UDEC/3DEC

→ How UDEC/3DEC works
→ Application examples

OVERVIEW OF DEM SOFTWARES

<u>Quasi-static metods</u> ← ***an <u>equilibrium state</u> is searched for***

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\mathbf{f}=\mathbf{0}"$$

→ Kishino, 1988
→ Bagi-Bojtár, 1991

circular, perfectly rigid elemets, deformable contacts

<u>Time-stepping methods</u> $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← ***a <u>process in time</u> is searched for***

simulate the motion of the system along small, but finite Δt timesteps

<u>Explicit timestepping methods:</u>

→ UDEC *deformable polyhedral elements, deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements, deformable contacts*

<u>Implicit timestepping methods:</u>

→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ contact dynamics models *rigid elements, non-deformable contacts*

UDEC / 3DEC

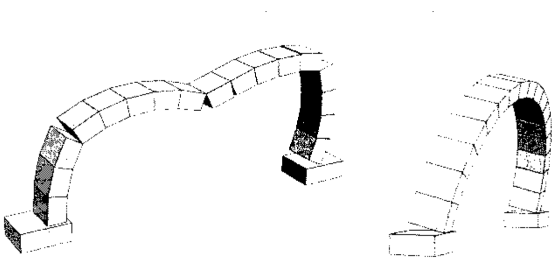
„**U**niversal **D**istinct **E**lement **C**ode”

P.A. Cundall, 1971;

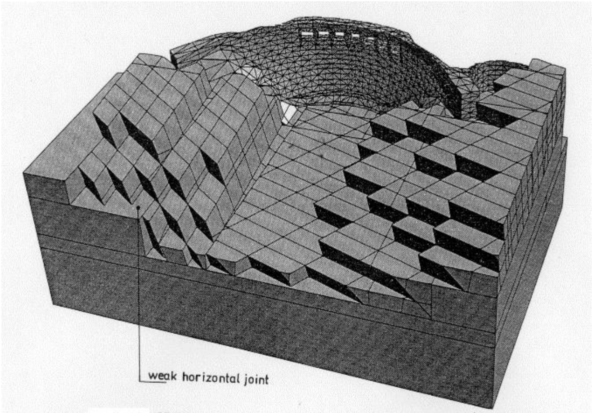
development through decades

Itasca Consulting Group

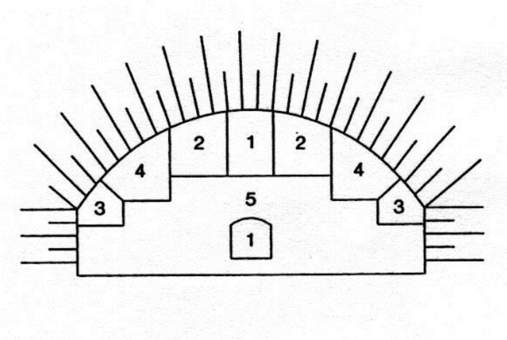
www.itascacg.com



MOST WIDESPREAD IN CIVIL ENGINEERING



3DEC representation of physical model of Cambambe dam.



UDEC / 3DEC

→ UDEC/3DEC basic principles

Elements: degrees of freedom; material characteristics

Contacts: types, material characteristics

Displacements: Numerical stability issues

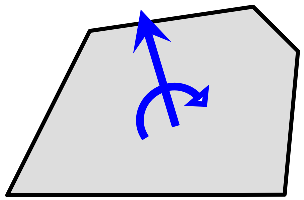
→ UDEC/3DEC practical applications

1. Underground ice hockey cavern, Norway
2. Dams
3. Masonry structures

BASIC PRINCIPLES – ELEMENTS

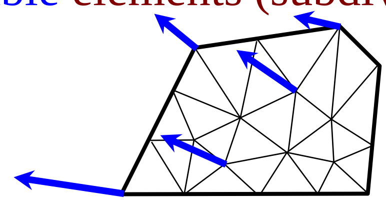
Elements: polygons / polyhedra (planar faces!);

– rigid elements



degrees of freedom:
translation of and rotation about the centroid

– deformable elements (subdivided into simplex zones)



„uniform strain” tetrahedral zones
((*10-node tetrahedra – not reliable*))
degrees of freedom: translations of the nodes

Material models for the elements:

e.g. marble: K = 37,2 GPa; G = 22,3 GPa
e.g. granite: K = 43,9 GPa; G = 30,9 GPa
e.g. sandstone: K = 26,8 GPa; G = 7 GPa; Mohr-Coulomb,
fric = 28°; coh = 27,2 MPa; tens = 1,17 MPa

(rigid) ↔ deformable:

– „null element” (no material in the element)

default → – linearly elastic, isotropic *(e.g. intact rock; metal)*

– lin. elast., with: Mohr-Coulomb or Prager-Drucker
(e.g. soils, concrete) *(e.g. clay)*
+ tensile strengh + cohesion + dilation angle

– ...

$$K = \frac{E}{3(1-2\nu)};\ G = \frac{E}{2(1+\nu)}$$
where
$$K \cdot (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

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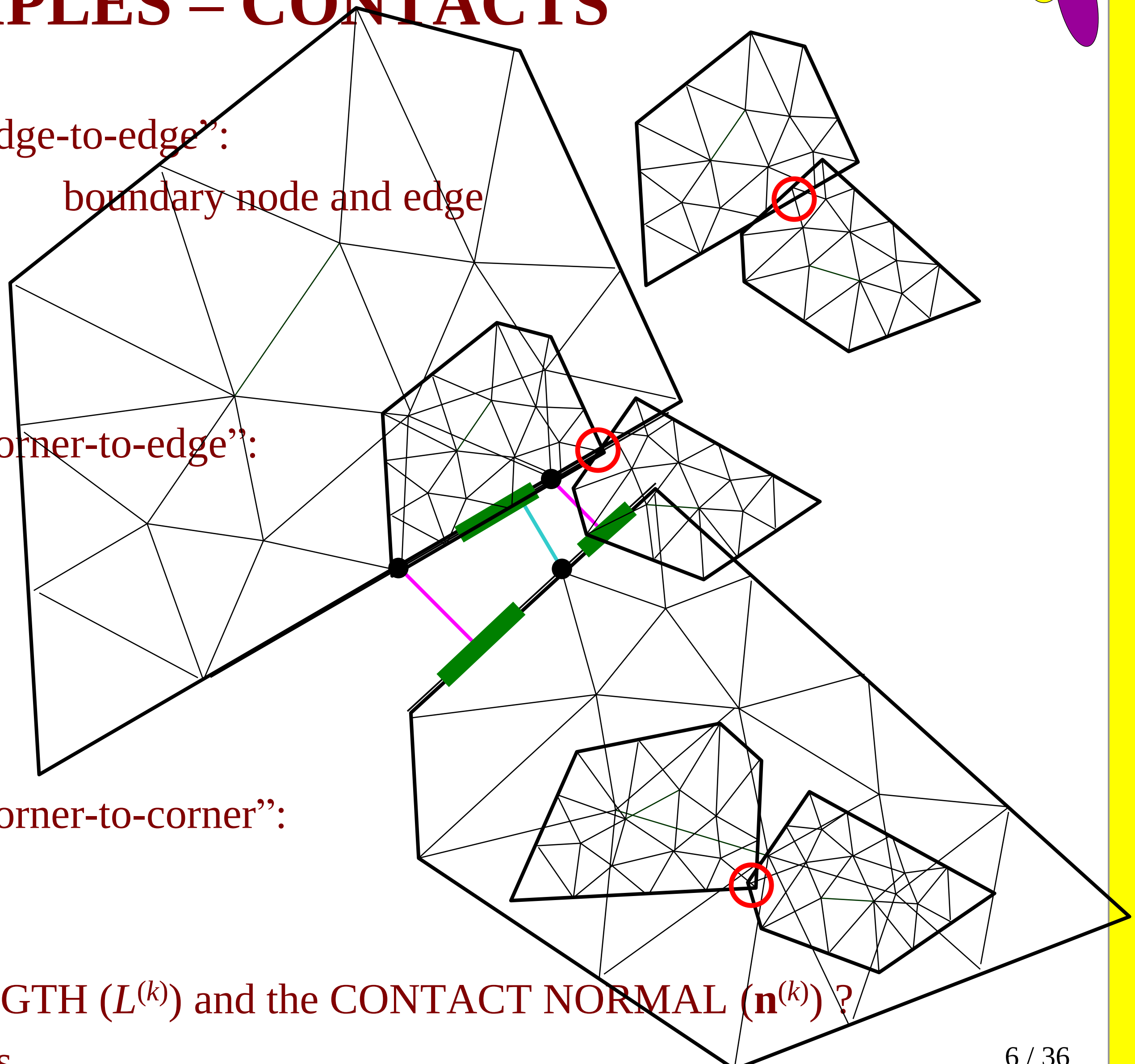
BASIC PRINCIPLES – CONTACTS

<u>Contacts in 2D:</u> → „edge-to-edge”:

boundary node and edge



→ „corner-to-edge”:

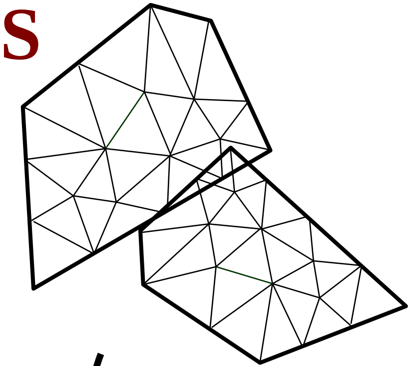
→ „corner-to-corner”:

What to mean by the LENGTH ($L^{(k)}$) and the CONTACT NORMAL ($\mathbf{n}^{(k)}$) ?

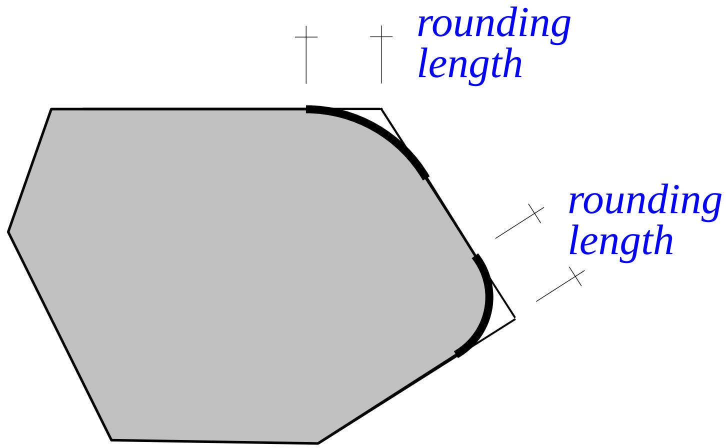
<u>Aim:</u> avoid abrupt changes

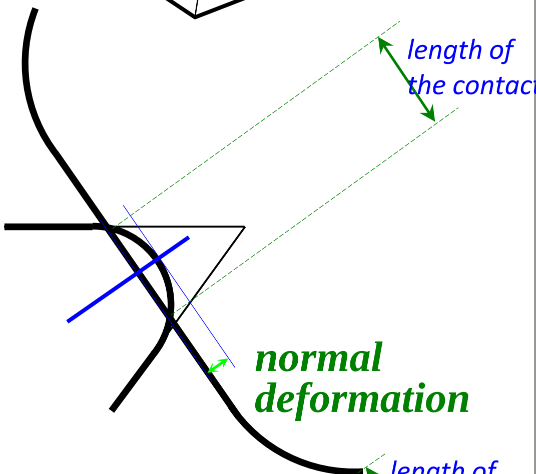
BASIC PRINCIPLES – CONTACTS

<u>Contacts in 2D:</u> → „edge-to-edge”:

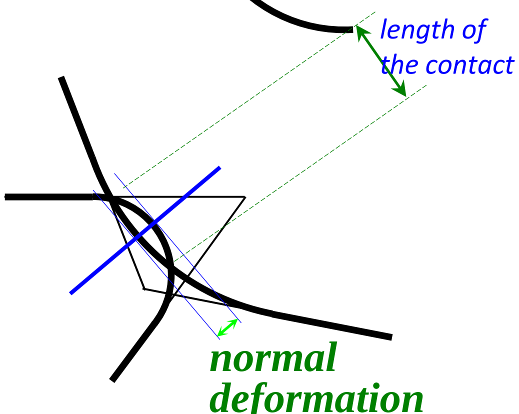


→ „corner-to-edge”:



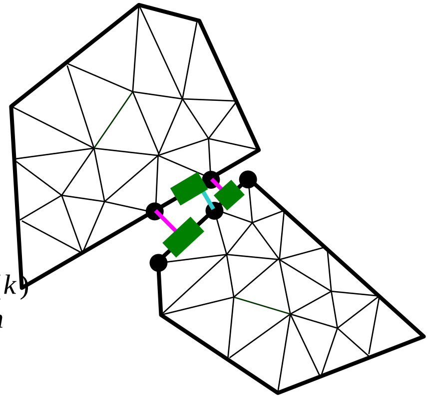


→ „corner-to-corner”:



BASIC PRINCIPLES – CONTACTS

Quantitative characteristics of the sub-contacts:



sub-contact length: $L^{(k)}$

normal direction: $\mathbf{n}^{(k)}$

normal and shear stiffness:

→ increment of contact normal stress: $\Delta\boldsymbol{\sigma}_n^{(k)} = -k_n \Delta u_n^{(k)}$

(**uniformly** distributed contact force)

→ increment of contact shear stress: $\Delta\boldsymbol{\sigma}_s^{(k)} = k_s \Delta u_s^{(k)}$

(**uniformly** distributed contact force)

maximal possible shear stress for Coulomb friction:

$$\left|\boldsymbol{\sigma}_s^{(k)}\right| \leq \ coh + \tan(\ fric) \cdot \boldsymbol{\sigma}_n^{(k)}$$

maximal posible tensile stress:

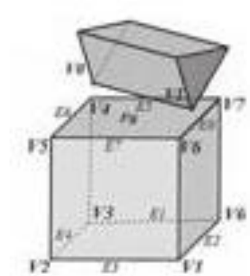
$$\boldsymbol{\sigma}_n^{(k)} \leq \ \text{tensile strength}$$

BASIC PRINCIPLES – CONTACTS

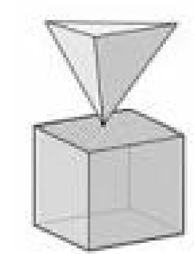
Contacts in 3D:

Types:

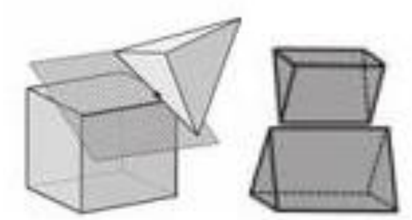
face-to-face



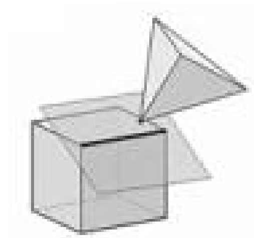
edge-to-face



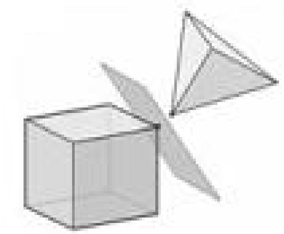
corner-to-face



edge-to-edge



corner-to-edge



corner-to-corner

The aim:

to define the AREA ($A^{(k)}$) and the CONTACT NORMAL ($\mathbf{n}^{(k)}$) of the contact in such a way that abrupt changes during block motions are avoided

→ **„common-plane" technique**;

contact recognized and then divided into sub-contacts

BASIC PRINCIPLES – CONTACTS

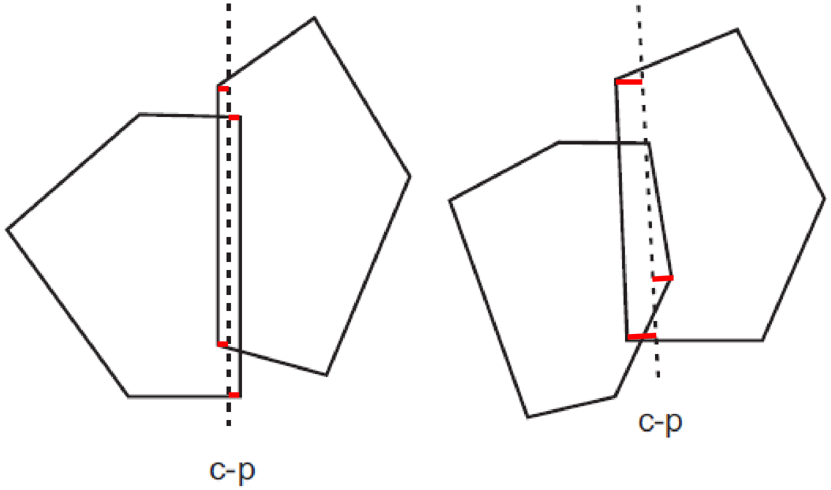
Contacts in 3D:

What to mean by the AREA ($A^{(k)}$) and the CONTACT NORMAL ($\mathbf{n}^{(k)}$) ?

„common-plane" concept :

„***Minimize*** the overlap between the common-plane and the node with the greatest overlap." or, equivalently:

„Locate the plane to have the ***smallest*** maximal distance between nodes on the other side, and the common-plane."



⇒ contact normal ✓

⇒ then separately for the two elements:

assign a sub-contact to every node

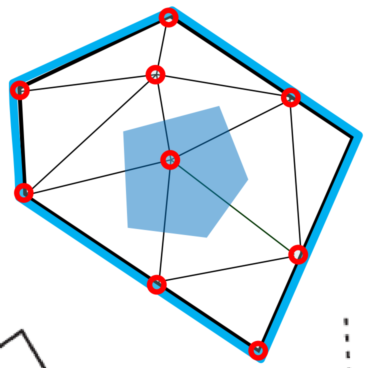
which is on the other side of the c-p

result: two sets of sub-contacts

BASIC PRINCIPLES – CONTACTS

<u>Contacts in 3D:</u>

What to mean by the AREA ($A^{(k)}$) and the CONTACT NORMAL ($\mathbf{n}^{(k)}$) ?



The definition of the sub-contact system:

[prepared twice, independently from both sides]

→ draw ⊥ from the nodes to the c-p

→ area assigned to a sub-contact: 1/3 [≈ !]

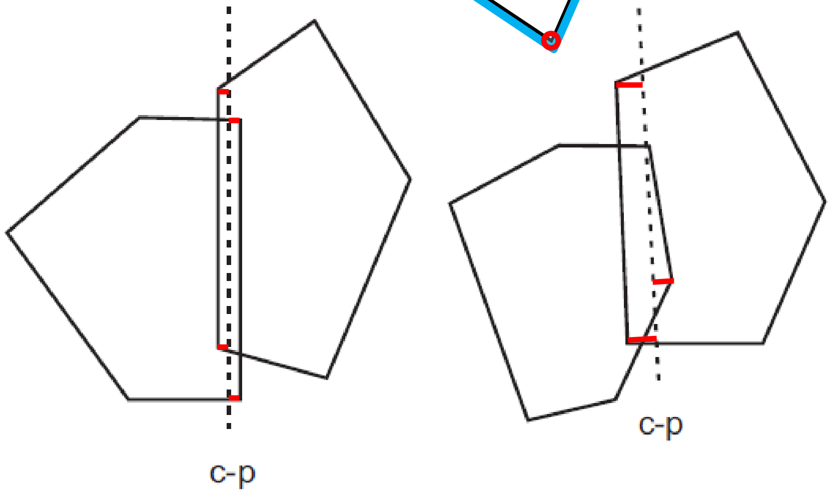
$\Rightarrow A^{(k)}$ ✓

→ special treatment at the edges

(not detailed here)

(Rigid blocks:

surface discretized; then similarly)



Sub-contact deformation increment during Δt:

relative vel. of a node and its projection on the other face: $\mathbf{v}_{rel}^{(k)} = \mathbf{v}_{node}^{(k)} - \mathbf{v}_{opposite\ face}^{(k)}$

during Δt : $\mathbf{v}_{rel}^{(k)} \cdot \Delta t \quad \Rightarrow \Delta u_n^{(k)}; \Delta u_s^{(k)}$

BASIC PRINCIPLES – CONTACTS

Distributed forces along the sub-contacts:

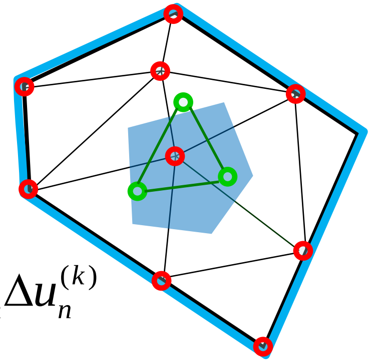
sub-contact area: $A^{(k)}$; normal direction: $\mathbf{n}^{(k)}$

Sub-contact deformation increment during Δt:

$$\mathbf{v}_{rel}^{(k)} = \mathbf{v}_{node}^{(k)} - \mathbf{v}_{opposite\ face}^{(k)}$$

$$\mathbf{v}_{rel}^{(k)} \cdot \Delta t \quad \Rightarrow \Delta u_n^{(k)}; \Delta u_s^{(k)}$$

by linear interpolation:



because of the normal and shear stiffness:

→ increment of contact normal stress: $\Delta\boldsymbol{\sigma}_n^{(k)} = -k_n \Delta u_n^{(k)}$

(**uniformly** distributed contact force)

→ increment of contact shear stress: $\Delta\boldsymbol{\sigma}_s^{(k)} = k_s \Delta u_s^{(k)}$

(**uniformly** distributed contact force)

Resultant force assigned to the node;

opposite resultant distributed among the three nodes on the opposite face

After doing the same also for all the face nodes of other block:

two sets of nodal („sub-contact”) forces are received for both blocks!

⇒ ½ („averaged”), for every node, on both faces

BASIC PRINCIPLES – CONTACTS

<u>Material models for the contacts:</u>

[calculate the increments of distrib. contact forces from the increments of rel. disps]

– if no material in the contacts: → k_n, k_s: numerical parameters, ∞; friction: real value

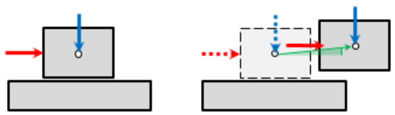
– if material in the joints: (modelled as length or area, with zero thickness):

default → – linear behaviour for compression and shear, Coulomb-friction,
+ cohesion and tensile strength

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion & tensile strength + softening + dilation angle

$$\Delta U_n(dil) = \Delta U_s \tan\psi$$

– others …



examples for characteristic values:

normal and shear stiffness: 10 – 100 MPa/m   …   …   100 GPa/m
(*soft, with clay*) … … (*hard rock, healed*)

friction angle: 10° … … 50°

cohesion and tensile strength: from 0 … … till the strength of intact rock… …

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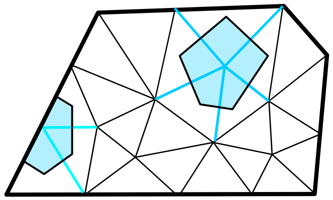
BASIC PRINCIPLES – TIME INTEGRATION

<u>Calculation of nodal displacements</u>

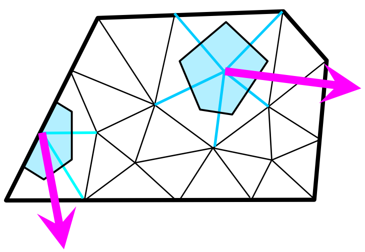
Newton II.: „ $m\,a = f$ ”

– mass assigned to the node:



Voronoi-cell

– force on the node: resultant of the forces acting on the Voronoi-cell of the node



← from the neighbouring element

← from external forces (e.g. self weight, drag force)

← from the stresses inside the simplexes

- force from the stress within a simplex:
  - nodal translations ⇒ simplex strain ✓
  - from this and material characteristics ⇒ uniform stress in the simplex ✓
  - stress vector acting on the face of the cell: $\sigma_{ij} n_j = p_i$; resultant ✓

BASIC PRINCIPLES – TIME INTEGRATION

Calculation of nodal displacements

Newton II.: „ $m\,a = f$ ”

– discretized form of the eqs of motion:

$$m\frac{\mathbf{v}(t_i+\Delta t/2)-\mathbf{v}(t_i-\Delta t/2)}{\Delta t}=\mathbf{f}(t_i)$$

or:

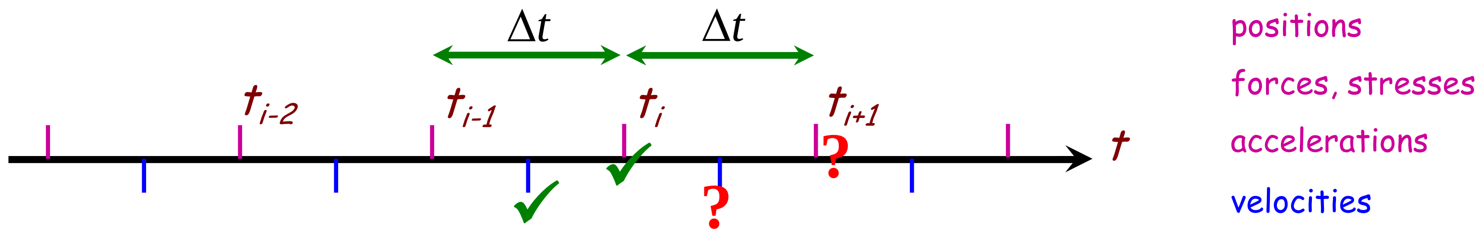
$$\boxed{\mathbf{v}(t_i+\Delta t/2)=\mathbf{v}(t_i-\Delta t/2)+\frac{\mathbf{f}(t_i)}{m}\Delta t}$$

– at t_i : the ***positions of the nodes*** and the ***forces and stresses*** are known;

at $t_i-\Delta t/2$: the ***nodal velocities*** are known;

determine the ***nodal velocities*** at $t_{i+1/2}=t_i+\Delta t/2$

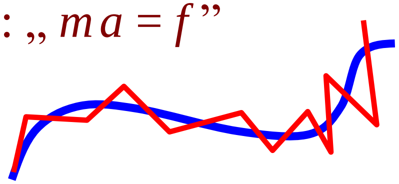
and the ***positions of the nodes*** at $t_{i+1}=t_i+\Delta t$



BASIC PRINCIPLES – TIME INTEGRATION

<u>Calculation of nodal displacements</u>

Newton II.: „ $m\,a = f$ ”



– series of small finite time steps:

– main disadvantages: explicit; no stiffness matrix!!!

$\Rightarrow$ numerical instabilities, convergence problems

– <u>to help numerical stability:</u>

1. estimate the longest allowed Δt

<u>ELASTIC SYSTEMS</u>:

requirement for deformation calculations: $\Delta t \le \Delta t_{nodes} = \min\limits_{(nodes)} \left\{ 2\sqrt{\dfrac{m_{node}}{k_{node}}} \right\}$

requirement for contact deformation calculations:

$$\Delta t \le \Delta t_{blocks} = \gamma \cdot \left\{ 2\sqrt{\dfrac{\min\limits_{(blocks)}\left(mass_{block}\right)}{\max\limits_{(\mathbf{joints})}\left(k_{\mathbf{joints}}\right)}} \right\}$$

$\uparrow$

default: $\gamma := 0.10$

$$\boxed{\Delta t := \min_{(p)} \left\{ \begin{array}{l} \Delta t_{nodes} \\ \Delta t_{blocks} \end{array} \right.}$$

NOT ELASTIC!!! $\rightarrow$ friction; damping; plastic yield; …

BASIC PRINCIPLES – TIME INTEGRATION

– to help numerical stability:

2. artifical dampings

Velocity-proportional damping:

$$\cancel{m\frac{\mathbf{v}(t_i+\Delta t/2)-\mathbf{v}(t_i-\Delta t/2)}{\Delta t}+c_V\mathbf{v}(t_i-\Delta t/2)=\mathbf{f}(t_i)}$$

$$\cancel{\boxed{\mathbf{v}(t_i+\Delta t/2)=\mathbf{v}(t_i-\Delta t/2)+\left(\mathbf{f}(t_i)-c_V\mathbf{v}(t_i-\Delta t/2)\right)\frac{\Delta t}{m}}}$$

Instead:

[help to find equilibrium:]

„local damping”: ← *default*

$$v_x(t_i+\Delta t/2)=v_x(t_i-\Delta t/2)+\left(f_x(t_i)-\alpha\cdot\left|f_x(t_i)\right|\frac{v_x(t_i-\Delta t/2)}{\left|v_x(t_i-\Delta t/2)\right|}\right)\frac{\Delta t}{m}$$

↑ *default:* $\alpha := 0.80$

advantageous if some parts of the system are already equilibrated, others are just collapsing

„adaptive global damping” or „auto damping”:

≅ velocity-proportional damping, with coefficients being adjusted, so that the change of kinetic energy during Δt is decreased (eg 50%)

advantageous if the whole system oscillates around the equilibrium

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http://www.itascacg.com/software/3dec/videos/bottle-conveyor-modeled-with-3dec

PRACTICAL APPLICATIONS

<u>Ice hockey cavern, Norway, Gjovick:</u>

The problem:

- ⇒ Fractured rock
- ⇒ Large dimensions

„The ice hockey cavern has a finished span of 62 m, a length 91 m and a height of 24 m. The spectator capacity is currently 5300, making it far the largest cavern for public use in the world. As is typical when one is extending the limits of experience and technology, the initial skepticism that had to be overcome was formidable.”

(completion: 1993)

PRACTICAL APPLICATIONS

Ice hockey cavern, Norway, Gjovick:

- Soil: fractured gneiss

  joint systems: 5

  no clay ☺
- Geological state wellknown:

  existing caves + drill

  ⇒ material characteristics ✓

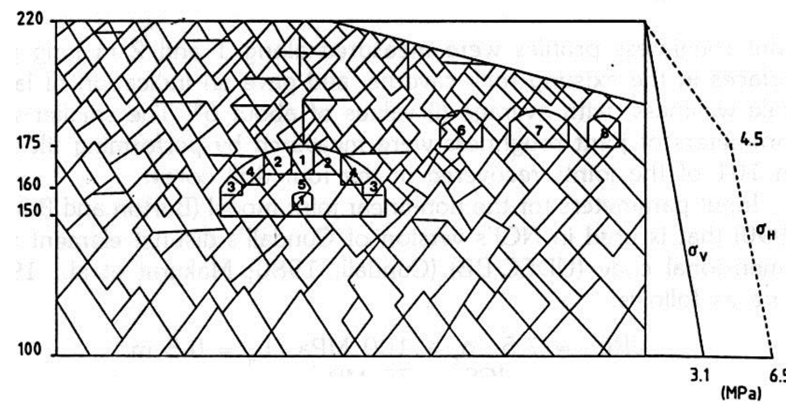
  ⇒ initial stress state ✓
- the structure:

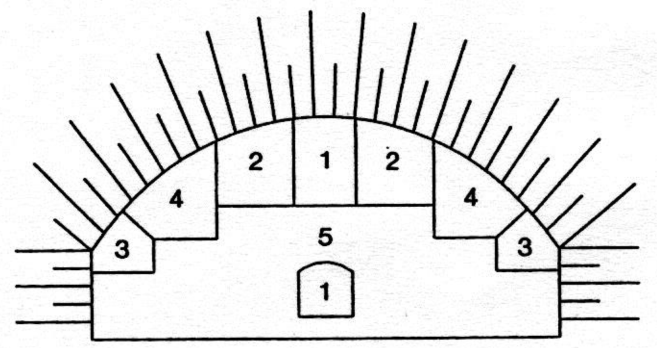
  cables / bars;

  shotcrete
- Numerical model:

  UDEC (2D)

  deformable elements

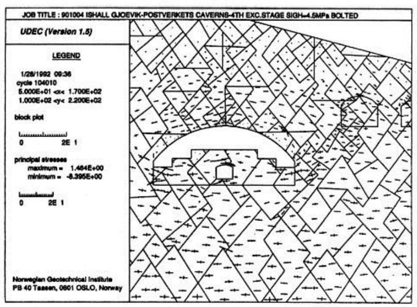


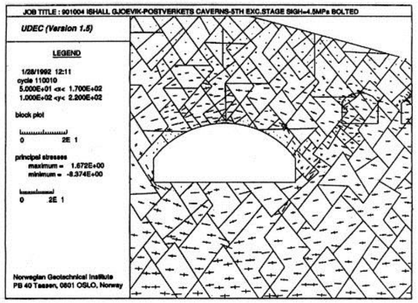


PRACTICAL APPLICATIONS

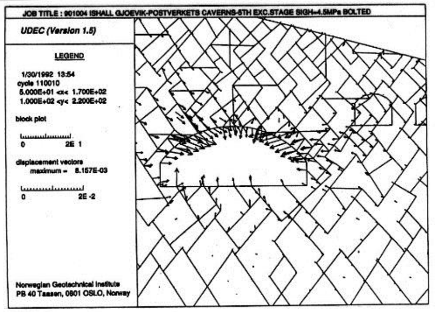
Ice hockey cavern, Norway, Gjovick:

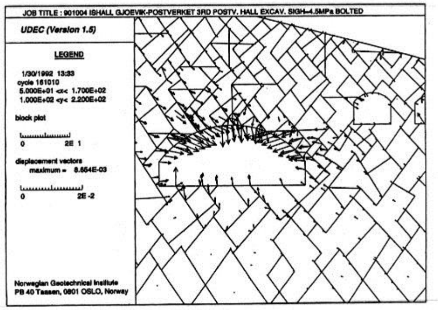
e.g. development of principal stresses:





e.g. translations:





PRACTICAL APPLICATIONS

Ice hockey cavern, Norway, Gjovick:

UDEC:

Table 1. Summary of Gjϕvik Olympic cavern run (with Postal service caverns)

Parameter	Step 1	Step 2	Step 3	Step 4	Step 5	Excav. of 1st cavern	Excav. of 2nd cavern	Excav. of 3rd cavern
Maximum principal stress MPa	9.29	11.49	9.91	8.39	8.37	8.56	8.71	8.83
Maximum displacement (mm)								
total	1.85	1.80	2.63	6.99	8.16	8.28	8.43	8.65
wall	—	—	—	1.33	3.78	3.88	3.92	3.97
crown (vertical component)	0.50	1.08	2.62	4.05	4.33	4.39	4.87	7.01
Maximum shear displacement (mm) along horizontal	1.11	1.54	2.49	3.51	4.67	5.67	5.54	5.56
joint crown	1.11	1.54	2.49	3.51	3.70	3.70	4.10	6.85
Maximum hydraulic aperture (mm) crown	0.69	1.01	1.62	2.64	2.86	3.68	3.72	4.13
Maximum axial forces on bolts (tnf)	7.0	25	25	25	25	25	25	25

Measured:

Table 2. Summary of Gjϕvik Olympic cavern *in situ* measurements for Location E4. The number given refer to total deformation. (NGI extensometers (E4) + SINTEF (S2) + surface subsidence).

Parameter	Step 1	Step 2	Step 3	Step 4	Step 5
Total deformation (mm)	0.65	1.31	2.86	6.56	8.55

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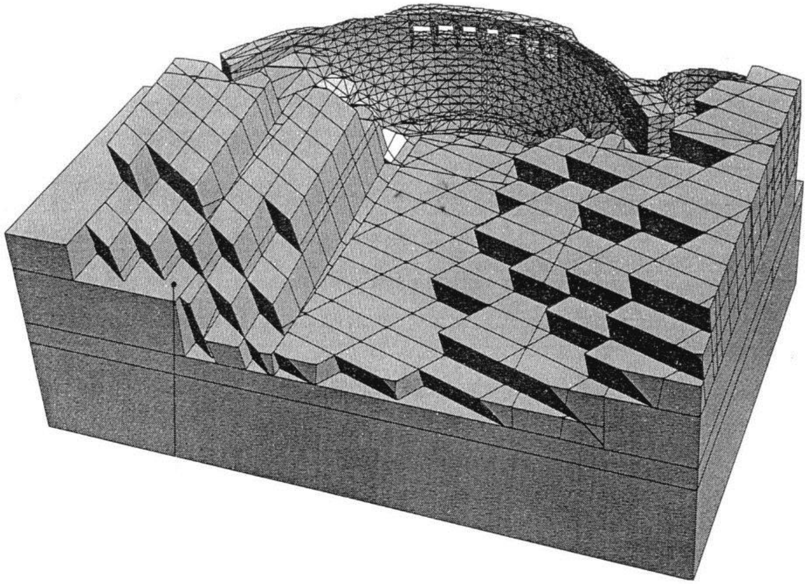
PRACTICAL APPLICATIONS

Cambambe dam 1995

Discrete element model: 3DEC + FEM

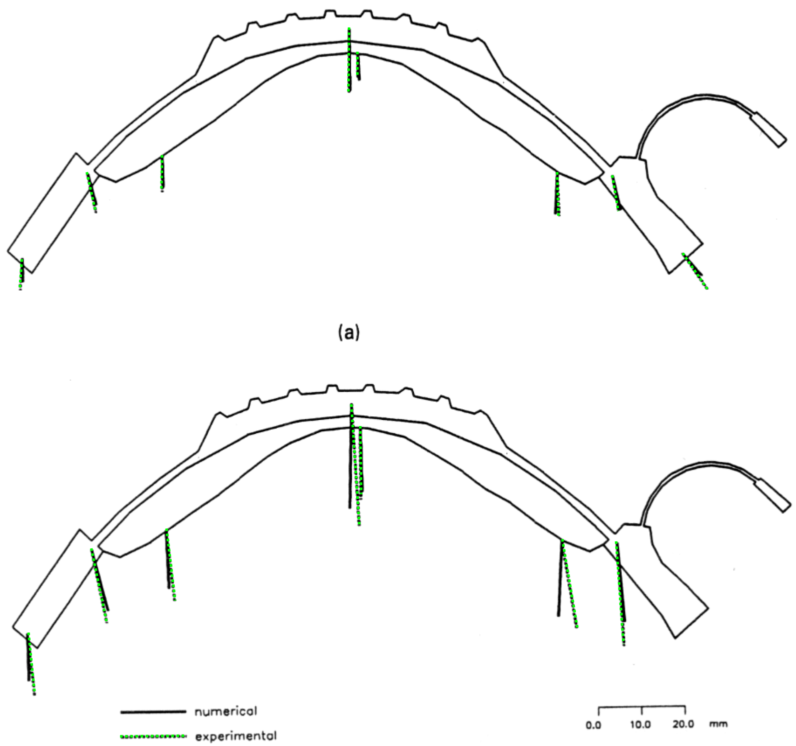


—— 3DEC

—— measured

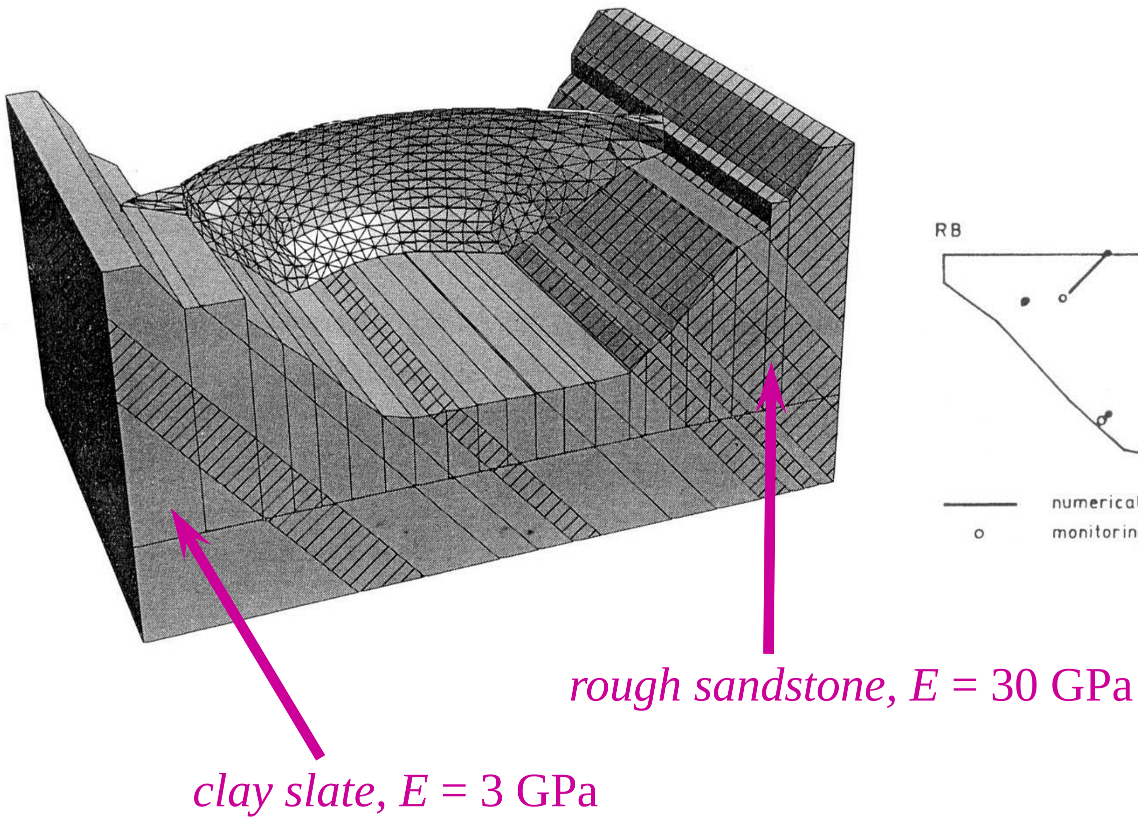
Measurements:

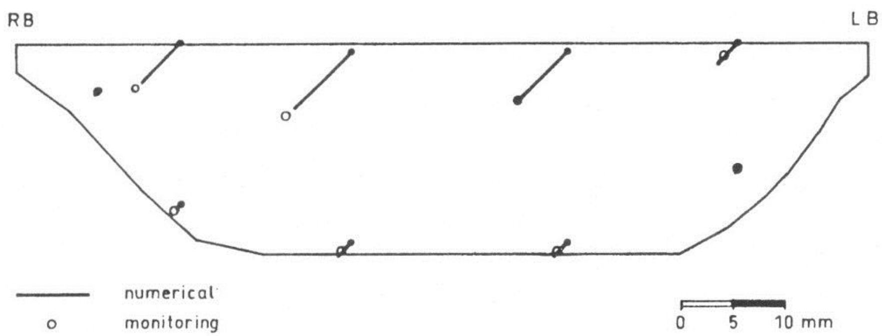
(translations at different water levels)



PRACTICAL APPLICATIONS

Funcho dam (Heterogeneous rock; + strongly unsymmetric)





UDEC / 3DEC

→ UDEC/3DEC basic principles

Elements: degrees of freedom; material characteristics

Contacts: types, material characteristics

Displacements: Numerical stability issues

→ UDEC/3DEC practical applications

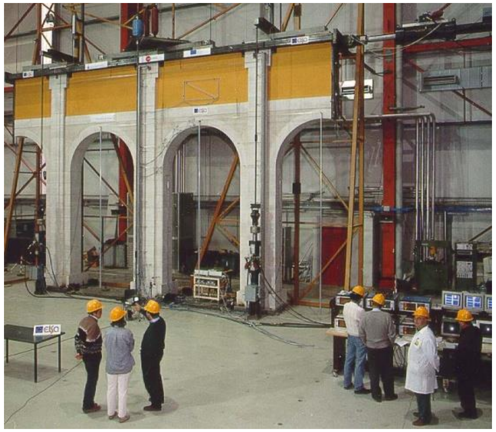
1. Underground ice hockey cavern, Norway
2. Dams
3. Masonry structures

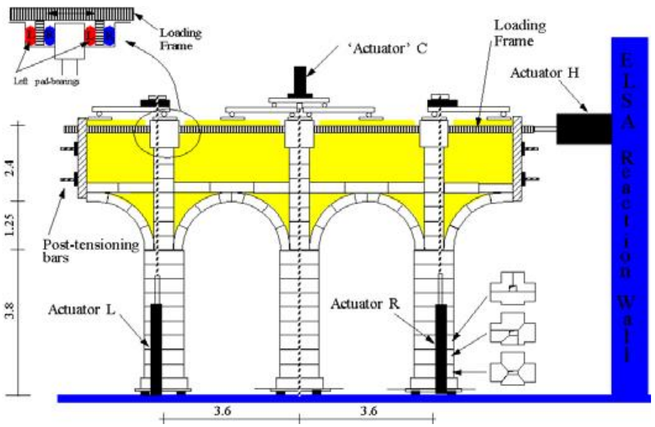
PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

previous studies:

- experiments
- FEM simulations





PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

Giordano et al, 2002: simulations with UDEC and with different FEM models

UDEC model:

→ geometry: 2D

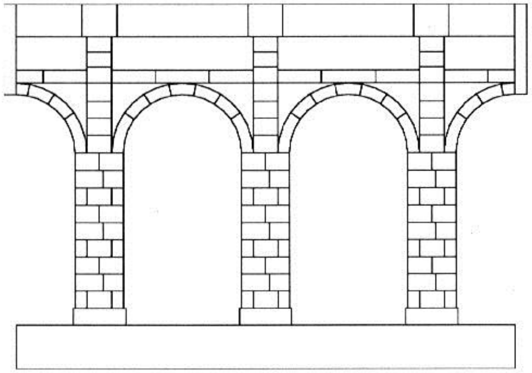


Fig. 18. UDEC discrete element model.

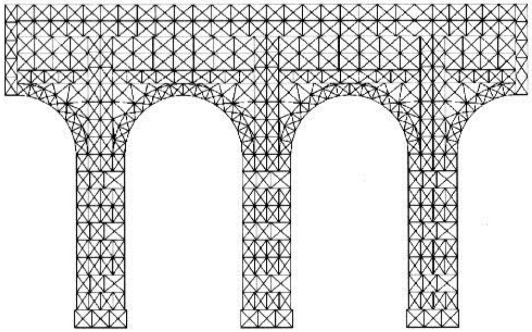


Fig. 19. UDEC internal finite element mesh.

PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

→ material parameters:

blocks:

Table 2
Parameter values for the CASTEM model

	Stones	Infill panels
Weight per unit volume (kg/m³)	2500	2500
Young's modulus (Gpa)	65	6.5
Poisson's ratio	0.2	0.2
k_n: normal stiffness (Gpa)	115	
k_s: shear stiffness (Gpa)	47.9	
N_t: tensile strength	0	
ϕ: friction angle	30	
μ: dilatancy angle	5°	

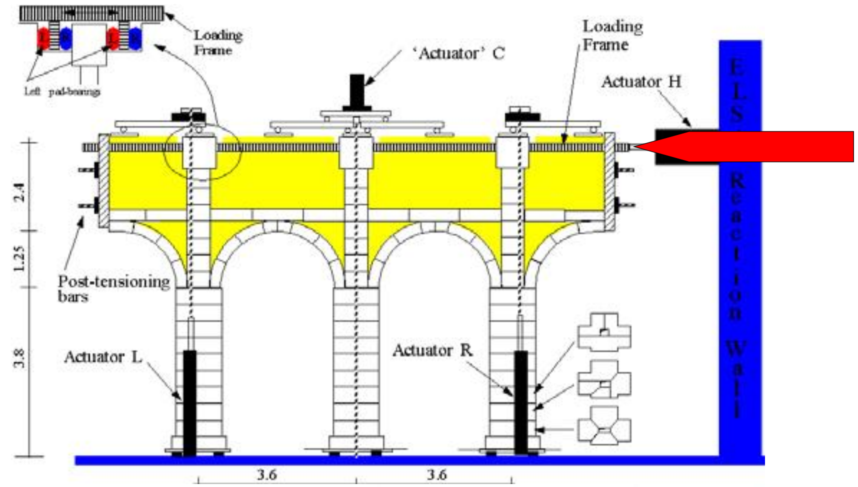
contacts:

Table 3
Parameter values for the UDEC model

k_n: normal stiffness (Gpa)	115
k_s: shear stiffness (Gpa)	46
N_t: tensile strength	0
ϕ: friction angle	35
μ: dilatancy angle	0
c: cohesion	0

PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

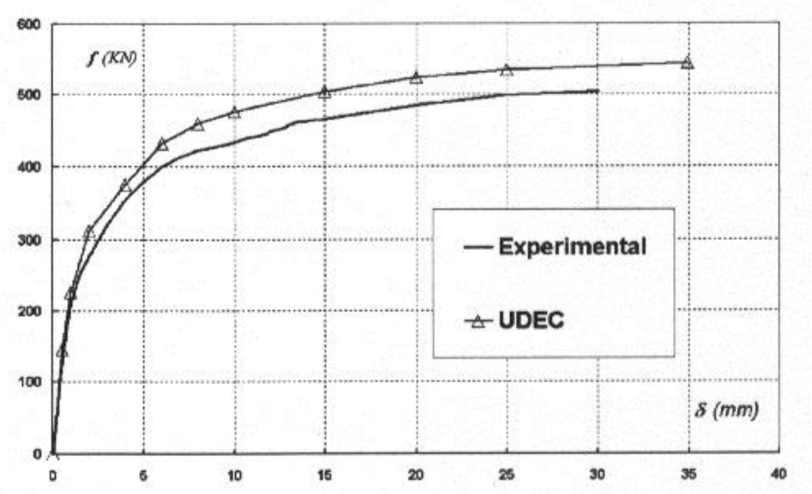


→ loading process:

constant vertical load;

lateral „force”: disp-controlled,

increasing translation



→ force-displacement-diagram:

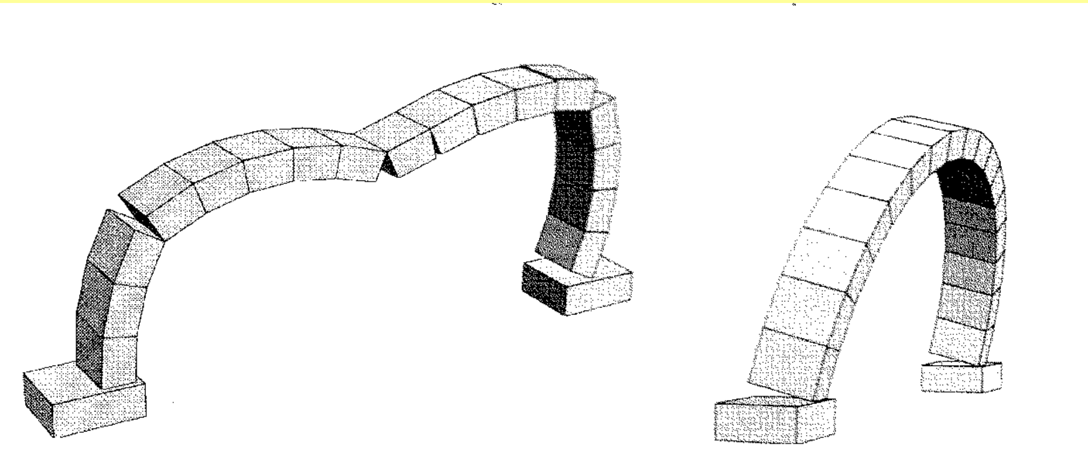
[filling: linear elastic,

isotropic model]

→ UDEC advantages:

large displacements O.K.,

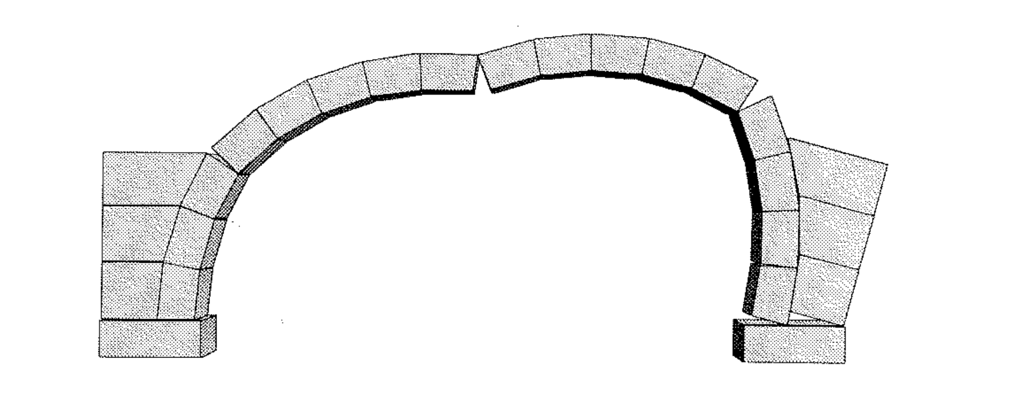
crack opening O.K.



Rigid or deformable blocks

Linear and non-linear force-displacement for discontinuities

Library of material models for deformable blocks and for discontinuities



Lemos

3DEC large displacements (slip and opening) along distinct surfaces in a discontinuous medium

N. Bicanic, 2003

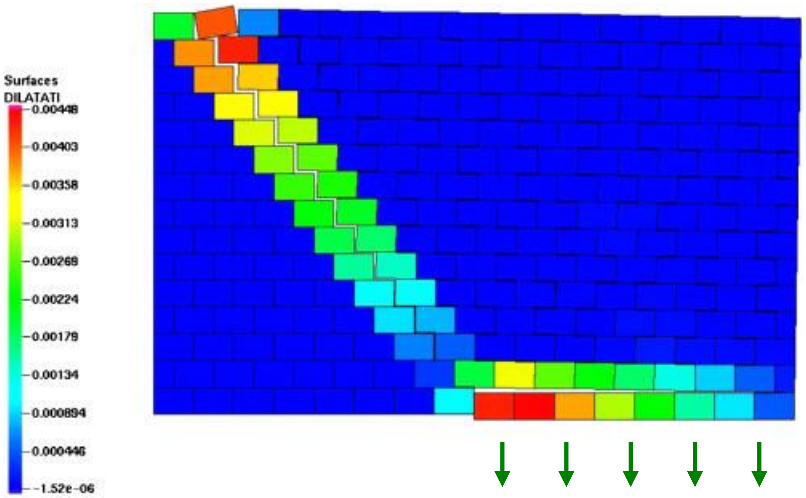
PRACTICAL APPLICATIONS

<u>Walls with soil motions</u>

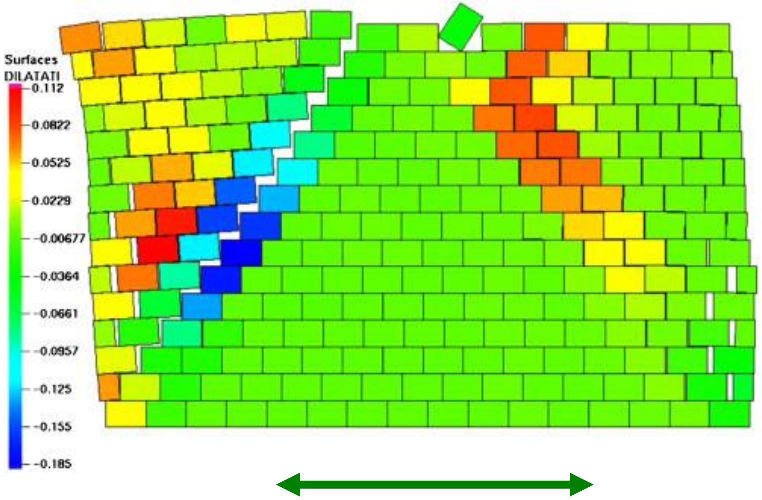
e.g. Chetouane et al, 2005:

how the blocks move?

downwards translation of the support



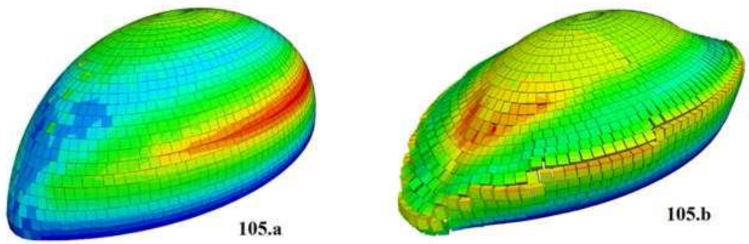
dynamic shaking

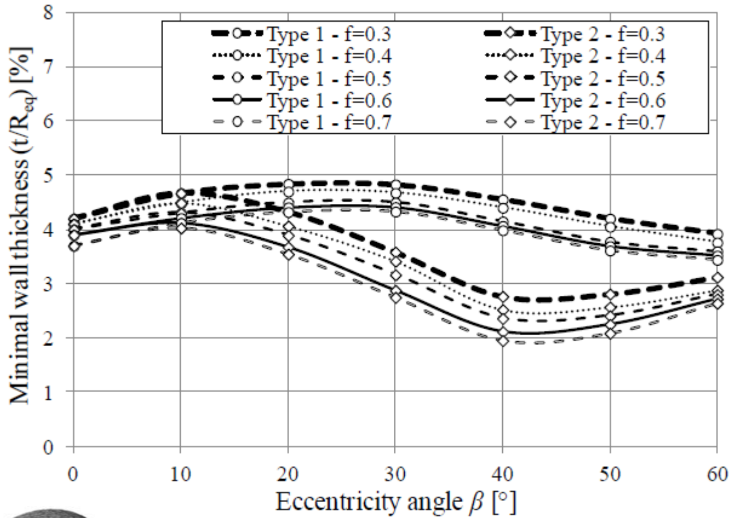


3DEC: OUR MSC AND PHD STUDENTS

<u>Mechanics of oval domes:</u>

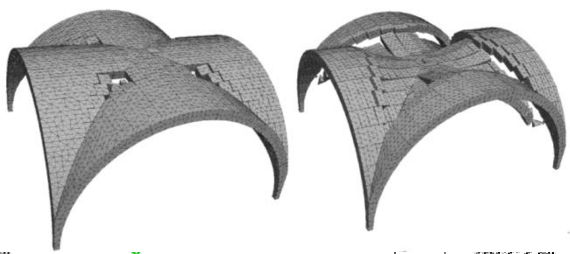
Simon, J., 2011

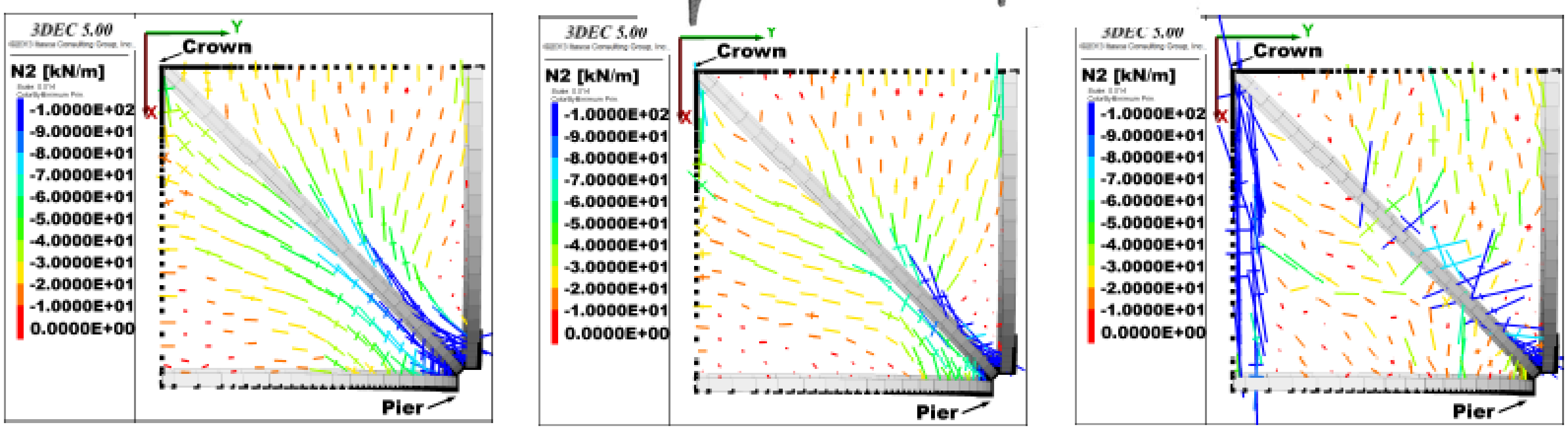




<u>Role of ribs in cross-vaults:</u>

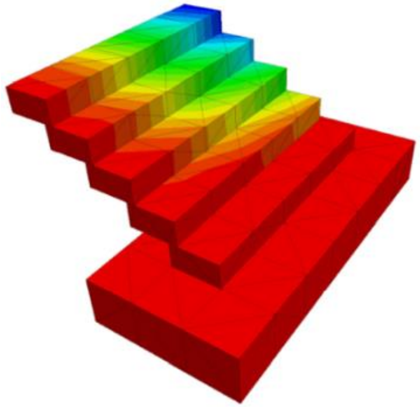
Lengyel, G., 2012





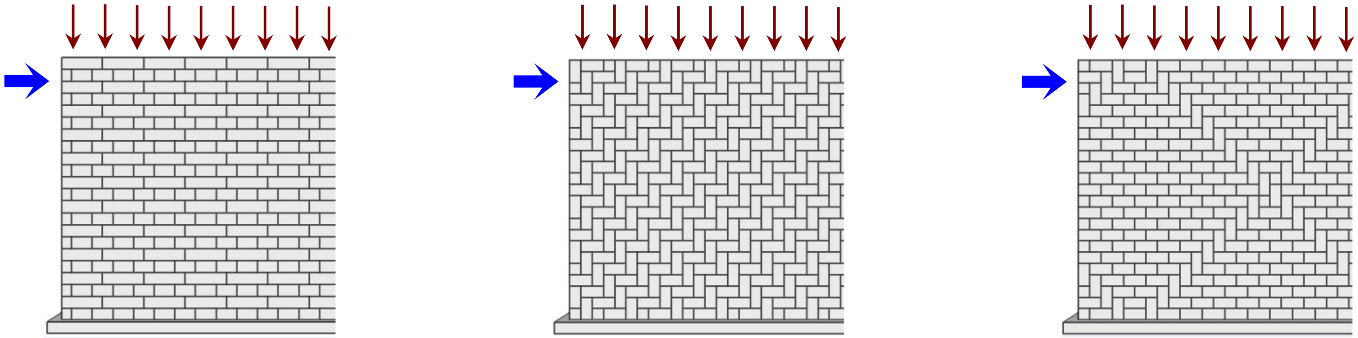
3DEC: OUR MSC AND PHD STUDENTS

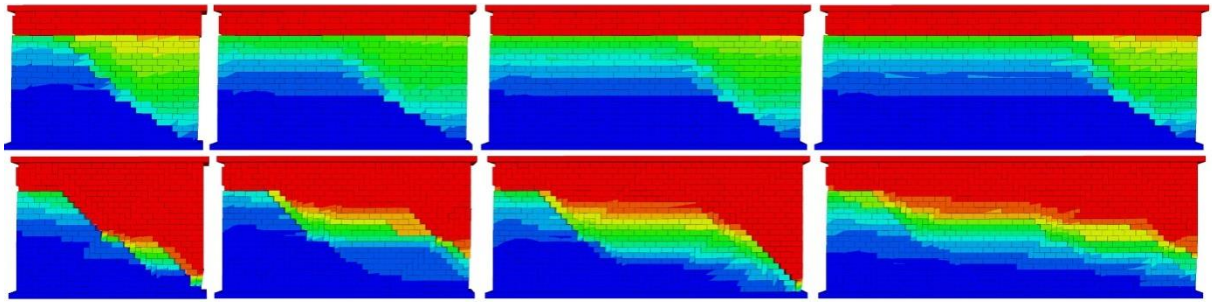
Cantilever stairs: *Rigó, B., 2013*



Shear strength of walls with different bond patterns:

Szakály, F.; 2015





QUESTIONS

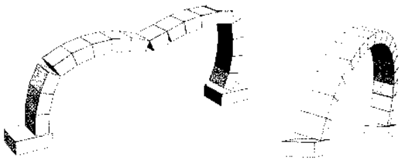
1. What are the degrees of freedom of deformable elements in UDEC/ 3DEC?

2. How can a contact be formed between two elements in the two-dimensional UDEC, and what is the size and the direction of the contact in the different cases ?

3. Explain the „common plane" method of 3DEC!

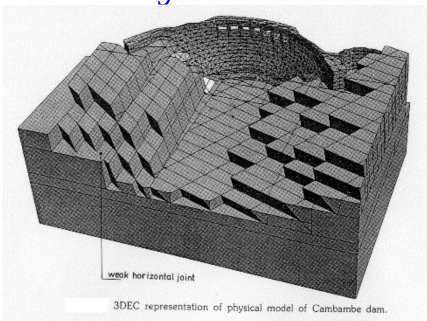
4. Introduce the most important mechanical types of contacts of UDEC/3DEC!

5. Explain how a time step is analysed in UDEC/3DEC!



6. What are the most important differences between UDEC and 3DEC?

7. What kinds of damping are used in UDEC/3DEC, and why are they necessary?



3DEC representation of physical model of Cambambe dam.