IMPLICIT TIMESTEPPING METHODS

→ Discontinuous Deformation Analysis

→ Contact Dynamics

OVERVIEW OF DEM SOFTWARES

Quasi-static metods ← ***an equilibrium state is searched for***

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\mathbf{f}=\mathbf{0}"$$

→ Kishino, 1988
→ Bagi-Bojtár, 1991
} *circular, perfectly rigid elemets, deformable contacts*

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← ***a process in time is searched for***

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ UDEC *deformable polyhedral elements, deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements, deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis") *deformable polyhedral elements*

→ contact dynamics models *rigid & deformable elements, non-deformable contacts*

CONTACT DYNAMICS

Jean & Moreau (1992): (2D, 3D) [mostly in physics]

Unger, T. – Kertész, J. (2003): The contact dynamics method for granular media. In: Modeling of Complex Systems, Melville, New York, American Institute of Physics, pp. 116-138

Software:

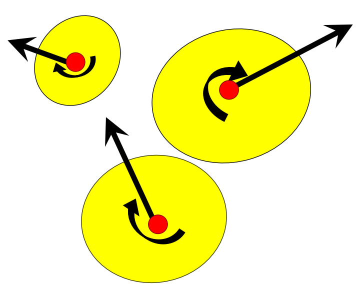
(1) LMGC91 (Dubois & Jean, 2006): **OPEN!**

rigid/deformable; spherical/polyhedral elements

(2) SOLFEC (Koziara & Bicanic, 2008):

rigid/deformable; polyhedral elements

– elements:



FIRST: for RIGID, SPHERICAL elements

CONTACT DYNAMICS

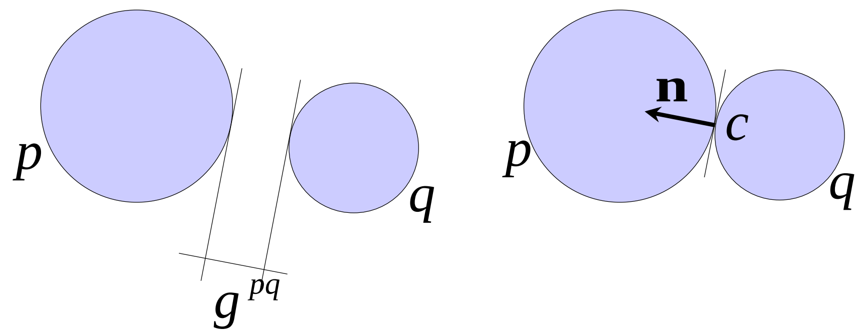
Basic entity of the analysis:

– ~~contacts~~

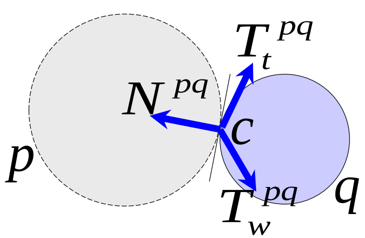
→ „pairs”:

– ~~element displacements~~

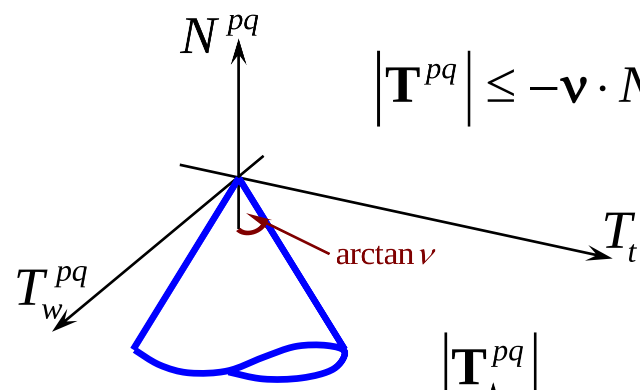
→ contact forces

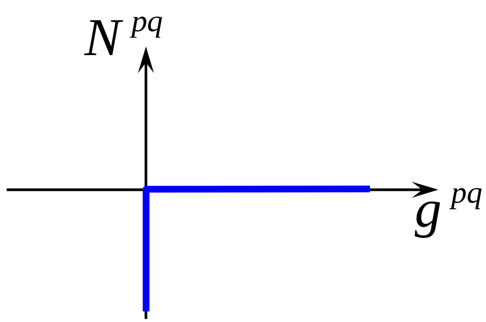


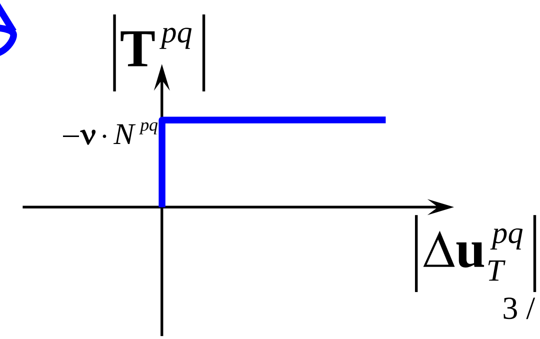
Mecanical conditions for the contact forces:



$$\left|\mathbf{T}^{pq}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}$$







CONTACT DYNAMICS

Equations of motion for a pair:

$$\begin{bmatrix} \mathbf{M}^p & \\ & \mathbf{M}^q \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \mathbf{v}^p(t) \\ \mathbf{v}^q(t) \end{bmatrix} = \begin{bmatrix} \mathbf{f}^p(t) \\ \mathbf{f}^q(t) \end{bmatrix}$$

forces and moments reduced to the reference points of the elements

translational and rotational velocity of the two elements

for spheres:

$$\mathbf{M}^p = \begin{bmatrix} m^p & & & & & \\ & m^p & & & & \\ & & m^p & & & \\ & & & I^p & & \\ & & & & I^p & \\ & & & & & I^p \end{bmatrix}$$

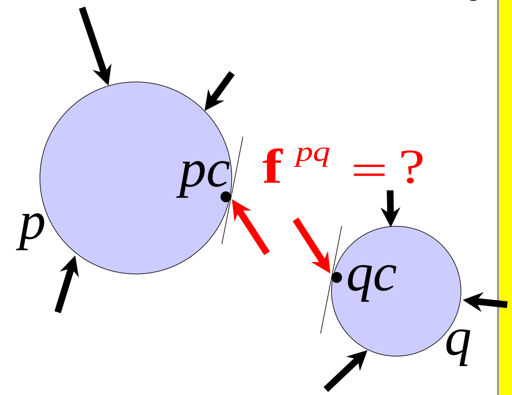
CONTACT DYNAMICS

Analysis of the (t_i, t_{i+1}) interval:

with the implicit version of the Euler-method:

$$\begin{bmatrix} \mathbf{v}^p_{i+1} \\ \mathbf{v}^q_{i+1} \end{bmatrix} := \begin{bmatrix} \mathbf{v}^p_{i} \\ \mathbf{v}^q_{i} \end{bmatrix} + \Delta t \cdot \begin{bmatrix} (\mathbf{M}^p)^{-1} & \\ & (\mathbf{M}^q)^{-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{f}^p_{i+1} \\ \mathbf{f}^q_{i+1} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{u}^p_{i+1} \\ \mathbf{u}^q_{i+1} \end{bmatrix} := \begin{bmatrix} \mathbf{u}^p_{i} \\ \mathbf{u}^q_{i} \end{bmatrix} + \Delta t \cdot \begin{bmatrix} \mathbf{v}^p_{i+1} \\ \mathbf{v}^q_{i+1} \end{bmatrix}$$



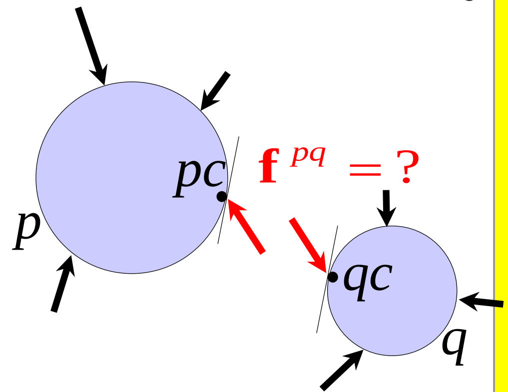
contain the basic unknowns i.e. the contact forces acting between the two elements, $\mathbf{f}^{pq}_{i+1}$

The core of the method:

The iterative solver: sweeps along all pairs, one-by-one in a random order; repeatedly determine the $\mathbf{f}^{pq}_{i+1}$ **contact forces** in every contact, so that at t_{i+1} the following conditions would be just met:

$$g^{pq}_{i+1} \geq 0\,; \quad N^{pq}_{i+1} \leq 0\,; \quad \left|\mathbf{T}^{pq}_{i+1}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}_{i+1}$$

CONTACT DYNAMICS



How to find the forces belonging to t_{i+1} :

The iterative solver:

→ Consider each pair individually, one after the other!

→ Analysis of a (*p*, *q*) pair:

– Compile the reduced forces $\mathbf{f}^{p}_{i+1}$ and $\mathbf{f}^{q}_{i+1}$,

but **WITHOUT** a force $\mathbf{f}^{pq}_{i+1}$ („no contact between *p* and *q*")

– assume constant acceleration during Δt, and calculate it from the reduced forces

⇒ the predicted position of *p* and *q* can be calculated

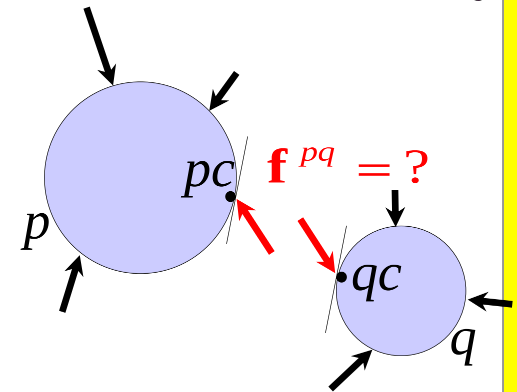
– check whether *p* and *q* are indeed not in contact:

⇒ if $g^{pq}_{i+1} > 0$: (i.e., no contact between *p* and *q*)

the contact force is indeed zero, the analysis of the pair is ready,

take the next pair!

CONTACT DYNAMICS



How to find the forces belonging to t_{i+1} :

The iterative solver

→ Consider each pair individually, one after the other!

→ Analysis of a (p, q) pair:

– Compile the reduced forces $\mathbf{f}^{p}_{i+1}$ and $\mathbf{f}^{q}_{i+1}$,

but **WITHOUT** a force $\mathbf{f}^{pq}_{i+1}$ („no contact between p and q")

– assume constant acceleration during Δt, and calculate it from the reduced forces

⇒ the predicted position of p and q can be calculated

– check whether p and q are indeed not in contact:

⇒ if $g^{pq}_{i+1} < 0$: (i.e., contact exists between p and q)

a non-zero p-q contact force exists; determine it! (*eqs. of motion*):

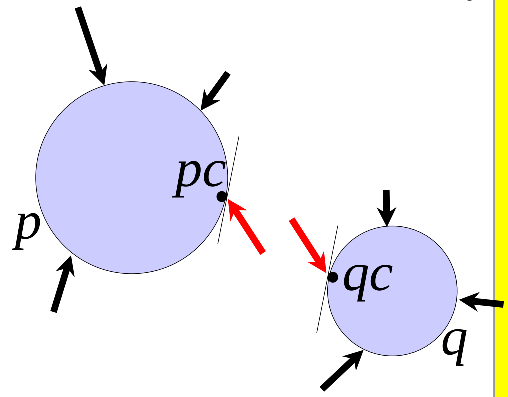
$\mathbf{f}^{pq}_{i+1}$ has to cause reduced forces which just lead to $g^{pq}_{i+1} = 0$ at t_{i+1} .

– Now check its tangential component; is $\left|\mathbf{T}^{pq}_{i+1}\right| \le -\nu \cdot N^{pq}_{i+1}$ satisfied?

⇒ if satisfied: the analysis of the pair is ready, take the next pair!

⇒ if not satisfied, i.e. if $\left|\mathbf{T}^{pq}_{i+1}\right| > -\nu \cdot N^{pq}_{i+1}$: truncate $\mathbf{T}$; take the next pair!

CONTACT DYNAMICS



<u>How to find the forces belonging to t_{i+1} :</u>

The iterative solver, overview:

→ Consider each pair individually, one after the other!

For the actual pair, find the suitable contact force;

• take the pairs in random order

• for a complete sweep: every pair taken once,

the next sweep: in a different order, …

→ Sweep along the pairs over and over again,

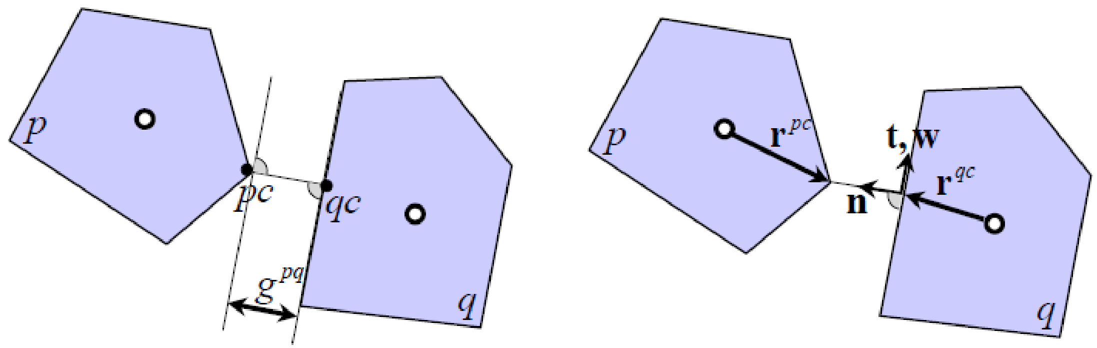
until the change in the forces becomes negligibly small;

<u>the forces belonging to t_{i+1} are received</u>

The next time step can be considered!

CONTACT DYNAMICS

Polyhedral elements:

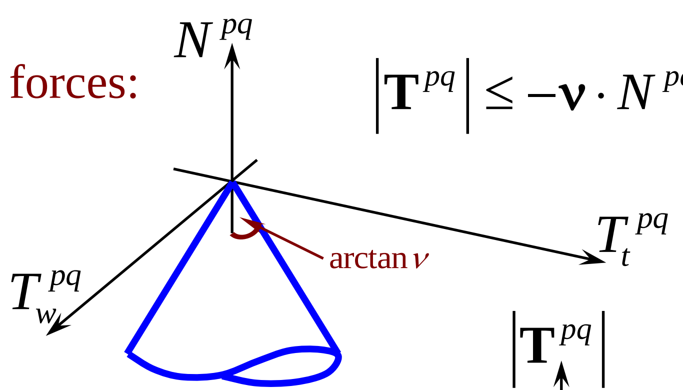


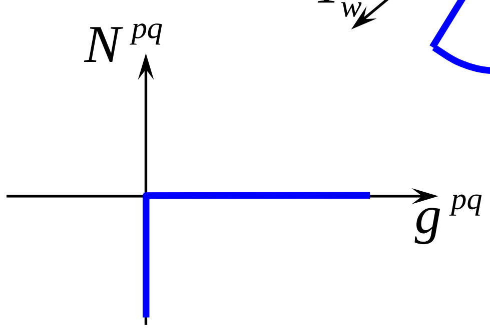
„**common plane concept**”

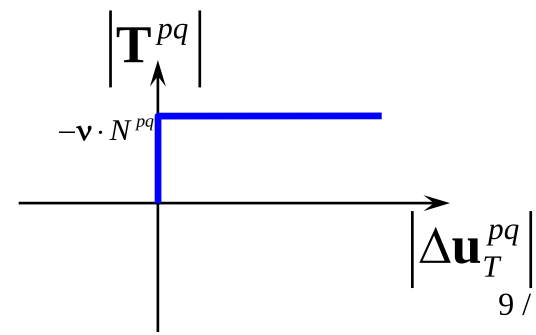
Mechanical conditions for the contact forces:

$$\left|\mathbf{T}^{pq}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}$$

[the same]

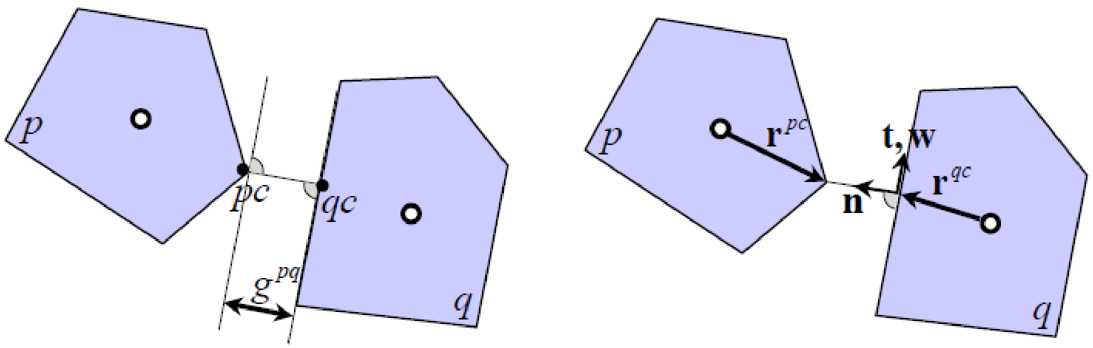






CONTACT DYNAMICS

Polyhedral elements:

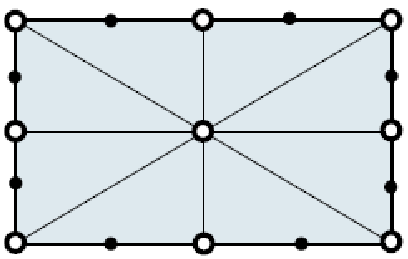


„**common plane concept**”

Deformable polyhedral elements: ~~constant strain~~ → unfavourable experiences

uniform-strain tetrahedral subdivision

The point of action of the contact force:



• : middle point of the face

„**approximated contact point**”

contact: if it touches another face

Masses: distributed to the **nodes**

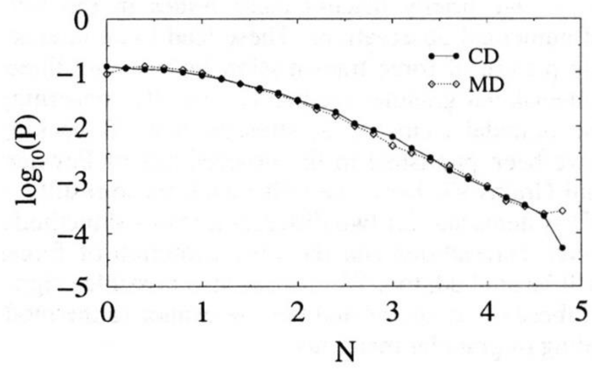
Equations of motion: for every **node** [no rotations considered]

CONTACT DYNAMICS

General remarks: [for rigid or deformable elements; for all shapes]

– advantages: very fast for motions in time

⇒ efficient for dynamic phenomena



– disadvantage: if an equilibrium state is searched for:

(slow convergence);

non-unique solution:

for rigid elements & rigid contacts:

gives one of the many statically admissible solutions of the statically indeterminate system!

for deformable elements and/or contacts:

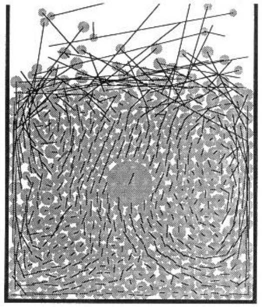
history-dependent behaviour →← random order of chosen pairs

Applications e.g. granular flows

e.g. vibration, mixing

e.g. dynamic, cyclically repeated loads

Simulation of segregation:



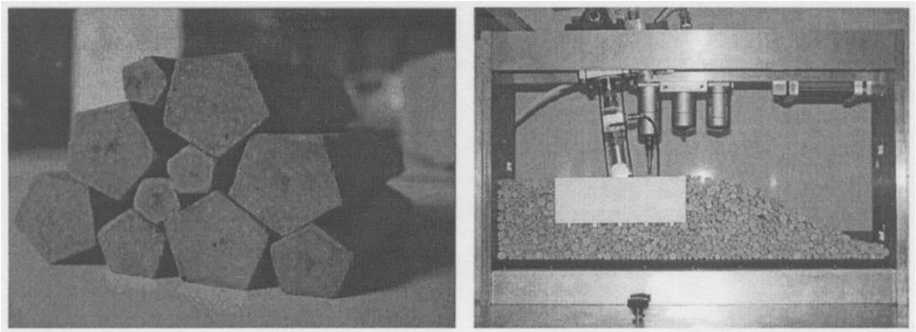
CONTACT DYNAMICS

Civil engineering applications

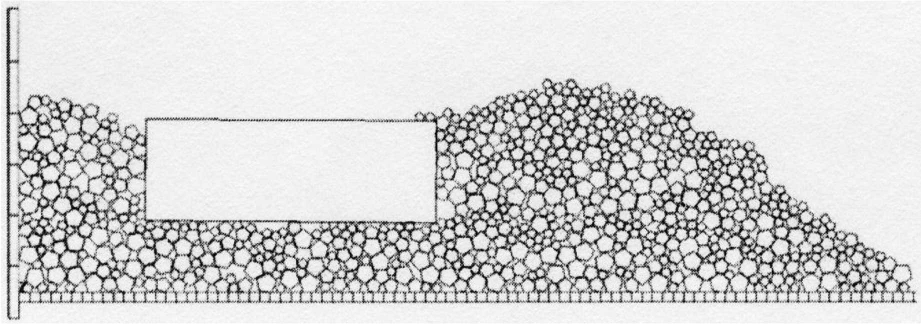
e.g. Saussine et al (2006):

„railway ballast”

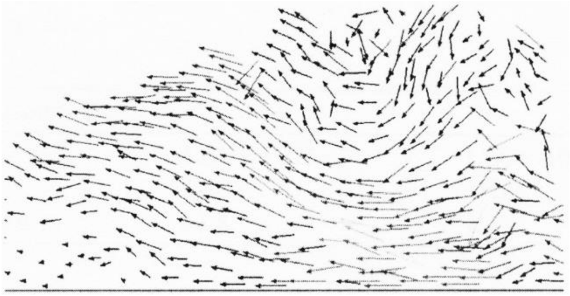
laboratory experiment („2D”):

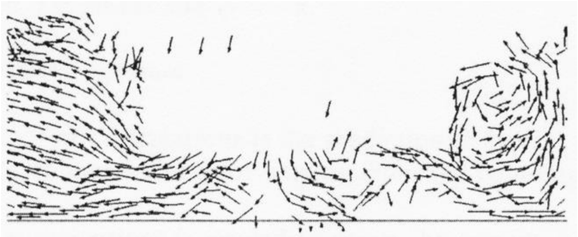


CD numerical model (2D):



results: densification due to cyclic loads



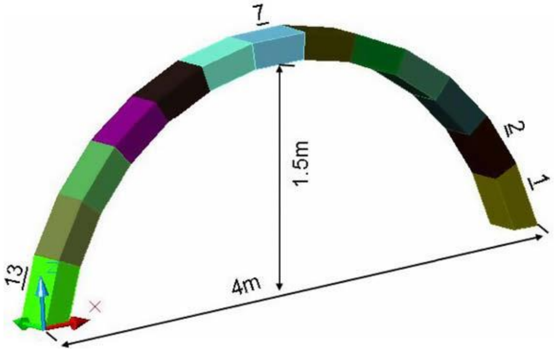


CONTACT DYNAMICS

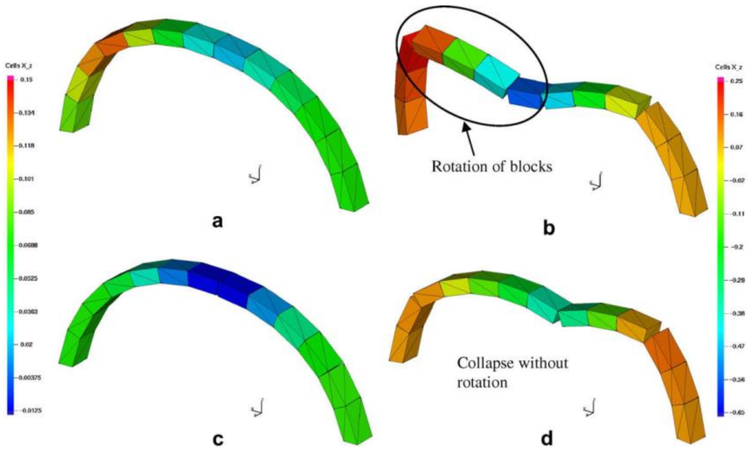
Civil engineering applications

e.g. Rafiee et al (2008):

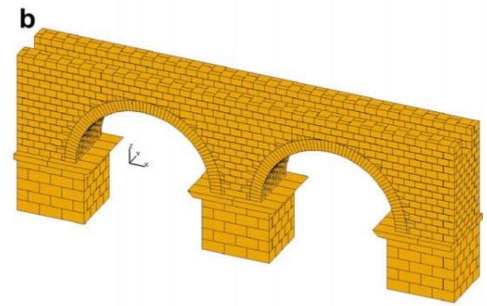
CD numerical model with deformable elements:



results: e.g. earthquake simulations



CONTACT DYNAMICS



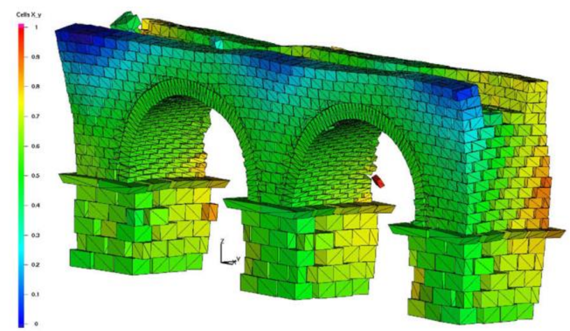
Civil engineering applications

e.g. Rafiee et al (2008):

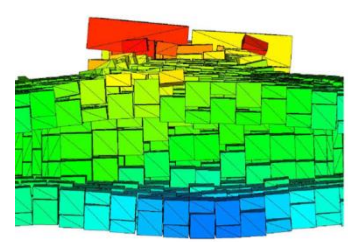
CD numerical model with deformable elements:

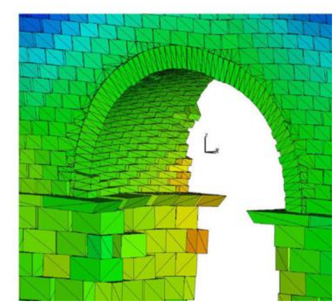
Arles, aqueduct

Earthquake simulations:

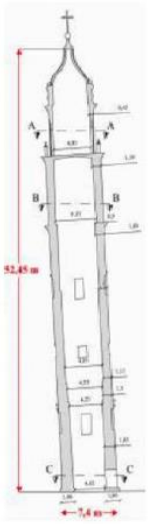


☹ Experimental verification?





CONTACT DYNAMICS



Civil engineering applications

e.g. Clementini et al (2018):

San Benedetto Church, Ferrara

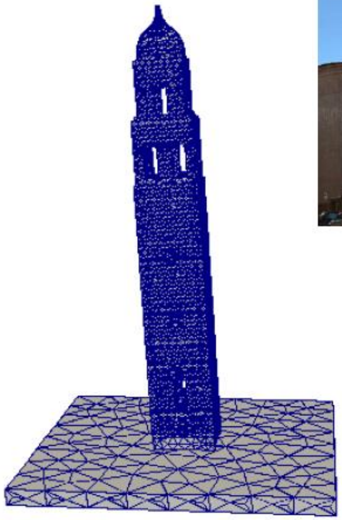
aim: analyse seismic behaviour

Model assumptions:

rigid blocks

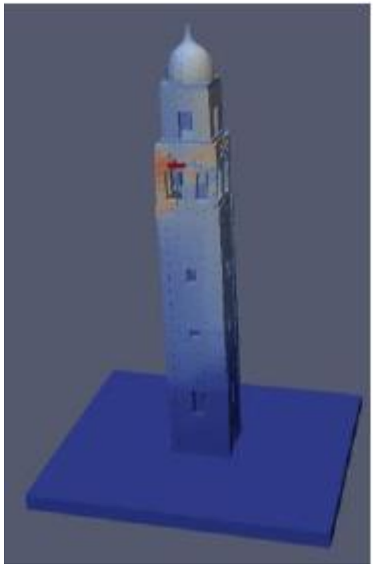
Coulomb-frictional contacts

perfectly plastic impact (no bouncing)



Load: basement oscillations $v(t) = C \sin(2\pi \cdot f \cdot t)$

$\equiv$ earthquake simulations



Model validation: compare first frequency to reality

Outcome: vulnerable regions of the structure

QUESTIONS

1. Write the equations of motion of a pair of spherical rigid elements, and shortly explain the meaning of the quantities in it.

2. Describe the mechanics of the contacts in CD: sketch the diagrams about the normal and tangential components of the contact force, and explain what can be seen on these diagrams.

3. Explain how an individual pair of rigid spherical elements is analyzed inside the analysis of a single time step (hint: Slides 6-7)

4. Explain the analysis of a time step in CD. How the “iterative solver” works? (hint: Slide 5 / lower part; Slide 8)

5. Summarize the main line of thought for using polyhedral deformable elements in CD. (hint: Slide 10)

6. The solution given by CD is non-unique. Why is it non-unique for rigid elements with rigid contacts? Why does the non-uniqueness maintains for deformable elements and deformable contacts? (hint: Slide 11, middle)