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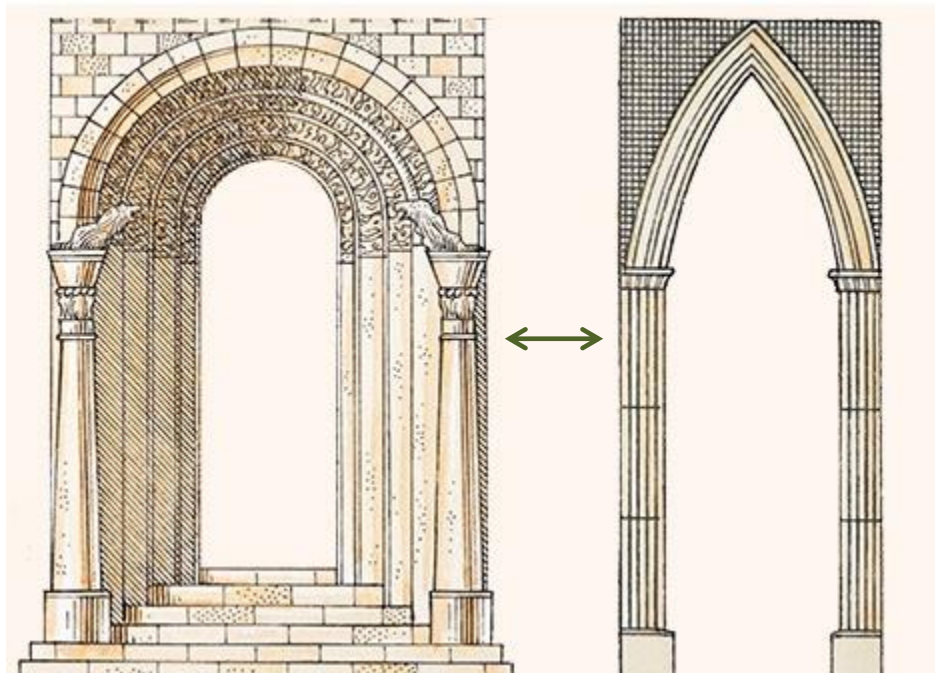
Budapest University of Technology and Economics

Department of Structural Mechanics

Numerical Models for Structures

DEM comparison of the horizontal reaction of Romanesque and Gothic arches

Homework 1 - 2013/2014/II.



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1. Introduction

1.1. Aim of the homework

The aim of my homework is to compare the behaviour and contact forces of a semi-circular (Romanesque) and a pointed vault (Gothic arch) with the help of a discrete element method, namely the software 3DEC of Itasca.

First of all I prepared a semi-circular barrel vault of 2 m inner span, 15 cm thickness and 1 m length, and after that I modelled a pointed vault having the same inner span, length and selfweight as the Romanesque arch.

I measured the horizontal reaction of both vaults under the selfweight. Afterwards I moved supports horizontally outwards and I analysed the changing of this horizontal reaction.

1.2. Romanesque and gothic architecture and arches

The Romanesque churches were very dark on the inside because the arches, ceiling needed high and very thick walls as a support and because of it these walls could not have large openings, holes for windows. The arches were rounded and they would not tolerate a lot of weight without crumbling [5].

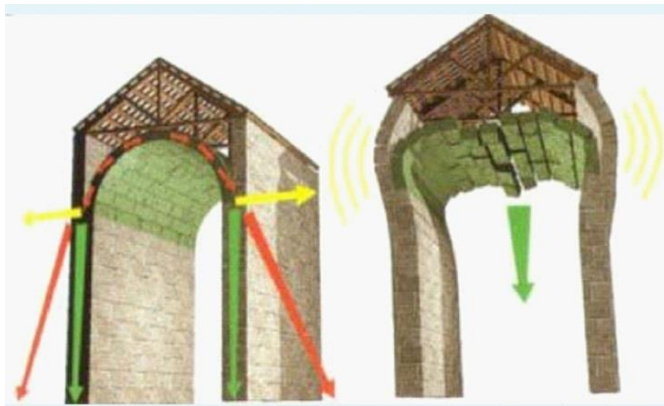


Figure 1: Reactions and failing mechanism of a Romanesque arch [5]

The main problem with these large Romanesque churches was that they kept falling down. In *Figure 1* we can see, that the horizontal reaction on the walls caused the ceiling to fall and the walls to crumble. In addition to this a lot of Romanesque churches burnt down, because they had timber roofs which were often struck by lightning or in some cases they burned along with other buildings when there was a major fire. There are very few surviving original Romanesque churches.

So as a conclusion we can say that several problems developed with Romanesque style churches:

- They easily fell down when the builder tried to make them larger.
- This rounded arch could not support the weight caused by the outward thrust of the rounded arches and ceilings pushing on the wall.
- The walls were the main support of the ceiling therefore they could only have small opening for windows. So that's why the Romanesque churches were dark and dreary.
- They had timber roofs, so they were easily burnt down because of lightning or sparks from other nearby buildings that were on fire.

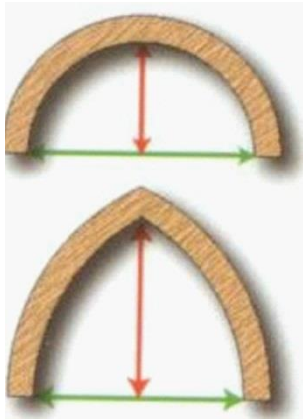


Figure 2: Romanesque and Gothic arch [6]

In 1140, Abbott Suger¹ began a renovation of St. Denis Abbey which was the beginning of what we now call Gothic architecture. Suger had a high theology of light and wanted the abbey filled with light and color. He first enlarged the *ambulatory*, the walkway behind the altar. He did this by using pointed arches instead of the former rounded arches of the Romanesque style. To these arches he linked flying buttresses and vaulted ceilings [6].

The use of the pointed arch rather than the rounded arch allowed for great flexibility in the interior design. The pointed arch could extend upward to great heights (*Figure 2*), and also allow for a greater distance between the columns or piers beneath them. The weight was not distributed to the wall, but to the flying buttresses. So, the walls were no longer load bearing which means they could have large openings for windows. Some churches were built with almost totally glass walls such a St. Chappell in Paris.

We can summarize the advantages of Gothic arches:

- The centerline of pointed arches follows more closely the compression forces which make it stronger.
- Many times the flying buttresses bear the weight, so the walls are no longer load bearing structural parts. This means they can have larger windows, so the gothic cathedrals are filled with light.
- The pointed arch can be higher, and there can be greater distance between the columns or piers, so the span can be bigger.

2. Solution strategy

First of all I had to define the geometries, the blocks of the semi-circular and a pointed arch. I determined the coordinates with help of Matlab in both cases, for the Romanesque arch I defined two concentric semi-circles, for the Gothic arch I defined a section of the circles with 60° angle, and after this I mirrored the points.

When I had the proper geometry I applied the gravity, examined the displacements and analyzed the contact forces. After this I could measure the horizontal reaction force.

My next task was to move the supports in a quasi-static way, and measure the contact shear force. To achieve the desired displacements I applied velocity to my support blocks, then I stopped them and equilibrated my structure.

¹ Abbott Suger (c. 1081 – 13 January 1151) was elected Abbott of St. Denis in Paris in 1122. Having served as a trusted councilor under his predecessor, Abbott Adam, he was able to organize the monastery finances, skillfully manage the abbey's immense estate and play an important political role in France.

3. Discrete element model

I built two 3D discrete element models in 3DEC 5.00.164., one for the Romanesque arch (see the code in Appendix B) and one for the Gothic arch (see the code in Appendix C). To the determination of the coordinates I used the Matlab program in both cases (see the code in Appendix A). To write the discrete element codes I used the help of the 3DEC Manual [7]. For the Matlab code the reference [9] was very useful.

3.1. Geometry

According to my homework description the geometry of the circular arch was given (*Figure 3*): 2 m inner span, 15 cm thickness and 1 m length. In the model I applied 17 blocks for the arch, and two additional ones as support blocks, with the width of 30 cm, height of 40 cm and length of 1 m.

After this I prepared a pointed vault having the same inner span, length and selfweight as the semi-circular vault (*Figure 3*). To achieve this I approximately calculated the value of the wall thickness, which is 11.75 cm.

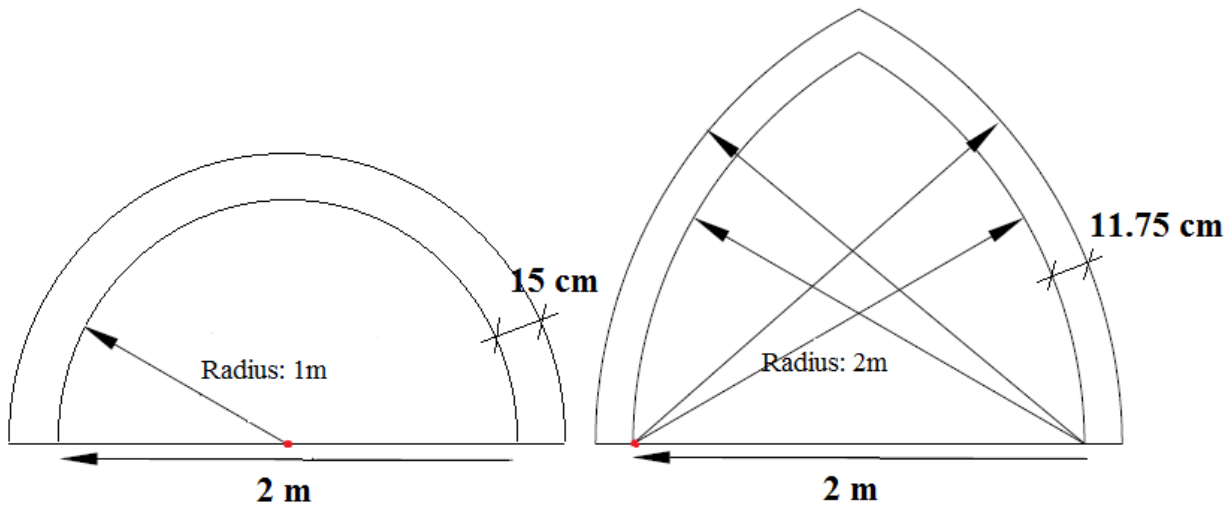


Figure 3: The geometry of the pointed and semi-circular arch

The calculation:

Area of the Romanesque arch: $\frac{(r_2^2 - r_1^2)\pi}{2} = \frac{((1.15m)^2 - (1m)^2)\pi}{2} = 0.5065818m^2$, where r_2 is the outer, r_1 is the inner radius of the arch.

Area of Gothic arch: $\frac{(R_2^2 - R_1^2)\pi}{6} \cdot 2 = \frac{(R_2^2 - (2m)^2)\pi}{3}$, where R_2 is the outer radius of the Gothic arch and R_1 is the inner radius. Here I took into account only the twice of a 60° section of a circle, so I neglected the upper part, the capstone (keystone).

The area of the two arches has to be the same: $\frac{(R_2^2 - (2m)^2)\pi}{3} = 0.5065818m^2 \Rightarrow R_2 = 2.1175m$

So the thickness is the difference between the inner and outer radius, which is 11.75 cm.

To the Gothic arch I also used 17 blocks, and two support blocks with the same geometry as before.

According to these data and with the help of the Matlab and Excel programs I built the geometry, the blocks of the models as showed in *Figure 4*.

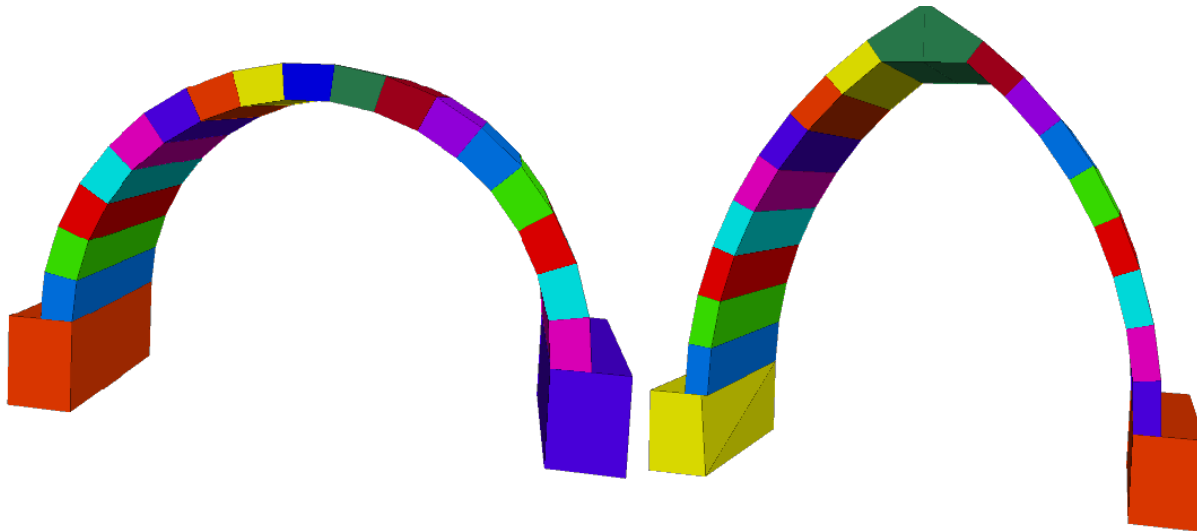


Figure 4: The 3DEC models of the arches

3.2. Material model of the blocks

In the models rigid elements were applied. According to [2] these cathedrals were generally made of limestone, sandstone or marble, so I chose a density value 2600 kg/m^3 which is fitting for every one of them, as we can see in *Figure 5*.

| | |
|-------------|-----------|
| Andesite | 2.5 - 2.8 |
| Basalt | 2.8 - 3.0 |
| Coal | 1.1 - 1.4 |
| Diabase | 2.6 - 3.0 |
| Diorite | 2.8 - 3.0 |
| Dolomite | 2.8 - 2.9 |
| Gabbro | 2.7 - 3.3 |
| Gneiss | 2.6 - 2.9 |
| Granite | 2.6 - 2.7 |
| Gypsum | 2.3 - 2.8 |
| Limestone | 2.3 - 2.7 |
| Marble | 2.4 - 2.7 |
| Mica schist | 2.5 - 2.9 |
| Peridotite | 3.1 - 3.4 |
| Quartzite | 2.6 - 2.8 |
| Rhyolite | 2.4 - 2.6 |
| Rock salt | 2.5 - 2.6 |
| Sandstone | 2.2 - 2.8 |
| Shale | 2.4 - 2.8 |
| Slate | 2.7 - 2.8 |

Figure 5: Rock densities in g/cm³ [3]

3.3. Contact properties

I used Coulomb frictional contacts in my models, so rigid, tensionless contacts with high friction. The shear stiffness (jks) and normal stiffness (jkn) are 10¹² N/m². According to the recommendation of [1] I used 50° as a friction angle.

Figure 6 illustrates the meaning of these values and their notation in the 3DEC coding.

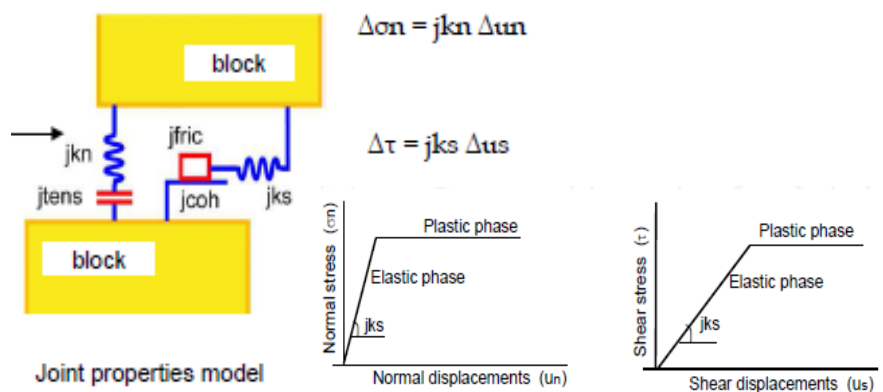


Figure 6: The normal stiffness, shear stiffness and friction of the contacts using 3DEC notations [8]

3.4. Boundary conditions and loading

The models were supported by the lowest blocks (called supportblock1 and supportblock2 in the code). I fixed these two support blocks of the models, so it means that these blocks cannot move, the velocities are zero.

As a load I applied the gravity to the structure (9.81 m/s^2 in the vertical, so in y direction).

3.5. Support displacements

After I found the equilibrium state for both structure under selfweight, my next task was to analyze the arches for several horizontal support displacements.

In discrete element modeling we cannot directly apply horizontal displacements, instead of that we can apply velocities to the lower support blocks. If we know the exact value of the velocity, time step and the number of steps (cycle or step command), then we also know the value of the horizontal displacements.

4. Results

4.1. Displacements

First of all let's see the displacements of the semi-circular arch.

After 10000 cycles the unbalanced force is $2.599 \times 10^{-7} \text{ N}$ as we can see in *Figure 7*.

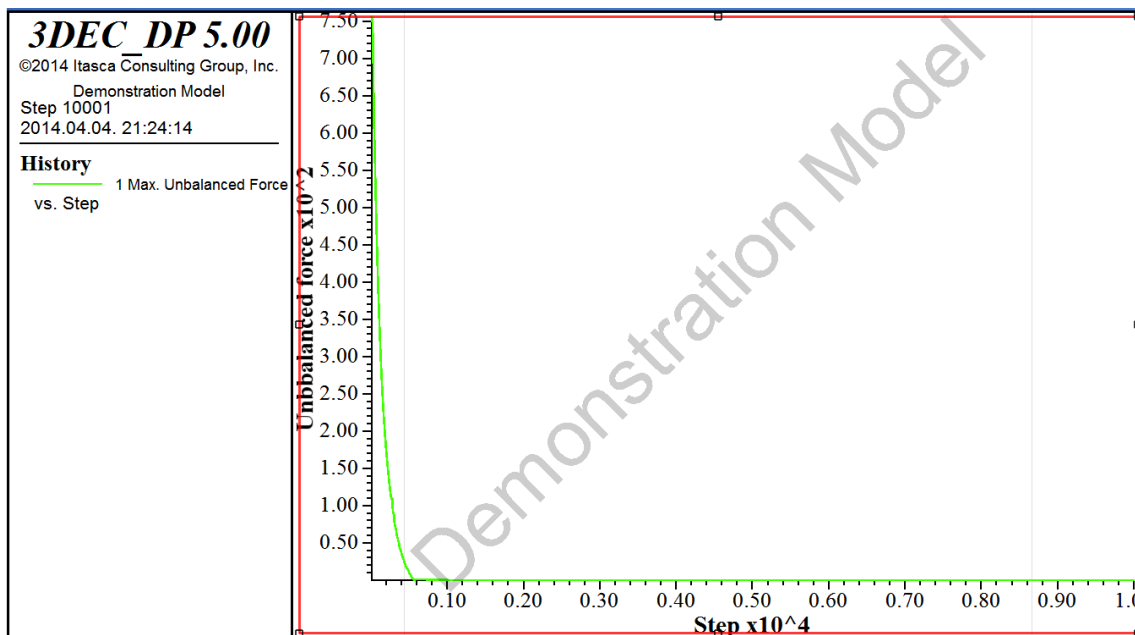


Figure 7: The unbalanced force

In *Figure 8* and *Figure 9* we can see the displacement contours of the arch and the displacement-step diagrams of the corresponding points.

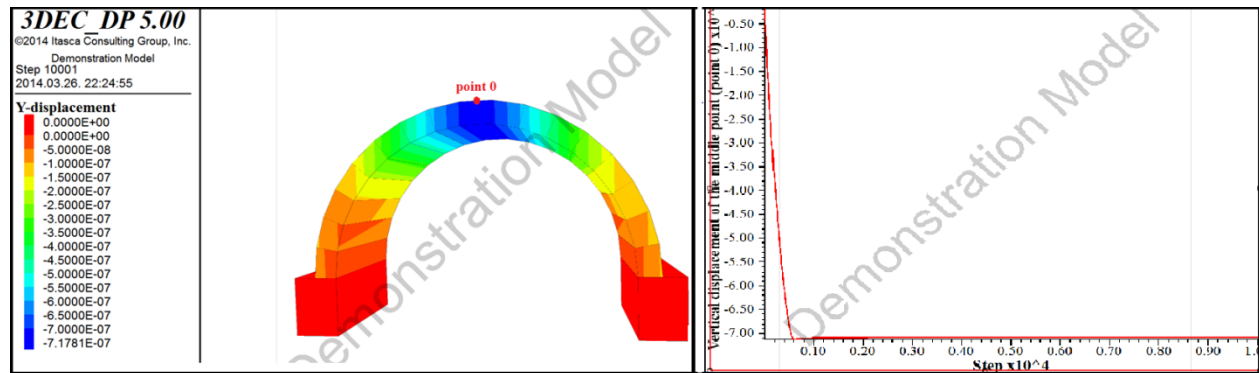


Figure 8: Contour of vertical displacements and the vertical displacement diagram of point 0

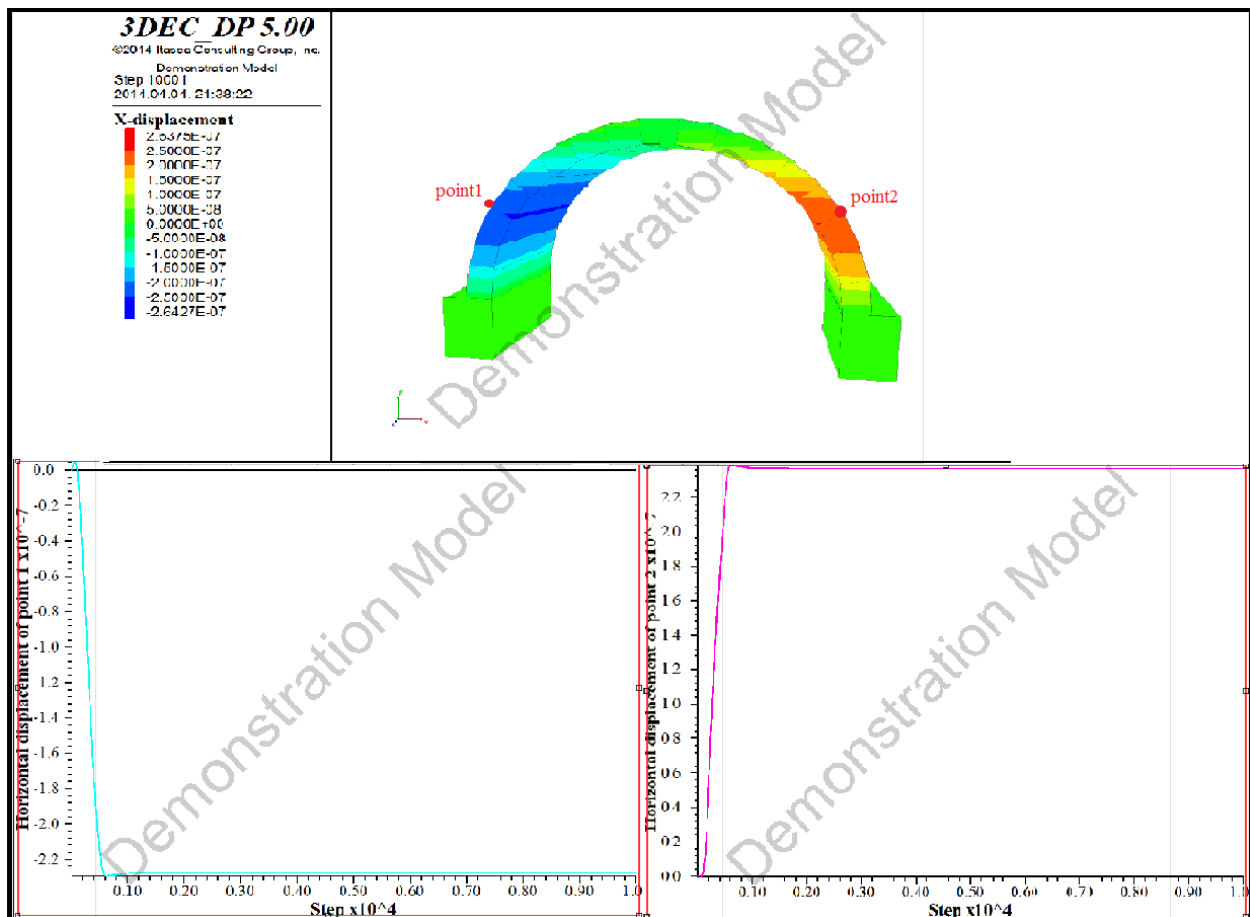


Figure 9: Contour of horizontal displacements and the horizontal displacement diagram of point1 and point2

After this we can plot the same diagrams for the Gothic arch. The unbalanced force is 8.436×10^{-6} N according to *Figure 10*. Furthermore just like before the *Figure 11* and *Figure 12* show the displacement contours of the arch and the displacement-step diagrams of the corresponding points.

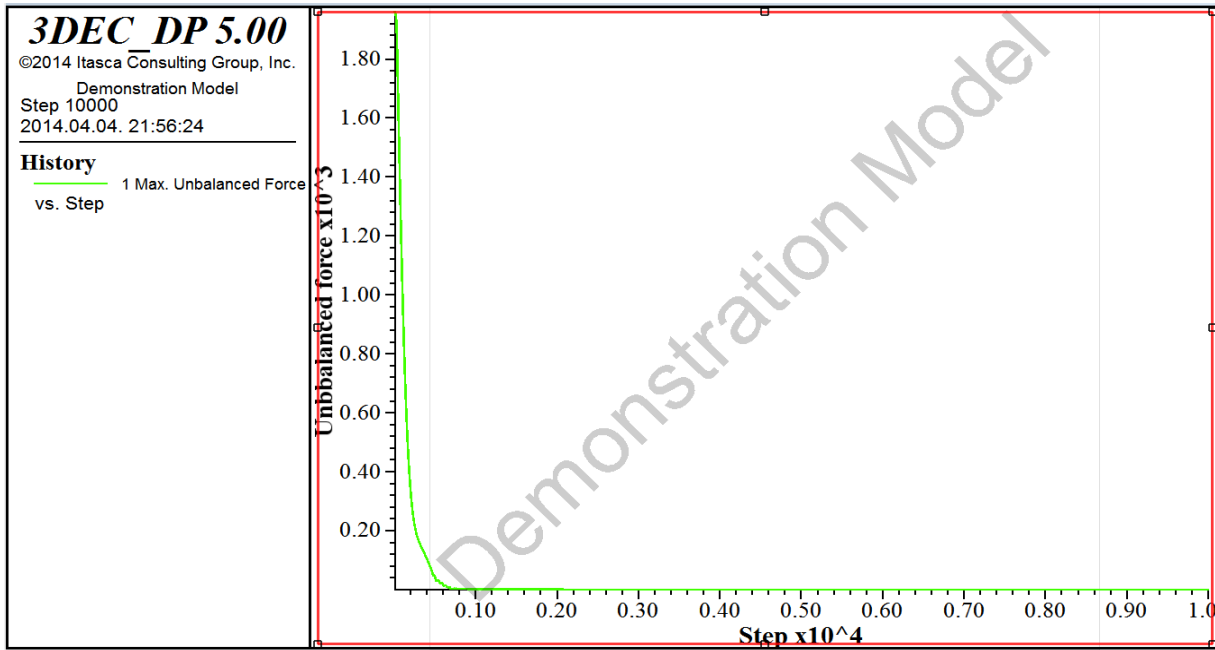


Figure 10: Unbalanced force of Gothic arch

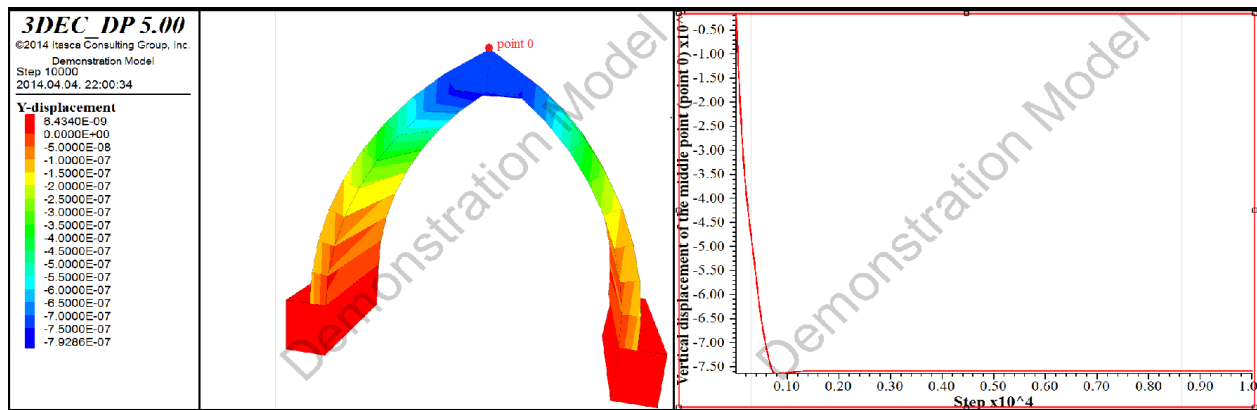


Figure 11: Contour of vertical displacements and the vertical displacement diagram of point 0 in case of Gothic arch

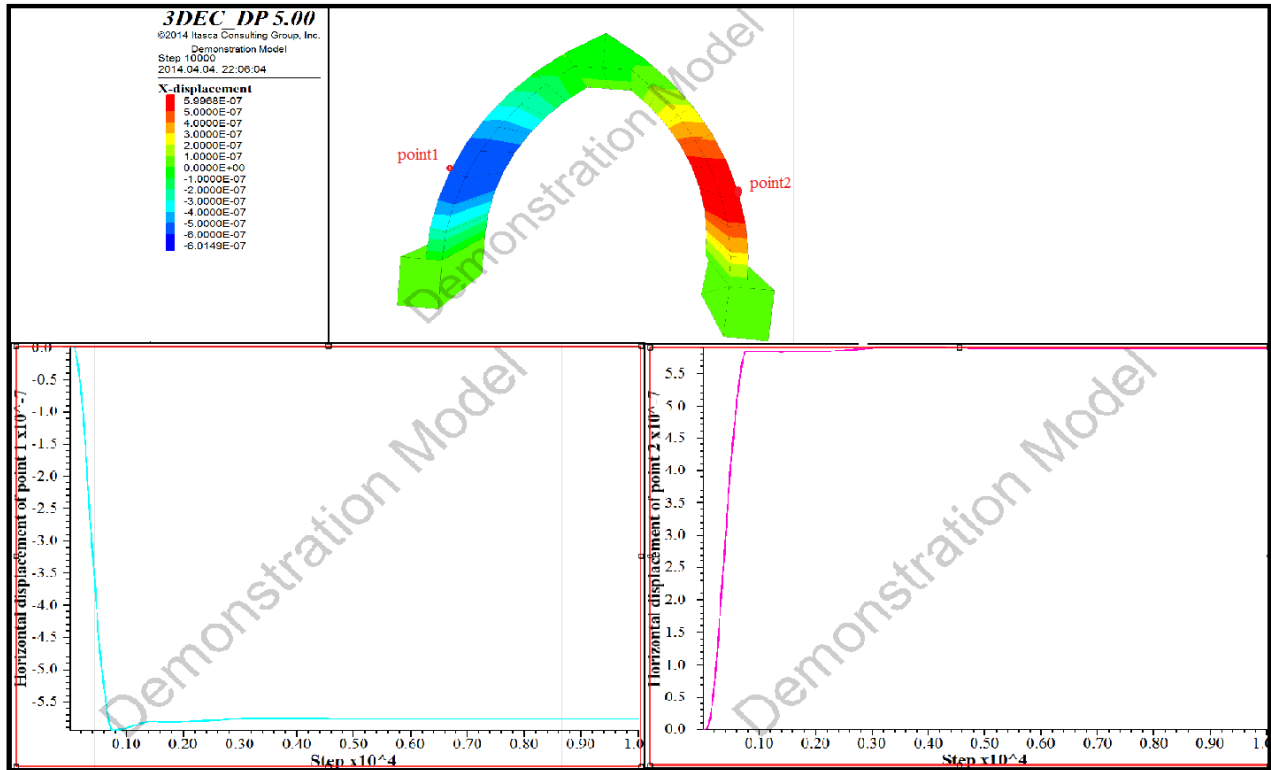


Figure 12: Contour of horizontal displacements and the horizontal displacement diagram of point1 and point2

We can compare the displacements of the two arches:

| | Romanesque arch | Gothic arch | Difference |
|-------------------------------------|-------------------------|-------------------------|------------|
| Maximum vertical displacement [m] | $-7.1781 \cdot 10^{-7}$ | $-7.9286 \cdot 10^{-7}$ | 9.47 % |
| Maximum horizontal displacement [m] | $-2.6427 \cdot 10^{-7}$ | $-6.0149 \cdot 10^{-7}$ | 56 % |

Table 1: Comparison of the displacements

According to *Table 1* we can observe that the displacements of the Gothic arch are bigger, especially in horizontal direction, but in both cases these displacements are very small.

4.2. The horizontal reaction

My next task was to measure the horizontal reaction which means the contact shear force between the support blocks and the arch. In this case the main problem was that I wasn't sure what the *contour* and *hist* diagrams are showing.

To check these diagrams, results first I analyzed the normal forces of the **Romanesque arch**. We know that the contact normal force between a support block and the arch has to be the half of the selfweight. To calculate it I listed the volumes of the blocks, and multiplied them with the density and gravity according to *Table 2*.

| | | |
|---------------------------|------------------------------|----------|
| Volumes (m ³) | | |
| 0,029579 | Density (kg/m ³) | 2600 |
| 0,029544 | Mass (kg) | 1308,746 |
| 0,029637 | Weight (N) | 12838,8 |
| 0,029756 | Half of the weight (N) | 6419,401 |
| 0,029605 | | |
| 0,029564 | | |
| 0,029603 | | |
| 0,029645 | | |
| 0,029502 | | |
| 0,029645 | | |
| 0,029603 | | |
| 0,029564 | | |
| 0,029605 | | |
| 0,029756 | | |
| 0,029637 | | |
| 0,029544 | | |
| 0,029579 | | |
| Sum of volumes | | |
| 0,503364 | | |

Table 2: Calculation of the selfweight

Respectively to this I should see this value when I plot the contact normal forces, but this is not the case. We can see entirely different values in *Figure 13*, they actually show the contact normal force of a subcontact. So to get the normal force of the whole contact I listed the subcontact stresses and areas, and calculated the force from these values. If we analyze the steps of the calculation (*Table 3*) we can get to the conclusion, that these diagrams in *Figure 13* are showing the subcontact force, so the subcontact stress multiplied with the subcontact area. For example the sixth subcontact force (933.56 N) in *Table 3* corresponds to the second diagram in *Figure 13*.

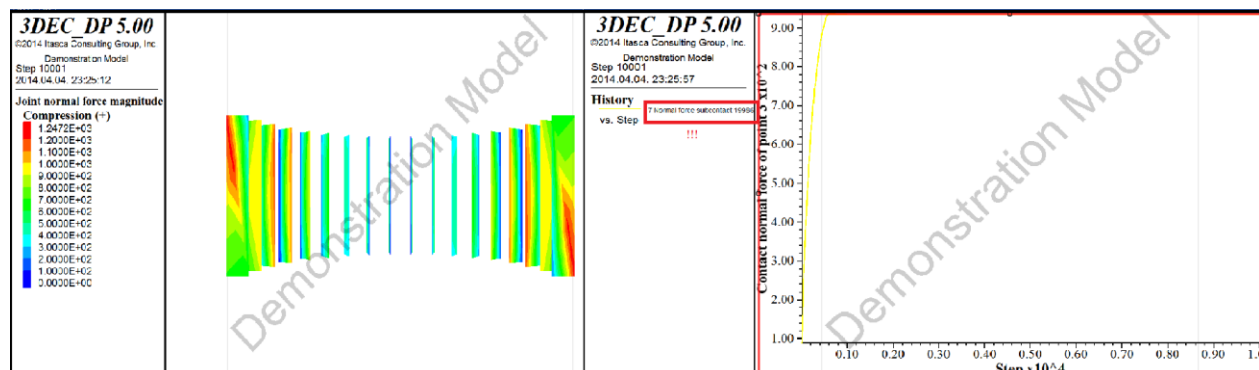


Figure 13: The contact normal force with plot contour and plot hist command

| Subareas (m ²) | Stresses (N/m ²) | Subcontact forces (N) |
|----------------------------|------------------------------|--------------------------|
| 0,012500 | 11780 | 147,25 |
| 0,025000 | 88520 | 2213,00 |
| 0,012500 | 110800 | 1385,00 |
| 0,025000 | 32400 | 810,00 |
| 0,003125 | 88520 | 276,63 |
| 0,009375 | 99580 | 933,56 |
| 0,018750 | 12520 | 234,75 |
| 0,012500 | 33510 | 418,88 |
| | | Contact normal force (N) |
| | | 6419,06 |

Table 3: Calculation of the contact normal force

In respect to this calculation and analysis, now I can get the contact shear forces. We can see the calculation and subcontact stresses, forces, areas in *Table 4*.

| Subareas (m ²) | Stresses (N/m ²) | Subcontact forces (N) |
|----------------------------|------------------------------|-------------------------|
| 0,012500 | 13800 | 172,50 |
| 0,025000 | 24750 | 618,75 |
| 0,012500 | 20580 | 257,25 |
| 0,025000 | 20590 | 514,75 |
| 0,003125 | 24750 | 77,34 |
| 0,009375 | 22670 | 212,53 |
| 0,018750 | 14680 | 275,25 |
| 0,012500 | 20590 | 257,38 |
| | | Contact shear force (N) |
| | | 2385,75 |

Table 4: Calculation of contact shear force of the Romanesque arch

We can do the same calculations for the **Gothic arch**; *Table 5* shows the half of the selfweight, *Table 6* shows the contact normal force and the contact shear force.

| | | |
|---------------------------|------------------------------|----------|
| Volumes (m ³) | | |
| 0,028177 | Density (kg/m ³) | 2600 |
| 0,028051 | Mass (kg) | 1323,717 |
| 0,028016 | Weight (N) | 12985,67 |
| 0,027997 | Half of the weight (N) | 6492,833 |
| 0,028185 | | |
| 0,028119 | | |
| 0,028037 | | |
| 0,028013 | | |
| 0,044145 | | |
| 0,043966 | | |
| 0,028013 | | |
| 0,028037 | | |
| 0,028119 | | |
| 0,028185 | | |
| 0,027997 | | |
| 0,028016 | | |
| 0,028051 | | |
| Sum of volumes | | |
| 0,509122 | | |

Table 5: Volume and selfweight of the Gothic arch

If we compare *Table 2* and *Table 5* we can see that there is a bit difference between the selfweight of the Gothic and Romanesque arch, it's because in the calculation is *Chapter 3.1*. I neglected the weight of the keystone (capstone).

| Subareas (m ²) | Normal stresses (N/m ²) | Subcontact normal forces (N) | Shear stresses (N/m ²) | Subcontact shear forces (N) |
|----------------------------|-------------------------------------|------------------------------|------------------------------------|-----------------------------|
| 0,009833 | 163300 | 1605,73 | 16057289 | 167,85 |
| 0,019670 | 41980 | 825,75 | 8257466 | 336,95 |
| 0,009833 | 9074 | 89,22 | 892246,42 | 101,77 |
| 0,019670 | 130000 | 2557,10 | 25571000 | 505,91 |
| 0,009833 | 42240 | 415,35 | 4153459,2 | 168,44 |
| 0,006851 | 143200 | 981,06 | 9810632 | 152,91 |
| 0,001934 | 130000 | 251,42 | 2514200 | 49,74 |
| 0,013700 | 9259 | 126,85 | 1268483 | 144,81 |
| | | Contact normal force (N) | | Contact shear force (N) |
| | | 6852,48 | | 1628,39 |

Table 6: Contact normal force and contact shear force of Gothic arch

If we compare *Table 4* and *Table 6* we can notice that the horizontal reaction is much smaller at the Gothic arch, the difference is 31%.

4.3. The horizontal reaction-displacement diagrams

I calculated the contact shear forces in the same way as before, so in the following I won't show the calculation process, only the final results. As I mentioned before I calculated the displacement from an applied velocity, time step and cycle number.

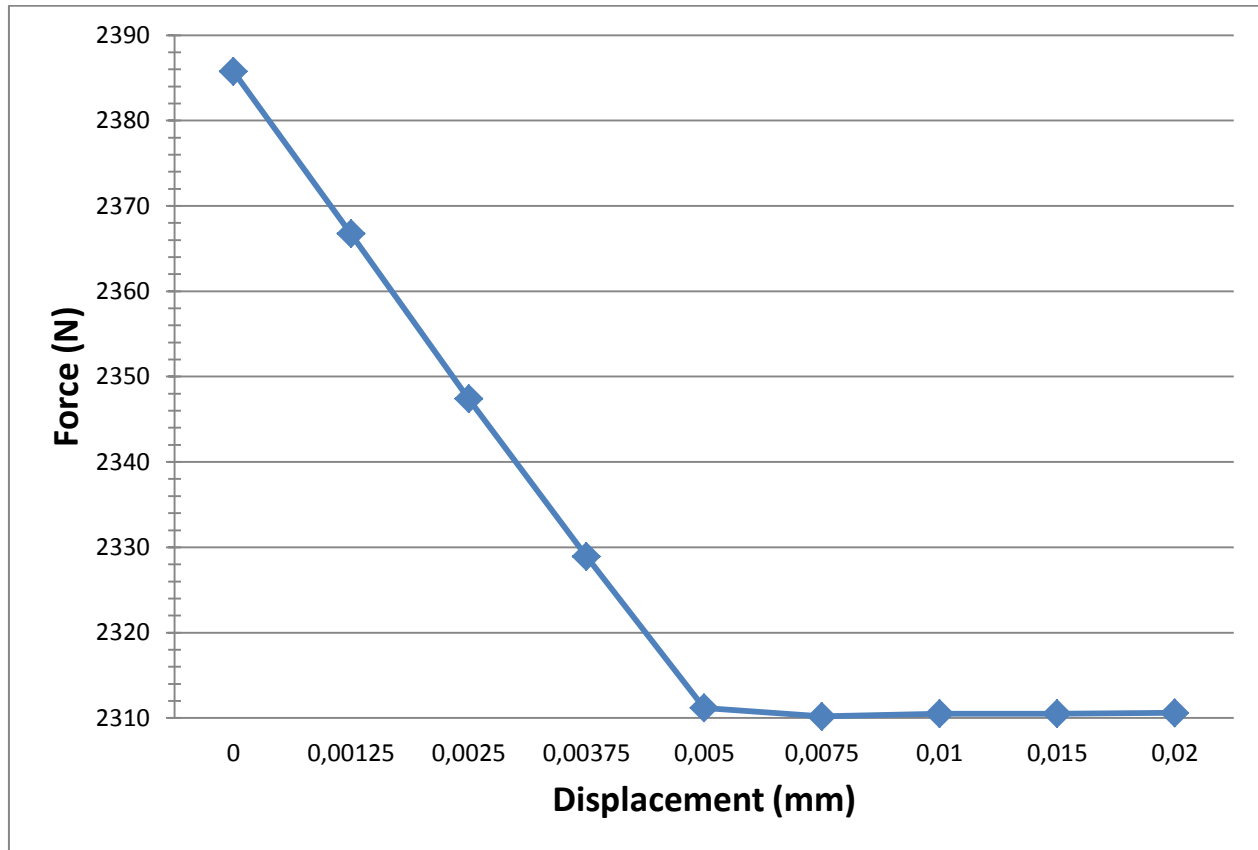


Figure 14: Horizontal reaction-displacement diagram of the Romanesque arch

We can see, that the force-displacement diagram of the semi-circular arch (*Figure 14*) has two parts, a linear and a constant. This means that the reaction force is decreasing till the structure becomes statically determinate, after this the reaction force won't change.

In case of the pointed arch we can observe almost the same phenomena (*Figure 15*). The function has two parts, a linear one with fast decreasing, but now the second part is also linear with slow decreasing (we can say it's almost constant).

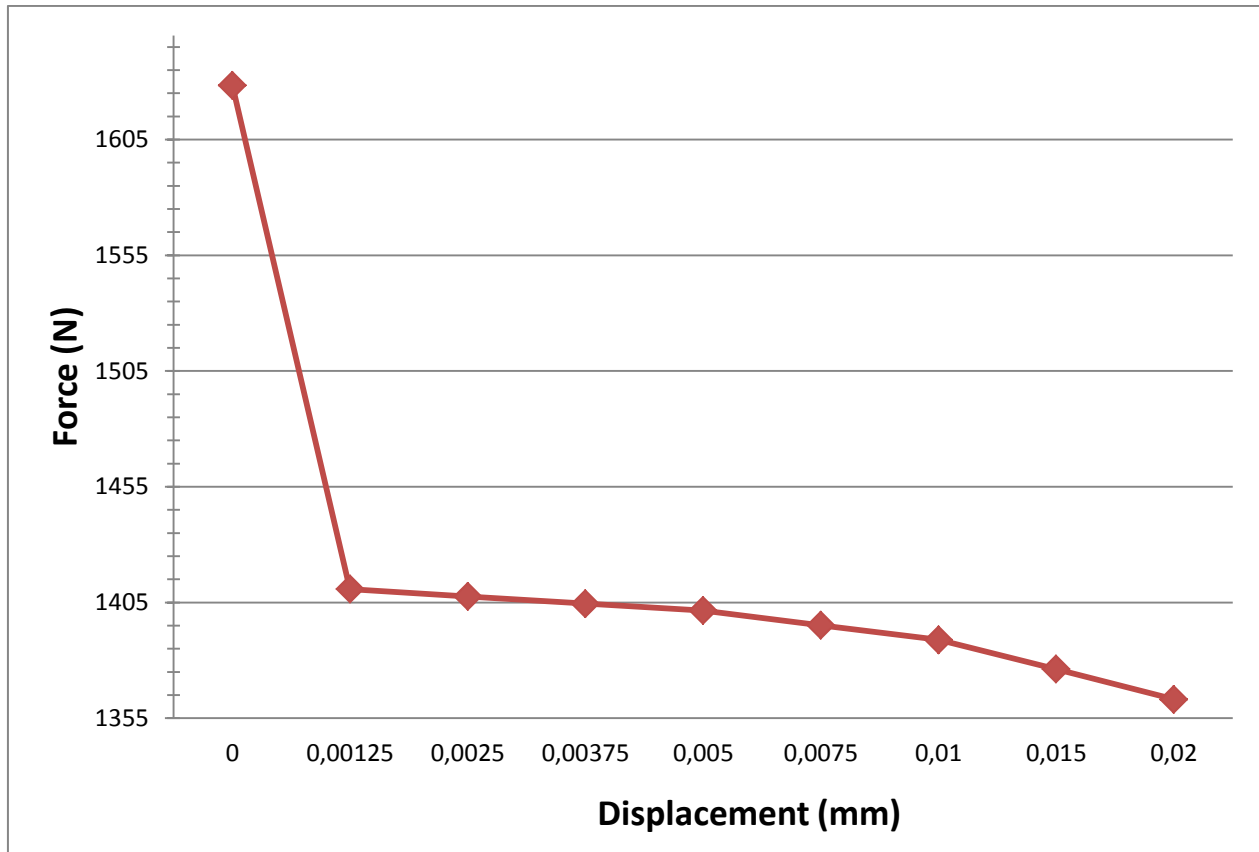


Figure 15: Horizontal reaction-displacement diagram of the Gothic arch

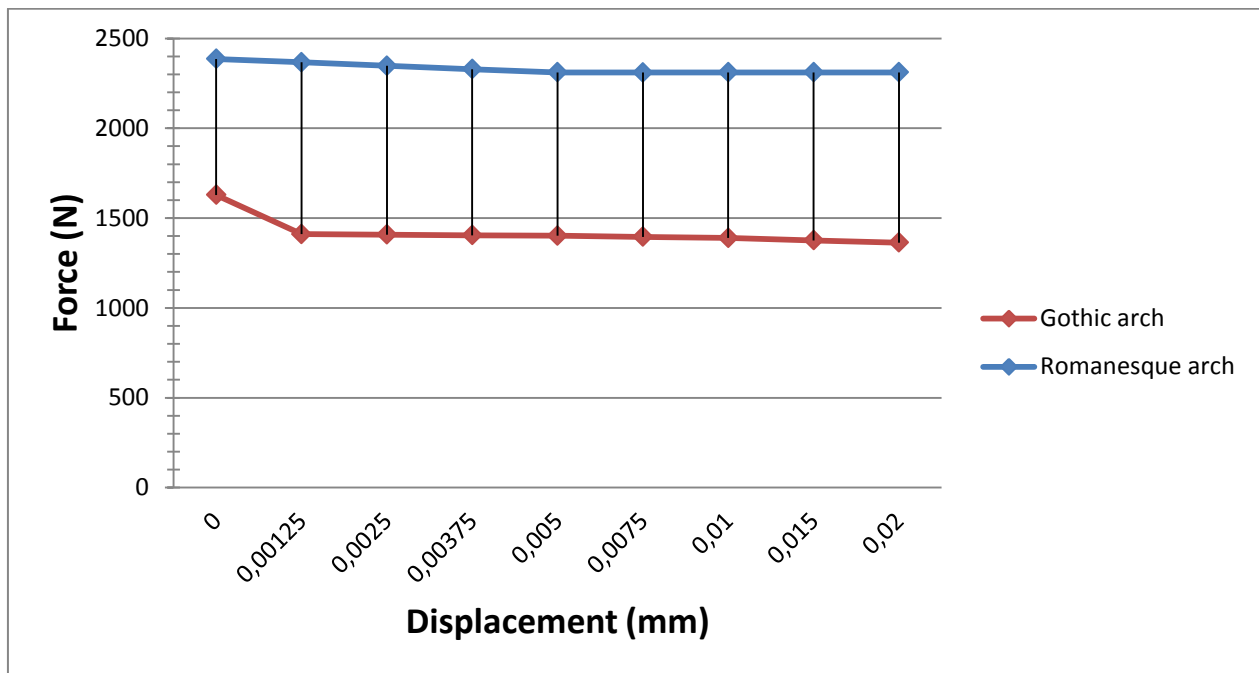


Figure 16: Comparison of the two force-displacement diagrams

We can compare the force-displacement diagrams of the two arches with the help of *Figure 16*. We can observe that the shear forces of the Gothic arch are much smaller. Furthermore in case of pointed arch the second part of the diagram starts sooner.

5. Conclusion

From the results we can observe that the horizontal reaction of the Gothic arch is much smaller than the reaction of the Romanesque arch. So the results really are showing the advantage of the pointed arch what I mentioned in the first chapter.

If the arches are under selfweight the horizontal reaction of the Gothic arch is 31% smaller. When we move the supports and the contact shear force nearly doesn't change anymore (we are on the constant section of the force-displacement diagram) the difference become even larger. In this case the pointed vault has about 40% smaller contact shear force than the semi-circular arch.

This difference is remarkable and we also have to mention that in these models the selfweight of the Gothic arch is a little larger than the weight of the Romanesque arch, because of the neglecting we made in the wall thickness calculation.

So we can say as a conclusion that the structure of the Gothic arch has a beneficial, advantageous effect on the support structure, because the horizontal force component is smaller. This means we have fewer problems with the stability of the supporting walls, so actually we can have thinner walls with bigger windows.

6. References

- [1] **Ganesh W. Rathod, Alex Varughese, A. K. Shrivastava, K.S. Rao**, American Society of Civil Engineers, Section: Analytical and Numerical Methods in Discontinuous Media, pp. 2382-2391
- [2] http://simple.wikipedia.org/wiki/Gothic_architecture#Building_materials
- [3] http://geology.about.com/cs/rock_types/a/aarockspeggrav.htm
- [4] <http://www.buzzle.com/articles/style-and-characteristics-of-gothic-architecture.html>
- [5] http://www.cathedralquest.com/romanesque_architecture.htm
- [6] http://www.cathedralquest.com/gothic_architecture.htm
- [7] 3DEC - 3 Dimensional Distinct Element Code - **Online Manual**
- [8] **Marwan Al-Heib**, *Ineris – Ecole des Mines de Nancy, Parc de Saurupt, France*, Distinct Element Method Applied on Old Masonry Structures, pp. 304-327
- [9] <http://www.matrixlab-examples.com/>

Appendix A – Matlab codes

Romanesque arch:

```
% The initial instructions clear the screen, delete all of the existing variables, and close any figure
clc; clear; close all
% This code draws two concentric semi-circle and divide it into 18 parts, so it gives the coordinates of the 17 blocks of the Romanesque arch, we can see the result in Figure 17.
% First it generates a 18-element vector, the angle is pi, so we will have a half circle
angle = linspace(0, pi, 18);
% After this it creates another vector with the value of the outer radius
r2 = linspace(1.15, 1.15, 18);
% We plot the circle and the chosen 18 points with little red circles (ro)
polar(angle, r2, 'ro')
% We name the figure 'Romanesque arch'
title('Romanesque arch')
% In the following we wouldn't like to delete this figure, so we have to keep it with the hold on command
hold on
% Now, we create another 18-element vector for the inner radius
r1 = linspace(1, 1, 18);
% And we also plot this circle and the chosen 18 points with little blue x-es (bx)
polar(angle, r1, 'bx')
% Finally, we mark the center of the circles with a black square (ks)
polar(0, 0, 'ks')
```

Gothic arch:

```
% The initial instructions clear the screen, delete all of the existing variables, and close any figure
clc; clear; close all
% This code draws two concentric section of circle with 60° angle and divide it into 10 parts, so it gives the coordinates of half of the blocks of the Gothic arch ( Figure 18).
% First it generates a 10-element vector, the angle is pi/3, so we will have a 60° angle section of circle
angle = linspace(0, pi/3, 10);
% Then, we create another 10-element vector containing only the outer radius
r2 = linspace(2.1175, 2.1175, 10);
% We plot the circle and the chosen 18 points with little red circles (ro)
polar(angle, r2, 'ro')
% We name the figure 'Gothic arch'
title('Gothic arch')
% We avoid the deletion of this figure
hold on
% Now, we create another 10-element vector for the inner radius
```

```

r1 = linspace(2, 2, 10);
% And we also plot this circle and the chosen 18 points with little blue x-es (bx)
polar(angle, r1, 'bx')
% Finally, we mark the center with a black square (ks)
polar(0, 0, 'ks')
    
```

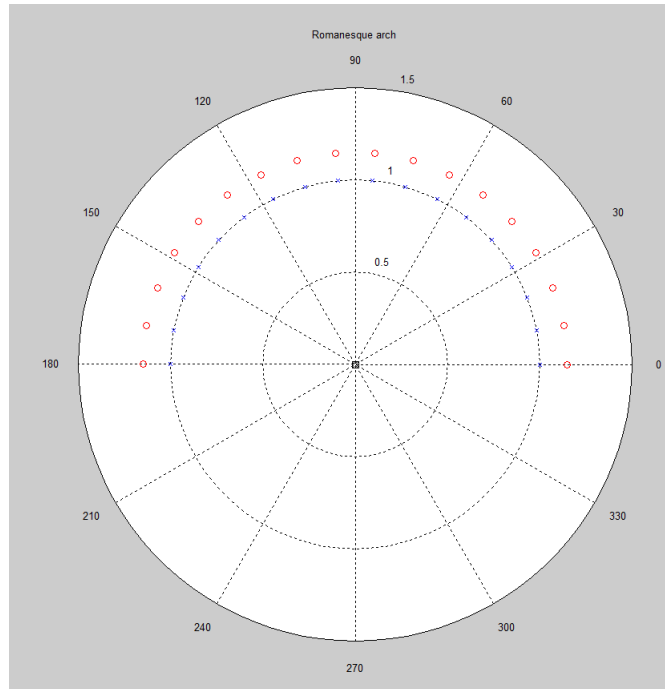


Figure 17: Points of the Romanesque arch

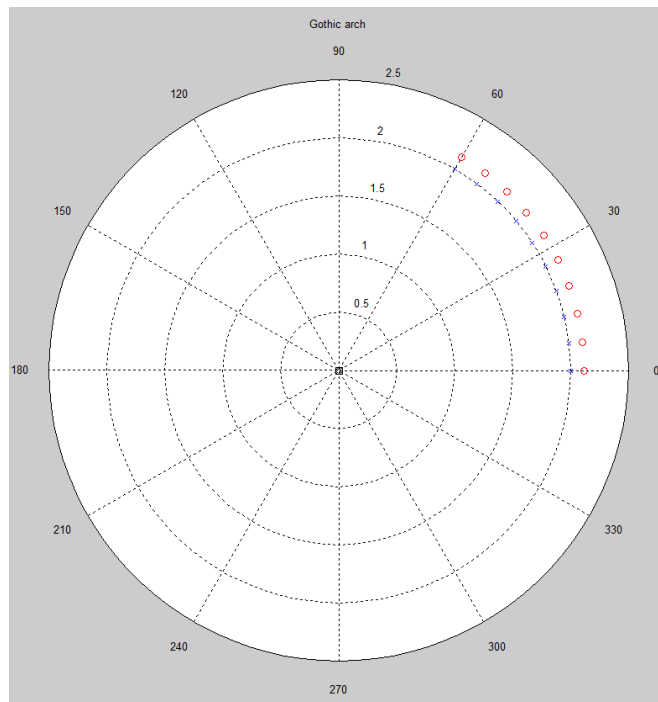


Figure 18: Half of the points of the Gothic arch

Appendix B – 3DEC code of the Romanesque arch

; We always have to start our *3ddat file with the new command
new

; Define the geometry with command prism, from the coordinates given by Matlab

; The prism command: The two parallel faces of the prism are defined by an arbitrary number of vertices. The opposing vertices on each face are then automatically connected to form the prism. The first face (face a) is defined by vertices entered in either a clockwise or counterclockwise order. The opposite face (face b) must have its vertices entered in the same order as the corresponding vertices for face a. Faces a and b must be planar and convex.

```
polyhedron prism a 2.280,0.211,0.000 2.300,0.000,0.000 2.150,0.000,0.000 2.133,0.184,0.000 b
2.280,0.211,1.000 2.300,0.000,1.000 2.150,0.000,1.000 2.133,0.184,1.000
polyhedron prism a 2.222,0.415,0.000 2.280,0.211,0.000 2.133,0.184,0.000 2.082,0.361,0.000 b
2.222,0.415,1.000 2.280,0.211,1.000 2.133,0.184,1.000 2.082,0.361,1.000
polyhedron prism a 2.128,0.605,0.000 2.222,0.415,0.000 2.082,0.361,0.000 2.000,0.526,0.000 b
2.128,0.605,1.000 2.222,0.415,1.000 2.082,0.361,1.000 2.000,0.526,1.000
polyhedron prism a 2.000,0.775,0.000 2.128,0.605,0.000 2.000,0.526,0.000 1.889,0.674,0.000 b
2.000,0.775,1.000 2.128,0.605,1.000 2.000,0.526,1.000 1.889,0.674,1.000
polyhedron prism a 1.843,0.918,0.000 2.000,0.775,0.000 1.889,0.674,0.000 1.753,0.798,0.000 b
1.843,0.918,1.000 2.000,0.775,1.000 1.889,0.674,1.000 1.753,0.798,1.000
polyhedron prism a 1.663,1.029,0.000 1.843,0.918,0.000 1.753,0.798,0.000 1.596,0.895,0.000 b
1.663,1.029,1.000 1.843,0.918,1.000 1.753,0.798,1.000 1.596,0.895,1.000
polyhedron prism a 1.465,1.106,0.000 1.663,1.029,0.000 1.596,0.895,0.000 1.424,0.962,0.000 b
1.465,1.106,1.000 1.663,1.029,1.000 1.596,0.895,1.000 1.424,0.962,1.000
polyhedron prism a 1.256,1.145,0.000 1.465,1.106,0.000 1.424,0.962,0.000 1.242,0.996,0.000 b
1.256,1.145,1.000 1.465,1.106,1.000 1.424,0.962,1.000 1.242,0.996,1.000
polyhedron prism a 1.044,1.145,0.000 1.256,1.145,0.000 1.242,0.996,0.000 1.058,0.996,0.000 b
1.044,1.145,1.000 1.256,1.145,1.000 1.242,0.996,1.000 1.058,0.996,1.000
polyhedron prism a 0.835,1.106,0.000 1.044,1.145,0.000 1.058,0.996,0.000 0.876,0.962,0.000 b
0.835,1.106,1.000 1.044,1.145,1.000 1.058,0.996,1.000 0.876,0.962,1.000
polyhedron prism a 0.637,1.029,0.000 0.835,1.106,0.000 0.876,0.962,0.000 0.704,0.895,0.000 b
0.637,1.029,1.000 0.835,1.106,1.000 0.876,0.962,1.000 0.704,0.895,1.000
polyhedron prism a 0.457,0.918,0.000 0.637,1.029,0.000 0.704,0.895,0.000 0.547,0.798,0.000 b
0.457,0.918,1.000 0.637,1.029,1.000 0.704,0.895,1.000 0.547,0.798,1.000
polyhedron prism a 0.300,0.775,0.000 0.457,0.918,0.000 0.547,0.798,0.000 0.411,0.674,0.000 b
0.300,0.775,1.000 0.457,0.918,1.000 0.547,0.798,1.000 0.411,0.674,1.000
polyhedron prism a 0.172,0.605,0.000 0.300,0.775,0.000 0.411,0.674,0.000 0.300,0.526,0.000 b
0.172,0.605,1.000 0.300,0.775,1.000 0.411,0.674,1.000 0.300,0.526,1.000
polyhedron prism a 0.078,0.415,0.000 0.172,0.605,0.000 0.300,0.526,0.000 0.218,0.361,0.000 b
0.078,0.415,1.000 0.172,0.605,1.000 0.300,0.526,1.000 0.218,0.361,1.000
polyhedron prism a 0.020,0.211,0.000 0.078,0.415,0.000 0.218,0.361,0.000 0.167,0.184,0.000 b
0.020,0.211,1.000 0.078,0.415,1.000 0.218,0.361,1.000 0.167,0.184,1.000
```

```
polyhedron prism a 0.000,0.000,0.000 0.020,0.211,0.000 0.167,0.184,0.000 0.150,0.000,0.000 b
0.000,0.000,1.000 0.020,0.211,1.000 0.167,0.184,1.000 0.150,0.000,1.000
```

; Define the support blocks with the same command

```
polyhedron prism a -0.150,-0.400,0.000 -0.150,0.000,0.000 0.150,0.000,0.000 0.150,-0.400,0.000
b -0.150,-0.400,1.000 -0.150,0.000,1.000 0.150,0.000,1.000 0.150,-0.400,1.000
polyhedron prism a 2.150,-0.400,0.000 2.150,0.000,0.000 2.450,0.000,0.000 2.450,-0.400,0.000
b 2.150,-0.400,1.000 2.150,0.000,1.000 2.450,0.000,1.000 2.450,-0.400,1.000
```

; Create ranges (if we name ranges, we can use them in other commands, so it will be much easier to handle these parts)

```
range name arch x=(0,10) y=(0,10) z=(0,10)
range name supportblock1 x=(-0.15,0.15) y=(-0.4,0.0) z=(0,1)
range name supportblock2 x=(2.15,2.450) y=(-0.4,0.0) z=(0,1)
```

; Fix the support blocks

```
fix range supportblock1
fix range supportblock2
```

; Define the material properties of the blocks and joints

```
prop mat=1 dens=2600
prop jmat=1 jkn 1e12 jks 1e12 jfri 50
```

; Apply gravity (in my model the y axis is the vertical one)

```
gravity 0,-9.81,0
```

; Listing the change of the unbalanced forces during the calculation: before we run the program we need to store these values

```
hist unbal id=1
```

; Listing the change of displacements during the calculation, we store the y (vertical) displacement of point 0, and the x (horizontal) displacements of point 1 and point2 (see the position of these points in *Figure 8* and *Figure 9*).

```
hist ydisp (1.150,1.150,0.0) id=2
hist xdisp (0.172,0.605,0.0) id=3
hist xdisp (2.128,0.605,0.0) id=4
```

; Before we store the contact shear forces we need to start the calculation

```
cycle 1
```

; After this we can store the shear force and normal force of point3 and point4 with hist command, point3 and point 4 are in the middle of the contacts.

```
hist sforce (0.075,0.00,0.5) id=5
hist sforce (2.075,0.00,0.5) id=6
hist nforce (0.075,0.00,0.5) id=7
hist nforce (2.075,0.00,0.5) id=8
```


; Calculation
cycle 10000

; Plot the unbalanced force, y displacement in the defined point (middle), and x displacements in the defined points and the contact shear and normal forces

```
plot hist 1 yaxis label 'Unbbalanced force'
plot hist 2 yaxis label 'Vertical displacement of the middle point (point 0)'
plot hist 3 yaxis label 'Horizontal displacement of point 1'
plot hist 4 yaxis label 'Horizontal displacement of point 2'
plot hist 5 yaxis label 'Contact shear force of point 3'
plot hist 6 yaxis label 'Contact shear force of point 4'
plot hist 7 yaxis label 'Contact normal force of point 3'
plot hist 8 yaxis label 'Contact normal force of point 4'
```

; Contour plot the y and x displacements and the distribution of contact forces

```
plot contour ydisp above au
plot contour xdisp above au
plot jointcontour sforce
plot jointcontour nforce
```

; After we analyzed the structure under selfweight we can move the support blocks, and start to create the force-displacement diagram.

; First of all we need to free the fixed blocks and after that we can apply velocities on them.

```
free range supportblock1
free range supportblock2
apply xvel -0.0001 range supportblock1
apply xvel 0.0001 range supportblock2
```

; We can set the length of time step, in this way we can easily calculate the displacement

```
set dt 0.0001
```

; After this we can run a given number of time steps, I changed this value from 125-2000 to get different values of displacements, as we can see in *Table 7*.

```
cycle 125
```

; After this we need to stop the motion, so we have to free the support blocks, and then apply zero velocity values on them. **Be aware:** here we cannot use the fix command to stop the motion, if we use the fix command it will fix the velocity value of the support.

```
free range supportblock1
free range supportblock2
apply xvel 0.000 range supportblock1
apply xvel 0.000 range supportblock2
```

; Now we can apply the gravity again and run cycles till we have a very little unbalanced force.

```
gravity 0,-9.81,0
cycle 100000
```

; First of all I listed the block velocities to check, that the support blocks are really fixed and don't have velocities, after that I listed the contact states (here we can find the subcontact areas, too) and the contact stresses.

; If we use the set log on command then the program creates a log file, and list everything there, but we can also apply these commands directly in the command bar, in this case the program lists the values in a different window.

list block vel

list contact state

list contact stress

Excel calculation of the shear forces:

| | | | | | | | |
|------------------|------------|--------------|----|--------------|---------|----------|----------|
| initial values | 0 | | | | | | 2385,75 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,36 | 4 | 13600 | 1,25 | 0,0125 | 170 |
| v | 0,0001 | 2,455 | 4 | 24550 | 2,5 | 0,025 | 613,75 |
| N | 125 | 2,046 | 4 | 20460 | 1,25 | 0,0125 | 255,75 |
| | | 2,048 | 4 | 20480 | 2,5 | 0,025 | 512 |
| ux(m) | 0,00000125 | 2,455 | 4 | 24550 | 3,125 | 0,003125 | 76,71875 |
| e(mm) | 0,00125 | 2,251 | 4 | 22510 | 9,375 | 0,009375 | 211,0313 |
| | | 1,448 | 4 | 14480 | 1,875 | 0,01875 | 271,5 |
| unbalanced force | 7,97E-13 | 2,048 | 4 | 20480 | 1,25 | 0,0125 | 256 |
| | | | | | | | 2366,75 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,336 | 4 | 13360 | 1,25 | 0,0125 | 167 |
| v | 0,0001 | 2,432 | 4 | 24320 | 2,5 | 0,025 | 608 |
| N | 250 | 2,039 | 4 | 20390 | 1,25 | 0,0125 | 254,875 |
| | | 2,04 | 4 | 20400 | 2,5 | 0,025 | 510 |
| ux(m) | 0,0000025 | 2,432 | 4 | 24320 | 3,125 | 0,003125 | 76 |
| e(mm) | 0,0025 | 2,235 | 4 | 22350 | 9,375 | 0,009375 | 209,5313 |
| | | 1,424 | 4 | 14240 | 1,875 | 0,01875 | 267 |
| unbalanced force | 4,80E-13 | 2,04 | 4 | 20400 | 1,25 | 0,0125 | 255 |
| | | | | | | | 2347,406 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,311 | 4 | 13110 | 1,25 | 0,0125 | 163,875 |
| v | 0,0001 | 2,407 | 4 | 24070 | 2,5 | 0,025 | 601,75 |
| N | 375 | 2,034 | 4 | 20340 | 1,25 | 0,0125 | 254,25 |
| | | 2,035 | 4 | 20350 | 2,5 | 0,025 | 508,75 |
| ux(m) | 0,00000375 | 2,407 | 4 | 24070 | 3,125 | 0,003125 | 75,21875 |
| e(mm) | 0,00375 | 2,221 | 4 | 22210 | 9,375 | 0,009375 | 208,2188 |
| | | 1,4 | 4 | 14000 | 1,875 | 0,01875 | 262,5 |
| unbalanced force | 6,98E-13 | 2,035 | 4 | 20350 | 1,25 | 0,0125 | 254,375 |
| | | | | | | | 2328,938 |

| | | shear stress | e [^] | shear stress | subarea | subarea | sforce |
|------------------|-----------|--------------|----------------|--------------|---------|----------|----------|
| dt | 0,0001 | 1,287 | 4 | 12870 | 1,25 | 0,0125 | 160,875 |
| v | 0,0001 | 2,383 | 4 | 23830 | 2,5 | 0,025 | 595,75 |
| N | 500 | 2,03 | 4 | 20300 | 1,25 | 0,0125 | 253,75 |
| | | 2,031 | 4 | 20310 | 2,5 | 0,025 | 507,75 |
| ux(m) | 0,000005 | 2,383 | 4 | 23830 | 3,125 | 0,003125 | 74,46875 |
| e(mm) | 0,005 | 2,207 | 4 | 22070 | 9,375 | 0,009375 | 206,9063 |
| | | 1,375 | 4 | 13750 | 1,875 | 0,01875 | 257,8125 |
| unbalanced force | 7,96E-13 | 2,031 | 4 | 20310 | 1,25 | 0,0125 | 253,875 |
| | | | | | | | 2311,188 |
| | | | | | | | |
| | | shear stress | e [^] | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,286 | 4 | 12860 | 1,25 | 0,0125 | 160,75 |
| v | 0,0001 | 2,382 | 4 | 23820 | 2,5 | 0,025 | 595,5 |
| N | 750 | 2,029 | 4 | 20290 | 1,25 | 0,0125 | 253,625 |
| | | 2,03 | 4 | 20300 | 2,5 | 0,025 | 507,5 |
| ux(m) | 0,0000075 | 2,382 | 4 | 23820 | 3,125 | 0,003125 | 74,4375 |
| e(mm) | 0,0075 | 2,206 | 4 | 22060 | 9,375 | 0,009375 | 206,8125 |
| | | 1,375 | 4 | 13750 | 1,875 | 0,01875 | 257,8125 |
| unbalanced force | 5,16E-07 | 2,03 | 4 | 20300 | 1,25 | 0,0125 | 253,75 |
| | | | | | | | 2310,188 |
| | | | | | | | |
| | | shear stress | e [^] | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,286 | 4 | 12860 | 1,25 | 0,0125 | 160,75 |
| v | 0,0001 | 2,382 | 4 | 23820 | 2,5 | 0,025 | 595,5 |
| N | 1000 | 2,03 | 4 | 20300 | 1,25 | 0,0125 | 253,75 |
| | | 2,031 | 4 | 20310 | 2,5 | 0,025 | 507,75 |
| ux(m) | 0,00001 | 2,382 | 4 | 23820 | 3,125 | 0,003125 | 74,4375 |
| e(mm) | 0,01 | 2,206 | 4 | 22060 | 9,375 | 0,009375 | 206,8125 |
| | | 1,374 | 4 | 13740 | 1,875 | 0,01875 | 257,625 |
| unbalanced force | 6,90E-07 | 2,031 | 4 | 20310 | 1,25 | 0,0125 | 253,875 |
| | | | | | | | 2310,5 |
| | | | | | | | |
| | | shear stress | e [^] | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,286 | 4 | 12860 | 1,25 | 0,0125 | 160,75 |
| v | 0,0001 | 2,382 | 4 | 23820 | 2,5 | 0,025 | 595,5 |
| N | 1500 | 2,03 | 4 | 20300 | 1,25 | 0,0125 | 253,75 |
| | | 2,031 | 4 | 20310 | 2,5 | 0,025 | 507,75 |
| ux(m) | 0,000015 | 2,382 | 4 | 23820 | 3,125 | 0,003125 | 74,4375 |
| e(mm) | 0,015 | 2,206 | 4 | 22060 | 9,375 | 0,009375 | 206,8125 |
| | | 1,374 | 4 | 13740 | 1,875 | 0,01875 | 257,625 |
| unbalanced force | 6,24E-07 | 2,031 | 4 | 20310 | 1,25 | 0,0125 | 253,875 |
| | | | | | | | 2310,5 |
| | | | | | | | |

| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
|------------------|----------|--------------|----|--------------|---------|----------|----------|
| dt | 0,0001 | 1,287 | 4 | 12870 | 1,25 | 0,0125 | 160,875 |
| v | 0,0001 | 2,383 | 4 | 23830 | 2,5 | 0,025 | 595,75 |
| N | 2000 | 2,029 | 4 | 20290 | 1,25 | 0,0125 | 253,625 |
| | | 2,03 | 4 | 20300 | 2,5 | 0,025 | 507,5 |
| ux(m) | 0,00002 | 2,383 | 4 | 23830 | 3,125 | 0,003125 | 74,46875 |
| e(mm) | 0,02 | 2,206 | 4 | 22060 | 9,375 | 0,009375 | 206,8125 |
| | | 1,375 | 4 | 13750 | 1,875 | 0,01875 | 257,8125 |
| unbalanced force | 6,26E-07 | 2,03 | 4 | 20300 | 1,25 | 0,0125 | 253,75 |
| | | | | | | | 2310,594 |

Table 7: Excel calculation of the displacements and horizontal reactions

Appendix C – 3DEC code of Gothic arch

In this case the details and commands are the same as before, so here I won't explain them again I only mark the main steps.

; Start a new 3DEC file
new

; Define the geometry

```
polyhedron prism a 2.221,0.246,0.000 2.235,0.000,0.000 2.118,0.000,0.000 2.104,0.232,0.000 b
2.221,0.246,1.000 2.235,0.000,1.000 2.118,0.000,1.000 2.104,0.232,1.000
polyhedron prism a 2.178,0.488,0.000 2.221,0.246,0.000 2.104,0.232,0.000 2.064,0.461,0.000 b
2.178,0.488,1.000 2.221,0.246,1.000 2.104,0.232,1.000 2.064,0.461,1.000
polyhedron prism a 2.107,0.724,0.000 2.178,0.488,0.000 2.064,0.461,0.000 1.997,0.684,0.000 b
2.107,0.724,1.000 2.178,0.488,1.000 2.064,0.461,1.000 1.997,0.684,1.000
polyhedron prism a 2.010,0.950,0.000 2.107,0.724,0.000 1.997,0.684,0.000 1.905,0.898,0.000 b
2.010,0.950,1.000 2.107,0.724,1.000 1.997,0.684,1.000 1.905,0.898,1.000
polyhedron prism a 1.887,1.164,0.000 2.010,0.950,0.000 1.905,0.898,0.000 1.788,1.099,0.000 b
1.887,1.164,1.000 2.010,0.950,1.000 1.905,0.898,1.000 1.788,1.099,1.000
polyhedron prism a 1.740,1.361,0.000 1.887,1.164,0.000 1.788,1.099,0.000 1.650,1.286,0.000 b
1.740,1.361,1.000 1.887,1.164,1.000 1.788,1.099,1.000 1.650,1.286,1.000
polyhedron prism a 1.571,1.540,0.000 1.740,1.361,0.000 1.650,1.286,0.000 1.490,1.455,0.000 b
1.571,1.540,1.000 1.740,1.361,1.000 1.650,1.286,1.000 1.490,1.455,1.000
polyhedron prism a 1.382,1.698,0.000 1.571,1.540,0.000 1.490,1.455,0.000 1.312,1.604,0.000 b
1.382,1.698,1.000 1.571,1.540,1.000 1.490,1.455,1.000 1.312,1.604,1.000
polyhedron prism a 1.118,1.868,0.000 1.382,1.698,0.000 1.312,1.604,0.000 1.118, 1.604,0.000 b
1.118,1.868,1.000 1.382,1.698,1.000 1.312,1.604,1.000 1.118, 1.604,1.000
polyhedron prism a 0.853,1.698,0.000 1.118,1.868,0.000 1.118, 1.604,0.000 0.923,1.604,0.000 b
0.853,1.698,1.000 1.118,1.868,1.000 1.118, 1.604,1.000 0.923,1.604,1.000
polyhedron prism a 0.664,1.540,0.000 0.853,1.698,0.000 0.923,1.604,0.000 0.745,1.455,0.000 b
0.664,1.540,1.000 0.853,1.698,1.000 0.923,1.604,1.000 0.745,1.455,1.000
polyhedron prism a 0.495,1.361,0.000 0.664,1.540,0.000 0.745,1.455,0.000 0.585,1.286,0.000 b
0.495,1.361,1.000 0.664,1.540,1.000 0.745,1.455,1.000 0.585,1.286,1.000
polyhedron prism a 0.348,1.164,0.000 0.495,1.361,0.000 0.585,1.286,0.000 0.447,1.099,0.000 b
0.348,1.164,1.000 0.495,1.361,1.000 0.585,1.286,1.000 0.447,1.099,1.000
```

```

polyhedron prism a 0.225,0.950,0.000 0.348,1.164,0.000 0.447,1.099,0.000 0.330,0.898,0.000 b
0.225,0.950,1.000 0.348,1.164,1.000 0.447,1.099,1.000 0.330,0.898,1.000
polyhedron prism a 0.128,0.724,0.000 0.225,0.950,0.000 0.330,0.898,0.000 0.238,0.684,0.000 b
0.128,0.724,1.000 0.225,0.950,1.000 0.330,0.898,1.000 0.238,0.684,1.000
polyhedron prism a 0.057,0.488,0.000 0.128,0.724,0.000 0.238,0.684,0.000 0.171,0.461,0.000 b
0.057,0.488,1.000 0.128,0.724,1.000 0.238,0.684,1.000 0.171,0.461,1.000
polyhedron prism a 0.014,0.246,0.000 0.057,0.488,0.000 0.171,0.461,0.000 0.131,0.232,0.000 b
0.014,0.246,1.000 0.057,0.488,1.000 0.171,0.461,1.000 0.131,0.232,1.000
polyhedron prism a 0.000,0.000,0.000 0.014,0.246,0.000 0.131,0.232,0.000 0.118,0.000,0.000 b
0.000,0.000,1.000 0.014,0.246,1.000 0.131,0.232,1.000 0.118,0.000,1.000

```

; Create a range from the upper blocks, because we have to glue, merge these two blocks together to create the keystone (capstone).

```
range name upperblocks x=(0.853,1.382) y=(1.60,1.99) z=(0,1)
```

;Glue the upperblocks range with join command, now these two blocks are becoming one.

```
join on range upperblocks
```

; Define the support blocks

```

polyhedron prism a -0.182,-0.400,0.000 -0.182,0.000,0.000 0.118,0.000,0.000 0.118,-0.400,0.000
b -0.182,-0.400,1.000 -0.182,0.000,1.000 0.118,0.000,1.000 0.118,-0.400,1.000
polyhedron prism a 2.118,-0.400,0.000 2.118,0.000,0.000 2.418,0.000,0.000 2.418,-0.400,0.000
b 2.118,-0.400,1.000 2.118,0.000,1.000 2.418,0.000,1.000 2.418,-0.400,1.000

```

; Create further ranges

```

range name arch x=(0,10) y=(0,10) z=(0,10)
range name supportblock1 x=(-0.182,0.118) y=(-0.4,0.0) z=(0,1)
range name supportblock2 x=(2.118,2.418) y=(-0.4,0.0) z=(0,1)

```

; Fix the support blocks

```

fix range supportblock1
fix range supportblock2

```

; Apply material models for the blocks and contacts

```

prop mat=1 dens=2600
prop jmat=1 jkn 1e12 jks 1e12 jfri 50

```

; Apply the gravity

```
gravity 0,-9.81,0
```

; Store the change of the unbalanced forces during the calculation

```
hist unbal id=1
```

; Store the change of displacements during the calculation, we store the y (vertical) displacement of point 0, and the x (horizontal) displacements of point 1 and point2 (see the position of these points in *Figure 11* and *Figure 12*).

```
hist ydisp (1.118,1.868,0.000) id=2
hist xdisp (0.128,0.724,0.000) id=3
hist xdisp (2.107,0.724,0.000) id=4
```

```
; Before we store the contact shear forces we need to start the calculation
cycle 1
```

```
; After this we can store the shear force and normal force of point3 and point4 with hist
command, point3 and point 4 are in the middle of the contacts.
```

```
hist sforce (0.075,0.00,0.5) id=5
hist sforce (2.075,0.00,0.5) id=6
hist nforce (0.075,0.00,0.5) id=7
hist nforce (2.075,0.00,0.5) id=8
```

```
; Calculation
cycle 10000
```

```
; Plot the unbalanced force, y displacement in the defined point (middle), and x displacements in
the defined points and the contact shear and normal forces
```

```
plot hist 1 yaxis label 'Unbbalanced force'
plot hist 2 yaxis label 'Vertical displacement of the middle point (point 0)'
plot hist 3 yaxis label 'Horizontal displacement of point 1'
plot hist 4 yaxis label 'Horizontal displacement of point 2'
plot hist 5 yaxis label 'Contact shear force of point 3'
plot hist 6 yaxis label 'Contact shear force of point 4'
plot hist 7 yaxis label 'Contact normal force of point 3'
plot hist 8 yaxis label 'Contact normal force of point 4'
```

```
; Contour plot the y and x displacements and the contact forces
```

```
plot contour ydisp above au
plot contour xdisp above au
plot jointcontour sforce
plot jointcontour nforce
```

```
; List the contact states and stresses
```

```
list contact state
list contact stress
```

```
; Move the supportblocks
```

```
free range supportblock1
free range supportblock2
apply xvel -0.0001 range supportblock1
apply xvel 0.0001 range supportblock2
```

```
;Set the length and number of time step
```

```
set dt 0.0001
```

cycle 125

; Stop the motion

free range supportblock1

free range supportblock2

apply xvel 0.000 range supportblock1

apply xvel 0.000 range supportblock2

; Apply gravity and balance the structure

gravity 0,-9.81,0

cycle 100000

; List the desired values of the blocks and contacts

list block vel

list contact state

list contact stress

The Excel calculation of the shear forces:

| | | | | | | | |
|------------------|----------|--------------|----|--------------|---------|----------|----------|
| initial values | 0 | | | | | | 1628,39 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,438 | 4 | 14380 | 9,833 | 0,009833 | 141,3985 |
| v | 0,0001 | 1,438 | 4 | 14380 | 1,967 | 0,019670 | 282,8546 |
| N | 125 | 1,673 | 4 | 16730 | 9,833 | 0,009833 | 164,5061 |
| | | 1,661 | 4 | 16610 | 1,967 | 0,019670 | 326,7187 |
| ux(m) | 1,25E-06 | 1,438 | 4 | 14380 | 9,833 | 0,009833 | 141,3985 |
| e(mm) | 0,00125 | 1,438 | 4 | 14380 | 6,851 | 0,006851 | 98,51738 |
| | | 1,553 | 4 | 15530 | 1,934 | 0,001934 | 30,03502 |
| unbalanced force | 9,09E-13 | 1,645 | 4 | 16450 | 1,37 | 0,013700 | 225,365 |
| | | | | | | | 1410,794 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,436 | 4 | 14360 | 9,833 | 0,009833 | 141,2019 |
| v | 0,0001 | 1,436 | 4 | 14360 | 1,967 | 0,019670 | 282,4612 |
| N | 250 | 1,668 | 4 | 16680 | 9,833 | 0,009833 | 164,0144 |
| | | 1,656 | 4 | 16560 | 1,967 | 0,019670 | 325,7352 |
| ux(m) | 2,5E-06 | 1,436 | 4 | 14360 | 9,833 | 0,009833 | 141,2019 |
| e(mm) | 0,0025 | 1,436 | 4 | 14360 | 6,851 | 0,006851 | 98,38036 |
| | | 1,549 | 4 | 15490 | 1,934 | 0,001934 | 29,95766 |
| unbalanced force | 9,60E-13 | 1,64 | 4 | 16400 | 1,37 | 0,013700 | 224,68 |
| | | | | | | | 1407,633 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,434 | 4 | 14340 | 9,833 | 0,009833 | 141,0052 |
| v | 0,0001 | 1,434 | 4 | 14340 | 1,967 | 0,019670 | 282,0678 |
| N | 375 | 1,663 | 4 | 16630 | 9,833 | 0,009833 | 163,5228 |
| | | 1,651 | 4 | 16510 | 1,967 | 0,019670 | 324,7517 |
| ux(m) | 3,75E-06 | 1,434 | 4 | 14340 | 9,833 | 0,009833 | 141,0052 |

| | | | | | | | |
|------------------|----------|--------------|----|--------------|---------|----------|----------|
| e(mm) | 0,00375 | 1,434 | 4 | 14340 | 6,851 | 0,006851 | 98,24334 |
| | | 1,545 | 4 | 15450 | 1,934 | 0,001934 | 29,8803 |
| unbalanced force | 9,09E-13 | 1,635 | 4 | 16350 | 1,37 | 0,013700 | 223,995 |
| | | | | | | | 1404,471 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,432 | 4 | 14320 | 9,833 | 0,009833 | 140,8086 |
| v | 0,0001 | 1,432 | 4 | 14320 | 1,967 | 0,019670 | 281,6744 |
| N | 500 | 1,658 | 4 | 16580 | 9,833 | 0,009833 | 163,0311 |
| | | 1,647 | 4 | 16470 | 1,967 | 0,019670 | 323,9649 |
| ux(m) | 0,000005 | 1,432 | 4 | 14320 | 9,833 | 0,009833 | 140,8086 |
| e(mm) | 0,005 | 1,432 | 4 | 14320 | 6,851 | 0,006851 | 98,10632 |
| | | 1,541 | 4 | 15410 | 1,934 | 0,001934 | 29,80294 |
| unbalanced force | 1,58E-12 | 1,63 | 4 | 16300 | 1,37 | 0,013700 | 223,31 |
| | | | | | | | 1401,507 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,428 | 4 | 14280 | 9,833 | 0,009833 | 140,4152 |
| v | 0,0001 | 1,428 | 4 | 14280 | 1,967 | 0,019670 | 280,8876 |
| N | 750 | 1,648 | 4 | 16480 | 9,833 | 0,009833 | 162,0478 |
| | | 1,637 | 4 | 16370 | 1,967 | 0,019670 | 321,9979 |
| ux(m) | 7,5E-06 | 1,428 | 4 | 14280 | 9,833 | 0,009833 | 140,4152 |
| e(mm) | 0,0075 | 1,428 | 4 | 14280 | 6,851 | 0,006851 | 97,83228 |
| | | 1,534 | 4 | 15340 | 1,934 | 0,001934 | 29,66756 |
| unbalanced force | 6,80E-13 | 1,62 | 4 | 16200 | 1,37 | 0,013700 | 221,94 |
| | | | | | | | 1395,204 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,424 | 4 | 14240 | 9,833 | 0,009833 | 140,0219 |
| v | 0,0001 | 1,424 | 4 | 14240 | 1,967 | 0,019670 | 280,1008 |
| N | 1000 | 1,638 | 4 | 16380 | 9,833 | 0,009833 | 161,0645 |
| | | 1,627 | 4 | 16270 | 1,967 | 0,019670 | 320,0309 |
| ux(m) | 0,00001 | 1,424 | 4 | 14240 | 9,833 | 0,009833 | 140,0219 |
| e(mm) | 0,01 | 1,424 | 4 | 14240 | 6,851 | 0,006851 | 97,55824 |
| | | 1,526 | 4 | 15260 | 1,934 | 0,001934 | 29,51284 |
| unbalanced force | 1,27E-12 | 1,61 | 4 | 16100 | 1,37 | 0,013700 | 220,57 |
| | | | | | | | 1388,881 |
| | | shear stress | e^ | shear stress | subarea | subarea | sforce |
| dt | 0,0001 | 1,416 | 4 | 14160 | 9,833 | 0,009833 | 139,2353 |
| v | 0,0001 | 1,416 | 4 | 14160 | 1,967 | 0,019670 | 278,5272 |
| N | 1500 | 1,618 | 4 | 16180 | 9,833 | 0,009833 | 159,0979 |
| | | 1,607 | 4 | 16070 | 1,967 | 0,019670 | 316,0969 |
| ux(m) | 0,000015 | 1,416 | 4 | 14160 | 9,833 | 0,009833 | 139,2353 |
| e(mm) | 0,015 | 1,416 | 4 | 14160 | 6,851 | 0,006851 | 97,01016 |
| | | 1,511 | 4 | 15110 | 1,934 | 0,001934 | 29,22274 |
| unbalanced force | 1,42E-12 | 1,59 | 4 | 15900 | 1,37 | 0,013700 | 217,83 |
| | | | | | | | 1376,256 |

| | | shear stress | e [^] | shear stress | subarea | subarea | sforce |
|------------------|----------|--------------|----------------|--------------|---------|----------|----------|
| dt | 0,0001 | 1,407 | 4 | 14070 | 9,833 | 0,009833 | 138,3503 |
| v | 0,0001 | 1,407 | 4 | 14070 | 1,967 | 0,019670 | 276,7569 |
| N | 2000 | 1,598 | 4 | 15980 | 9,833 | 0,009833 | 157,1313 |
| | | 1,587 | 4 | 15870 | 1,967 | 0,019670 | 312,1629 |
| ux(m) | 0,00002 | 1,407 | 4 | 14070 | 9,833 | 0,009833 | 138,3503 |
| e(mm) | 0,02 | 1,407 | 4 | 14070 | 6,851 | 0,006851 | 96,39357 |
| | | 1,495 | 4 | 14950 | 1,934 | 0,001934 | 28,9133 |
| unbalanced force | 7,96E-13 | 1,57 | 4 | 15700 | 1,37 | 0,013700 | 215,09 |
| | | | | | | | 1363,149 |

Table 8: Excel calculation of the displacements and horizontal reactions