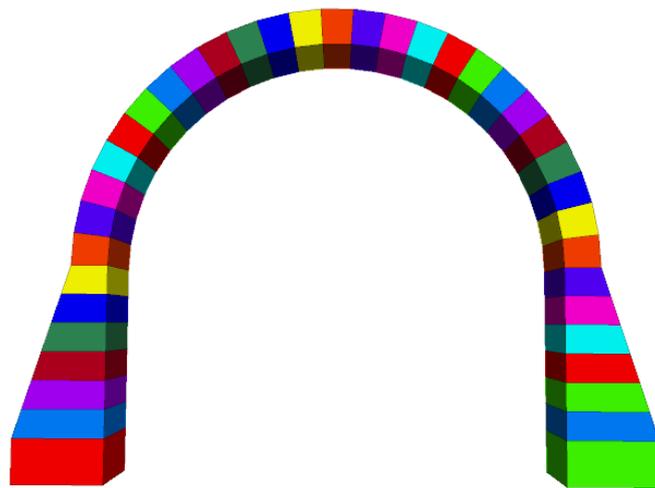




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# COMPARISON OF THE INTERNAL FORCES IN STILTED ARCH AND GOTHIC ARCH



NUMERICAL MODELS FOR STRUCTURES – BMEEOTMMB06

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## 1. Introduction

### 1.1. Aim of the homework and initial parameters

The aim of the homework within the subject 'Numerical Models for Structures' is to learn the principles of discrete element modeling, solving a discrete element problem using any available software.

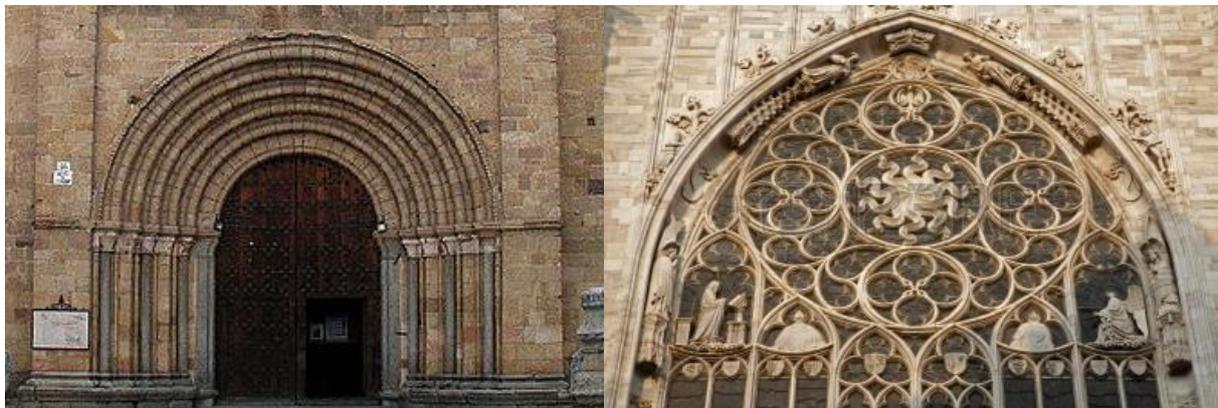
My task is to compare the static behavior and contact forces of two historic type of vaults: The stilted arch and the Gothic arch. In both cases the free span at the bottom of the vaults need to be 5 meters, the width is 90 cm and the thickness of the arches is an equal 40 cm.

Both arches are built of discrete elements of limestone with dry joints, the friction angle is set to  $40^\circ$ .

### 1.2. Stilted and Gothic arches

Stilted arches are arches in which the springing line is some distance above the impost. The arch itself is a half circle having the radius equal to half of the distance between the supports.

Gothic arches are pointed arches. In the homework circular arches have been chosen with a radius equal to the distance between the supports. An example of both arches is shown on *Figure 1*.



**Figure 1: Stilted Romanesque arch, Spain (left), Gothic arch, Dom of Milan (right)**

Stilted arches are found first in Classical architecture, but they were more frequently used in the Romanesque era. From static approach it is obvious that stilted Romanesque arches with the same span at the bottom are need to be built with larger dimensions compared to Gothic arches. This yields to the historic light and aerial architecture of Gothic cathedrals. Therefore the task to be proven is that the horizontal force will be smaller at the Gothic arch than the stilted Romanesque one if we use similar geometry and material properties.

A more detailed comparison of Romanesque and Gothic architecture can be read in *Reference* [2] and [3] with an excellent explanation of the developing of Gothic arches in European architecture of Catholic cathedrals, starting from the first half of the 12<sup>th</sup> century.

## 2. Steps of the solution

First a definition had to be made to the geometry of the vaults, the coordinates for the discrete elements were defined using Matlab. For both arches self-weight was added, displacements and contact forces were analyzed. Afterwards the horizontal reaction forces were measured.

Then the supports were moved in quasi-static way in order to measure the contact shear force. After reaching the desired displacements, the uniform velocity of the supports was eliminated and the structure was equilibrated.

## 3. Discrete element model

For building the discrete element model the coordinates were determined using Matlab R2015a and the model was finished in 3DEC 5.2. Both codes are attached to this document as Appendix. Solving the problem a previous homework task “*DEM comparison of the horizontal reaction of Romanesque and Gothic arches*” by Zsuzsa Borbála Pap, the Matlab Help and the 3DEC Manual provided sufficient help.

### 3.1. Geometry

As described in the previous chapter “*1. Introduction*” the geometry of the vaults was given. In both cases an inner span of 5 m had to be provided, the thickness of the arches is 40 cm and the depth of the structures is 90 cm. In the model 25 blocks were used for both arches. Supporting blocks were defined with a width of 80 cm, height of 50 cm, length of 90 cm. The geometry is shown on *Figure 2*.

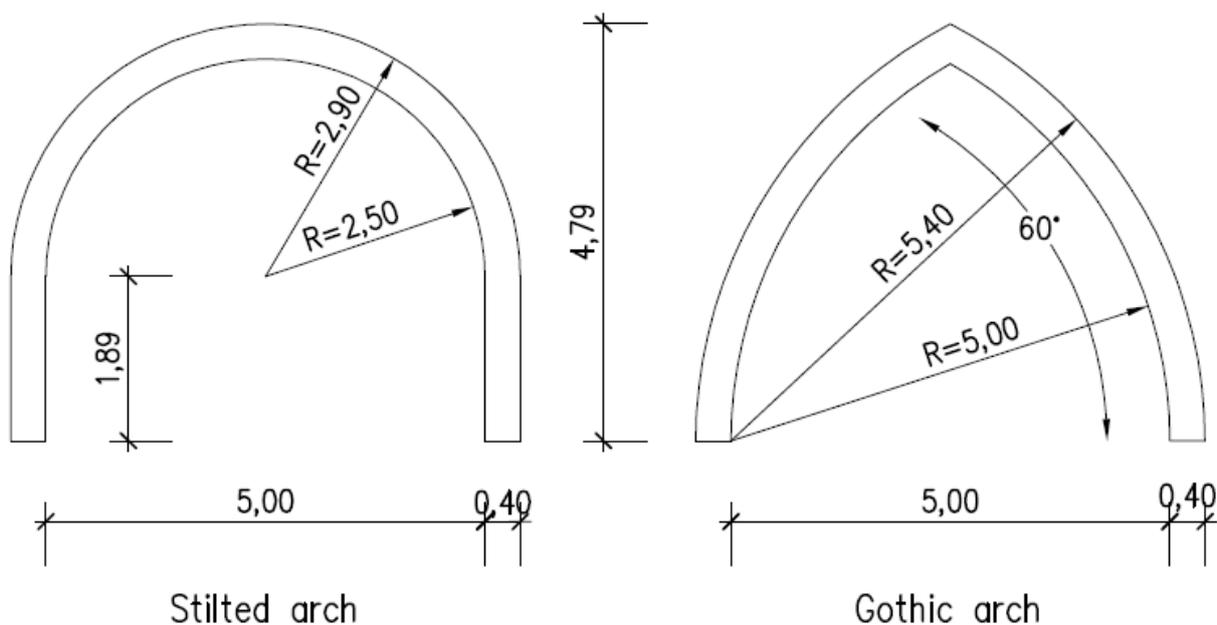
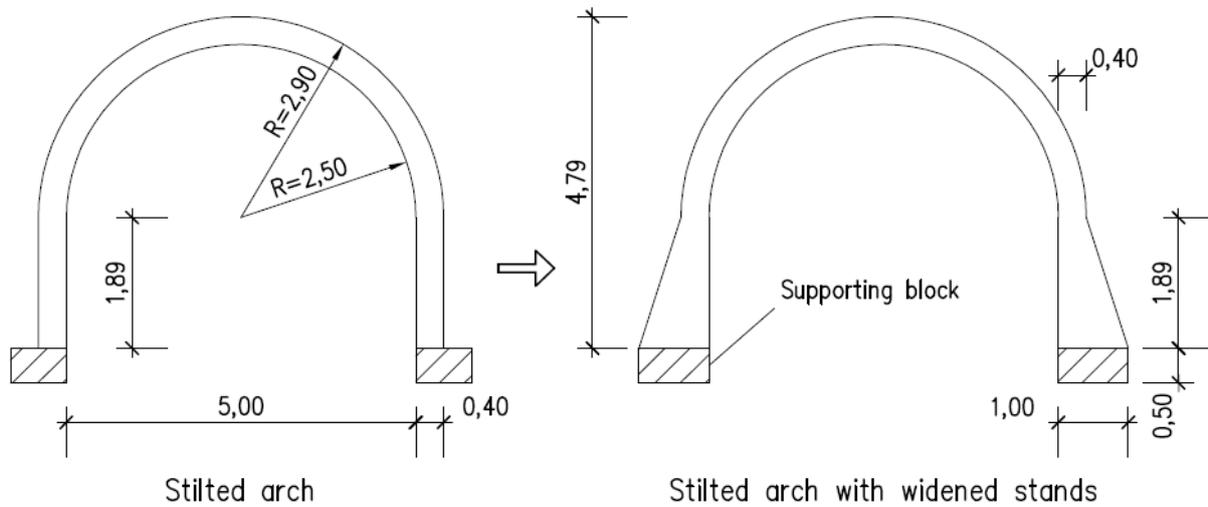


Figure 2: Initial geometry

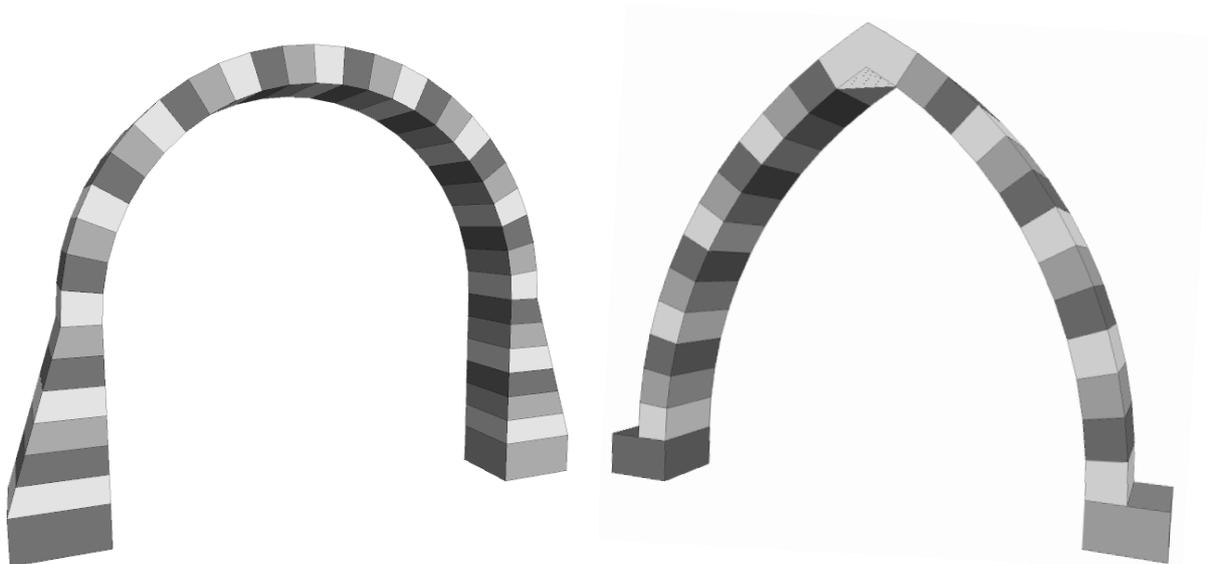
With the given geometry the Gothic arch was fully equilibrated, but the Stilted arch collapsed. For creating a well-comparable model the Romanesque arch itself was not modified, but the stilting walls of the vault were thickened along a linear function. The modified model is

presented on *Figure 3*. This way the given height and span of the arch could have been provided, and the wall thickness of the arch could have been also left untouched.



**Figure 3: Modified geometry for the stilted arch**

With the help of Matlab the models could have been built in 3DEC as shown on *Figure 4*.



**Figure 4: 3DEC model of the arches**

### 3.2. Material model of the blocks

In both models blocks of rigid elements were applied with the material properties of limestone referring to the *Assignment*. Historical architecture of cathedrals and other structures consisting of vaults used limestone commonly, therefore it is a suitable material for the task. First an average value for the density of limestone is  $2500 \text{ kg/m}^3$ , it has been used for solving the problem. However, with the given parameters the self-weight of the two arches differs, which can be resolved by defining distinct density values. The structure was initially drawn in

AutoCAD 2015 Student. The areas of the objects were easy to measure from the software and thus the self-weight could have been calculated:

	In-plane area (m <sup>2</sup> )	Volume (m <sup>3</sup> )	Density kg/m <sup>3</sup>	Self-weight kN
Stilted arch	5.577	5.02	<b>2230</b>	112
Gothic arch	4.448	4.00	<b>2700</b>	112

Therefore in the case of the stilted arch 2230 kg/m<sup>3</sup>, in the case of the Gothic arch 2700 kg/m<sup>3</sup> density has been defined, which are both valid values for regular sandstone according to *Reference* [1].

Blocks have been divided to deformable elements with edge size of 355 mm, Young modulus of 10<sup>9</sup> N/m<sup>2</sup> and Poisson’s ratio of 0.2. The definition of the size of elements was governed by the limits of the trial version of 3DEC software used in the task, allowing a sum of 40 blocks per model including the supporting members.

### 3.3. Contact properties

Contacts are rigid, tensionless connections with high friction values originating from the Coulomb theory. The shear stiffness (jks) and normal stiffness (jkn) are defined 10<sup>12</sup> Pa/m. According to the *Assignment*, a friction angle of 40 ° was used. The schematic *Figure 5* shows the meaning and notation of the two stiffnesses. In the *Assignment* having defined dry masonry, tension and cohesion are neglected.

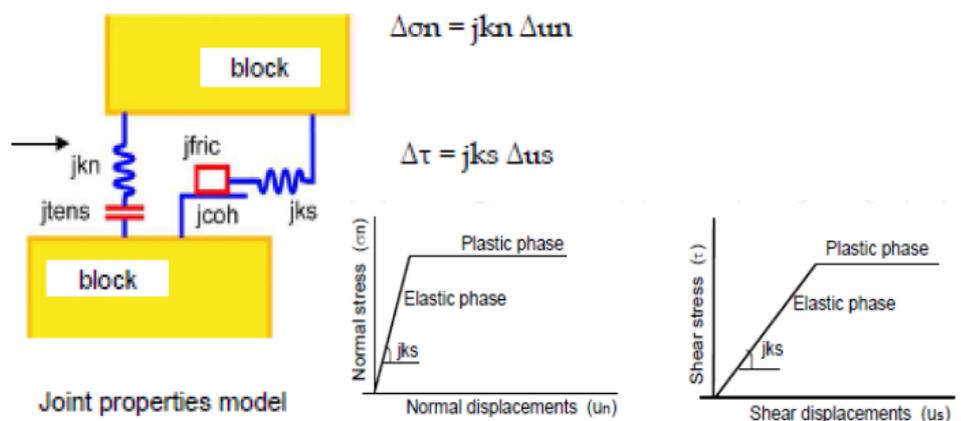


Figure 5: Contact properties in 3DEC

### 3.4. Boundary conditions, loading

Supports have been defined at the bottom of the arches as two frictional foundation blocks fixed against any translation. These block have the sizes 40x50x90 cm.

Only self-weight was added to the model, by applying gravity to the whole structure with 9.81 m/s<sup>2</sup> vertically, in  $-y$  direction.

## 4. Results

### 4.1. Collapse of the stilted arch

As mentioned in 3.1. *Geometry*, the geometry prescribed in the *Assignment* does not provide sufficient reaction forces for the stilted vault to reach a stable equilibrium. The self-weight cannot be equilibrated, displacements grow and the vault collapses. The growing vertical displacement of the topmost point of the arch and a spectacular view of the collapse can be observed at step number 700001 of the iteration on *Figure 6*. This was the reason why a modified geometry (shown on *Figure 3* was applied for the stilted arch).

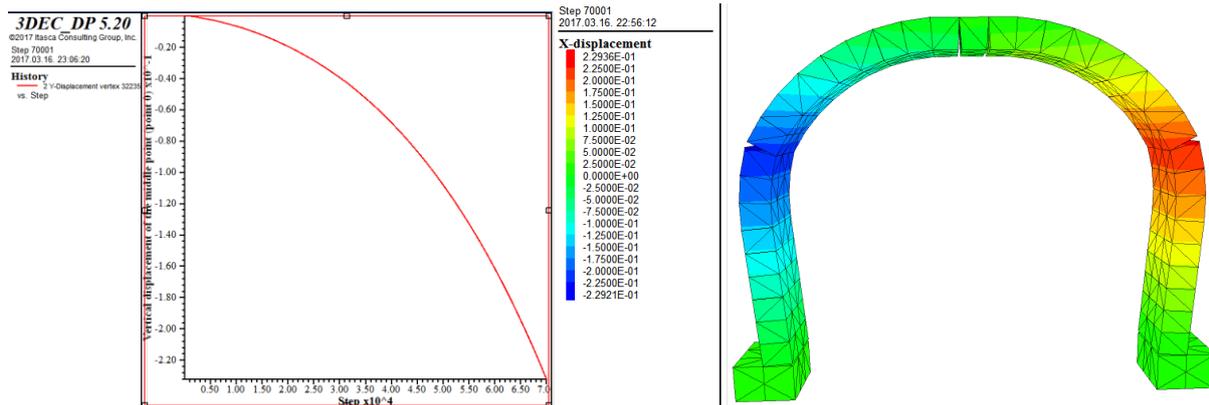


Figure 6: Horizontal displacements of the collapsing stilted arch

### 4.2. Equilibrium

The equilibrated state was reached much quicker in the model of the Gothic arch requiring around 10 000 steps. In the case of the modified stilted arch the convergence was much slower requiring nearly 35 000 steps of iteration. *Figure 7* compare the unbalanced force at the two cases.

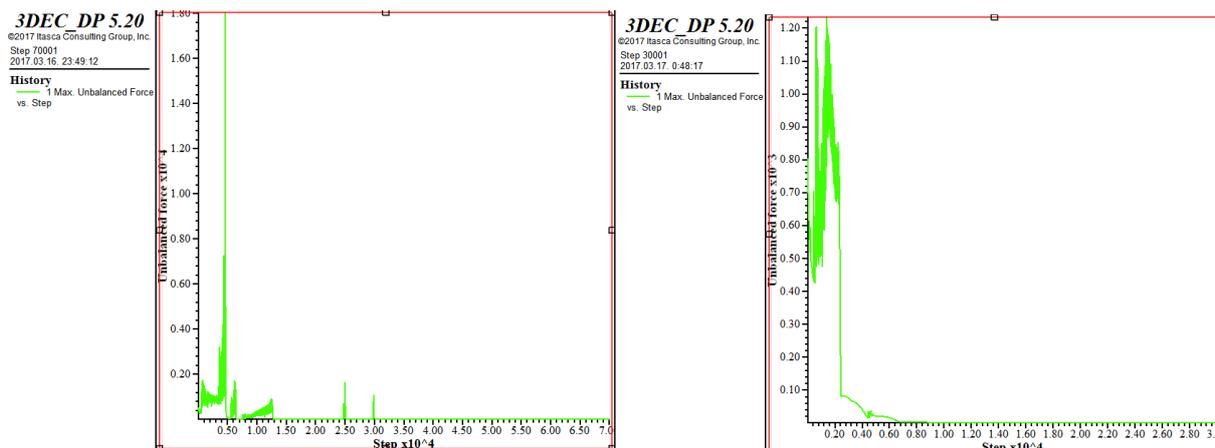


Figure 7: Unbalanced force iteration (left: Stilted, right: Gothic)

### 4.3. Displacements

Both arches reached equilibrium and suffered minor displacements, the largest of which was 1.4 mm (vertical deflection of the topmost point of the stilted arch). Maximum displacements are concluded in the following table, *Figure 8* provides a visual explanation. It is clear from the difference percentage that the Gothic arch suffers less deflection in both directions.

	Stilted arch	Gothic arch	Difference
Max. vertical displacement (m)	$1.400 \cdot 10^{-3}$	$6.379 \cdot 10^{-4}$	54.4 %
Max. horizontal displacement (m)	$8.169 \cdot 10^{-4}$	$2.033 \cdot 10^{-4}$	75.1 %

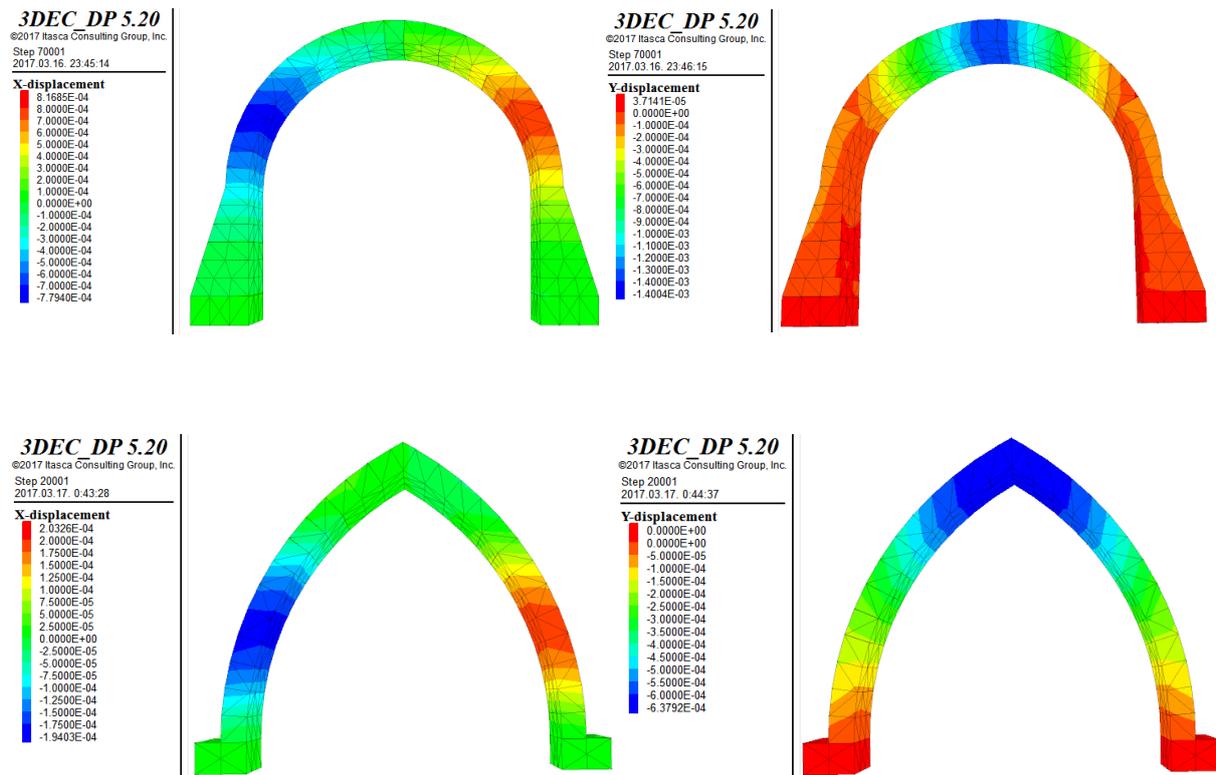


Figure 8: X- and y-displacements of the stilted (upper) and the Gothic (lower) arch

### 4.4. Contact forces, stress trajectories

Unlike to the displacements, the contact forces appear to be slightly larger at the Gothic arch. Although the self-weight of the arches is approximately equal, the model is not entirely correct. From one point, the alteration of the stilted arch affected only the vertical stiling wall, resulting no increase in the horizontal action. From the other hand, the larger standing surface of the stilted arch provides a larger surface for distribution of the contact forces at the feet of the arch. The data are summarized in the following table.

	Stilted arch	Gothic arch	Difference
Max. normal cont. force (N)	6117.7	8415.4	27.3 %
Max. shear cont. force (N)	1624.7	1775.5	8.5 %

The maximal contact forces (arising at the bottoms of the arches' center lines) are in close relation with the reactions at the supporting blocks. Another interesting task would be the calculation of the horizontal and vertical reaction forces at the supports.

Figure 9 shows the normal and shear distributed contact forces in the two arches.

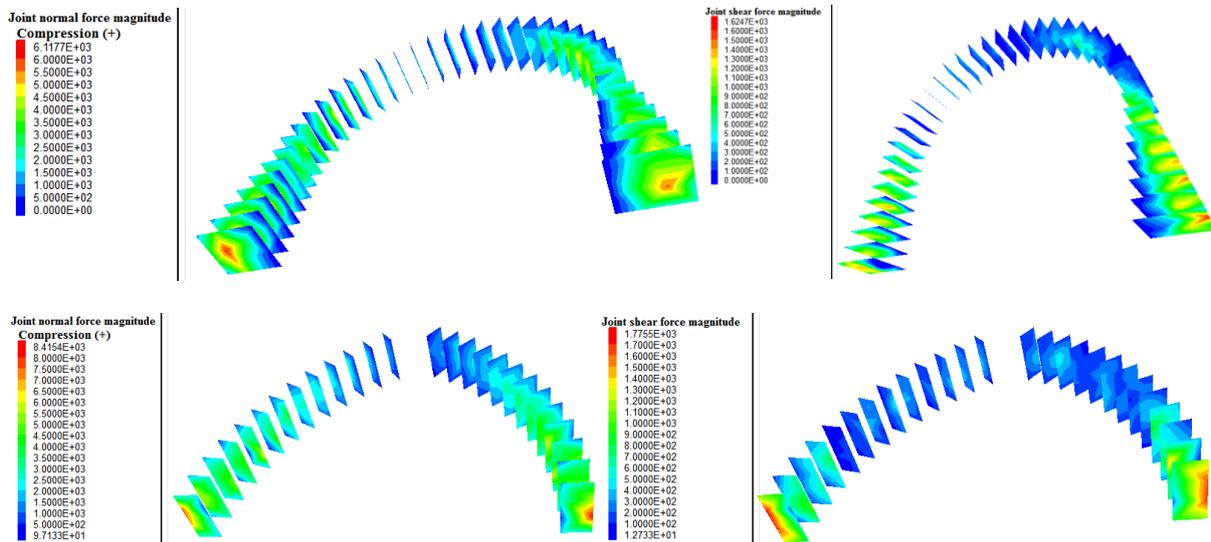


Figure 9: Normal (left) and shear (right) distributed contact forces of the stilted (upper) and the Gothic (lower) arch

The contact forces have been analyzed at the feet of the arches, in the region of the maximum values. In the following (Figure 10, Figure 11, Figure 12, Figure 13), the contact force – time history diagrams are collected for the central point of the arches on each side.

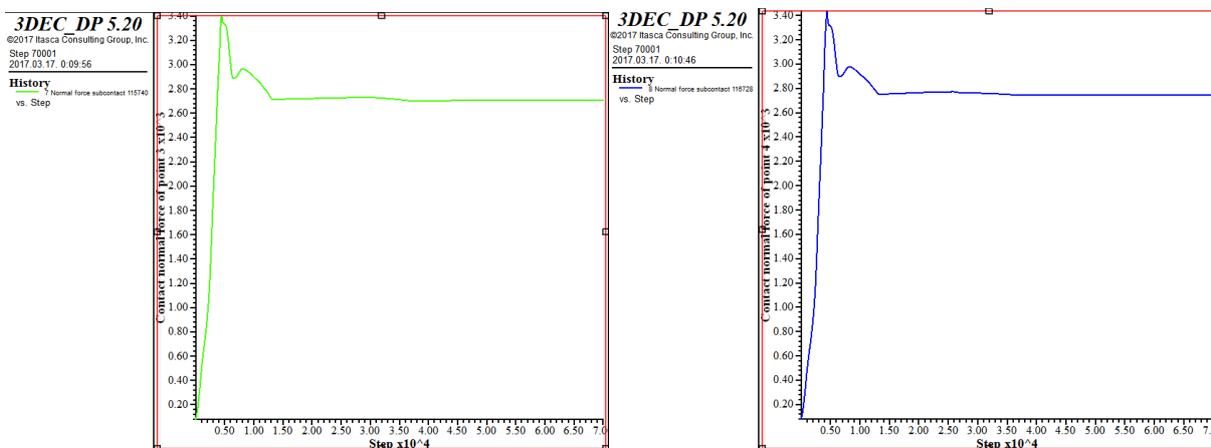


Figure 10: Contact normal force at the bottom left (left) and right (right) midpoint of the stilted arch

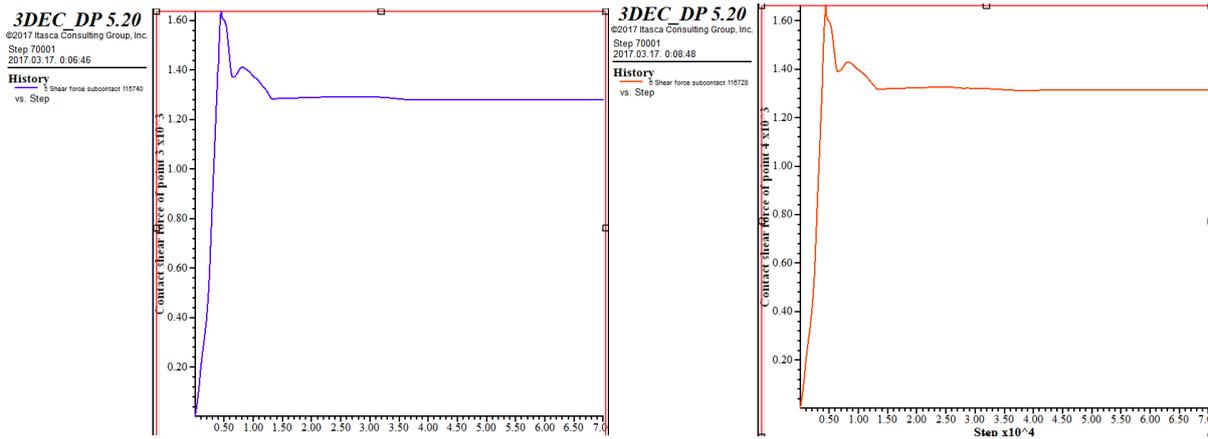


Figure 11: Contact shear force at the bottom left and right midpoint of the stilted arch

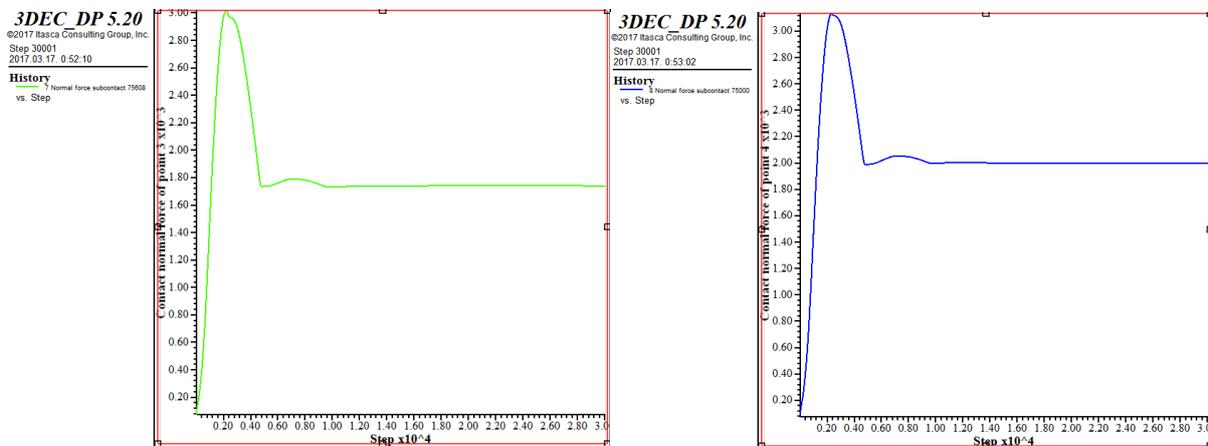


Figure 12: Contact normal force of the bottom left and right midpoints of the Gothic arch

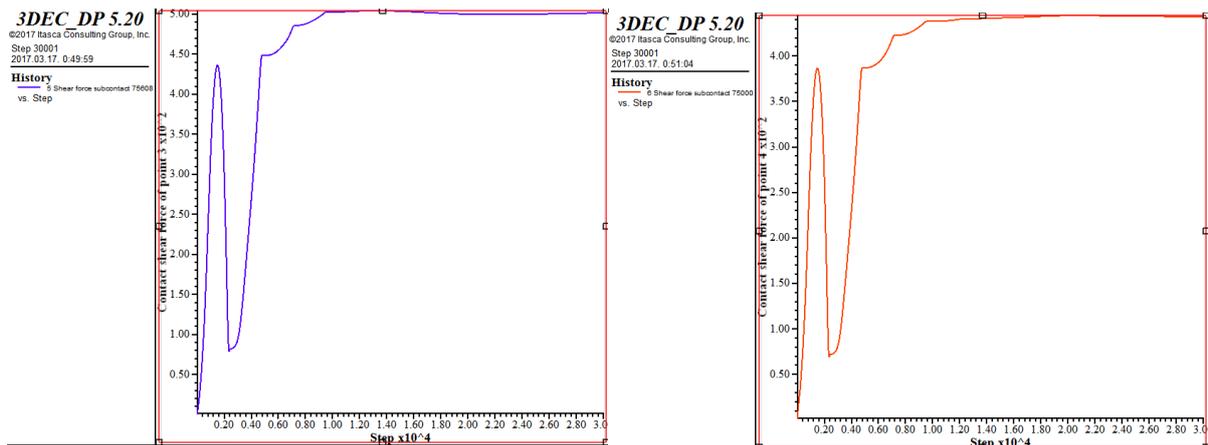
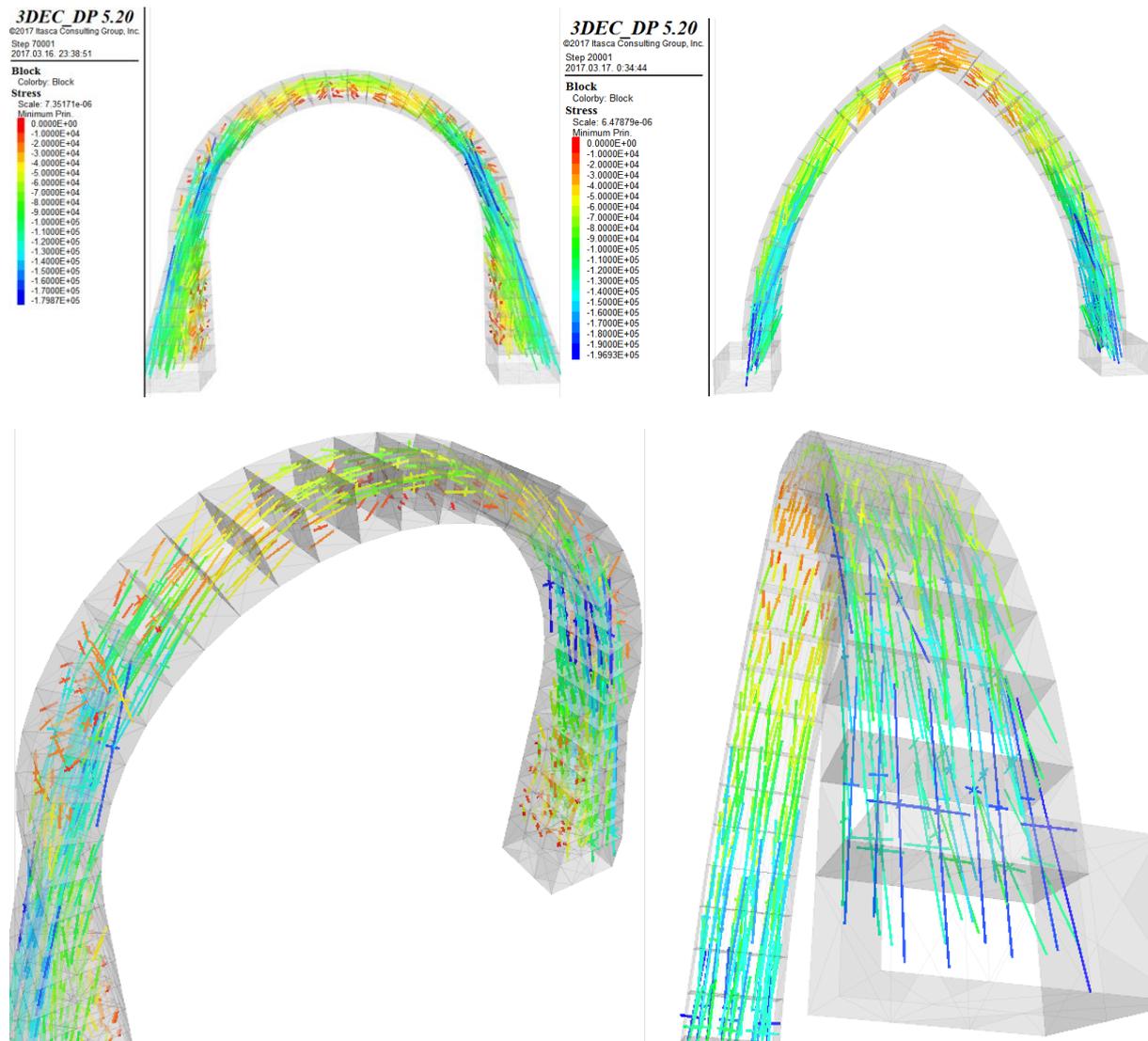


Figure 13: Contact shear force of the bottom left and right midpoint of the Gothic arch

The normal stress trajectories of the arches show the stress flow within the structure giving a very suggestive view of the static behavior. *Figure 14* compares the trajectories of the Stilted and the Gothic arch. For clear view a closer aspect has also been generated.



**Figure 14: Stress trajectory lines of the stilted (left) and the Gothic (right) arch**

Interesting to see how close the trajectory lines are to the breaking point of the stilted arch at the connection to the wall. It means a nearly total utilization of the section therefore the smallest possible geometry has been found. It is also visible that at some parts of the arch the defined width of the section is redundant for the loading of self-weight.

From the trajectories we can also see that the stilted arch as an architectural element is not efficient, unless a sequence of arches are put side by side supporting each other. Either a connecting material is needed with load-bearing capacity for bending moment, or sufficient width of the wall is necessary to provide a hidden continuation of the arch it is bearing.

Historical structures containing vaults frequently face the damage of support displacement. In such cases the displacement might reach a measure when the trajectory line leaves the contour line of the material. Then the force disbands and the structure suffers a partial or an absolute collapse. It might happen due to earth movement, roots of trees, a nearby civil engineering project or any major effect on the structure. It is a reason to design arches with adequate safety in the section size and to use tensional supports where necessary.

## 5. Conclusion

From the calculations it is clearly visible that the Gothic arch is more suitable for covering a higher opening. The stilted arch did not bear its self-weight with the given parameters, so its geometry had to be altered. After the thickening of its vertical wall the equilibrium was reached. As expected, the Romanesque-type arch suffers greater deflections than the pointed vault. From the contact force data it is derivable, that the Gothic arch has a better behavior for horizontal reaction forces (and shear contact forces). The stress trajectories showed that the stilted arch in reality is an arch with a support having a skew stress flow, the reason why the thin, 40 cm wall thickness was not sufficient for bearing the horizontal action. As conclusion it is declarable that the Gothic arch spread quickly in the medieval ages because it is more efficient, large inner spaces can be created and light interiors can be built.

## 6. References

- [1] Zsuzsa Borbála PAP: DEM comparison of the horizontal reaction of Romanesque and Gothic arches, BME – Department of Structural Mechanics, 2014.
- [2] [http://geology.about.com/cs/rock\\_types/a/aarockspeggrav.htm](http://geology.about.com/cs/rock_types/a/aarockspeggrav.htm), Internet, 2017.
- [3] [http://www.cathedralquest.com/romanesque\\_architecture.htm](http://www.cathedralquest.com/romanesque_architecture.htm), Internet, 2017.
- [4] [http://www.cathedralquest.com/gothic\\_architecture.htm](http://www.cathedralquest.com/gothic_architecture.htm), Internet, 2017.
- [5] 3DEC - 3 Dimensional Distinct Element Code - Online Manual, Internet, 2017.

## 7. Reference of images

Figure 1 Gothic arch, Milan:

<https://us.123rf.com/400wm/400/400/sedmak/sedmak1208/sedmak120800224/15016759-milano--roseton-gotico-de-la-catedral-del-duomo.jpg>, Internet, 2017.

Figure 1 Stilted Romanesque arch, Spain:

[https://upload.wikimedia.org/wikipedia/commons/thumb/1/11/Avila\\_San\\_Pedro\\_View.jpg/440px-Avila\\_San\\_Pedro\\_View.jpg](https://upload.wikimedia.org/wikipedia/commons/thumb/1/11/Avila_San_Pedro_View.jpg/440px-Avila_San_Pedro_View.jpg), Internet, 2017.

## APPENDIX A – MATLAB CODES

## 1) Stilted arch

```

% The initial instructions clear the screen, delete all of the existing variables, and close any figure
clc; clear; close all
% This code draws two concentric semi-circle and divide it into 26 parts, so it gives the coordinates of
the 25 blocks of the Romanesque arch.
% First it generates a 26-element vector, the angle is pi, so we will have a half circle
angle = linspace(0, pi, 26);
% After this it creates another vector with the value of the outer radius
r2 = linspace(2.9, 2.9, 26);
% We plot the circle and the chosen 26 points with little red circles (ro)
polar(angle, r2, 'ro')
% We name the figure 'Stilted arch'
title('Stilted arch')
% In the following we wouldn't like to delete this figure, so we have to keep it with the hold on
command
hold on
% Now, we create another 26-element vector for the inner radius
r1 = linspace(2.5, 2.5, 26);
% And we also plot this circle and the chosen 26 points with little blue x-es (bx)
polar(angle, r1, 'bx')
% Finally, we mark the center of the circles with a black square (ks)
polar(0, 0, 'ks')

```

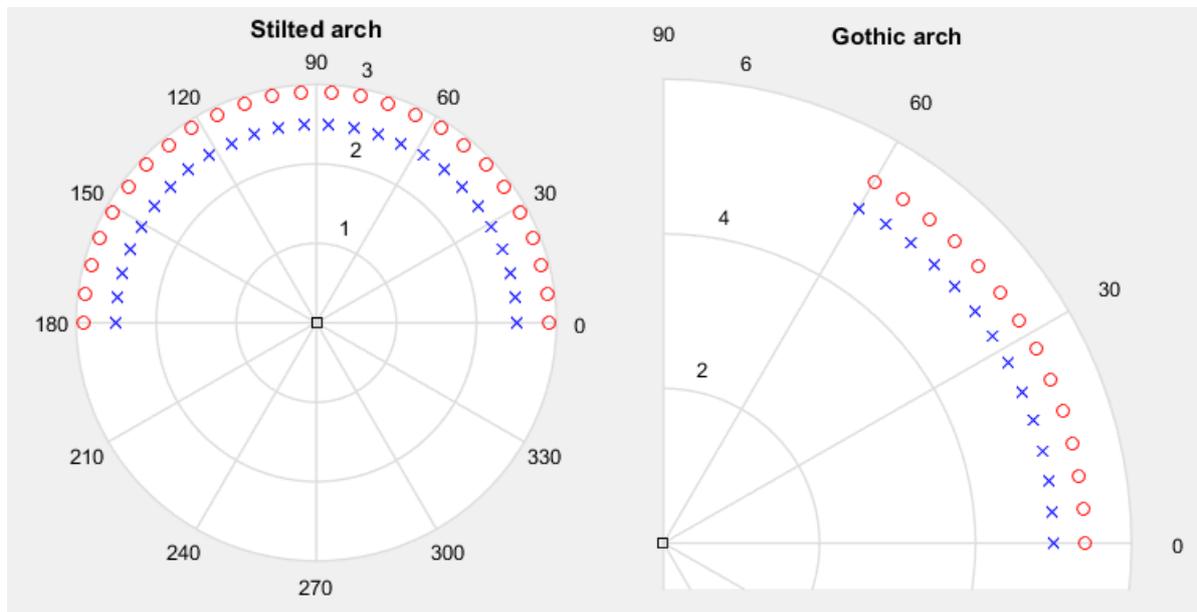
## 2) Gothic arch

```

% The initial instructions clear the screen, delete all of the existing variables, and close any figure
clc; clear; close all
% This code draws two concentric section of circle with 60° angle and divide it into 14 parts, so it
gives the coordinates of half of the blocks of the Gothic arch.
% First it generates a 14-element vector, the angle is pi/3, so we will have a 60° angle section of circle
angle = linspace(0, pi/3, 14);
% Then, we create another 14-element vector containing only the outer radius
r2 = linspace(5.4, 5.4, 14);
% We plot the circle and the chosen 26 points with little red circles (ro)
polar(angle, r2, 'ro')
% We name the figure 'Gothic arch'
title('Gothic arch')
% We avoid the deletion of this figure
hold on
% Now, we create another 10-element vector for the inner radius
r1 = linspace(5, 5, 14);
% And we also plot this circle and the chosen 26 points with little blue x-es (bx)
polar(angle, r1, 'bx')
% Finally, we mark the center with a black square (ks)
polar(0, 0, 'ks')

```

3) Matlab plots



APPENDIX B – 3DEC CODES

1) Stilted arch

```

; STILTED ARCH WIDENED STILTING WALL
; Always start *3ddat file with the new command
new
; Define the geometry with command prism, from the coordinates given by Matlab
; The prism command: The two parallel faces of the prism are defined by an arbitrary number of
vertices. The opposing vertices on each face are then automatically connected to form the prism. The
first face (face a) is defined by vertices entered in either a clockwise or counterclockwise order. The
opposite face (face b) must have its vertices entered in the same order as the corresponding vertices for
face a. Faces a and b must be planar and convex.
polyhedron prism a 6.400,0.000,-0.450 6.300,0.315,-0.450 5.400,0.315,-0.450 5.400,0.000,-0.450 b
6.400,0.000,0.450 6.300,0.315,0.450 5.400,0.315,0.450 5.400,0.000,0.450
polyhedron prism a 6.300,0.315,-0.450 6.200,0.630,-0.450 5.400,0.630,-0.450 5.400,0.315,-0.450 b
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polyhedron prism a 6.200,0.630,-0.450 6.100,0.945,-0.450 5.400,0.945,-0.450 5.400,0.630,-0.450 b
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6.100,0.945,0.450 6.000,1.260,0.450 5.400,1.260,0.450 5.400,0.945,0.450
polyhedron prism a 6.000,1.260,-0.450 5.900,1.575,-0.450 5.400,1.575,-0.450 5.400,1.260,-0.450 b
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 polyhedron prism a 2.004,4.648,-0.450 1.665,4.514,-0.450 1.836,4.152,-0.450 2.127,4.268,-0.450 b  
 2.004,4.648,0.450 1.665,4.514,0.450 1.836,4.152,0.450 2.127,4.268,0.450  
 polyhedron prism a 1.665,4.514,-0.450 1.346,4.339,-0.450 1.560,4.001,-0.450 1.836,4.152,-0.450 b  
 1.665,4.514,0.450 1.346,4.339,0.450 1.560,4.001,0.450 1.836,4.152,0.450  
 polyhedron prism a 1.346,4.339,-0.450 1.051,4.124,-0.450 1.306,3.816,-0.450 1.560,4.001,-0.450 b  
 1.346,4.339,0.450 1.051,4.124,0.450 1.306,3.816,0.450 1.560,4.001,0.450  
 polyhedron prism a 1.051,4.124,-0.450 0.786,3.875,-0.450 1.078,3.601,-0.450 1.306,3.816,-0.450 b  
 1.051,4.124,0.450 0.786,3.875,0.450 1.078,3.601,0.450 1.306,3.816,0.450  
 polyhedron prism a 0.786,3.875,-0.450 0.554,3.595,-0.450 0.877,3.359,-0.450 1.078,3.601,-0.450 b  
 0.786,3.875,0.450 0.554,3.595,0.450 0.877,3.359,0.450 1.078,3.601,0.450  
 polyhedron prism a 0.554,3.595,-0.450 0.359,3.287,-0.450 0.709,3.094,-0.450 0.877,3.359,-0.450 b  
 0.554,3.595,0.450 0.359,3.287,0.450 0.709,3.094,0.450 0.877,3.359,0.450  
 polyhedron prism a 0.359,3.287,-0.450 0.204,2.958,-0.450 0.576,2.810,-0.450 0.709,3.094,-0.450 b  
 0.359,3.287,0.450 0.204,2.958,0.450 0.576,2.810,0.450 0.709,3.094,0.450  
 polyhedron prism a 0.204,2.958,-0.450 0.091,2.611,-0.450 0.479,2.512,-0.450 0.576,2.810,-0.450 b  
 0.204,2.958,0.450 0.091,2.611,0.450 0.479,2.512,0.450 0.576,2.810,0.450  
 polyhedron prism a 0.091,2.611,-0.450 0.023,2.253,-0.450 0.420,2.203,-0.450 0.479,2.512,-0.450 b  
 0.091,2.611,0.450 0.023,2.253,0.450 0.420,2.203,0.450 0.479,2.512,0.450  
 polyhedron prism a 0.023,2.253,-0.450 0.000,1.890,-0.450 0.400,1.890,-0.450 0.420,2.203,-0.450 b  
 0.023,2.253,0.450 0.000,1.890,0.450 0.400,1.890,0.450 0.420,2.203,0.450  
 polyhedron prism a 0.000,1.890,-0.450 -0.100,1.575,-0.450 0.400,1.575,-0.450 0.400,1.890,-0.450 b  
 0.000,1.890,0.450 -0.100,1.575,0.450 0.400,1.575,0.450 0.400,1.890,0.450  
 polyhedron prism a -0.100,1.575,-0.450 -0.200,1.260,-0.450 0.400,1.260,-0.450 0.400,1.575,-0.450 b -  
 0.100,1.575,0.450 -0.200,1.260,0.450 0.400,1.260,0.450 0.400,1.575,0.450  
 polyhedron prism a -0.200,1.260,-0.450 -0.300,0.945,-0.450 0.400,0.945,-0.450 0.400,1.260,-0.450 b -  
 0.200,1.260,0.450 -0.300,0.945,0.450 0.400,0.945,0.450 0.400,1.260,0.450

```

polyhedron prism a -0.300,0.945,-0.450 -0.400,0.630,-0.450 0.400,0.630,-0.450 0.400,0.945,-0.450 b -
0.300,0.945,0.450 -0.400,0.630,0.450 0.400,0.630,0.450 0.400,0.945,0.450
polyhedron prism a -0.400,0.630,-0.450 -0.500,0.315,-0.450 0.400,0.315,-0.450 0.400,0.630,-0.450 b -
0.400,0.630,0.450 -0.500,0.315,0.450 0.400,0.315,0.450 0.400,0.630,0.450
polyhedron prism a -0.500,0.315,-0.450 -0.600,0.000,-0.450 0.400,0.000,-0.450 0.400,0.315,-0.450 b -
0.500,0.315,0.450 -0.600,0.000,0.450 0.400,0.000,0.450 0.400,0.315,0.450
; Define the support blocks with the same command
polyhedron prism a 6.400,-0.500,-0.450 6.400,0.000,-0.450 5.400,0.000,-0.450 5.400,-0.500,-0.450 b
6.400,-0.500,0.450 6.400,0.000,0.450 5.400,0.000,0.450 5.400,-0.500,0.450
polyhedron prism a -0.600,0.000,-0.450 -0.600,-0.500,-0.450 0.400,-0.500,-0.450 0.400,0.000,-0.450 b
-0.600,0.000,0.450 -0.600,-0.500,0.450 0.400,-0.500,0.450 0.400,0.000,0.450
; Create ranges (if we name ranges, we can use them in other commands, so it will be much easier to
handle these parts)
range name arch x=(-10.0,10.0) y=(-10.0,10.0) z=(0.0,10.0)
range name supportblock1 x=(-0.6,0.4) y=(-0.5,0.0) z=(-0.45,0.45)
range name supportblock2 x=(5.4,6.4) y=(-0.5,0.0) z=(-0.45,0.45)
; Fix the support blocks
fix range supportblock1
fix range supportblock2
; Define the material properties of the blocks and joints
gen edge 0.355
prop mat=1 dens=2230.0 ymod 1e9 prat 0.2
prop jmat=1 jkn 1e12 jks 1e12 jfri 40.0
; Apply gravity (here the y axis is the vertical one)
gravity 0.0,-9.81,0.0
; Listing the change of the unbalanced forces during the calculation: before we run the program we
need to store these values
hist unbal id=1
; Listing the change of displacements during the calculation, we store the y (vertical) displacement of
point 0, and the x (horizontal) displacements of point 1 and point2 (see the position of these points in
Figure 8 and Figure 9.
hist ydisp (2.900,4.790,0.0) id=2
hist xdisp (0.786,3.875,0.0) id=3
hist xdisp (5.014,3.875,0.0) id=4
; Before we store the contact shear forces we need to start the calculation
cycle 1
; After this we can store the shear force and normal force of point3 and point4 with hist command,
point3 and point 4 are in the middle of the contacts.
hist sforce (-0.600,0.00,0.0) id=5
hist sforce (6.400,0.00,0.0) id=6
hist nforce (-0.600,0.00,0.0) id=7
hist nforce (6.400,0.00,0.0) id=8
; Calculation
cycle 70000
; Plot the unbalanced force, y displacement in the defined point (middle), and x displacements in the
defined points and the contact shear and normal forces
plot hist 1 yaxis label 'Unbalanced force'
plot hist 2 yaxis label 'Vertical displacement of the middle point (point 0)'
plot hist 3 yaxis label 'Horizontal displacement of point 1'
plot hist 4 yaxis label 'Horizontal displacement of point 2'
; Following two plots are the HORIZONTAL REACTION FORCES
plot hist 5 yaxis label 'Contact shear force of point 3'
plot hist 6 yaxis label 'Contact shear force of point 4'
plot hist 7 yaxis label 'Contact normal force of point 3'
plot hist 8 yaxis label 'Contact normal force of point 4'

```

```

; Contour plot the y and x displacements and the distribution of contact forces
plot contour ydisp above au
plot contour xdisp above au
plot jointcontour sforce
plot jointcontour nforce
plot block color white
list contact stress

```

## 2) Gothic arch

```

; GOTHIC ARCH
; Start a new 3DEC file
new
; Define the geometry
polyhedron prism a 5.400,0.000,-0.450 5.382,0.435,-0.450 4.984,0.402,-0.450 5.000,0.000,-0.450 b
5.400,0.000,0.450 5.382,0.435,0.450 4.984,0.402,0.450 5.000,0.000,0.450
polyhedron prism a 5.382,0.435,-0.450 5.330,0.866,-0.450 4.935,0.802,-0.450 4.984,0.402,-0.450 b
5.382,0.435,0.450 5.330,0.866,0.450 4.935,0.802,0.450 4.984,0.402,0.450
polyhedron prism a 5.330,0.866,-0.450 5.243,1.292,-0.450 4.855,1.197,-0.450 4.935,0.802,-0.450 b
5.330,0.866,0.450 5.243,1.292,0.450 4.855,1.197,0.450 4.935,0.802,0.450
polyhedron prism a 5.243,1.292,-0.450 5.122,1.710,-0.450 4.743,1.583,-0.450 4.855,1.197,-0.450 b
5.243,1.292,0.450 5.122,1.710,0.450 4.743,1.583,0.450 4.855,1.197,0.450
polyhedron prism a 5.122,1.710,-0.450 4.968,2.117,-0.450 4.600,1.960,-0.450 4.743,1.583,-0.450 b
5.122,1.710,0.450 4.968,2.117,0.450 4.600,1.960,0.450 4.743,1.583,0.450
polyhedron prism a 4.968,2.117,-0.450 4.781,2.510,-0.450 4.427,2.324,-0.450 4.600,1.960,-0.450 b
4.968,2.117,0.450 4.781,2.510,0.450 4.427,2.324,0.450 4.600,1.960,0.450
polyhedron prism a 4.781,2.510,-0.450 4.564,2.886,-0.450 4.226,2.672,-0.450 4.427,2.324,-0.450 b
4.781,2.510,0.450 4.564,2.886,0.450 4.226,2.672,0.450 4.427,2.324,0.450
polyhedron prism a 4.564,2.886,-0.450 4.317,3.244,-0.450 3.997,3.004,-0.450 4.226,2.672,-0.450 b
4.564,2.886,0.450 4.317,3.244,0.450 3.997,3.004,0.450 4.226,2.672,0.450
polyhedron prism a 4.317,3.244,-0.450 4.042,3.581,-0.450 3.743,3.316,-0.450 3.997,3.004,-0.450 b
4.317,3.244,0.450 4.042,3.581,0.450 3.743,3.316,0.450 3.997,3.004,0.450
polyhedron prism a 4.042,3.581,-0.450 3.741,3.894,-0.450 3.464,3.606,-0.450 3.743,3.316,-0.450 b
4.042,3.581,0.450 3.741,3.894,0.450 3.464,3.606,0.450 3.743,3.316,0.450
polyhedron prism a 3.741,3.894,-0.450 3.415,4.183,-0.450 3.162,3.873,-0.450 3.464,3.606,-0.450 b
3.741,3.894,0.450 3.415,4.183,0.450 3.162,3.873,0.450 3.464,3.606,0.450
polyhedron prism a 3.415,4.183,-0.450 3.068,4.444,-0.450 2.840,4.115,-0.450 3.162,3.873,-0.450 b
3.415,4.183,0.450 3.068,4.444,0.450 2.840,4.115,0.450 3.162,3.873,0.450
;TOP BLOCK OF THE ARCH:
polyhedron prism a 3.068,4.444,-0.450 2.700,4.677,-0.450 2.500,4.790,-0.450 2.300,4.677,-0.450
1.932,4.444,-0.450 2.160,4.115,-0.450 2.500,4.330,-0.450 2.840,4.115,-0.450 b 3.068,4.444,0.450
2.700,4.677,0.450 2.500,4.790,0.450 2.300,4.677,0.450 1.932,4.444,0.450 2.160,4.115,0.450
2.500,4.330,0.450 2.840,4.115,0.450
polyhedron prism a 1.932,4.444,-0.450 1.585,4.183,-0.450 1.838,3.873,-0.450 2.160,4.115,-0.450 b
1.932,4.444,0.450 1.585,4.183,0.450 1.838,3.873,0.450 2.160,4.115,0.450
polyhedron prism a 1.585,4.183,-0.450 1.259,3.894,-0.450 1.536,3.606,-0.450 1.838,3.873,-0.450 b
1.585,4.183,0.450 1.259,3.894,0.450 1.536,3.606,0.450 1.838,3.873,0.450
polyhedron prism a 1.259,3.894,-0.450 0.958,3.581,-0.450 1.257,3.316,-0.450 1.536,3.606,-0.450 b
1.259,3.894,0.450 0.958,3.581,0.450 1.257,3.316,0.450 1.536,3.606,0.450
polyhedron prism a 0.958,3.581,-0.450 0.683,3.244,-0.450 1.003,3.004,-0.450 1.257,3.316,-0.450 b
0.958,3.581,0.450 0.683,3.244,0.450 1.003,3.004,0.450 1.257,3.316,0.450
polyhedron prism a 0.683,3.244,-0.450 0.436,2.886,-0.450 0.774,2.672,-0.450 1.003,3.004,-0.450 b
0.683,3.244,0.450 0.436,2.886,0.450 0.774,2.672,0.450 1.003,3.004,0.450

```

```

polyhedron prism a 0.436,2.886,-0.450 0.219,2.510,-0.450 0.573,2.324,-0.450 0.774,2.672,-0.450 b
0.436,2.886,0.450 0.219,2.510,0.450 0.573,2.324,0.450 0.774,2.672,0.450
polyhedron prism a 0.219,2.510,-0.450 0.032,2.117,-0.450 0.400,1.960,-0.450 0.573,2.324,-0.450 b
0.219,2.510,0.450 0.032,2.117,0.450 0.400,1.960,0.450 0.573,2.324,0.450
polyhedron prism a 0.032,2.117,-0.450 -0.122,1.710,-0.450 0.257,1.583,-0.450 0.400,1.960,-0.450 b
0.032,2.117,0.450 -0.122,1.710,0.450 0.257,1.583,0.450 0.400,1.960,0.450
polyhedron prism a -0.122,1.710,-0.450 -0.243,1.292,-0.450 0.145,1.197,-0.450 0.257,1.583,-0.450 b -
0.122,1.710,0.450 -0.243,1.292,0.450 0.145,1.197,0.450 0.257,1.583,0.450
polyhedron prism a -0.243,1.292,-0.450 -0.330,0.866,-0.450 0.065,0.802,-0.450 0.145,1.197,-0.450 b -
0.243,1.292,0.450 -0.330,0.866,0.450 0.065,0.802,0.450 0.145,1.197,0.450
polyhedron prism a -0.330,0.866,-0.450 -0.382,0.435,-0.450 0.016,0.402,-0.450 0.065,0.802,-0.450 b -
0.330,0.866,0.450 -0.382,0.435,0.450 0.016,0.402,0.450 0.065,0.802,0.450
polyhedron prism a -0.382,0.435,-0.450 -0.400,0.000,-0.450 0.000,0.000,-0.450 0.016,0.402,-0.450 b -
0.382,0.435,0.450 -0.400,0.000,0.450 0.000,0.000,0.450 0.016,0.402,0.450
; Create a range from the upper blocks, because we have to glue, merge these two blocks together to
create the keystone (capstone).
range name upperblocks x=(0.853,1.382) y=(1.60,1.99) z=(0.0,1.0)
;Glue the upperblocks range with join command, now these two blocks are becoming one.
join on range upperblocks
; Define the support blocks
polyhedron prism a -0.800,0.000,-0.450 -0.800,-0.500,-0.450 0.000,-0.500,-0.450 0.000,0.000,-0.450 b
-0.800,0.000,0.450 -0.800,-0.500,0.450 0.000,-0.500,0.450 0.000,0.000,0.450
polyhedron prism a 5.800,-0.500,-0.450 5.800,0.000,-0.450 5.000,0.000,-0.450 5.000,-0.500,-0.450 b
5.800,-0.500,0.450 5.800,0.000,0.450 5.000,0.000,0.450 5.000,-0.500,0.450
; Create further ranges
range name arch x=(-10.0,10.0) y=(0.0,10.0) z=(-10.0,10.0)
range name supportblock1 x=(-0.800,0.000) y=(-0.5,0.0) z=(0.0,0.9)
range name supportblock2 x=(5.000,5.800) y=(-0.5,0.0) z=(0.0,0.9)
; Fix the support blocks
fix range supportblock1
fix range supportblock2
; Apply material models for the blocks and contacts
gen edge 0.355
prop mat=1 dens=2700 ymod 1e9 prat 0.2
prop jmat=1 jkn 1e12 jks 1e12 jfri 40
; Apply the gravity
gravity 0,-9.81,0
; Store the change of the unbalanced forces during the calculation
hist unbal id=1
; Store the change of displacements during the calculation, we store the y (vertical) displacement of
point 0, and the x (horizontal) displacements of point 1 and point2 (see the position of these points in
Figure 11 and Figure 12.
hist ydisp (2.500,4.790,0.0) id=2
hist xdisp (0.219,2.510,0.0) id=3
hist xdisp (4.781,2.510,0.0) id=4
; Before we store the contact shear forces we need to start the calculation
cycle 1
; After this we can store the shear force and normal force of point3 and point4 with hist command,
point3 and point 4 are in the middle of the contacts.
hist sforce (-0.200,0.00,0.0) id=5
hist sforce (5.200,0.00,0.0) id=6
hist nforce (-0.200,0.00,0.0) id=7
hist nforce (5.200,0.00,0.0) id=8
; Calculation
cycle 30000

```

```
; Plot the unbalanced force, y displacement in the defined point (middle), and x displacements in the
defined points and the contact shear and normal forces
plot hist 1 yaxis label 'Unbalanced force'
plot hist 2 yaxis label 'Vertical displacement of the middle point (point 0)'
plot hist 3 yaxis label 'Horizontal displacement of point 1'
plot hist 4 yaxis label 'Horizontal displacement of point 2'
; Following two plots are the HORIZONTAL REACTION FORCES
plot hist 5 yaxis label 'Contact shear force of point 3'
plot hist 6 yaxis label 'Contact shear force of point 4'
plot hist 7 yaxis label 'Contact normal force of point 3'
plot hist 8 yaxis label 'Contact normal force of point 4'
; Contour plot the y and x displacements and the distribution of contact forces
plot contour ydisp above au
plot contour xdisp above au
plot jointcontour sforce
plot jointcontour nforce
plot block color white
list contact stress
```

Course Homework for

## Bálint Szedenik

to partially fulfil the requirements of the subject

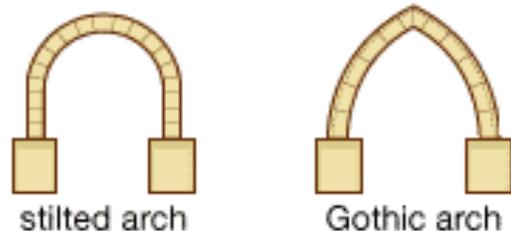
# Numerical Models for Structures

Title:

## Comparison of the Internal Forces in Stilted Arch and Gothic Arch

Short description:

The middle lines of the arches shown on the right consist of different circular curves. Analyse the differences between the internal forces under selfweight!



Details:

- The free span of the arches should equally be 5 m. Set the thickness of both arches to 40 cm. The size perpendicularly to the plane of the arch should be 90 cm. (Take care for the symmetry: the origin of the coordinate system should be located on the planes of symmetry.)
- The two walls in the stilted structure should be as high as necessary to have equal total height with the gothic arch.
- Use deformable blocks in the model, with the properties of a usual limestone. The joints should be dry, with a friction angle  $40^\circ$ .
- Support the structures from below with two frictional foundation blocks fixed against any translation.
- Equilibrate the structures under selfweight.
- Visualize the distributed contact forces and the stress trajectories. Compare!

Submission deadline:

27 March 2017

/Prof. Katalin Bagi/  
lecturer