BALL-TYPE MODELS

→ Overview

→ PFC: fundamentals and applications

→OVAL, EDEM, YADE

→ other applications

OVERVIEW OF DEM SOFTWARES

Quasi-static metods ← ***an equilibrium state is searched for***

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\mathbf{f}=\mathbf{0}"$$

→ Kishino, 1988
→ Bagi-Bojtár, 1991

} *circular, perfectly rigid elemets, deformable contacts*

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← ***a process in time is searched for***

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ UDEC *deformable polyhedral elements, deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements, deformable contacts*

Implicit timestepping methods:

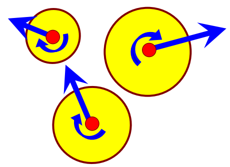
→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ contact dynamics models *rigid elements, non-deformable contacts*

BALL-TYPE MODELS

<u>BALL:</u> P.A. Cundall, 1979

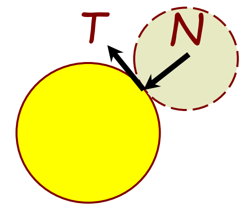
<u>elements:</u>



2D, perfectly rigid;

$$\mathbf{u}^p(t) = \begin{bmatrix} u_x^p(t) \\ u_y^p(t) \\ \varphi_z^p(t) \end{bmatrix}$$

<u>contacts:</u> point-like;
Coulomb friction



<u>eqs of motion:</u>

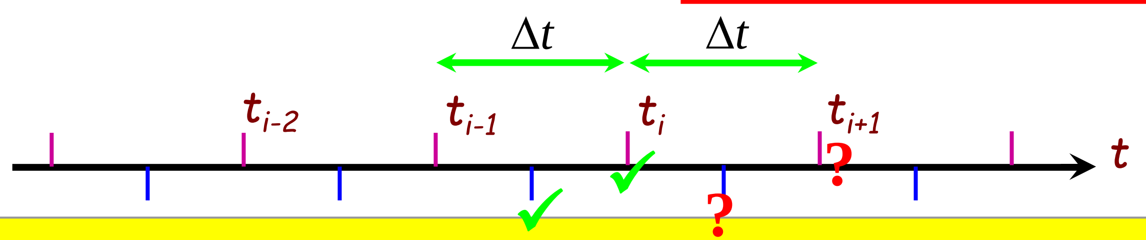
$$\mathbf{M}^p(t)\mathbf{a}^p(t) = \mathbf{f}^p(t, \mathbf{u}(t), \mathbf{v}(t))$$

$$\mathbf{M}^p = \begin{bmatrix} m^p & & \\ & m^p & \\ & & I^p \end{bmatrix}$$

$$\begin{cases} m^p a_x^p = f_x^p \\ m^p a_y^p = f_y^p \\ I^p \beta_z^p = m_z^p \end{cases}$$

numerical solution:
method of central differences

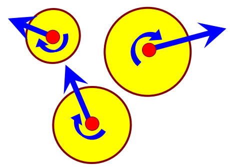
$$\mathbf{v}^p(t_i + \Delta t/2) = \mathbf{v}^p(t_i - \Delta t/2) + \left(M^p\right)^{-1} \mathbf{f}^p(t_i)\Delta t$$



positions
forces, stresses
accelerations
velocities

BALL-TYPE MODELS

BALL: P.A. Cundall, 1979

elements:  2D, perfectly rigid;

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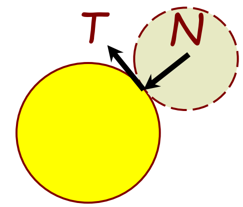
Coulomb friction



eqs of motion:

$$\mathbf{M}^p(t)\mathbf{a}^p(t) = \mathbf{f}^p(t, \mathbf{u}(t), \dot{\mathbf{u}}(t))$$

numerical solution:
method of central differences

$$v_x^p(t_i + \Delta t/2) = v_x^p(t_i - \Delta t/2) + \frac{f_x^p(t_i)}{m^p}\Delta t \qquad u_x^p(t_i + \Delta t) = u_x^p(t_i) + v_x^p(t_i + \Delta t/2)\cdot\Delta t$$

$$v_y^p(t_i + \Delta t/2) = v_y^p(t_i - \Delta t/2) + \frac{f_y^p(t_i)}{m^p}\Delta t \qquad u_y^p(t_i + \Delta t) = u_y^p(t_i) + v_y^p(t_i + \Delta t/2)\cdot\Delta t$$

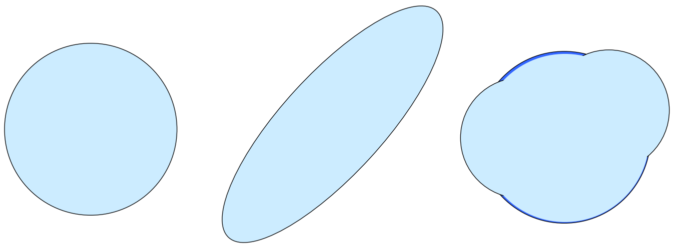
$$\omega_z^p(t_i + \Delta t/2) = \omega_z^p(t_i - \Delta t/2) + \frac{m_z^p(t_i)}{I^p}\Delta t \qquad \varphi_z^p(t_i + \Delta t) = \varphi_z^p(t_i) + \omega_z^p(t_i + \Delta t/2)\cdot\Delta t$$

BALL-TYPE MODELS

BALL: P.A. Cundall, 1979

3D version: 1983, TRUBAL
NSF grant, free for anyone who asks for
⇒ huge effect on granular mechanics researches !!!

„BALL-type" models:



→ perfectly rigid elements

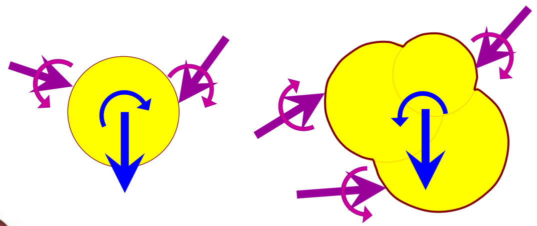
→ shape: to have point-like contacts

→ eqs of motion: $\mathbf{M}^p(t)\mathbf{a}^p(t)=\mathbf{f}^p(t,\mathbf{u}(t),\dot{\mathbf{u}}(t))$

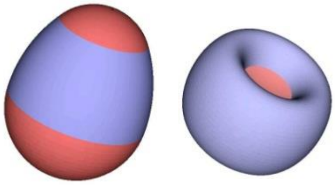
→ numerical solution of the eqs of motion: explicit time integration,
(mostly: central difference scheme)

BALL-TYPE MODELS

most important codes:

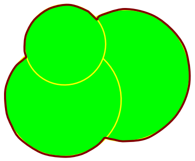


PFC-2D, PFC-3D, P.A. Cundall, Itasca Consulting



OVAL, M.R. Kuhn, Portland Catholic University

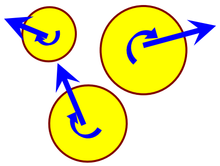
EDEM, J. Favier, Edinburgh



YADE (some versions), French origin, very active international community

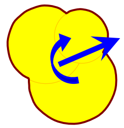
PFC BASIC PRINCIPLES

Elements:



perfectly rigid cylinders (2D) or spheres (3D)

m: mass, *I*: inertia



„clump": rigid group of elements, fixed together

m: sum of masses, *I*: sum of inertia about the „centre"

Degrees of freedom:

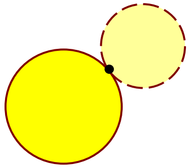
translation of the „centre" (i.e. reference point);

rotation about the „centre"

PFC BASIC PRINCIPLES – THE CONTACTS

Contact types:

frictional

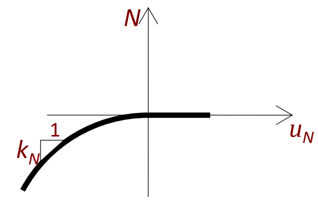


$$\Delta N^c = k_N \Delta u_N^c \qquad N^c \le 0$$

$$\Delta T^c = k_T \Delta u_T^c \qquad \left|T^c\right| \le -\nu \cdot N^c$$

→ linear: $k_N = \text{konst.};\ k_T = \text{konst.}$ ← *default*

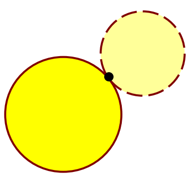
→ Hertz-Mindlin:



$$k_N = \frac{\langle G\rangle \sqrt{2\langle R\rangle}}{1-\langle \mu\rangle}\sqrt{u_N^c}$$

$$k_S = \frac{2\sqrt[3]{3\langle G\rangle^2 (1-\langle \mu\rangle)\langle R\rangle}}{2-\langle \mu\rangle}\sqrt[3]{-N^c}$$

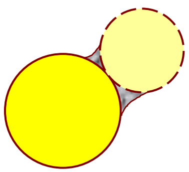
cemented → point-like:



$$N^c \le N_{\max}$$

$$\left|T^c\right| \le T_{\max}$$

→ extended:



$$\Delta\sigma_N = \tilde{k}_N \Delta u_N \ (lin.) \qquad \sigma_N \le \sigma_N^{\max}$$

$$\Delta\sigma_T^c = \tilde{k}_T \Delta u_T^c \qquad \left|\sigma_T\right| \le \sigma_T^{\max}$$

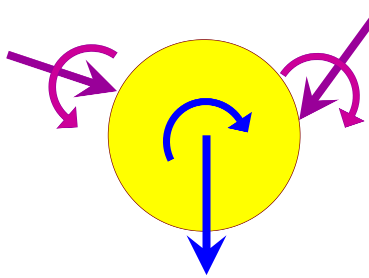
Viscoelastic contacts, Softening models, other possibilities , C/C++

PFC BASIC PRINCIPLES – TIME INTEGRATION

Displacement calculations

Newton II.: „ $m\,a = f$ ”

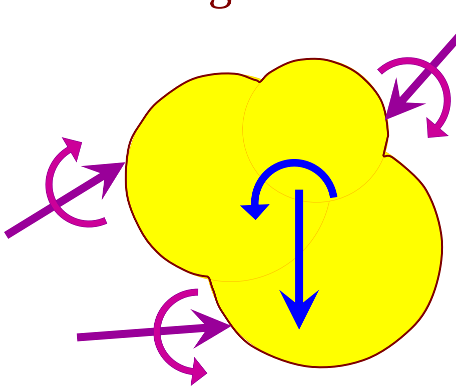
– forces acting on the spherical elements:



← from the contacts of the element
← from the external loads (weight, drag force)
← from damping

} → f, M

– forces acting on the clumps:



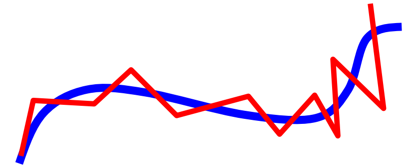
← from the contacts of the element
← from the external loads (weight, drag force)
← from damping

} → f, M

PFC BASIC PRINCIPLES – TIME INTEGRATION

Displacement calculations

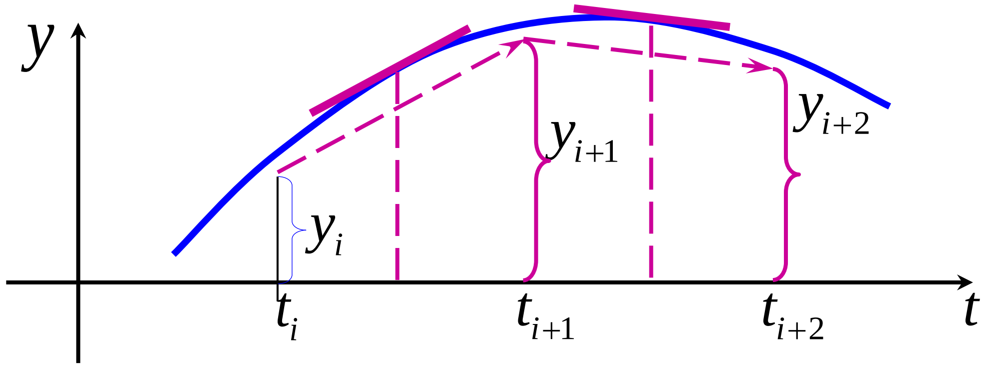
Newton II.: „ $m\,a = f$ ”



Method of Central Differences

– the eqs of motion, discretized: $\mathbf{v}^p(t_i + \Delta t/2) = \mathbf{v}^p(t_i - \Delta t/2) + \left(\mathbf{M}^p\right)^{-1}\mathbf{f}^{\,p}(t_i)\Delta t$

– from this: $\mathbf{u}^p(t_i + \Delta t) = \mathbf{u}^p(t_i) + \mathbf{v}^p(t_i + \Delta t/2)\Delta t$



– to ensure numerical stability, and to help fast convergence:

1. estimation for the longest allowed Δt : $\Delta t_{crit} = \min \begin{cases} \sqrt{m/k^{transl}} \\ \sqrt{I/k^{rotat}} \end{cases}$
2. density scaling: to modify masses/inertia

   for time-dependent problems: never use it!
3. damping

PFC BASIC PRINCIPLES – TIME INTEGRATION

3. Damping:

the eqs. of motion including velocity-proportional damping:

$$\mathbf{v}(t_i + \Delta t/2) = \mathbf{v}(t_i - \Delta t/2) + \mathbf{M}^{-1}\left(\mathbf{f}(t_i) - c_u \mathbf{v}(t_i - \Delta t/2)\right) \cdot \Delta t$$

→ 1. Local damping ← *default*

→ 2. Contact viscous damping

PFC BASIC PRINCIPLES – TIME INTEGRATION

3. Damping:

a. *Local damping*: e.g. for a sphere:

$$v_x(t_i+\Delta t/2)=v_x(t_i-\Delta t/2)+\left(f_x(t_i)-\alpha\cdot|f_x(t_i)|\,\mathrm{sign}(v_x(t_i-\Delta t/2))\right)\frac{\Delta t}{m}$$

$$\omega_z(t_i+\Delta t/2)=\omega_z(t_i-\Delta t/2)+\left(M_z(t_i)-\alpha\cdot|M_z(t_i)|\,\mathrm{sign}(\omega_z(t_i-\Delta t/2))\right)\frac{\Delta t}{I_z}$$

default: $\alpha := 0.70$

- *unequilibrated motions are damped only;*
- *does not depend on the magnitude of the velocities;*
- *very good for systems in which:*
  parts are already in equilibrium, while parts are still far from the eq.

b. *Contact viscous damping*:

viscous force added to the contact force:

its direction: opposite to the relative motion

$$\begin{cases} |D_N| = c_N \left|\dfrac{\Delta u_N}{\Delta t}\right| \\ |D_T| = c_T \left|\dfrac{\Delta u_T}{\Delta t}\right| \end{cases}$$

PFC BASIC PRINCIPLES

Further details:

e.g. application results, publications, courses, conferences, …

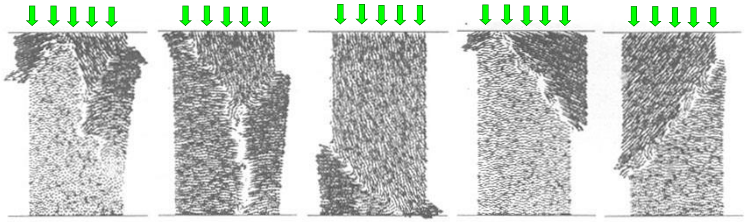
www.itascacg.com

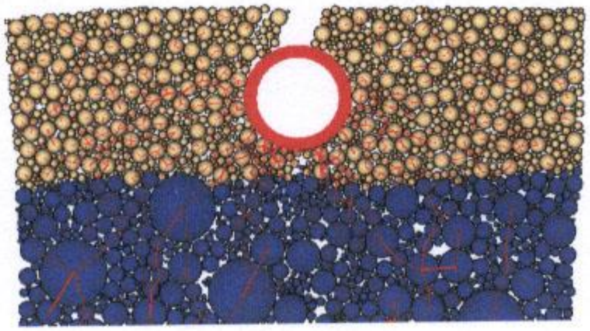
→ Software → Downloads → Demonstration versions

PFC PRACTICAL APPLICATIONS

Usual applications:

loose and cemented granular materials





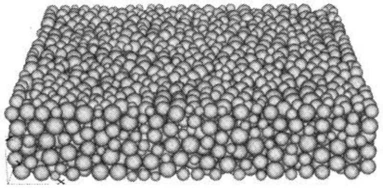
1. Weathering of cemented granular rock under buildings
2. Railway ballast modelling

PFC PRACTICAL APPLICATIONS

<u>Weathering of cemented granular rock</u>

Nova et al, 2004

PFC-3D



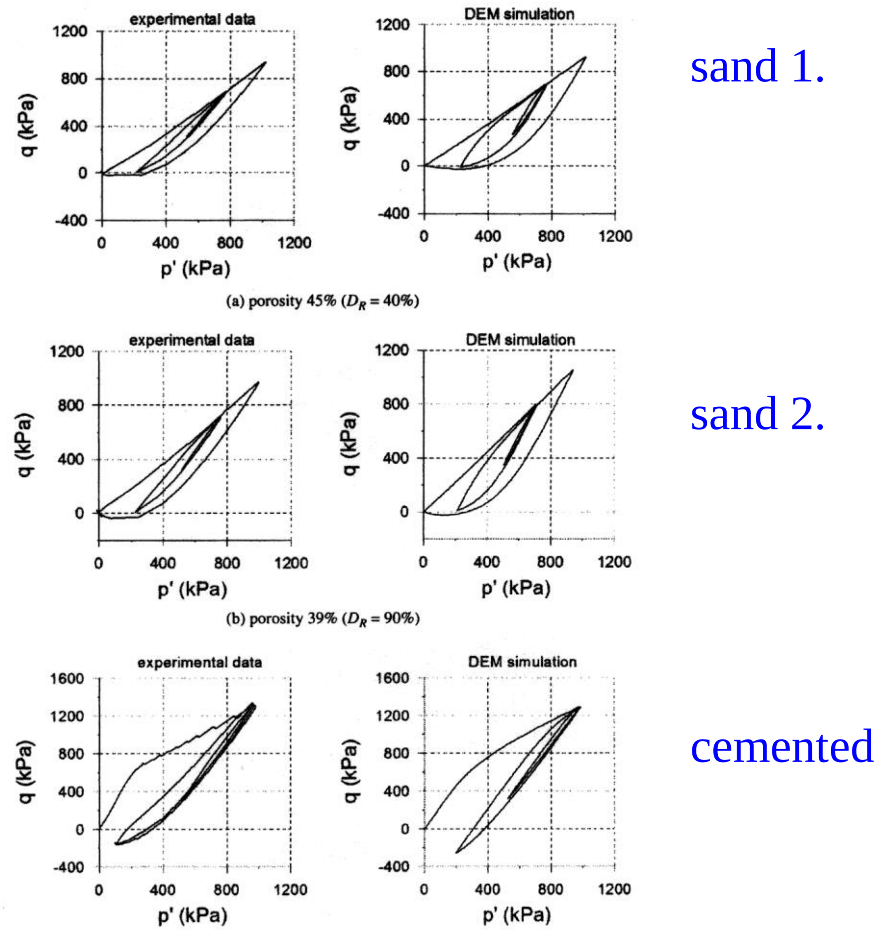
triaxial loading

3500 elements

size distribution: 1:2

analyzed problem:

weathering of rock under buildings



(a) porosity 45% ($D_R = 40\%$)

(b) porosity 39% ($D_R = 90\%$)

(c) porosity 49% (cemented)

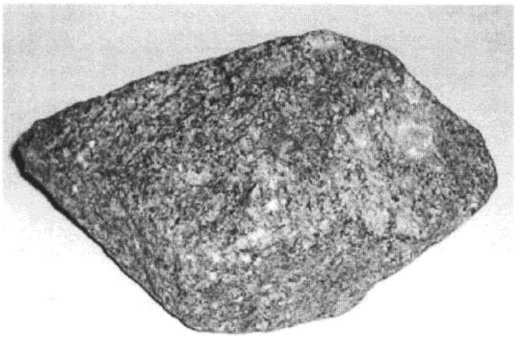
k_n	k_s	μ	c_b	σ_b^t	χ	ΔU^p_{lim}
225 KN/m	135 KN/m	0.2	1500 kPa	250 kPa	0.6	0.2

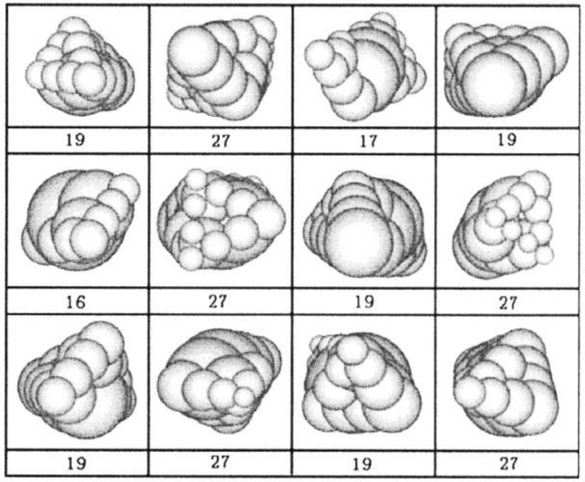
(d) DEM parameters

PFC PRACTICAL APPLICATIONS

<u>Railway ballast modelling</u>

e.g. effect of stone shape: Lu & McDowell, 2007, PFC-3D





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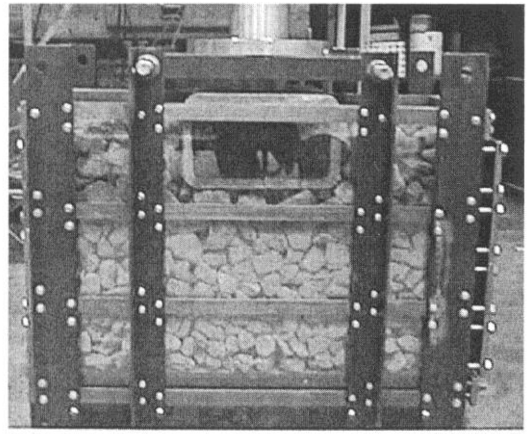

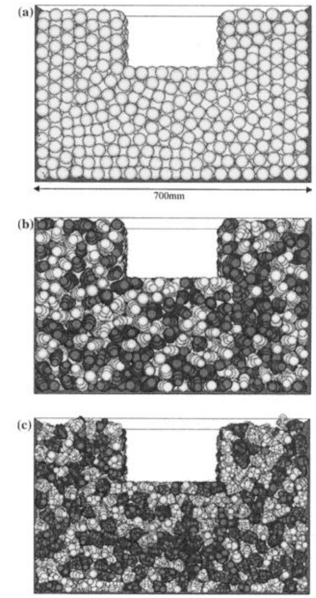

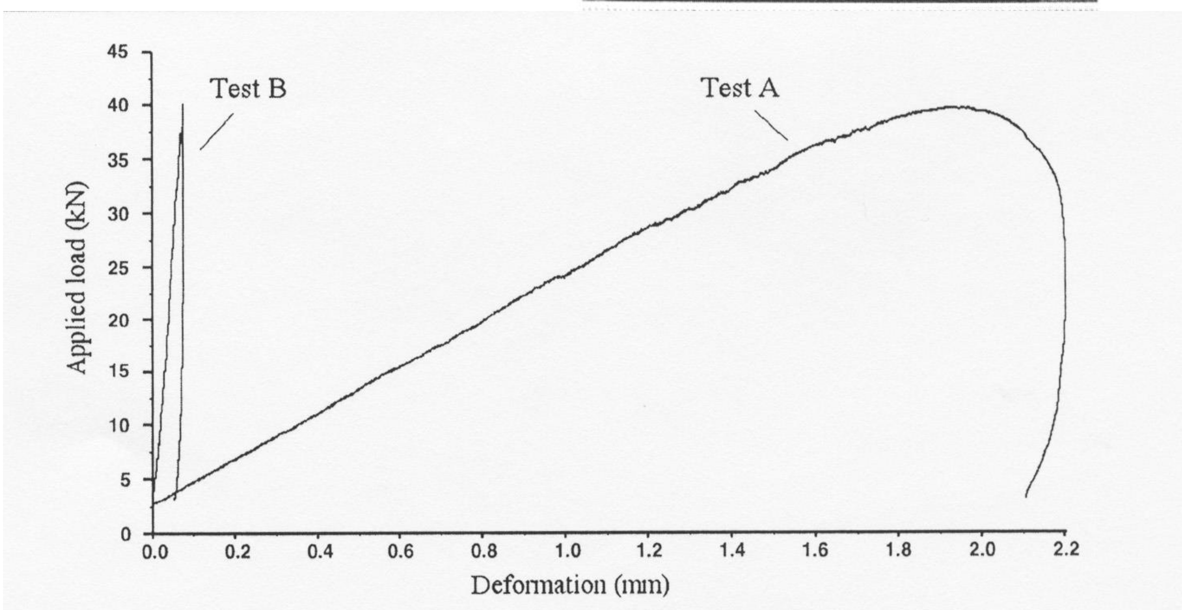

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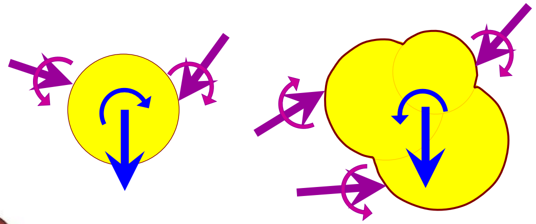
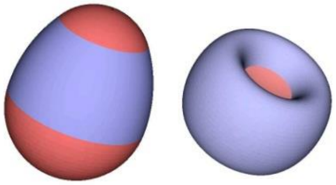
BALL-TYPE MODELS

most important codes:

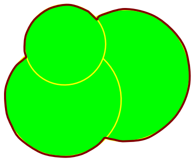
PFC-2D, PFC-3D, P.A. Cundall, Itasca Consulting



OVAL, M.R. Kuhn, Portland Catholic University



EDEM, J. Favier, Edinburgh

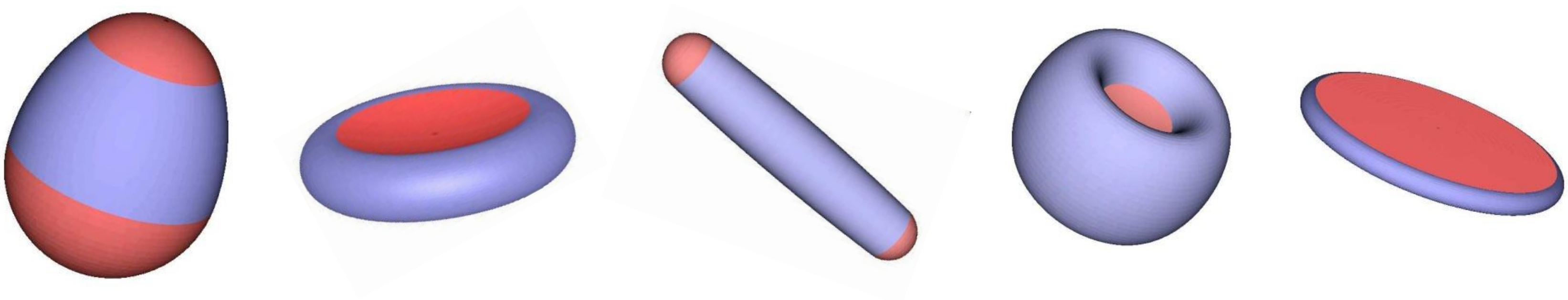


YADE (some versions), French origin, very active international community

OVAL

Matthew R. Kuhn, USA, Research code!

→ elements: surface composed of cylindical/spherical/toroidal etc. surfaces



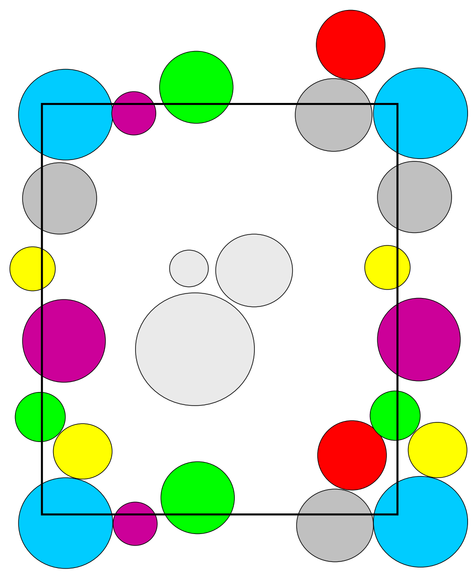
→ contacts:

frictional

→ boundaries: walls; or:

periodic boundaries

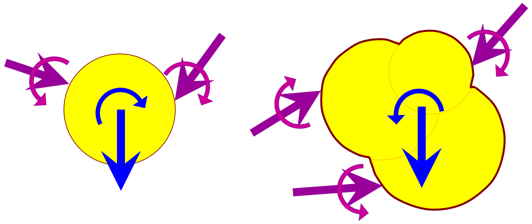
→ input: command files,
output: data files, (figures)



kuhn@up.edu

BALL-TYPE MODELS

PFC-2D, PFC-3D, P.A. Cundall, Itasca Consulting



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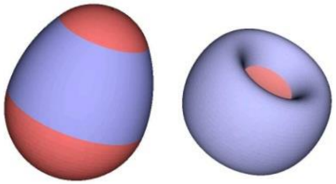

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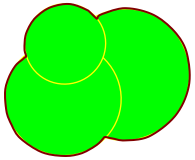


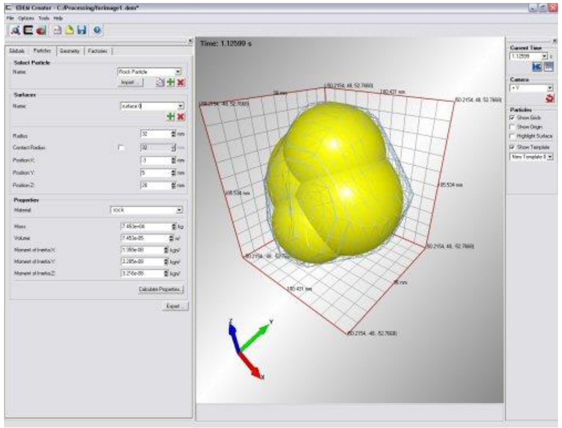
YADE (some versions), French origin, very active international community

EDEM *www.dem-solutions.com*

J. Favier, Edinburgh

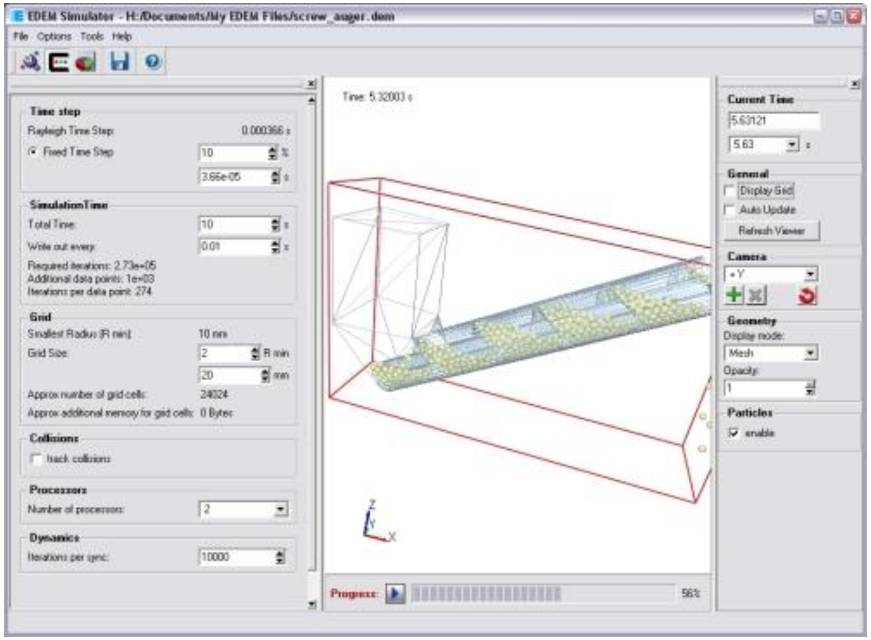
→ elements: spheres / composed of spheres

→ contacts: frictional, linearly elastic
Hertz-Mindlin
cohesional
cemented
individually coded



→ boundaries: several types!

→ special features of the
displacement calcultions:
- easy to connect to
FEM or CFD
(solids, fluids)
- fast, \\



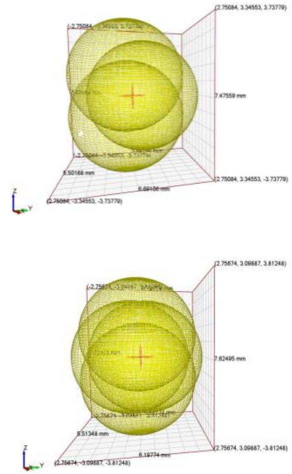
→ output: rich (videos, pictures, etc.)

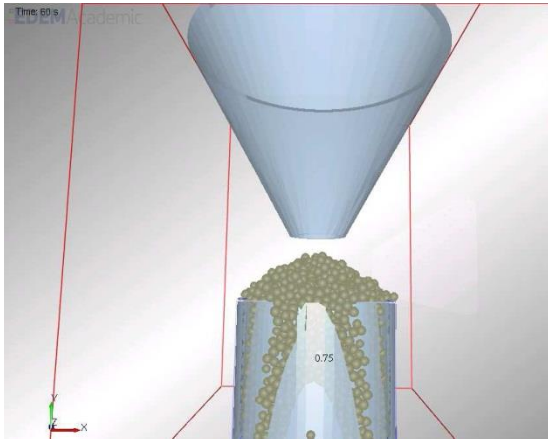
EDEM *www.dem-solutions.com*

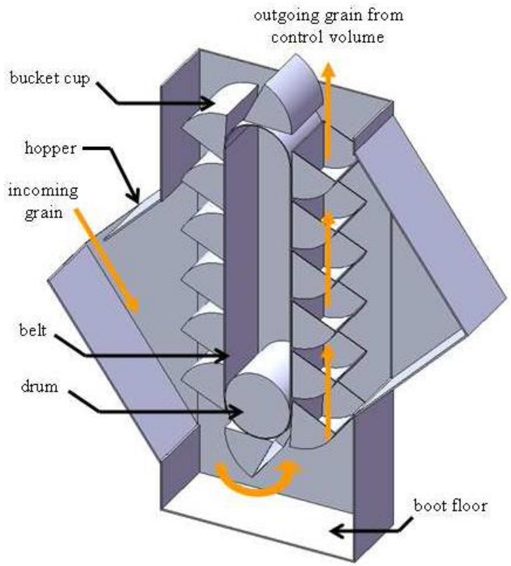
applications:

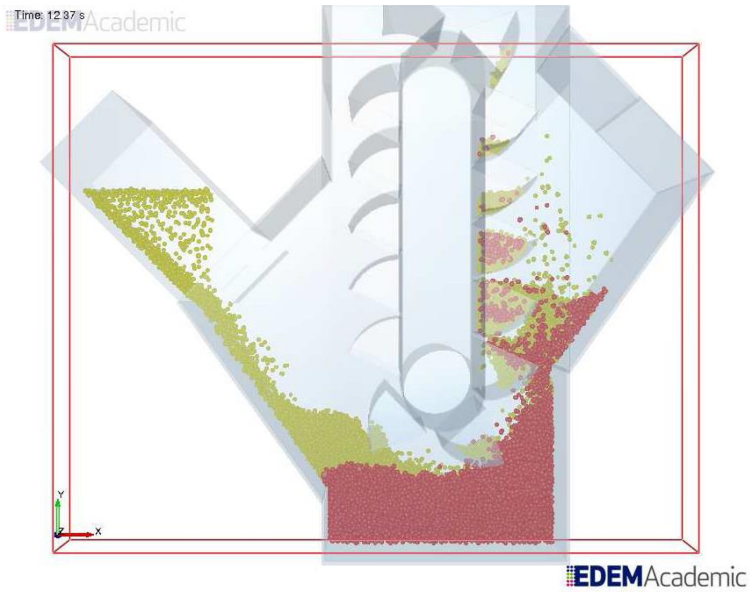
e.g. J. Boac, 2010:
assemblies of soy bean & corn

result:
well-calibrated model
for machine design





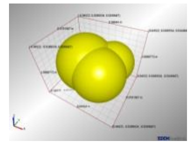


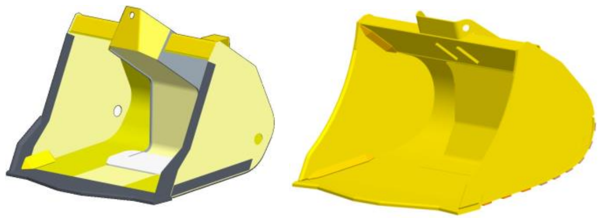


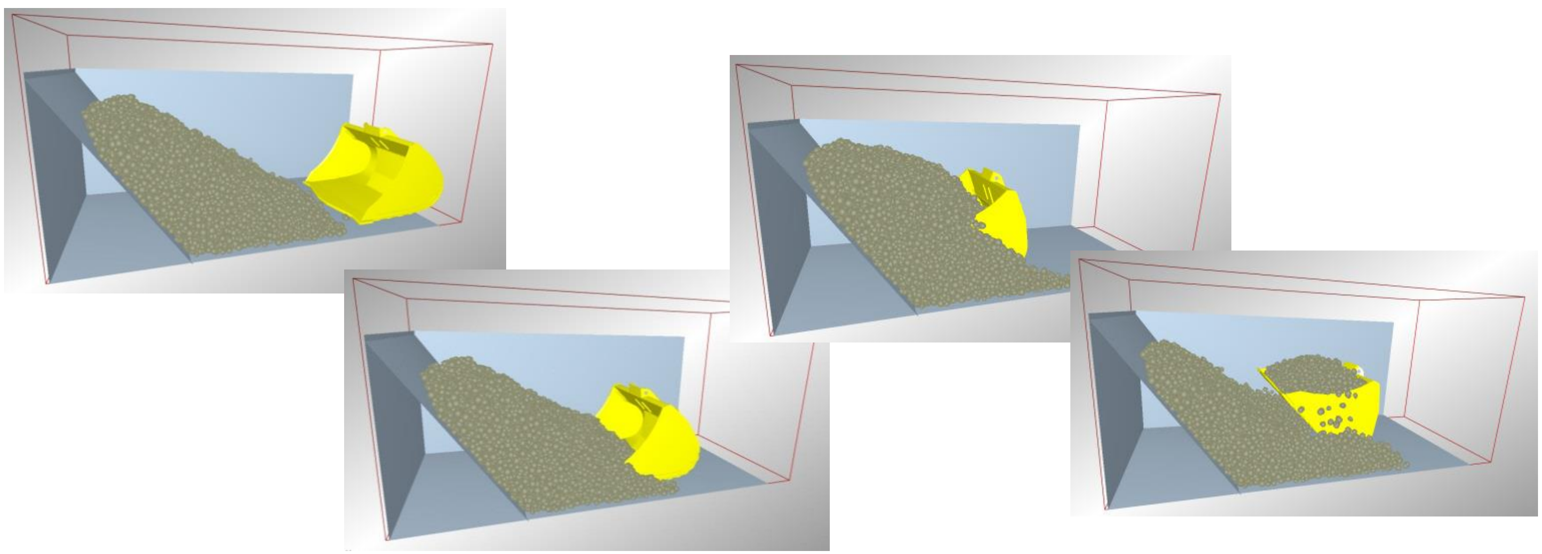
EDEM *www.dem-solutions.com*

applications:

e.g. J. Helgesson, 2010:





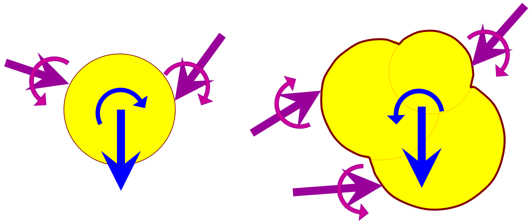


result:

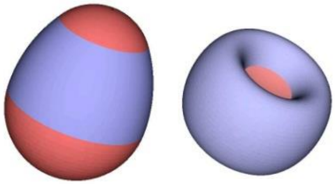
optimize the geometry of the spoon

BALL-TYPE MODELS

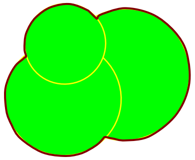
PFC-2D, PFC-3D, P.A. Cundall, Itasca Consulting



OVAL, M.R. Kuhn, Portland Catholic University

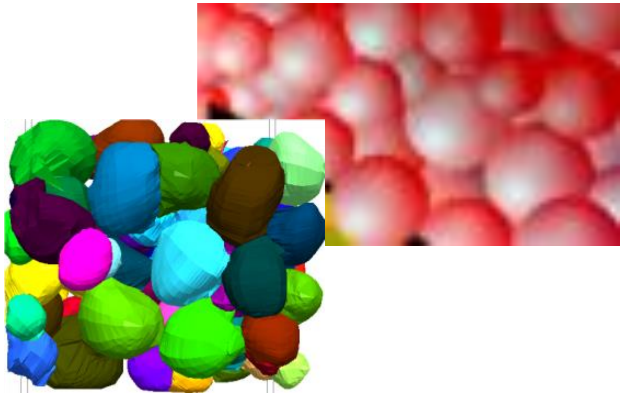


EDEM, J. Favier, Edinburgh



YADE (some versions), French origin, very active international community

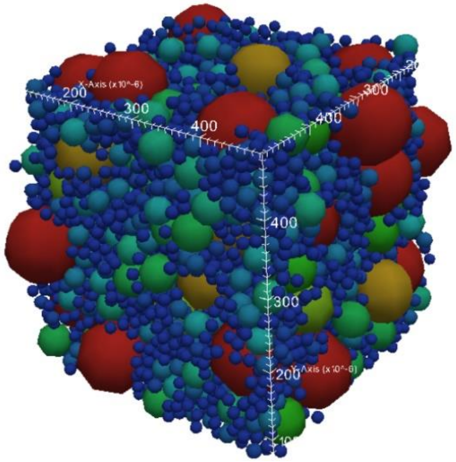
YADE



elements: spheres;
complex shapes consisting of spheres;
polyhedra;
[anything personal can be coded]

contacts:
[several models, individual codes shared]

applications:
researchers!
e.g. simulation of lunar regolith: Modenese (2013)

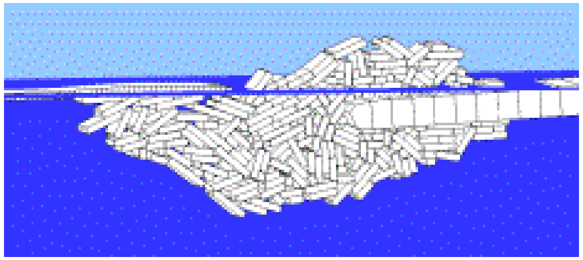


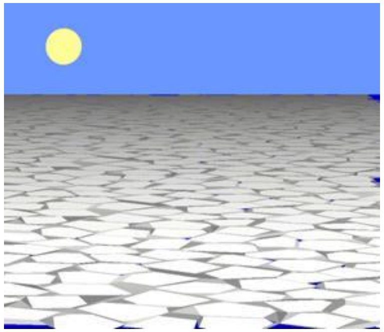
https://yade-dem.org

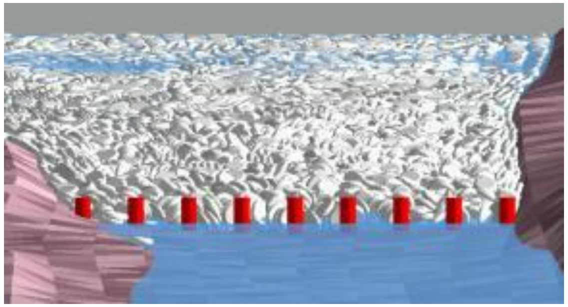
BALL-TYPE MODELS:
OTHER APPLICATIONS

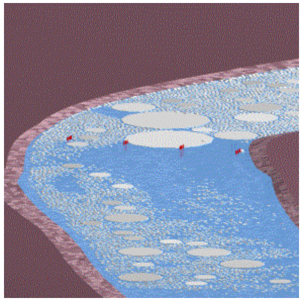
Floating ice blocks: effect on military vessels and structures

US Army Research Institute,
Cold Regions Department:

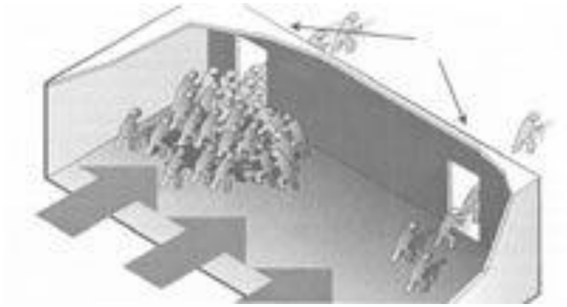




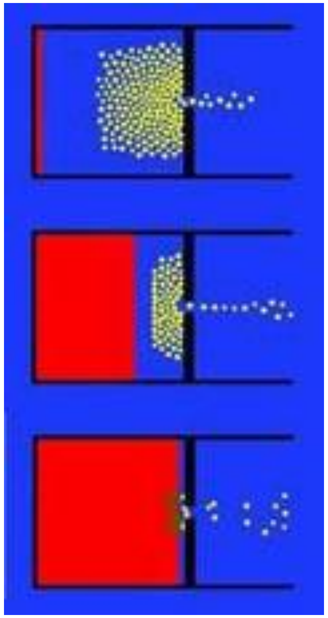


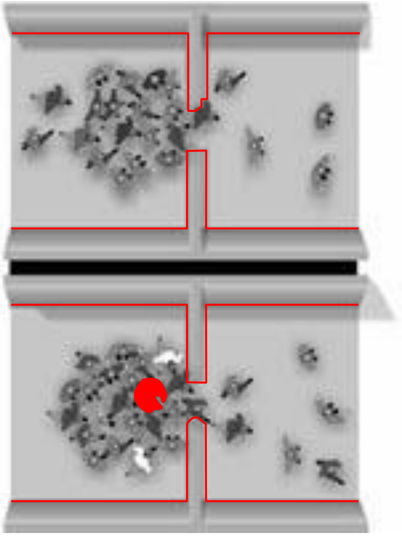


BALL-TYPE MODELS:
OTHER APPLICATIONS



Architectural design: Simulation of a crowd in panic





QUESTIONS

1. Under what conditions does a discrete element model belong to the family of BALL-type models?
2. What types of contact models are applied in the BALL-type models? Shortly describe them!
3. Explain the calculation of a single time step in PFC! Why is it important to set a limit to the length of the time step, and how can this limit be estimated?
4. Why should damping be used in BALL-type models? Introduce local damping and contact viscous damping!