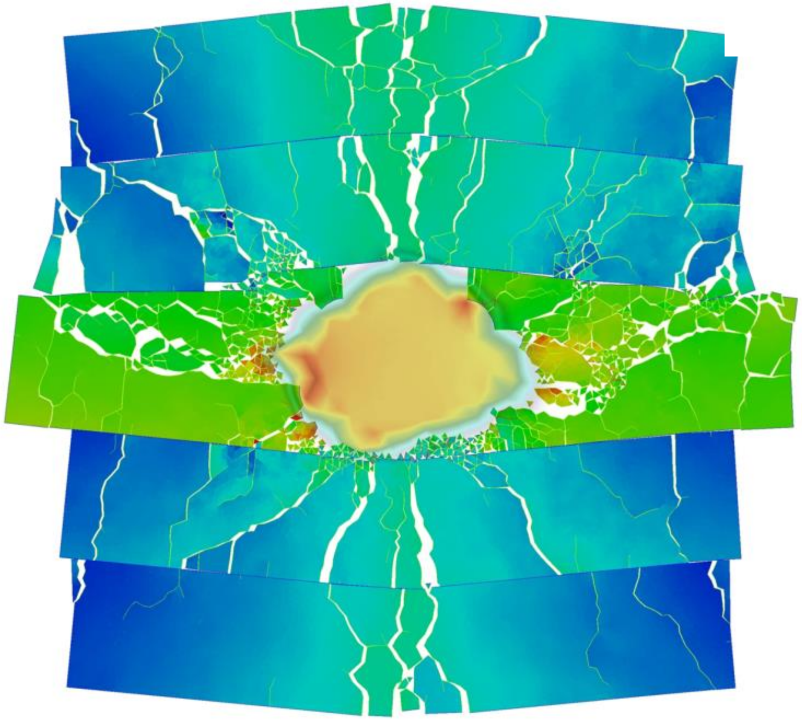


MUNJIZA'S FEM/DEM METHOD

→ Aim and the main idea

→ The elements; The equations of motion; The main steps

→ Contact detection algorithm

→ Contact interaction model; Fracture & fragmentation model

→ Applications

OVERVIEW OF DEM SOFTWARES

Quasi-static metods ← ***an equilibrium state is searched for***

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K} \cdot \Delta\mathbf{u} + \mathbf{f} = \mathbf{0}"$$

→ Kishino, 1988
→ Bagi-Bojtár, 1991

} *circular, perfectly rigid elemets, deformable contacts*

Time-stepping methods $"\mathbf{M} \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))"$ ← ***a process in time is searched for***

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods

→ UDEC *deformable polyhedral elements, deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements, deformable contacts*

Implicit timestepping methods

→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ contact dynamics models *rigid & deformable elements, non-deformable contacts*

AIM & THE MAIN IDEA

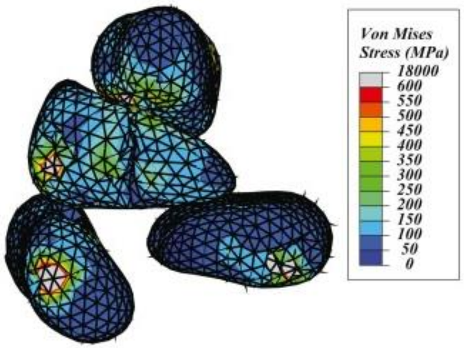
Ante Munjiza (1999), (2004), …: (2D, 3D)

→ to simulate fracture and fragmentation of discrete elements

Recent years:

→ further development of several algorithmic details

→ applications to historic masonry



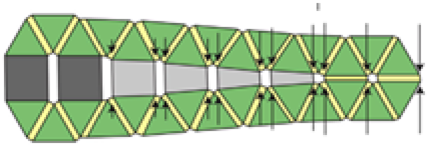
Main features:

→ deformable, polyhedral discrete elements ;
deformable contacts between them

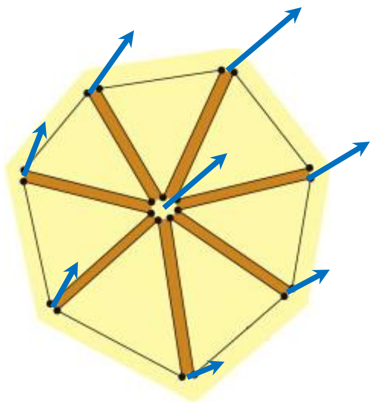
→ discrete elements are subdivided into uniform-strain FEM tetrahedra

→ „joint elements”:

inside the discrete elements,
between the FEM tetrahedra:
able to soften and crack



THE ELEMENTS



Degrees of freedom:

translations of the nodes

→ like in 3DEC

Strain in the finite element tetrahedra:

different possibilities available:

small strain tensor; right or left Cauchy-Green strain tensor

Stress options: Cauchy stress tensor; Ist or IInd Piola-Kirchhoff stress tensor

→ more options than in 3DEC

Constitutive model of the elements:

Hooke law, no plasticity of the finite elements [very simple]

→ in 3DEC: plastic yield and user-defined constitutive relations can be used

THE EQUATIONS OF MOTION

Diagonal mass matrix (Voronoi cells of nodes)

stress field is reduced to FE nodes

forces transmitted through joint elements reduced to FE nodes

$$\mathbf{Ma} + \mathbf{Cv} + \mathbf{F}_{\mathbf{int}} - \mathbf{F}_{\mathbf{ext}} - \mathbf{F}_{\mathbf{joint}} = \mathbf{0}$$

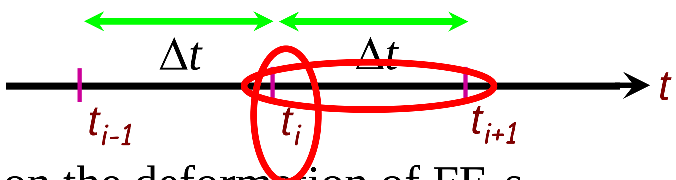
Diagonal damping matrix (no artificial damping)

Reduced vector of external forces

mass matrix: masses of the Voronoi cells of the nodes → like in 3DEC

stress field inside the tetrahedra: reduced to the nodes → like in 3DEC

Solution: central difference method → like in 3DEC



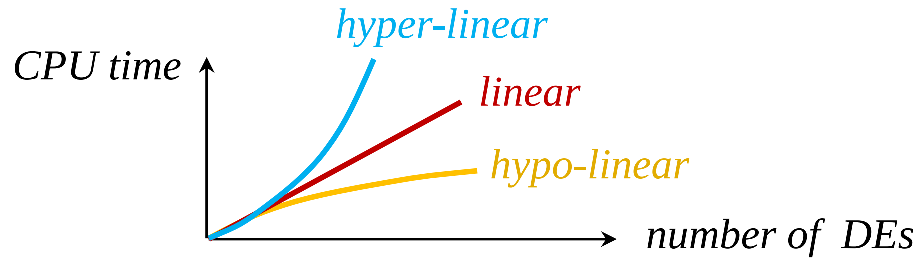
Analysis of a time step:

1. Evaluation of stresses → the internal forces, based on the deformation of FE-s
2. Evaluation of joint forces based on the deformation of joint elements
3. Fracture of joints: open the cracks and upgrade forces if needed
4. Run the contact detection algorithm: find candidate pairs for contact
5. Run the contact interaction algorithm: evaluate contact forces
6. Apply the external forces; finalize the reduced force vector
7. Solution of the equation of motion: apply the Central Difference Method

CONTACT DETECTION ALGORITHM

<u>Aim:</u> to detect pairs close to each other → possible overlap for detailed analysis

contact detection algorithms in general:



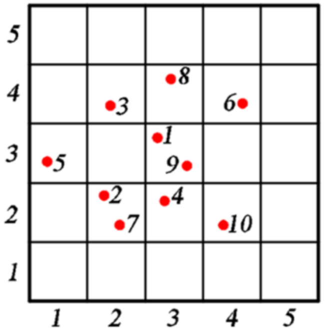
Different techniques are available in FEM/DEM

e.g. Munjiza's NBS algorithm (1995) : time $\propto N$

e.g. „screening array algorithm" : time $\propto N$ but too large RAM requirement

e.g. „sorting algorithm" : very small RAM but time $\propto N \log_2 N$ (hyper-linear)

Munjiza's NBS algorithm:



→ grid assigned to the system (!!! correct choice of grid size !!!)

→ scan DE-s: assign to the cell containing its centroid ($\propto N$)

→ scan DE-s: check neighbouring cells only ($\propto N$)

important: no loop over cells!

CONTACT INTERACTION ALGORITHM

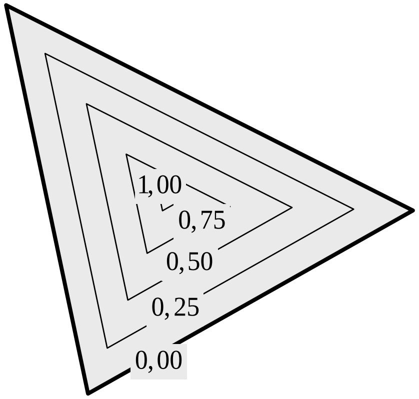
aim: to produce the contact forces transmitted between partly-overlapping DE-s

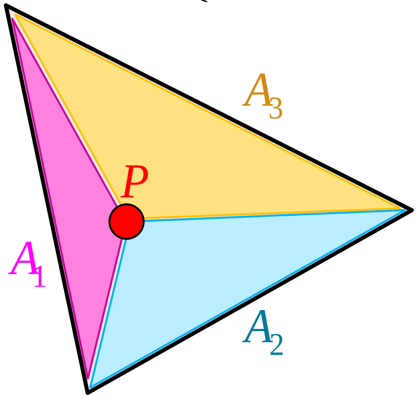
How it works in 2D: [$\rightarrow$ similar in 3D]

„potential function" inside every FE:

$$\varphi(P) = \min \begin{cases} 3A_1 / A \\ 3A_2 / A \\ 3A_3 / A \end{cases}$$

(„*P*": ● arbitrary point in the FE)





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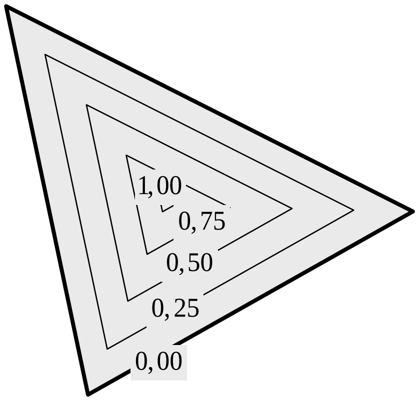
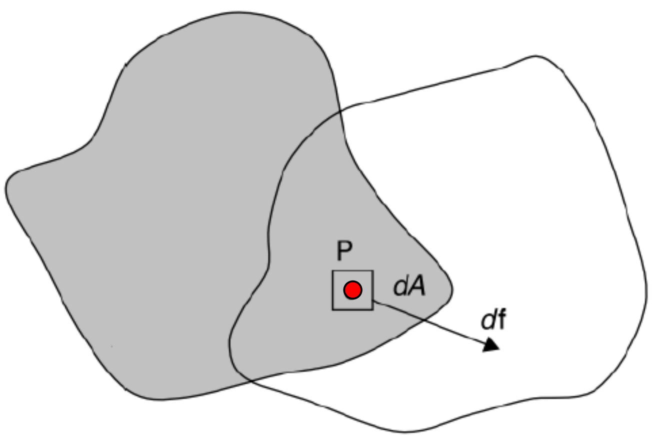


Case of two overlapping discrete elements:

P scans over the total overlap



Potential functions of the two FE-s

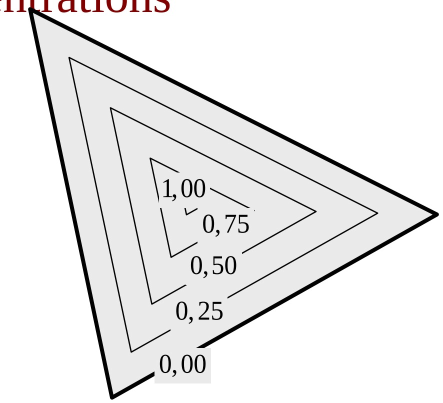
$$\mathrm{d}\boldsymbol{f} = \left[\mathrm{grad}\,\varphi_1(P) - \mathrm{grad}\,\varphi_2(P)\right]\mathrm{d}A$$

$\Rightarrow$ distributed force along the overlap: then reduced to the nodes

CONTACT INTERACTION ALGORITHM

<u>Advantageous features:</u>

- → distributed contact forces: no unrealistic stress concentrations
- → complicated contact behaviour (sliding, plasticity, cohesion etc): easy to incorporate
- → energy conservation satisfied!
- → computationally relatively efficient



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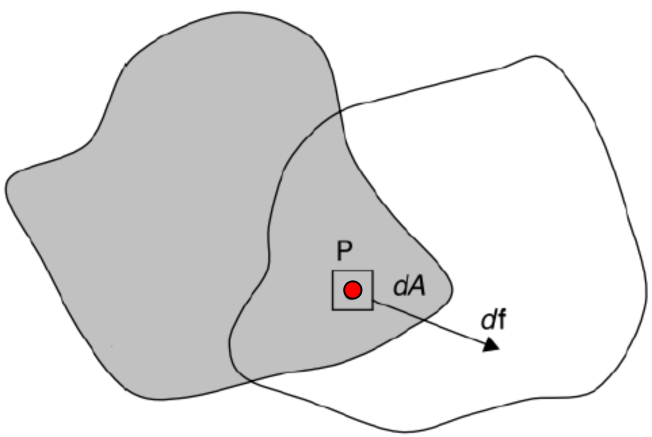

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⇒ distributed force along the overlap: then reduced to the nodes

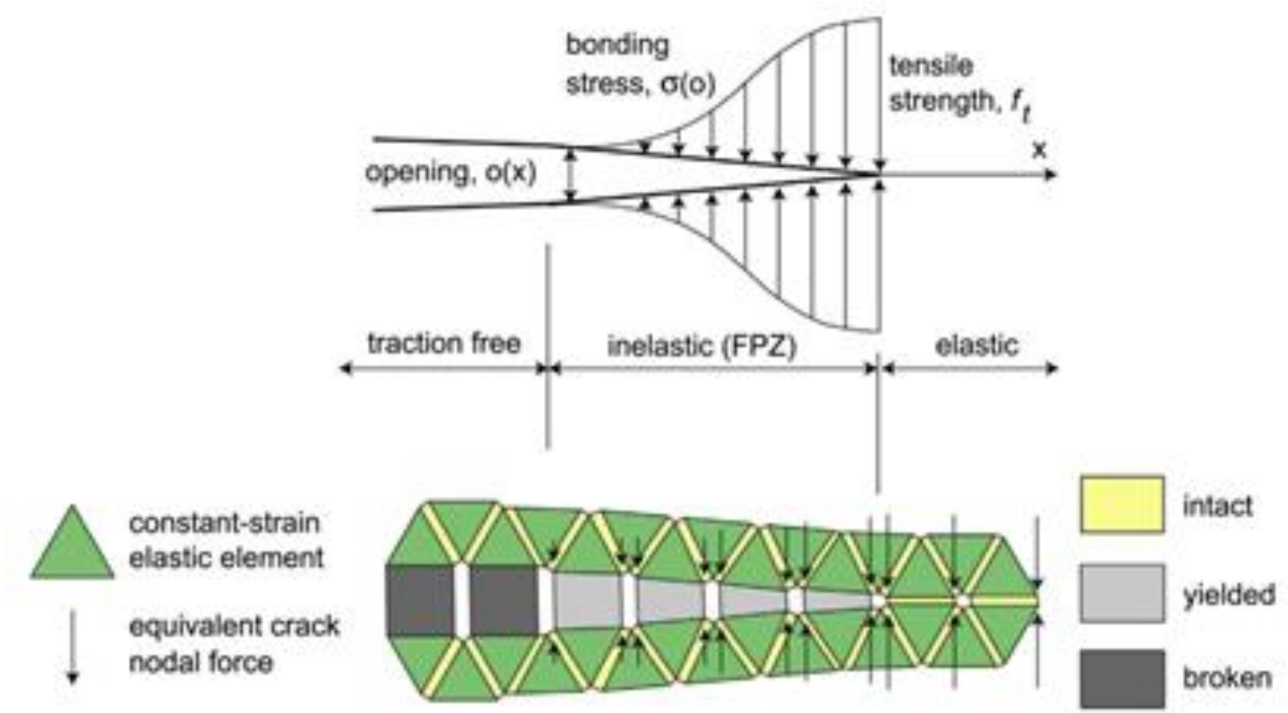
FRACTURE & FRAGMENTATION ALGORITHM

aims: → to define crack initiation

→ to describe how cracks propagate,

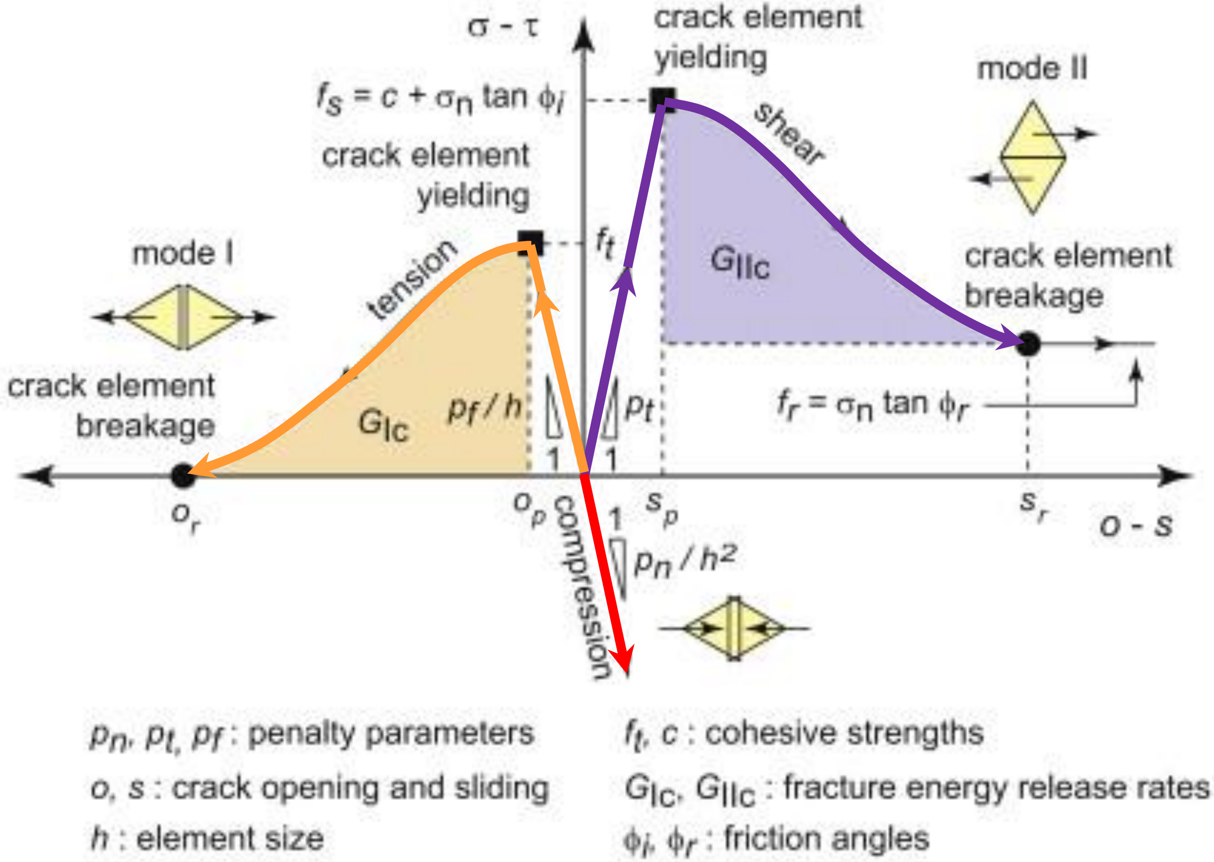
→ to replace the released internal forces with new contact forces

„joint elements”: the common surfaces between FE-s ! in the interior of DE-s !



THE JOINT ELEMENTS

Mechanical behaviour of joints:



Disadvantage:

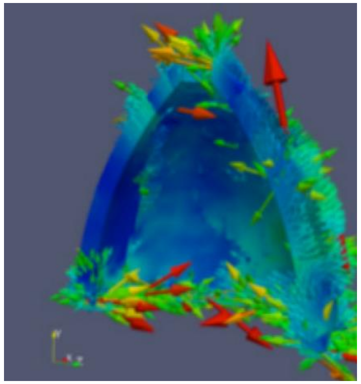
simulated fracture behaviour is very sensitive to mesh density & orientation

⇒ very dense subdivision of the DE-s is needed

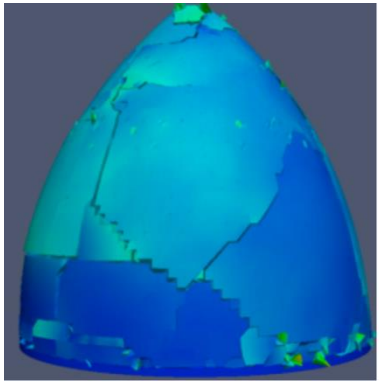
APPLICATIONS

e.g. Rougier et al (2014):

Seismic analysis of the Dome of the Santa Maria del Fiore cathedral



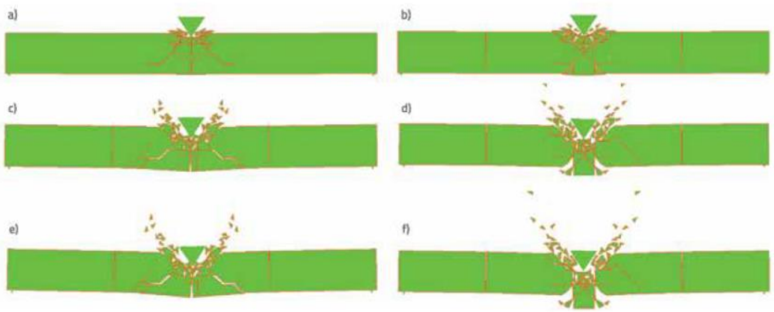
stress vawe propagation



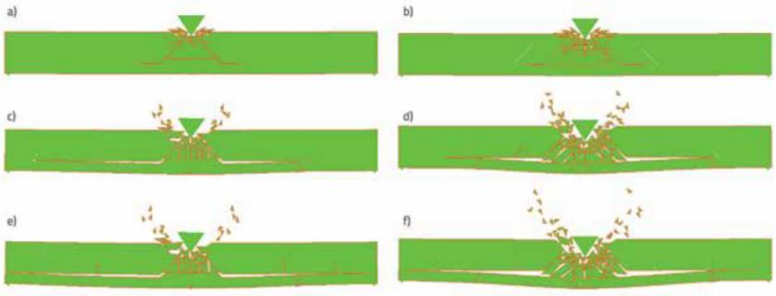
cracked final state

e.g. Zivaljic et al (2014):

Impact loading of a concrete beam



unreinforced



reinforced

QUESTIONS

1. How are the discrete elements made deformable in FEM/DEM? What are the degrees of freedom in the FEM/DEM model? (Hint: Slides 2-3)

2. How is the strain distribution over the discrete elements? Which strain tensors and which stress tensors can be used? (Hint: Slides 2-3)

3. List five similarities between 3DEC and FEM/DEM. Tell two advantages of 3DEC over FEM/DEM, and four advantages of FEM/DEM over 3DEC. List two similarities and three differences between FEM/DEM and DDA.

4.* What are the 7 main steps when analysing a timestep in FEM/DEM? Name these steps, and for each, shortly descibe what happens in it. (Hint: start with Slide 4)

5. What does it mean that a contact detection algorithm is hyper-linear / linear?

6. What is a „joint elment", and what are the main tasks of the „fracture and fragmentation algorithm"? (Hint: Slide 9)

Extra question for +2 bonus points:
Explain how the „potential function" method works for determining the contact forces between two elements. What are the main advantages of this contact interaction model?