Surveying I – Practical 11
Fundamental tasks of plane surveying calculations

In this lesson, the two fundamental tasks carried out in plane surveying calculations are described with examples.

I. fundamental task of surveying

We have a point with known easting and northing coordinates (or simply called a known point) in our reference system that we use as a station (point S on Fig. 1). We would like to find the coordinates of unknown point P. To this end, we measured (or calculated) the horizontal distance d_{SP} and calculated the whole circle bearing from our station to the unknown point (WCB$_{SP}$). The WCB of a direction is always measured clockwise from the north direction and can have a value between 0° and 360°.

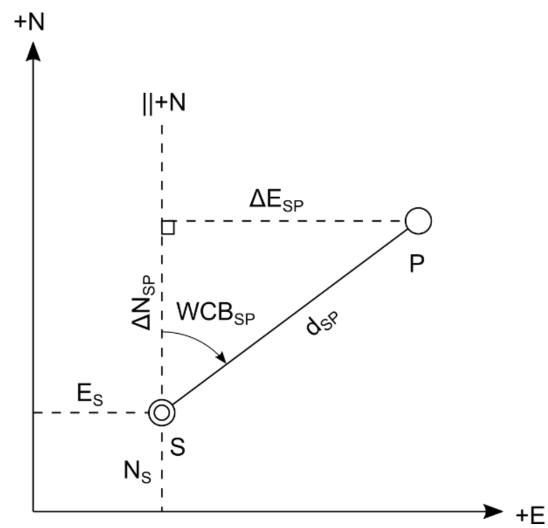

Figure 1. The fundamental tasks of surveying.

We can obtain the coordinates of point P by taking the coordinates of our station and adding the ΔE_{SP}, ΔN_{SP} coordinate differences to them. Using the right triangle in Fig. 1, we can compute the coordinate changes from the horizontal distance and the WCB mentioned above.

1. We first compute the ΔE_{SP} and ΔN_{SP} coordinate differences.

$$\Delta E_{SP} = d_{SP} \cdot \sin(\mathrm{WCB}_{SP})$$
$$\Delta N_{SP} = d_{SP} \cdot \cos(\mathrm{WCB}_{SP})$$

2. By adding the coordinate differences to the coordinates of the station, we receive the coordinates of the unknown point P.

$$E_P = E_S + \Delta E_{SP}$$
$$N_P = N_S + \Delta N_{SP}$$

Example 1: calculating the coordinates of point B using the following data.

Coordinates of point A: $E_A = 624\ 523.18$ m
$N_A = 247\ 639.55$ m

Horizontal distance between point A and B: $d_{AB} = 677.36$ m

Whole circle bearing from point A to B: $\mathrm{WCB}_{AB} = 237 - 45 - 58$

Coordinate differences: $\Delta E_{AB} = d_{AB} \cdot \sin(\mathrm{WCB}_{AB}) = 677.36 \cdot \sin(237 - 45 - 58) = -572.964$ m
$\Delta N_{AB} = d_{AB} \cdot \cos(\mathrm{WCB}_{AB}) = 677.36 \cdot \cos(237 - 45 - 58) = -361.288$ m

Coordinates of point B: $E_B = E_A + \Delta E_{AB} = 624\,523.18 + (-572.964) = 623\,950.216 \approx 623\,950.22$ m
$N_B = N_A + \Delta N_{AB} = 247\,639.55 + (-361.288) = 247\,278.262 \approx 247\,278.26$ m

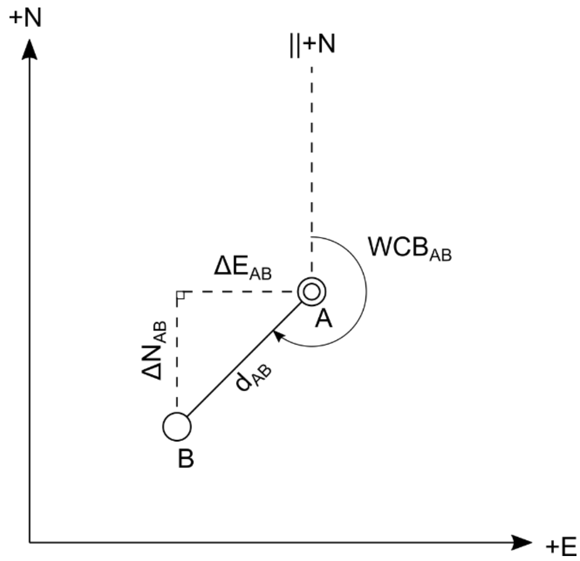

Figure 2. The layout of the points in example 1.

II. fundamental task of surveying

The second fundamental task of surveying is the inverse of the I. fundamental task. In this case, we have two known points (points with known coordinate values) and we want to find the horizontal distance and the whole circle bearing between the two points.

1. Looking at the right triangle in Fig. 1, we can calculate the easting and northing coordinate differences (ΔE_{SP}, ΔN_{SP}) by simply subtracting the respective coordinates of S from the coordinates of P.

$$\Delta E_{SP} = E_P - E_S$$
$$\Delta N_{SP} = N_P - N_S$$

   We have to be careful about the order of the points when subtracting as the change in the sign of these values changes the value of the whole circle bearing. A rule of thumb is that we always take the coordinate of the target point and subtract from it the coordinate of the station.

2. The horizontal distance between the points can be found by simply using the Pythagorean theorem with the coordinate differences.

$$d_{SP} = \sqrt{\Delta E_{SP}^2 + \Delta N_{SP}^2}$$

3. To calculate the whole circle bearing, we have to take the inverse tangent of the easting coordinate difference over the northing coordinate difference. However, as the range of the inverse tangent function is only between -90° and +90°, we do the calculation in two steps.

   a. We calculate an angle α using the absolute value of the coordinate differences.

$$\alpha = \tan^{-1}\left(\left|\frac{\Delta E_{SP}}{\Delta N_{SP}}\right|\right)$$

b. Using the sign of the coordinate differences, we decide in which quadrant the WCB lies and change the value of α accordingly.

Table 1. Finding the quadrant of the WCB from the coordinate differences.

Quadrant	Sign of ΔE	Sign of ΔN	WCB
1.	+	+	$\text{WCB} = \alpha$
2.	+	–	$\text{WCB} = 180° - \alpha$
3.	–	–	$\text{WCB} = 180° + \alpha$
4.	–	+	$\text{WCB} = 360° - \alpha$

Fig. 3 shows the value of α and the WCB in the different quadrants.

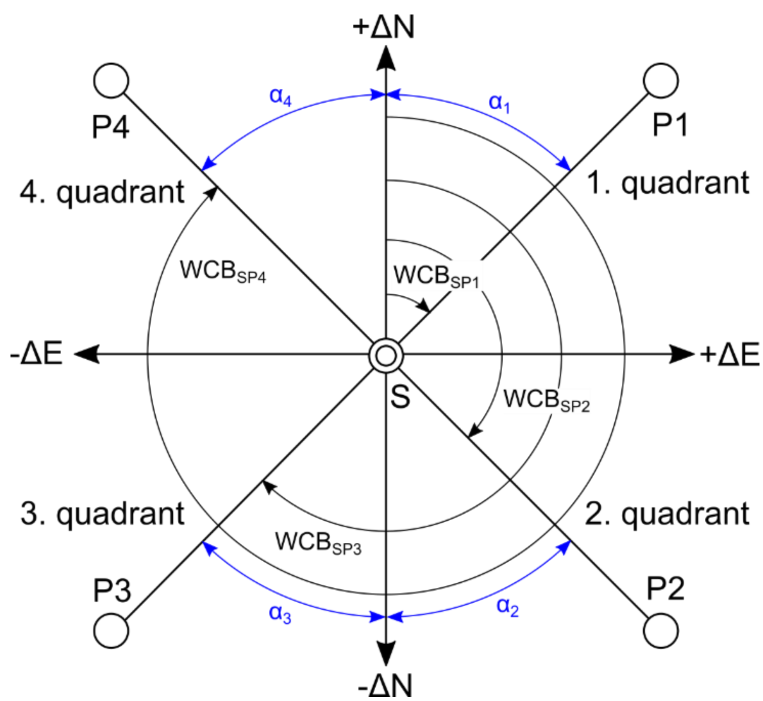

Figure 3. The connection between the α angles and the WCB in the different quadrants.

Another, simpler way to solve the second fundamental task is to use the calculator's polar (POL) function. These are typically denoted on modern scientific calculators as Pol(or $\rightarrow r\Theta$. When using these functions, we first specify the value of ΔN_{SP}, then the value of ΔE_{SP}. The calculator computes the distance and the WCB and stores the results in two memory locations (A-B, E-F or X-Y are the most typical). We can recall the values from these memory slots for further calculations. If the value of the whole circle bearing is negative, we first have to change it to a positive value by adding 360°.

Example 2: computing the distance and the whole circle bearing between points Q and R using the following data.

Coordinates of point Q: $E_Q = 614\,588.78$ m
$N_Q = 238\,656.36$ m

Coordinates of point R: $E_R = 614\,010.77$ m
$N_R = 239\,035.37$ m

Coordinate differences between the two points:

$$\Delta E_{QR} = E_R - E_Q = 614\,010.77 - 614\,588.78 = -578.01 \text{ m}$$
$$\Delta N_{QR} = N_R - N_Q = 239\,035.37 - 238\,656.36 = 379.01 \text{ m}$$

Horizontal distance between the two points:

$$d_{QR} = \sqrt{\Delta E_{QR}^2 + \Delta N_{QR}^2} = \sqrt{(-578.010)^2 + 379.010^2} = 691.190 \approx 691.19 \text{ m}$$

Whole circle bearing from point Q to R:

$$\alpha = \tan^{-1}\left(\left|\frac{\Delta E_{QR}}{\Delta N_{QR}}\right|\right) = \tan^{-1}\left(\left|\frac{-578.01}{379.01}\right|\right) = 56 - 44 - 48$$

According to Table 1, we are in the fourth quadrant, as our ΔE is negative and our ΔN is positive, so we have to change the value of α:

$$\text{WCB}_{QR} = 360° - \alpha = 360° - (56 - 44 - 48) = 303 - 15 - 12$$

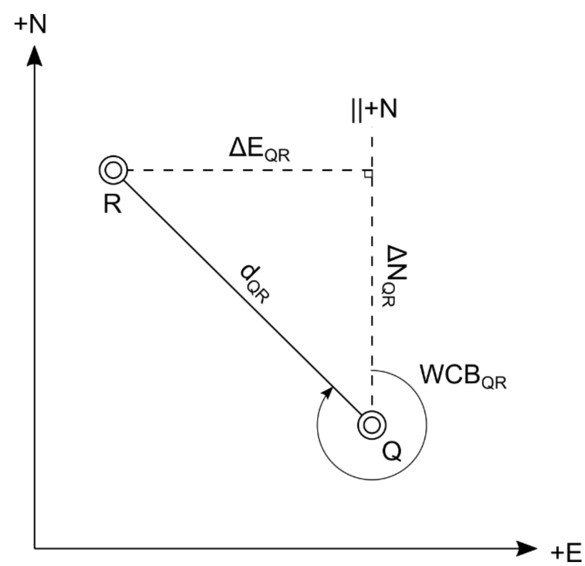

Figure 4. The layout of the points in example 2.

Whole circle bearing of opposing directions

If points A and B are two points in a coordinate system and we know the WCB from A to B (δ_{AB}), the WCB of the opposite direction (δ_{BA}) can be computed by adding or subtracting 180° the original WCB (see figure below):

$$\text{WCB}_{BA} = \text{WCB}_{AB} \pm 180°$$

Similarly to other surveying calculations, if the results becomes negative, we add 360°, if it becomes larger than 360°, we can subtract 360°.

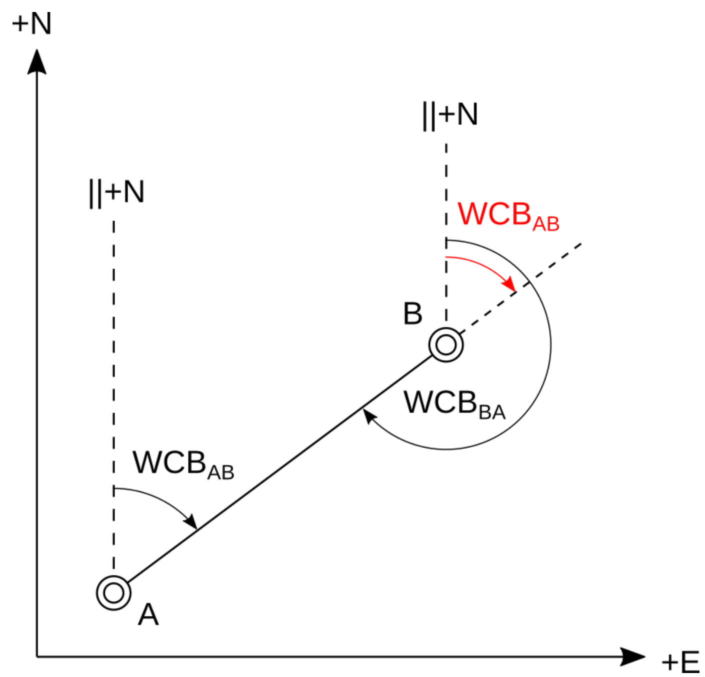

Figure 5. WCB of opposing direction.

Computing the deflection angle from the whole circle bearings

In the figure below, the deflection angle (β) between to directions, AB and AC can always be computed using the whole circle bearing of the directions. We first have to take the WCB corresponding to the right leg of the deflection angle and subtract from it the WCB corresponding to the left leg. If the result is a negative angle, we simply have to add 360°.

$$\beta = \mathrm{WCB}_{AC} - \mathrm{WCB}_{AB}$$

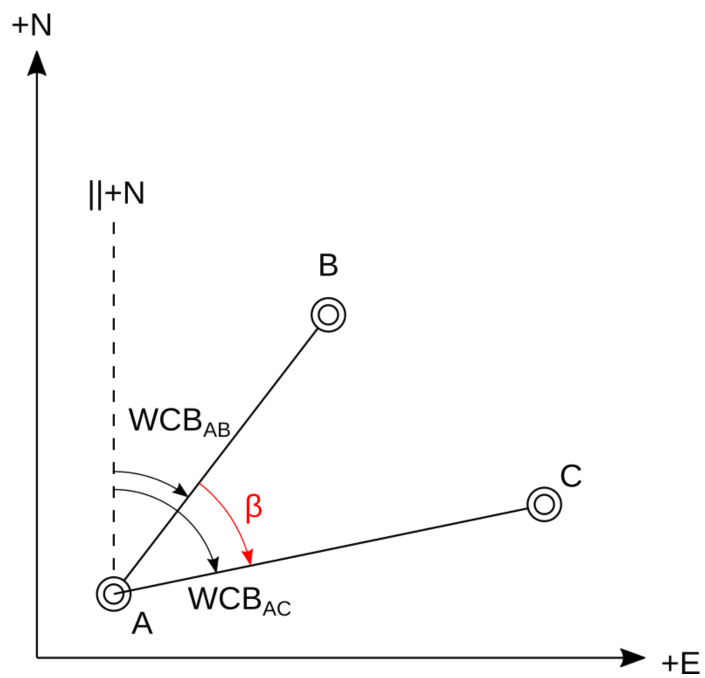

Figure 6. Computing a deflection angle from the WCBs.

Transferring the whole circle bearing

In the following example shown in the figure below, we know the WCB of the direction AB and deflection angles β (the angle $\angle CBA$) and γ (the angle $\angle CBD$). Our task is to find the WCB from B to D (δ_{BD}).

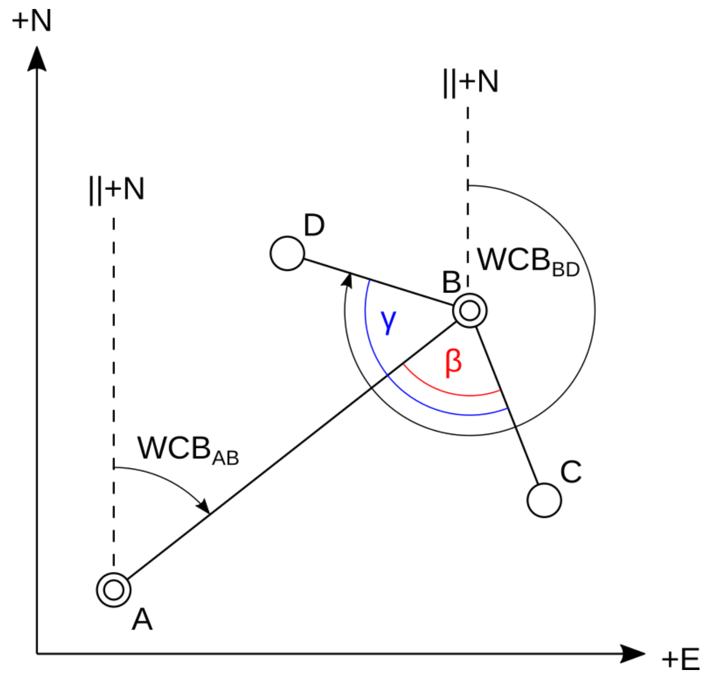

Figure 7. Transferring the WCB using deflection angles.

First, we can compute the WCB of the opposing direction from AB, that is, δ_{BA}:

$$\mathrm{WCB}_{BA} = \mathrm{WCB}_{AB} \pm 180°$$

Using δ_{BA} and the deflection angle α, we can compute the WCB from B to C:

$$\mathrm{WCB}_{BC} = \mathrm{WCB}_{BA} - \beta$$

Now, we have can add the deflection angle β to our computed WCB and find the WCB from B to D:

$$\mathrm{WCB}_{BD} = \mathrm{WCB}_{BC} + \gamma$$