THE

Discrete
Element
Method
WHAT IS DEM?

The aim: to model materials or structures having discrete internal builtup „what does it do if loads are put on it?”

This presentation:
→ phenomena which are not properly reflected by continuum modelling
→ the definition of Discrete Element Modelling
→ the main steps of Discrete Element Modelling
NON-CONTINUOUS PHENOMENA

Collapse of masonry structures:
NON-CONTINUOUS PHENOMENA

Tunnels in fractured rock soils:

D. Borbély, MSc Thesis
NON-CONTINUOUS PHENOMENA

Silos:

Problems e.g.:

→ Pressure acting on the walls?

→ Emptying: sudden large forces

→ Arching
NON-CONTINUOUS PHENOMENA

Segregation: „Brasil nut effect”
NON-CONTINUOUS PHENOMENA

Segregation:
useful application:

when harmful:
   e.g. pharmaceutical industry

Microstructural explanation:  ????
NON-CONTINUOUS PHENOMENA

Soil mechanics:

e.g. Large scatter in the measured data:

also for cemented materials!
NON-CONTINUOUS PHENOMENA

Soil mechanics:
e.g. Stress dip under sand piles:

depends on:

→ deposition technique

→ grain size distribution

→ stiffness of the subsoil

Microstructual explanation:
„internal arches”
NON-CONTINUOUS PHENOMENA

Microgravity environment:

e.g. earthquakes:

\[ g = 9.81 \text{ m/sec}^2 \Rightarrow \]

San Francisco, 1989

project question:
How granular assemblies behave under nearly-zero gravity?

NASA, 1996; 1998; [2003]
NON-CONTINUOUS PHENOMENA

Microgravity environment:

→ very low confining pressure
→ triaxial compression procedures
→ until different stress levels
→ fixed with thin resin
→ after coming home: CT

The 3D microstructure during the process:

Theoretical results:
extremely complicated continuum models
NON-CONTINUOUS PHENOMENA

Microgravity environment:

e.g. modelling of „rubble pile”-asteroids:

„rubble-pile”-type:
porosity > 30%

e.g. Tunguska event (?),
1908, Siberia
NON-CONTINUOUS PHENOMENA

The surface of the Moon:

use it for protection:

„regolith”:
NON-CONTINUOUS PHENOMENA

Landing on a comet:

😊 !!!

← Churyumov – Gerasimenko
WHAT IS DEM?

The aim: to model materials or structures having discrete internal built-up

„what does it do if loads are put on it?”

„There are no good continuum models, only good curve fits.”

/unknown soil mechanican
from the XXth century/

This presentation:
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WHAT IS DEM?

The aim: to model materials or structures having discrete internal built-up

„what does it do if loads are put on it?”

The components of the model:

- separate elements + their contacts

- mechanical models for the material of the elements: → rigid
  → deformable

- contacts → recognition
  → mechanical models for the contacts:
    → non-deformable
    → deformable: e.g. point-like, deformable
  } e.g. frictional,
  e.g. finite size, deformable
  e.g. cemented
WHAT IS DEM?

Definition:

A discrete element model is a numerical model which satisfies the following conditions:

→ consists of clearly separated elements and contacts between them;
→ the elements have their own independent degrees of freedom e.g. translational, rotational, deformational
  (→↔ FEM: „C0-continuous”, „C1-continuous”)
→ the displacements are finite,
→ elements can be separated and new contacts can be formed between them, so that the creation of new contacts are automatically recognised
  (→↔ frame models, FEM fracture models: no new contacts)
WHAT IS DEM?

History overview

→ end of 1960ies:

Peter A Cundall, Imperial College:

**UDEC**
(„Uniform Distinct Element Code”)

model for fractured rocks

→ 1970ies: Molecular Dynamics methods, physics literature
not really DEM
WHAT IS DEM?

History overview

→ end of 1970ies: Cundall & Strack, 1979: BALL

→ from the 1980ies:
  → several new codes, already in 3D
  → general element shapes
  → different mathematical tools

→ from the 1990ies: practical applications in engineering
EXAMPLE

1. Define the geometry:
   - ball id 1 x 0.10 y 0.20 rad 0.10
   - ball id 2 x 0.55 y 0.20 rad 0.15
   - ball id 3 x 0.30 y 0.40 rad 0.15
   - wall id 1 nodes 0.0 0.0 0.7 0.0
   - wall id 2 nodes 0.7 0.0 0.7 0.5
   - wall id 3 nodes 0.0 0.5 0.0 0.0

2. Specify the material parameters:
   - property density 10.0
   - property kn 1.e4 ks 0.5e4 friction 0.2
   - wall id 1 kn 1.e12 ks 0. friction 0.
   - wall id 2 kn 1.e12 ks 0. friction 0.
   - wall id 3 kn 1.e12 ks 0. friction 0.

3. Specify the loads:
   - set gravity 0.0 -9.81

4. Calculate the displacements [series of small increments]
WHAT IS DEM?

Main steps of the analysis of an engineering problem:

- the model: collection of separate elements (‘discrete elements’)
  {1 body ↔ 1 element} or {several bodies ↔ few elements}
  **Step 1.: define the initial geometry**

- rigid or deformable *elements*; rigid or deformable *contacts*
  **Step 2.: specify the material characteristics**

- the loading process:
  ( e.g. external forces acting on the elements; e.g. prescribed displacements)
- calculation of the state changing: *series of small increments, based on* „ f = ma ”
  **Step 3.: calculation of the actual displacement increments**
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THE GEOMETRY

Element shapes:

polygon, polyhedron  circle, sphere  ellipse, ellipsoid  complex shapes

Matsushima, 2005:

e.g. Lu & McDowell, 2007, PFC-3D:
    Railway ballast

(a) Toyoura sand model  (b) Ottawa sand model
THE GEOMETRY

Contact recognition: a point of an element is in the interior of another element

circles/spheres: contact if:
\[ |\mathbf{r}^q - \mathbf{r}^p| \leq R^p + R^q \]

Elements composed of circles/spheres: can be treated similarly to circles/spheres

Polygons/polyhedra: contact if:
-- a node of an element is inside another element;
-- an edge of an element intersects the faces of another element

Elements defined with the help of the equation of their surface: contact if:
the two equations have common solutions
THE GEOMETRY

Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

the time consuming part: to check the existence of a contact with exact calculations

Trick #1: avoid checking every element with every other element:

→ „body based search” technique: consider only those others which are in the vicinity of the analyzed element; then take the next element to analyze, …

→ „space based search”:
  divide the domain into „windows” (overlapping); collect which elements are in which windows; analyze those pairs only where both elements belong to the same window
THE GEOMETRY

Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

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Trick #2:

Simple surrounding domains checked first (instead of the elements having complicated shapes)

the idea: „surrounding domain” assigned to each element (simple shape: brick; sphere)

→ Phase 1.: intersection between the surrounding domains? (fast)

→ if necessary: Phase 2.: detailed, exact calculations (slow)
THE GEOMETRY

Common problem often faced:

Prepare an initial arrangement so that the elements touch each other!

→ Regular arrangements:

unrealistic mechanical behaviour!

PVC beads
2-4 mm
THE GEOMETRY

Common problem often faced:

Prepare an initial arrangement
so that the elements touch each other!

→ Regular arrangements:
  unrealistic
  mechanical
  behaviour!

→ Dynamic techniques
→ Constructive techniques
Common problem often faced:

Prepare an initial arrangement so that the elements touch each other!

→ Dynamic techniques: apply the DEM code itself!

- e.g. gravity deposition:
- e.g. isotropic compression:
- e.g. grow the elements into the domain as a container:
THE GEOMETRY

Common problem often faced:

   Prepare an initial arrangement
   so that the elements touch each other!

→ Constructive techniques: purely geometric calculations!

  e.g. SSI: „Simple Sequential Inhibition”

  then dynamic densification is needed!

  e.g. „sedimentation” techniques:

  e.g. Inwards Packing:
THE GEOMETRY

Common problem often faced:

    Prepare an initial arrangement
    so that the elements touch each other!

Summary:

→ Regular arrangements:
    unrealistic results

→ Dynamic techniques:
    slow, but easy to use

→ Constructive techniques:
    very fast, but no commercial codes available at the moment
Main steps of the analysis of an engineering problem:

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MECHANICAL PROPERTIES

Mechanical behaviour of the elements:

*role: to specify how to calculate the stresses from the deformations of the elements*

→ perfectly rigid elements: deformability concentrated into the contacts
→ deformable elements:
  stress-strain-relations have to be specified
  [e.g. $E$, $\mu$, …]

Mechanical behaviour of the contacts:

*role: to specify how to calculate the contact forces from the relative displacements at the contact*

→ usually: „deformable” contacts  (relative displ. at the contact regions)
→ sometimes: infinitely rigid contacts: no overlap or any other deformation
WHAT IS DEM?

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  **Step 3.: calculation of the actual displacement increments**
QUESTIONS

1. What are the conditions to consider a numerical technique a discrete element model?

2. What is the role of the constitutive relations of the elements? From the point of view of mechanical behaviour, what basic types of elements are used in the different DEM models?

3. What is the role of the constitutive relations of the contacts? From the point of view of mechanical behaviour, what basic types of contacts are used in the different DEM models?

4. How can we prepare an initial arrangement of touching elements? What is the difference between dynamic, constructive and collective rearrangement techniques?

5. Introduce the aim and the basic idea of the body-based technique! Introduce the aim and the basic idea of the space-based technique!

6. What are the basic steps of discrete element modelling?