

MASONRY DOMES







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This lecture

Earliest domes

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses criticism: a masonry dome is not in membrane state typical crack pattern; usual protections against cracking tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul Brunelleschi's dome in Florence

Questions

Heyman (1995): "a rounded vault forming a roof" [no generally accepted definition]

Mezhyrich / Межиріч, Ukraine: remains of four mammoth bone huts ≈ 15.000-14.000 BC; diameters 4-6 m



A Gregorovich, http://209.82.14.226/history/inventions/



http://images.fineartamerica.com/imag es-medium-large/mammoth-bone-hutexcavation-ukraine-ria-novosti.jpg

similar other sites Dobranichivka, Gontsy, Ioudinovo ? Cracow, Poland ?

Maltar Corbel domes from Neolith from ≈3600 BC e.g. Hagar Qim: bending (instead of ≈ pure compression)



Malta: Corbel domes from Neolith from ≈3600 BC





Reconstruction of the north temple roof of megalithic temple complex Hagar Qim (3600–3200 BC) Malta. https://commons.wikimedia.org/wiki/File:Couverture_du_temple_Hagar_Qim.jpg?uselang=fr

Malta: Corbel domes from Neolith from ≈3600 BC







Barratt (2022) (hypothesis): sometimes stone roofing, sometimes wood, sometimes combined





Everywhere in the Mediterranean:

Corbel dome huts, from Neolith till today

Martynenko, 2017:





Greece: Tholos, Thessaly



Croatia: Kazun, Istria



Italy: Casella, Apula 7/46

Everywhere in the Mediterranean:

Corbel dome huts, from Neolith:

Martynenko, 2017:











CORBEL DOMES

Static analysis:

Line of thrust Profile of dome Profile of corbel domes? given: density; height

[thrust line is a very wrong approximation]

1. "Classic Corbelling Theory" (CT) Benvenuto & Corradi, 1987

- 2. "Modified Corbelling Theory" (MCT) Rovero & Tonietti, 2014
- 3. "New Formulation of MCT" (NFMCT) *Foti et al, 2016*
- 4. "Further Refinement of the Corbelling Theory" *Fraddosio et al, 2019*



CORBEL DOMES

Profile of corbel domes? Static analysis: given: density; height

- 1. "Classic Corbelling Theory" (*CT*) Benvenuto & Corradi, 1987
 - \rightarrow separate lunes, no hoop forces
 - \rightarrow the lune consists of horizontal layers
 - \rightarrow rigid blocks
 - \rightarrow no sliding

Outcome: Differential eqs for r_{int} and r_{ext}

2. "Modified Corbelling Theory" (MCT) Rovero & Tonietti, 2014

 \rightarrow Classic Corbelling Theory + hoop forces (,, ϕ '')



4. "Further Refinement of the Corbelling Theory" *Fraddosio et al, 2019*→ upfill also taken into account

CORBEL DOMES

Static analysis: Profile of corbel domes? given: density; height

4. "Further Refinement of the Corbelling Theory" $z = \frac{1}{2} \int z Fraddosio \ et \ al, \ 2019$ \rightarrow upfill also taken into account

TRUE DOME SHAPES

Heyman (1995): "a rounded vault forming a roof" [no generally accepted definition]

Shapes: huge variety

Oval

Hemispherical Segmental

Faceted

Pointed

Bulbous etc.

How to support it:

This lecture

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Oval domes

Most famous masonry domes:

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Questions

Repetition: Membrane solution for spherical domes $\sqrt[q]{}^q$

 σ_m

 $\sigma_{\scriptscriptstyle h}$

m: meridional direction*h*: hoop direction

Predicted crack pattern:

lateral thrust! (

Repetition: Membrane solution for spherical domes $\int_{a}^{q} \checkmark$

m: meridional direction*h*: hoop direction

Importance of boundaries:

membrane state

 σ_m

 $\sigma_{\scriptscriptstyle h}$

membrane state

Criticism: Not a membrane state

The ideal membrane dome shape for pure selfweight & uniform thickness:

$$y = \frac{1}{2} x_0 \left[\int_0^{x/x_0} e^{t^2} dt - \int_0^{x/x_0} e^{-t^2} dt \right]$$

$$y = (0.7206) [x^{3} + (0.3338) x^{7} + (0.0496) x^{11} + (0.0041) x^{15} + (0.0002) x^{19} + \cdots]$$
(12)

Consequence:

- \Rightarrow In real domes the thrust does
 - not run along the middle surface
 - \Rightarrow membrane solution:

only a poor first approximation!

(Stephen Ressler, West Point)

Typical crack pattern under selfweight

(Stephen Ressler, West Point)

 $https://www.youtube.com/watch?v=cgzh0YfESbA\&list=PLKd0u75kvQExZb0jrI-lr_CpBQegfPo-m&index=1Q_{9/46}$

Typical crack pattern under selfweight

Atamturktur et al (2012)

\Rightarrow crack pattern depends on the stiffness of bottom support

20 / 46

Remark: Direction of the reactions

also depends on the **stiffness of bottom support**: Chen&Bagi, 2023:

e.g. under the ground: resistance of neighbouring earth mass

\rightarrow Atreus treasury, BC 1250 :

e.g. iron rings or chains:

St Peter's Basilica, Rome, strengthened by Poleni

Aoki et al (2004)

Blasi et al (2014)

e.g. iron rings or chains:

Chen&Bagi, 2023:

e.g. stabilizing effect of a tiburium:

Beatini et al (2018)

e.g. more sophisticated structural solutions:

Hagia Sofia:

Protection against typical cracking: Tensile resistance due to crosswise compression

Protection against typical cracking: Tensile resistance due to crosswise compression

Chen & Bagi (2020): Hemisphere: brick $\approx 1:4 \dots 1:5 \Rightarrow$ no cracking Simon & Bagi (2016): smaller value found for bricks in bond: $t_{\min} = 0,037 \cdot R_{middle}$ Beatini et al (2018): $\tau \leq \mu \cdot (-\sigma_{merid})$ $2 \cdot (\frac{b}{2}\tau) = h \cdot 2\sigma_{hoop}$ $h \cdot 2\sigma_{hoop} \leq b \cdot \mu \cdot (-\sigma_{merid})$ $\sigma_{hoop} \leq \frac{b}{2h} \cdot \mu \cdot (-\sigma_{merid})$ $\sigma_{hoop} \leq \frac{b}{2h} \cdot \mu \cdot (-\sigma_{merid})$

This lecture

What is a masonry dome?

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses typical crack pattern; usual protection against cracking criticism: a masonry dome is not in membrane state tension resistance due to crosswise compression

Oval domes

Most famous masonry domes: Hagia Sofia, Istanbul Brunelleschi's dome in Florence

← widely applied in the Renaissance & Baroque

"Oval":

- \rightarrow closed, convex, smooth curve having
 - two axes of symmetry
- $\rightarrow \approx$ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

"Oval": Method 1: compose it from circular arcs!

,,eccentricity angle", β : deviation from the circle:

→ b = 0 or $\beta = 0$: no deviation from the circle → $b = \sqrt{3}a$ or $\beta = 60^\circ$: largest possible deviation

"Oval":

 → closed, convex, smooth curve having two axes of symmetry
 → ≈ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

"Oval": Method 2: elongate a circle!

Leonardo da Vinci, Codex Atlanticus, ≈1510

Albrecht Dürer, 1525

[\Rightarrow ellipse received; not used at that time for construction]

The middle surface of oval domes:

<u>Type 1:</u> "flat domes" *Rotate the groundplan about the longer axis!*

<u>Type 2:</u> "high domes" *Rotate the groundplan about the shorter axis!*

The middle surface of oval domes:

Type 1: "flat domes"

Rotate the groundplan about the longer axis!

<u>Type 2:</u> "high domes" *Rotate the groundplan about the shorter axis!* Simon & Bagi (2016)

Some of the conclusions:

 \rightarrow Both types may require smaller thickness than a semispherical dome.

 \rightarrow Type 2 (,,high") domes are stronger than Type 1 (,,flat") domes.

33 / 46

Failure modes:

<u>Type 1:</u> "flat domes" *Rotate the groundplan about the longer axis!*

(a) Type 1 dome

Type 2: "high domes"Rotate the groundplanabout the shorter axis!

failure for too small t

failure for too low fric

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Main Dome of Hagia Sofia, Istanbul

http://www.istanbulturkeybook.com

Cathedral originally built: 537 537 – 1453 Christian church 1453 – 1935 Moslim mosque 1935 – museum

The main dome: 558: previous dome collapsed in earthquake 562: the recent dome (made much higher) is ready survived several earthquakes

[overview of changes since 537: https://www.youtube.com/watch?v=rFOlOZzO3jY]

Replacement of the main dome:

Dome of Hagia Sofia, Istanbul

Material: brick and stone

Span: 31 m (largest masonry dome until Brunelleschi's dome in Florence)

<u>Support system:</u> innovation: ,,pendentives" [sections from a larger sphere] ; complex system of structural units carry the lateral thrust

https://www.youtube.com/watch?v=uEKtWii7Vns

https://www.youtube.com/watch?v=XfpusWEd2jE

Dome of Hagia Sofia, Istanbul

<u>To prevent cracking due to hoop stress:</u> → 40 brick ribs forming the dome → between them: arched windows

"suspended from heaven"

https://www.youtube.com/watch?v=XfpusWEd2jE

https://www.youtube.com/watch?v=XfpusWEd2jE

Suggested videos:

https://www.youtube.com/watch?v=5DTh1c-f1uc (long, history & structural) https://www.youtube.com/watch?v=XfpusWEd2jE (cooperating struct. units) https://www.youtube.com/watch?v=S90SMOKeVpA (short, supporting) https://www.youtube.com/watch?v=uEKtWii7Vns (short, structural system)38/46

Beginning of Renaissance, flourishing & competing cities:
Florence cathedral planned still in 14th century, then it was nearly ready but missed a dome ⁽²⁾
44 m span [nearly the span of the Pantheon]

https://www.youtube.com/watch?v= _IOPlGPQPuM&feature=youtu.be

Main challenges: HUGE size

- → no external buttressing allowed [no space around]
- → no scaffolding allowed [not enough trees]

aislesalvotimeingh

Can it be solved at all? public competition launched, 1419 won: Filippo Brunelleschi, a goldsmith; childhood: mathematics and arts; stubborn, self-confident; kept his ideas top secret

39 / 46

https://www.youtube.com/watch?v=_IOPlGPQPuM&t=11s

Brunelleschi's answers:

- \rightarrow to decrease the lateral thrust:
 - a) make the dome *pointed*
 - b) make it *light*:

build two domes instead of a single one

- a thick inner, sandstone and marble dome
- a thin outer, brick dome interconnected:

http://florencedome.com/blog.html

http://www.yousubtitles.com/13-Three-Great-Domes-Rome-tothe-Renaissance-id-1373387

Brunelleschi's answers:

- \rightarrow to decrease hoop stress & strain:
 - 3 stone & 1 wooden "chains"

http://www.yousubtitles.com/13 -Three-Great-Domes-Rome-tothe-Renaissance-id-1373387

www.teggelaar.com

http://www.digitalmediaworld.tv/indepth/226-brunelleschi-s-dome

e.g. the lowest stone ring:

http://florencedome.com/blog.html

Brunelleschi's answers:

 \rightarrow to build the dome without scaffolding:

a self-supporting construction method is needed!

https://www.youtube.com/watch?v=kkBaxFuh40E

[herringbone pattern is found in the dome]

www.teggelaar.com

Brunelleschi's answers:

 \rightarrow to build the dome without scaffolding:

a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN \Rightarrow dome successfully built between 1420-1436

ytaba36.wordpress.com

Experiment by Jones, Sereni & Massimo Ricci (2010):

https://www.youtube.com/watch?v=kkBaxFuh40E

[Ricci's dome is unfinished]

Brunelleschi's answers:

 \rightarrow to build the dome without scaffolding:

a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN

How the idea came?

- \rightarrow may origin from seljukids (moslim architecture)
- \rightarrow Venice: applied too, from XIth century, but for very small niches

Askarov(2004)

Brunelleschi may had proved that the idea works: [small theatre dome nearby, found in 2012]

https://www.youtube.com/watch?v=kkBaxFuh40E 44/46

Suggested videos:

https://www.khanacademy.org/humanities/renaissance-reformation/earlyrenaissance1/sculpture-architecture-florence/v/brunelleschi-dome-of-thecathedral-of-florence-1420-36 short

https://www.youtube.com/watch?v=_IOPIGPQPuM&feature=youtu.be ③ short https://www.youtube.com/watch?v=QWz90KdrDBs short, on Ricci's results

Home study:

on technical details:

https://www.youtube.com/watch?v=RUBnNDloGHg,

51:55 [entitled "Great Cathedral Mystery" ©]

on the nearby small "test dome":

https://www.youtube.com/watch?v=RUBnNDlo GHg

https://www.youtube.com/watch?v=ex rD5RwjlIo&feature=youtu.be&t=93

Questions

- 1. Recognize from a figure: *hemispheral / oval / pointed dome*. Explain the following terms: *pendentive*; *drum*; *tiburium*; *corbel dome*; *herringbone pattern*.
- 2. What is a *corbel dome*? What theorical predictions exist about the shape of their intrados and extrados?
- 3. Introduce the typical crack pattern of a hemispherical masonry dome without or with a tension ring at its bottom. How can you *protect* a spherical dome against its typical cracking modes?
- 4. How to calculate the *tensile resistance* in horizontal direction in a dry masonry wall with simple running bond pattern, caused by the vertical compression stress and friction resistance in the horizontal contacts?
- 5. What are the *two main geometrical types* of oval domes? Explain the meaning of the diagram that shows the relation between *minimally necessary wall thickness* and the *eccentricity angle*.