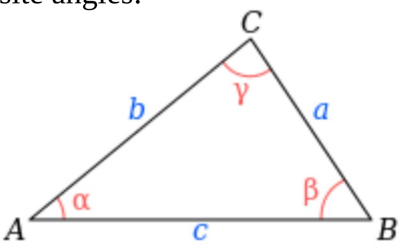
Surveying II – Practical 2
Trigonometric Point Determination

In this lesson trigonometric co-ordinate calculations are covered.
Trigonometric co-ordinate calculation are mostly based on the principal tasks and calculations in triangles (e.g. sine rule and cosine rule).

Sine rule: the ration of the sides is equal the ration of the sine of opposite angles:



$$\frac{a}{b}=\frac{\sin\alpha}{\sin\beta} \quad \text{or} \quad \frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

Cosine rule: the square of a side can be calculated from the two other sides and the internal angle.

$$c^2=a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma$$

Intersection

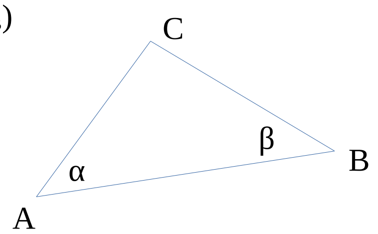
In order to determine the co-ordinates of a new point, the directions from two known points are measured.

Intersection using internal angles of the triangle

We have two known points (A and B) and two internal angles were measured (α and β).
We are looking for the easting and northing for point C.
Restriction: A and B have to be visible from each other (no blocking object in the direction).

Solution:

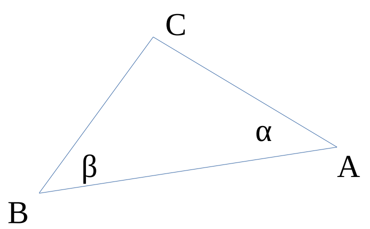
1. Calculate the distance and the whole circle bearing (WCB) between the two known points using the second principal task of surveying (WCB_{AB} and d_{AB})
2. Calculate the distance between A and C using sine rule
   $$d_{AC}=d_{AB}\cdot\frac{\sin\beta}{\sin\gamma}=d_{AB}\cdot\frac{\sin\beta}{\sin(\alpha+\beta)}$$
3. Calculate WCB fromA to C
   $$WCB_{AP}=WCB_{AB}\pm\alpha$$
   You need a sketch to decide to add α or subtract.
4. Solve the first principal task of surveying using the distance and the WCB between A and C.
   $$E_C=E_A+d_{AC}\cdot\sin WCB_{AC}$$
   $$N_C=N_A+d_{AC}\cdot\sin WCB_{AC}$$



Example:

Point ID	Easting	Northing	Internal angle
A	658077.70	247431.38	α = 81-34-45
B	657310.23	247123.54	β = 66-45-57

1. d_{AB} = 826.91
   WCB_{AB} = 248-08-38
2. d_{AC} = 1447.87
3. WCB_{AC} = 329-43-23   (+α from the sketch)
4. E_P = 657347.71 N_P = 248681.76



Intersection with whole circle bearing (WCB)

We have two known points (A and B) and two WCB from the known points to the unknown point.
WCBs are calculated by the orientation on the two known points.
We are looking for the easting and northing for point C.
Not necessary that A and B be visible from each other.

Solution:

1. Calculate the distance and the whole circle bearing (WCB) between the two known points using the second principal task of surveying (WCB_{AB} and d_{AB})
2. Calculate internal angles as the difference of WCB
   $\alpha = WCB_{AB} - WCB_{AC}$ or $\alpha = WCB_{AC} - WCB_{AB}$
   $\beta = WCB_{BA} - WCB_{BC}$ or $\beta = WCB_{BC} - WCB_{BA}$ use a sketch to decide
3. Now we have all data to calculate intersection with internal angles (see previous topic)

Example:

Point ID	Easting	Northing	WCB to C
A	657310.23	247123.54	6-30-47
B	657638.80	247759.38	290-09-00

1. d_{AB} = 715.72
   WCB_{AB} = 27-19-39
2. $\alpha = WCB_{AB} - WCB_{AC}$ = 20-48-52
   $\beta = WCB_{BP} - WCB_{BA}$ = 82-50-21
3. d_{AC} = 730.71
4. E_C = 657393.11 N_C = 247849.53

Side section with internal angle

We have two known points (A and B) and two internal angles were measured (α and γ). This method is is used instead of intersection when an instrument can't beset up on one known point (e.g. it is a tower)
We are looking for the easting and northing for point C.
Restriction: A and C have to be visible from each other (no blocking object in the direction).

Solution:

1. Calculate the distance and the whole circle bearing (WCB) between the two known points using the second principal task of surveying (WCB_{AB} and d_{AB})
2. $\beta = 180 -(\alpha + \gamma)$
3. Now we have all data to calculate intersection with internal angles (see previous topic)

Example:

Point ID	Easting	Northing	Internal angle
A	658077.70	247431.38	α = 63-23-45
B	657310.23	247123.54	γ = 51-01-49

1. d_{AB} = 826.91
   WCB_{AB} = 248-08-38
2. β = 65-35-26

3. $WCB_{AC} = WCB_{AB} + \alpha = 311\text{-}32\text{-}23$
   $d_{AC} = 968.38$
4. $E_C = 657352.87$ $N_C = 248073.55$

Arc section

We have two known points (A and B) and two distances were measured (d_{AC} and d_{BC}).
We are looking for the easting and northing for point C.

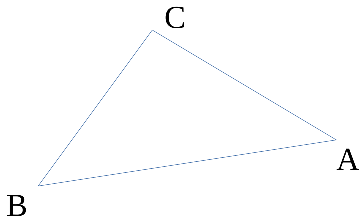
Solution:

1. Calculate the distance and the whole circle bearing (WCB) between the two known points using the second principal task of surveying (WCB_{AB} and d_{AB})
2. Calculate α using cosine rule
   $$\alpha = \arccos \frac{d_{AB}^2 + d_{AC}^2 - d_{BC}^2}{2 \cdot d_{AB} \cdot d_{AC}}$$
3. Now we have all data to calculate intersection with internal angles (see previous topic)

Example:

Point ID	Easting	Northing	Distance
A	654653.23	232456.39	$d_{AC} = 967.34$
B	654234.92	232167.47	$d_{BC} = 846.45$

1. $d_{AB} = 508.39$
   $WCB_{AB} = 235\text{-}22\text{-}04$
2. $\alpha = 60\text{-}56\text{-}28$
3. $WCB_{AC} = WCB_{AB} + \alpha$
4. $E_C = 653786.09$ $N_C = 232885.13$

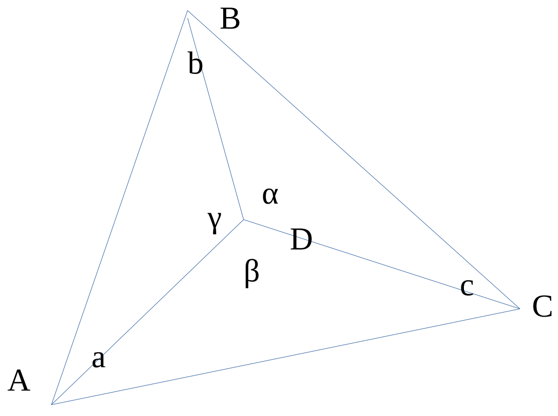


Resection

We have three known points (A, B, C) and the angles (α and β) to the known point are measured from an unknown point (D).
We are looking for the easting and northing for point D.
There are more solutions to find the coordinates of D, the Tienstra formula is introduced here.

Solution:

1. Calculate the whole circle bearings between known points
   WCB_{AB}, WCB_{BC} and WCB_{CA}
   $WCB_{BA} = WCB_{AB} \pm 180$
   $\gamma = 360 - (\alpha + \beta)$
   $a = WCB_{AC} - WCB_{AB}$
   $b = WCB_{BA} - WCB_{BC}$
   $c = WCB_{CB} - WCB_{CA}$



2. Calculate

$$K_1 = \frac{1}{\cot(a) - \cot(\alpha)}$$
$$K_2 = \frac{1}{\cot(b) - \cot(\beta)}$$
$$K_3 = \frac{1}{\cot(c) - \cot(\gamma)}$$

3. Calculate the co-ordinates

$$E_D = \frac{K_1 \cdot E_A + K_2 \cdot E_B + K_3 \cdot E_C}{K_1 + K_2 + K_3}$$
$$N_D = \frac{K_1 \cdot N_A + K_2 \cdot N_B + K_3 \cdot N_C}{K_1 + K_2 + K_3}$$

Example:

Point ID	Easting	Northing	Angle
A	3810.80	7997.25	α = 136-33-55
B	2959.39	7487.09	β = 140-58-51
C	2876.24	8754.11	

1. WCB_{AB} = 239-04-13 WCB_{BC} = 356-14-43 WCB_{CA} = 129-00-09
   γ = 82-28-14
   a = 69-55-56   b = 62-49-30   c = 47-14-34 (check a + b + c = 180)
2. K_1 = 0.7037878
   K_2 = 0.5722685
   K_3 = 1.2619110
3. E_D = 3154.15 N_D = 8258.54

Examples for practice:

Co-ordinates:

Point ID	Easting	Northing
11	91515.440	2815.220
12	90661.580	1475.280
13	84862.540	3865.360
14	91164.160	4415.080
15	86808.180	347.660
16	90050.240	3525.120
231	88568.240	2281.760
232	88619.860	3159.880

Observations:

Station number	Target number	Horizontal angle	Horizontal distance
11	12	295-54-35	
11	5004	327-22-03	
11	5002	339-45-58	954.730
11	14	71-01-11	
12			
12	231	232-53-54	
12	5004	271-50-42	
12	5002	298-02-00	1117.280
12	11	334-20-10	
231	15	341-58-03	
231	13	52-48-11	
231	5002	200-58-58	
231	5004	212-37-10	
16	14	290-57-39	
16	11	355-25-59	
16	5002	29-41-41	1078.440
16	5004	51-11-51	
5001	14	175-34-56	
5001	11	224-29-01	
5001	12	265-25-02	
5001	231	330-11-39	
5001	232	358-30-20	
5001	13	26-17-24	
5001	14	175-34-58	
5003	14	99-10-24	
5003	11	140-58-30	
5003	12	187-53-01	
5003	231	291-20-12	
5003	232	348-21-01	
5003	13	335-34-21	
5003	14	99-10-18	

1. Calculate the co-ordinates of point 5004 using intersection from points 231 and 12!
2. Calculate the co-ordinates of point 5002 using arc section from points 11 and 16!
3. Calculate the coordinates of point 5001 using resection from points 14, 232 and 13
4. Calculate the coordinates of point 5002 using intersection from points 11 and 12!
5. Calculate the coordinates of point 5003 using resection from point 12, 13 and 14!
6. Find more intersections, arc sections and resections and calculate them!

Coordinates of the unknown points:

Point ID	Easting	Northing
5001	89562.447	3587.503
5002	90587.624	2590.112
5003	89398.545	2775.181
5004	90246.238	2195.168