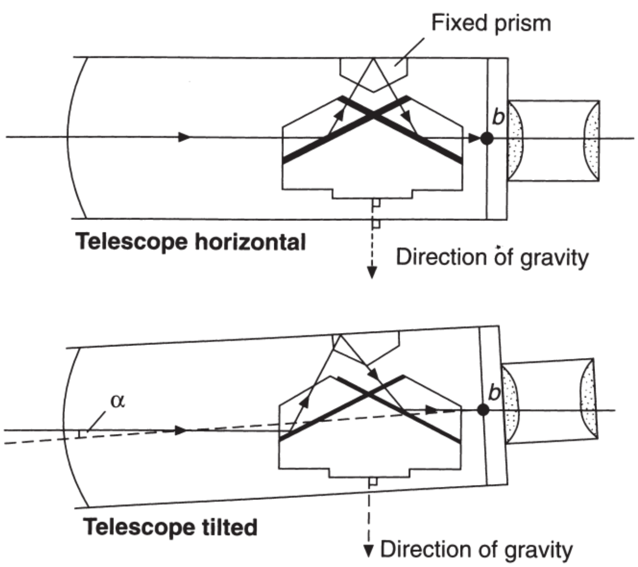
Fixed to the scope, we have the bubble tube, which is much more sensitive than the circular bubble on the alidade. The line of sight of the scope can be precisely levelled by first approximately levelling the alidade with the foot screws, then the using the levelling screw to adjust the bubble in the bubble tube. The bubble tube has to be adjusted before taking each reading, because when we turn the instrument from the BS point to the FS point, the line of sight diverges from the horizontal due to the tilting of the alidade (remember, we only set the alidade approximately horizontal with the circular bubble!).

The eyepiece can be adjusted to sharpen the image of the crosshairs inside the scope, while the parallax screw on the scope can be used to sharpen the image of the staff. The scope always creates a magnified image in order to make it possible to take accurate readings on the levelling staves. The horizontal crosshair inside the scope is used to take the readings, while the stadia lines can be used to calculate the distance between the instrument and the staff.

Automatic levels

Automatic levels are the most common instruments used in everyday surveying tasks concerning vertical control. An automatic level does not contain a bubble tube and a levelling screw. A compensator (a set of prisms) is built into the instrument which can compensate for moderate amounts of tilting. This means, that if we approximately level the instrument using the foot screws and the circular bubble on the alidade, the compensator can take care of the residual tilt due to the low sensitivity of the circular bubble.

Sokkia C320 automatic level



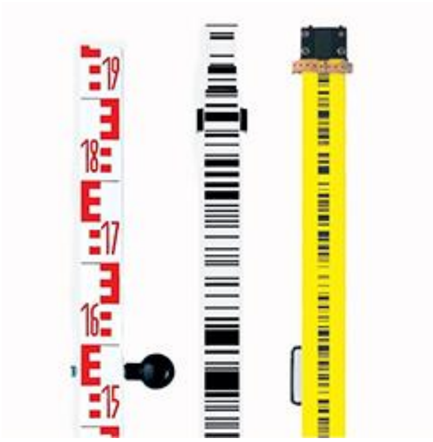
Source: W. Schofield, M. Breach: Engineering Surveying

The suspended prism is always forced into a vertical position due to the force of gravity. Combined with a fixed prism, this creates a compensation effect, so even when the telescope is tilted the line of sight remains horizontal. Naturally, the amount of tilt the compensator can handle is fixed, so the telescope has to be at least roughly horizontal.

The advantage of the automatic level is that the measurements can be carried out much faster as there is no need to adjust the bubble tube before each reading. This also means that there is less chance of committing errors by forgetting to set the bubble tube.

The disadvantage of the automatic level is that under some circumstances (vibration due to traffic/construction or wind) the compensator prisms start to oscillate which prevents us from taking readings. The compensator can also get stuck so it is advisable to check whether it is working correctly before taking a reading. This can be done by carefully tapping the side of scope while looking through it and confirming that the image in the scope starts to shake slightly before become still again.

Electronic levels



Normal levelling staff (left) and barcoded staves (center and right).

Today's state-of-the-art technology in levelling is the electronic level. Using an electronic level, the reading is not carried out by the observer, it is all done by the instrument automatically. Instead of regular levelling staves, we have to use barcoded staves. The reading is carried out by first taking an image of the portion of the levelling staff inside the scope. The instrument's memory contains the full image of the barcoded staff, using this and the image of the portion of the staff inside the scope, it can determine on which part of levelling staff the reading is located. The location from the tip of staff (also called the index) is converted into a reading value and saved in the instrument's memory.

There are numerous advantages to these levels. Automatic readings eliminate the chance of committing a reading error by the observer, while saving the reading values in the memory of the level eliminates the chance of bookkeeping blunders. Moreover, the measurements can be carried out faster than with a human observer.

The disadvantages of automatic levels are the battery dependency, the need for special barcoded staves (usually barcoded staves are different for each manufacturer, so they cannot be mixed and matched) and the sensitivity for bright illumination or lack of illumination on the staff, which restricts their use under certain conditions.

Error sources in leveling and how to eliminate them:

Equipment errors:

Collimation error: the line of sight is not horizontal when the bubbles are adjusted. Its effect is eliminated when the instrument is at equal distance from the BS and the FS points.

Parallax error: The image of the staff is not created in the same plane as the crosshairs inside the scope. This effectively means that the reading changes when looking into the scope from different angles. The parallax has to be compensated using the parallax screw.

Index error of the staff: the index of the staff ("the zero mark") does not coincide with the bottom plane of the staff. The effect cancels out if only one staff is used. If two staves are used, the number of stations has to be even. Staff graduation can also be adjusted by testing and then taking the error into account during calculations.

Staff graduation: the scales on the staff are not equally spaced due to the deterioration of the staff. The staff can be calibrated, or if the graduation is too extensive, replaced.

Observational errors:

Tilting of the staff: the staff is not held vertically. The effect can be minimized using a staff bubble.

Settlement of the tripod and the staves: the levelling should be carried out in both directions and the mean values of the height differences have to be used in the calculations. Whenever an intermediate point is measured, a change plate has to be used to support the staff.

Natural errors:

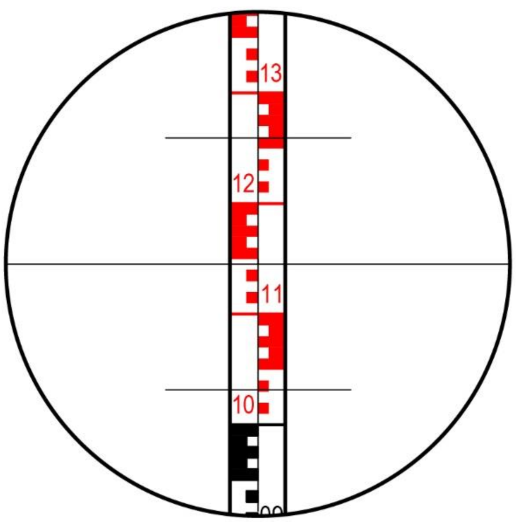
Effect of the Earth's curvature: the instrument has to be at equal distance from the BS and the FS points.

Refraction: This effect is mostly considered in precise levelling and neglected when using the surveyors' level. As a rule, precise levelling should only be carried out when the effect of refraction is minimal (the best times are the 2 hour interval starting half an hour after sunrise and the 2 hour interval ending half an hour before sunset).

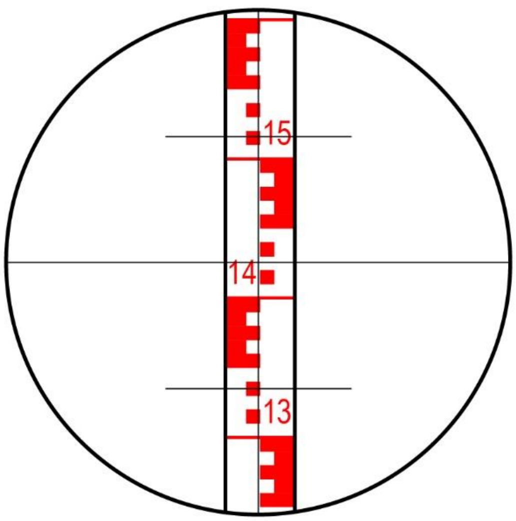
Calculation examples:

Example 1: calculate the height difference between points A and B! Calculate the distance between the level and the staff on point B!

Reading on point A:



Reading on point B:



Solution:

Reading on point A: $l_A = 1145$

Reading on point B: $l_B = 1427$

Height difference: $\Delta h_{AB} = l_A - l_B = 1145 - 1427 = -282$ mm $= -0.282$ m

Distance from point B:

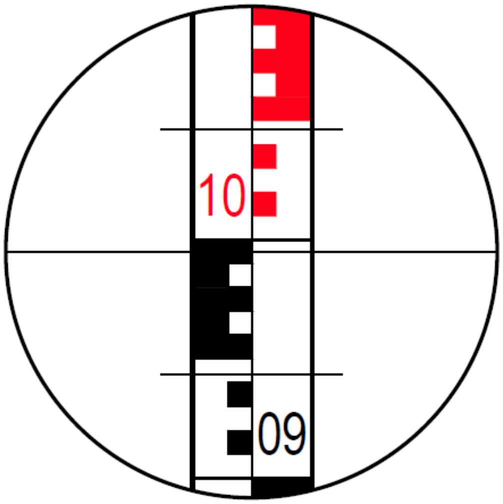
Reading on the upper stadia line: $U = 1516$

Reading on the lower stadia line: $L = 1335$

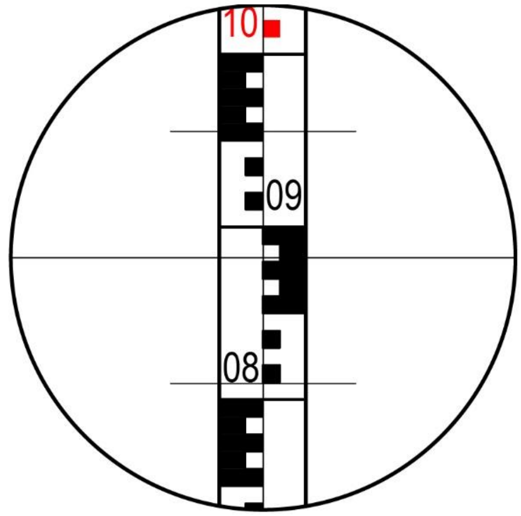
Distance: $d = (U - L)/10 = (1516 - 1335)/10 = 18.1 \rightarrow 18.1$ m

Example 2: calculate the height difference between point C and point D! Calculate the distance between the instrument and the staff on point D!

Reading on point C



Reading on point D



Solution:

Reading on point C: $l_C = 0995$

Reading on point D: $l_D = 0882$

Height difference: $\Delta h_{CD} = l_C - l_D = 0995 - 0882 = 113$ mm $= +0.113$ m

Distance from point D:

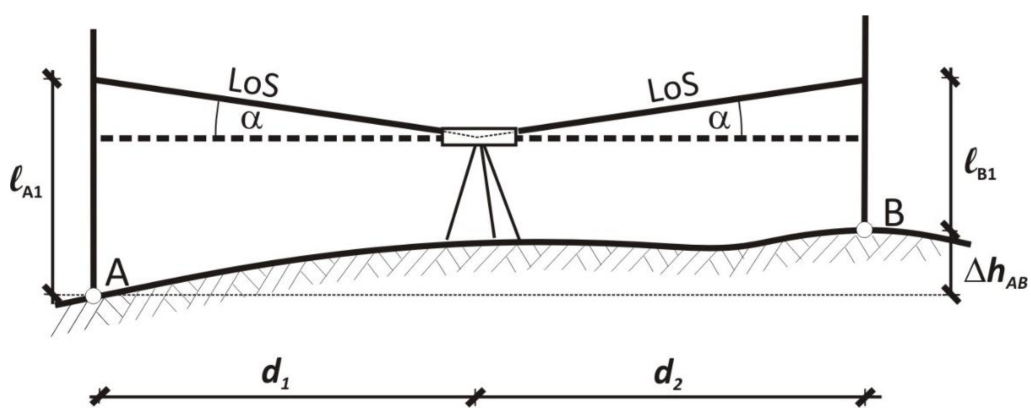
Reading on the upper stadia line: $U = 0955$

Reading on the lower stadia line: $L = 0810$

Distance: $(U - L)/10 = (0955 - 0810)/10 = 14.5 \rightarrow 14.5$ m

Example 3: adjustment for the collimation error (two-peg test).

The distances d_1 and d_2 between the instrument and the staves are both 10 m.

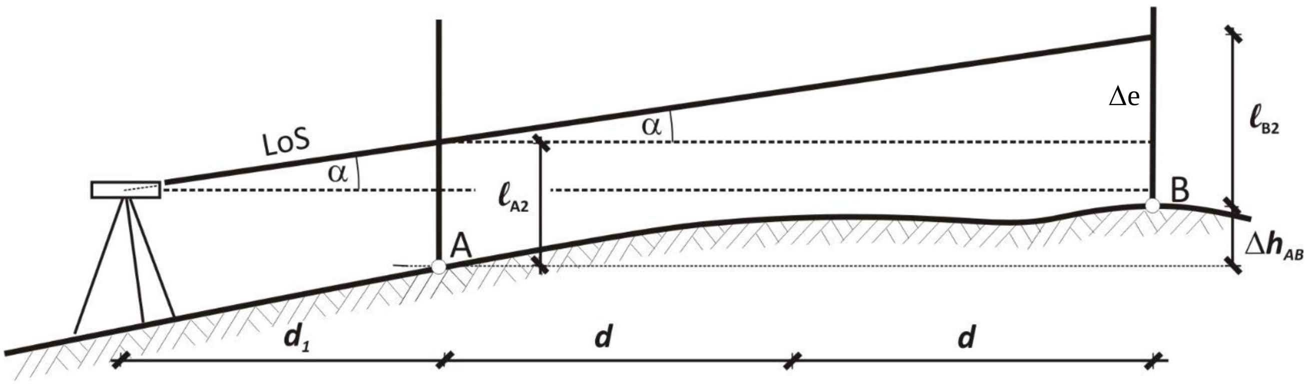


Position 1: the instrument is between the two staves.

Reading on the backsight staff (A): $l_{A1} = 1543$

Reading on the foresight staff (B): $l_{B1} = 1695$

The true height difference between the backsight and the foresight point: $\Delta h_{AB} = l_{A1} - l_{B1} = -152\ \text{mm}$



Position 2: the instrument is behind the backsight staff.

Reading on the backsight staff (A): $l_{A2} = 1652$

Reading on the foresight staff (B): $l_{B2} = 1807$

The value of Δe can be calculated as follows: $\Delta e = l_{B2} + \Delta h_{AB} - l_{A2} = \Delta h_{AB} - \underbrace{(l_{A2} - l_{B2})}_{\Delta h'_{AB}} =$

$= \Delta h_{AB} - \Delta h'_{AB}$

The value of $\Delta h'_{AB} = l_{A2} - l_{B2} = 1652 - 1807 = -155\ \text{mm}$

The value of $\Delta e = \Delta h_{AB} - \Delta h'_{AB} = (-152\ \text{mm}) - (-155\ \text{mm}) = +3\ \text{mm}$

The solution for the collimation error (α), using the distance between the two staves from the figure (20 m):

$$\alpha = \arctan\frac{\Delta \text{e}}{20\ \text{m}} = \arctan\frac{+3\ \text{mm}}{20000\ \text{mm}} = 0.00859437° \rightarrow 30.94" \approx 31"$$

Let d_1 on the second figure be equal to 10 m. Using the collimation error, we can calculate the correct reading on the backsight staff:

$$l_{A2}^{c} = l_{A2} - \tan\alpha \cdot d_1 = 1652 - \tan 31" \cdot 10000\ \text{mm} = 1652 - 2 = 1650$$

We can adjust the telescope until we see the correct value and then using the bubble's adjustment screws, level the bubble.