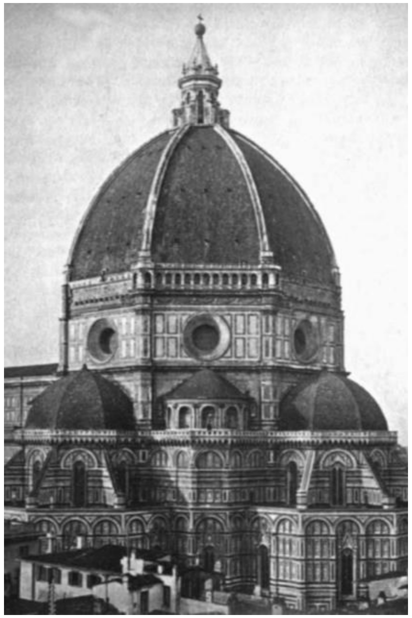
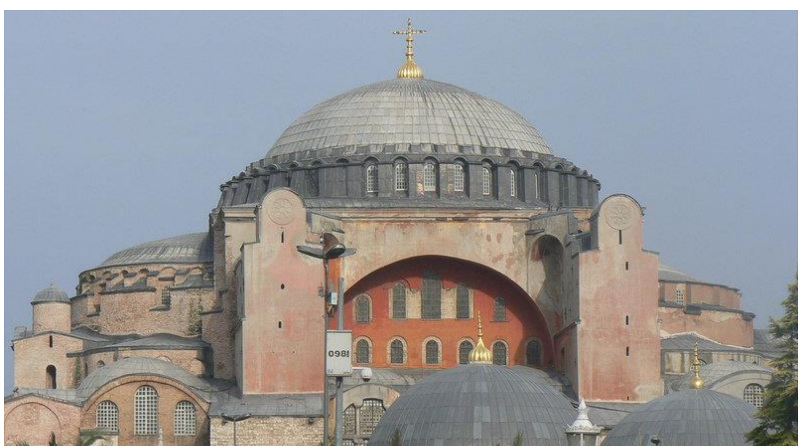
MASONRY DOMES





This lecture

What is a masonry dome?

Spherical domes

- repetition of membrane solution:
  - distribution of meridian and hoop stresses
- criticism: a masonry dome is not in membrane state
- typical crack pattern; usual protection against cracking
- tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

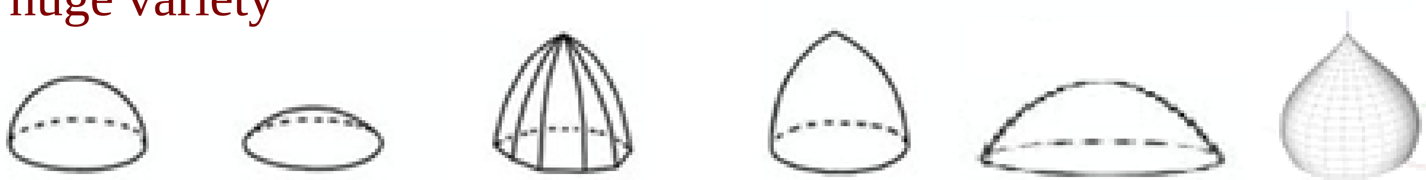
- Hagia Sofia, Istanbul
- Brunelleschi's dome in Florence

WHAT IS A MASONRY DOME?

Heyman (1995): „a rounded vault forming a roof”

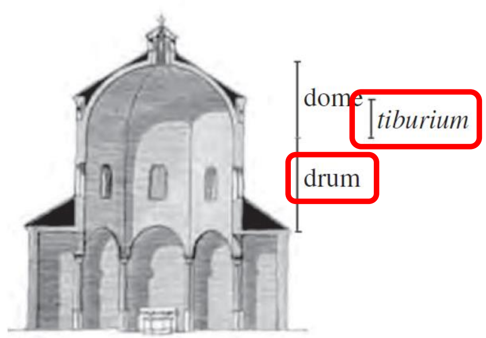
[no generally accepted definition]

<u>Shapes:</u> huge variety

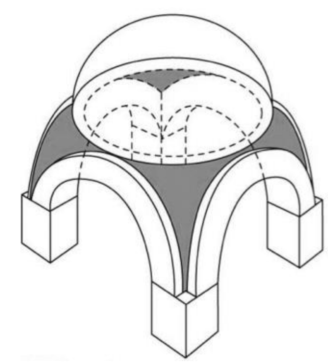


Hemispherical *Segmental* *Faceted* *Pointed* *Oval* *Bulbous* *etc.*

<u>How to support it:</u> e.g. dome rests on a drum e.g. dome rests on pendentives



Beatini et al (2018)



http://sridhistory.blogspot.com/

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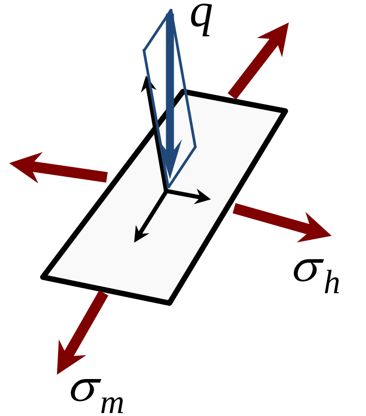
Repetition: Membrane solution for spherical domes

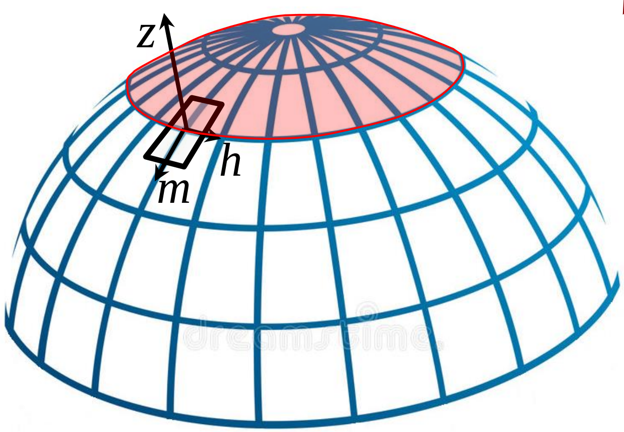
m: meridional direction

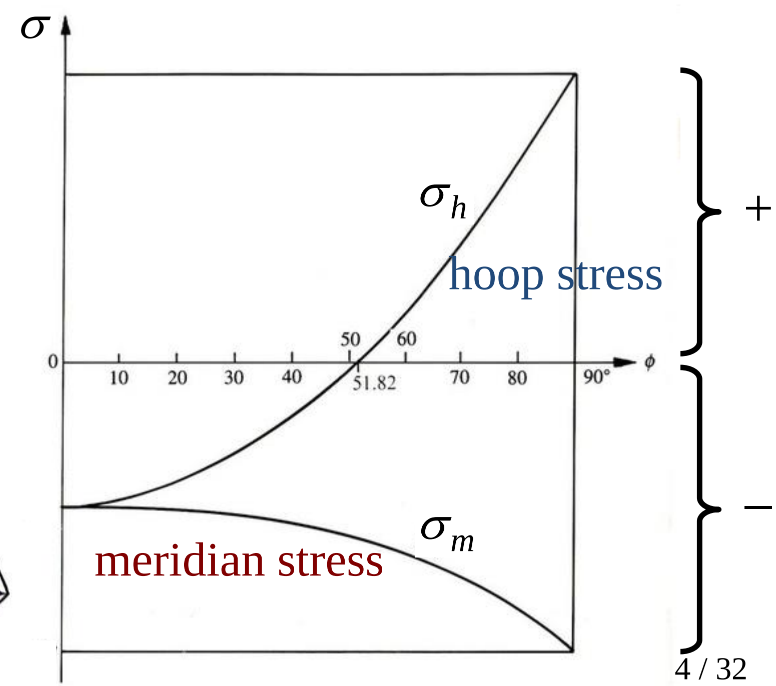
h: hoop direction



$$\sigma_m = -\frac{q}{t} R \frac{1}{1+\cos\varphi}$$

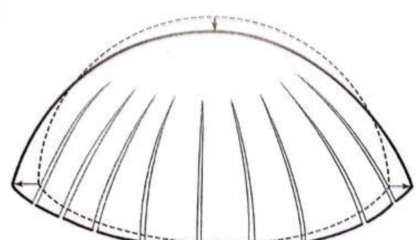
$$\sigma_h = \frac{q}{t} R \left(\cos\varphi - \frac{1}{(1+\cos\varphi)} \right)$$





Predicted crack pattern:

lateral thrust! ⇐



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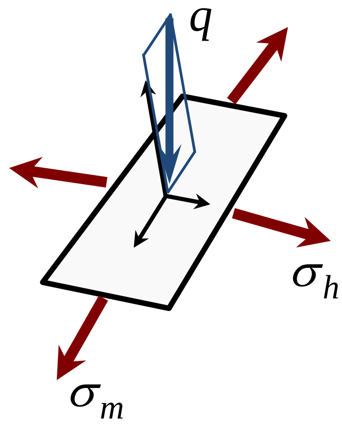
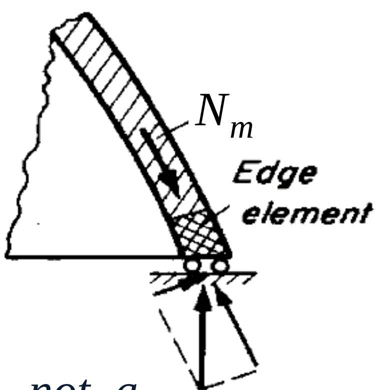


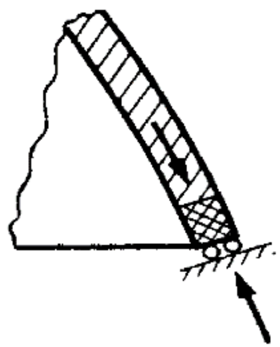
$$\sigma_m = -\frac{q}{t} R \frac{1}{1+\cos\varphi}$$

$$\sigma_h = \frac{q}{t} R \left(\cos\varphi - \frac{1}{(1+\cos\varphi)} \right)$$

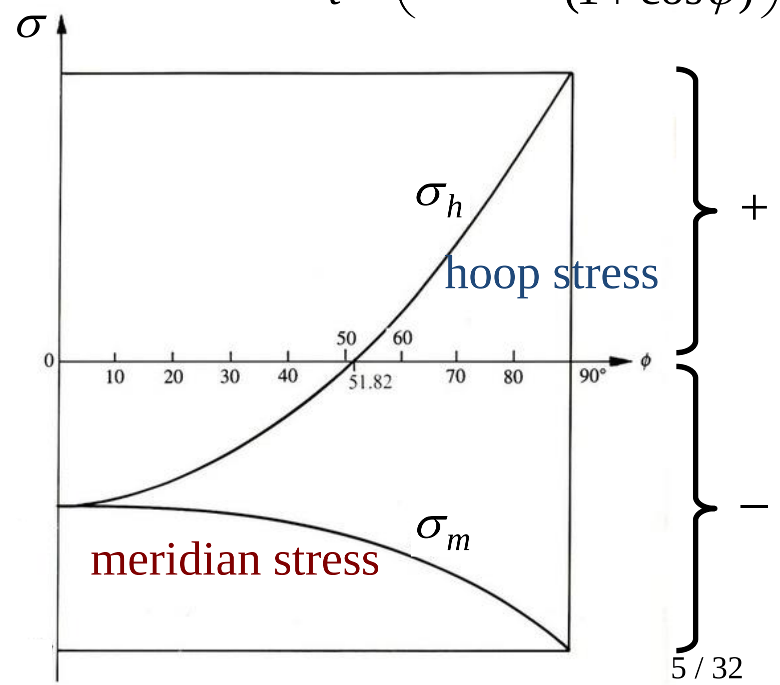
Importance of boundaries:



not a membrane state



membrane state



Criticism: Not a membrane state

The ideal dome shape for pure selfweight & uniform thickness:

$$y = \frac{1}{2} x_0 \left[\int_0^{x/x_0} e^{t^2} dt - \int_0^{x/x_0} e^{-t^2} dt \right]$$

$$y = (0.7206) [x^3 + (0.3338) x^7 + (0.0496) x^{11} + (0.0041) x^{15} + (0.0002) x^{19} + \cdots] \quad (12)$$

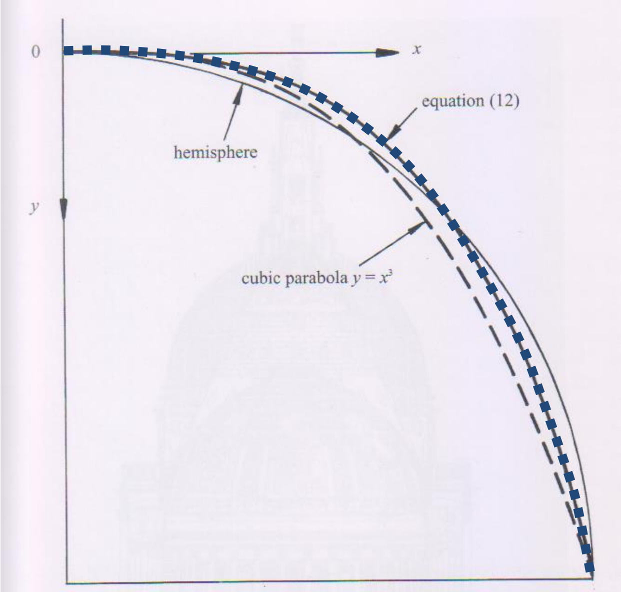


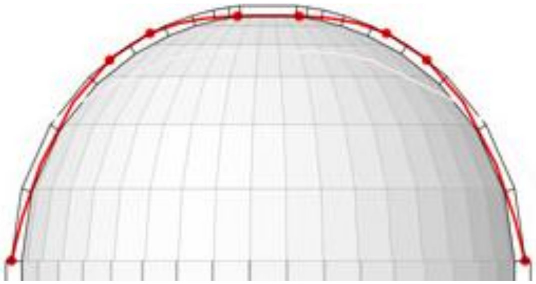
FIGURE 4. The 'perfect' dome compared with the cubic parabola and the hemisphere.

Heyman (1998)

Consequence:

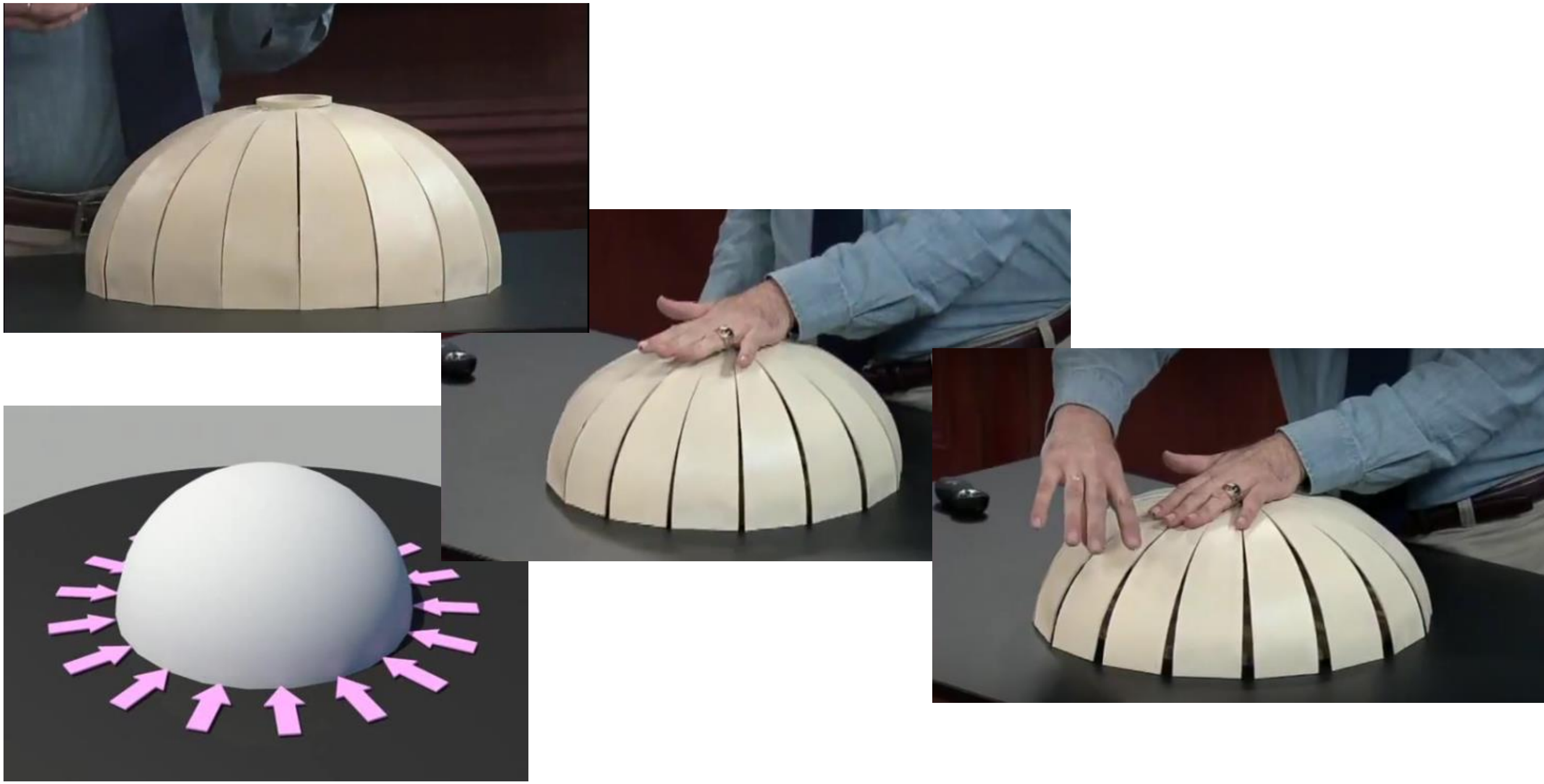
⇒ In real domes the thrust does not run along the middle surface

⇒ membrane solution: only a poor first approximation!



e.g Beatini et al (2018)

Typical crack pattern under weight



(Stephen Ressler, West Point)

http://www.yousubtitles.com/13-Three-Great-Domes-Rome-to-the-Renaissance-id-1373387

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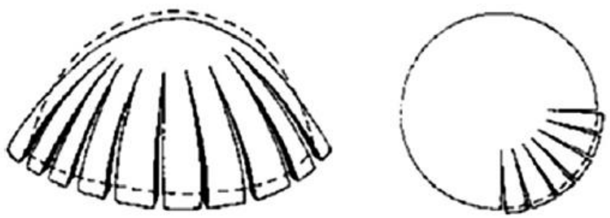
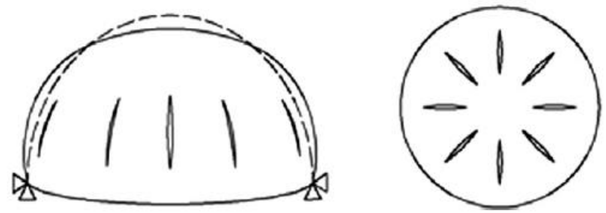
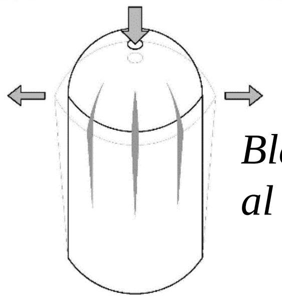


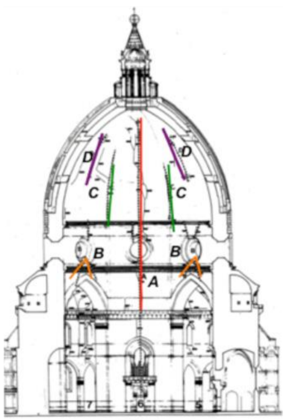


Atamturktur et al (2012)



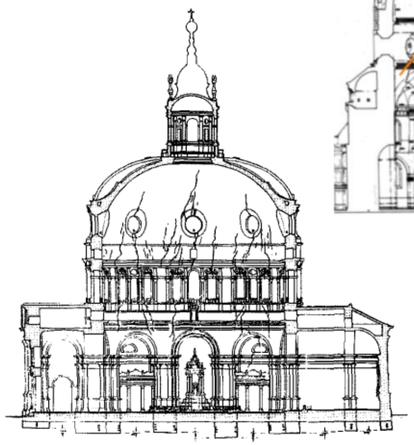
Blasi et al (2014)

Ottoni (2014):



Santa Maria del Fiore, Florence

Garro (1962):



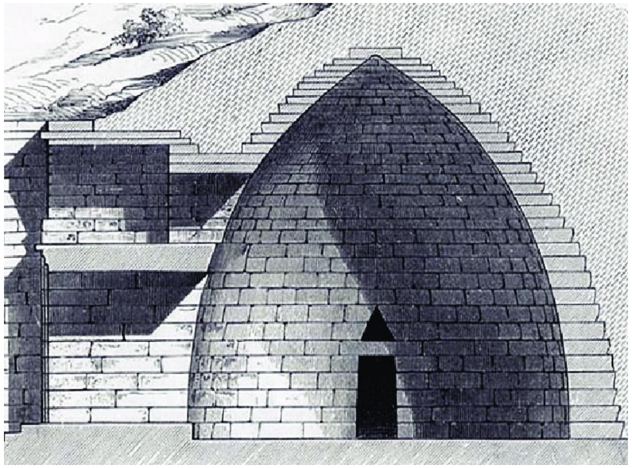
Sanctuary of Vicoforte

Protection against typical cracking

e.g. resistance of the dome neighbourhood:

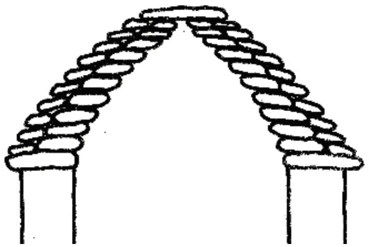
works even for corbel domes:

→ Atreus treasury, BC 1250 :



[simply the neighbouring earth mass takes the lateral forces]

→ „Cardenha" in Northern Portugal, used for shelter since neolith:

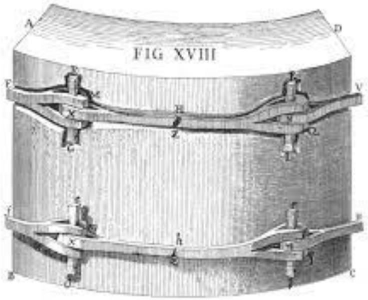


Martynenko (2017)

[heavy, thick dome & support]

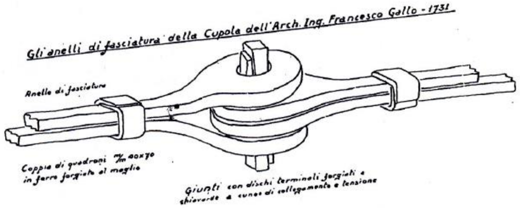
Protection against typical cracking

e.g. iron rings or chains:



St Peter's Basilica, Rome, strengthened by Poleni

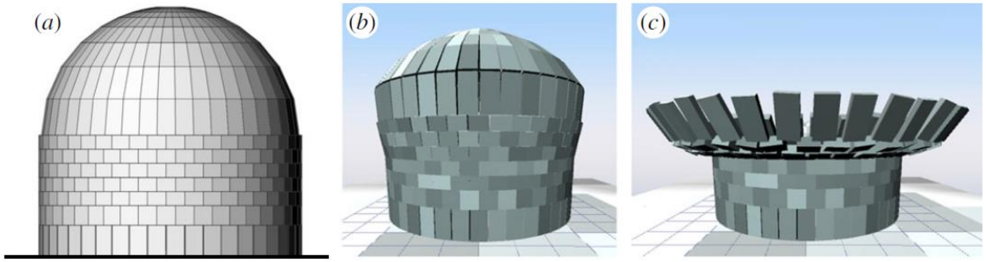
Aoki et al (2004)

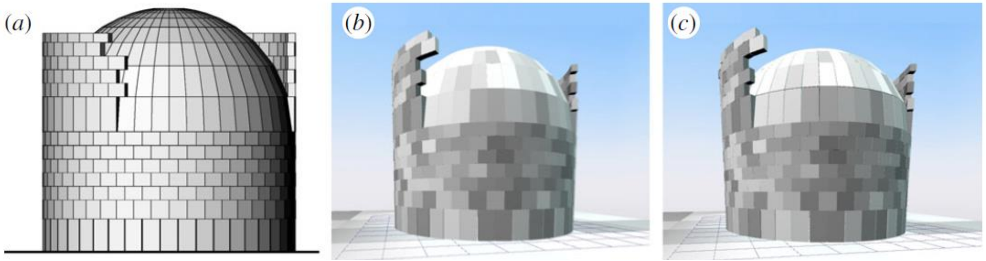


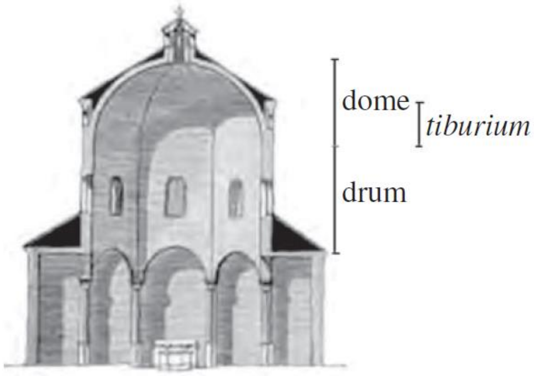
Blasi et al (2014)

Protection against typical cracking

e.g. stabilizing effect of a tiburium:





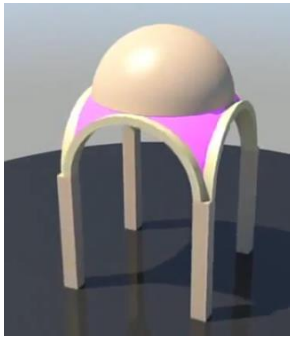
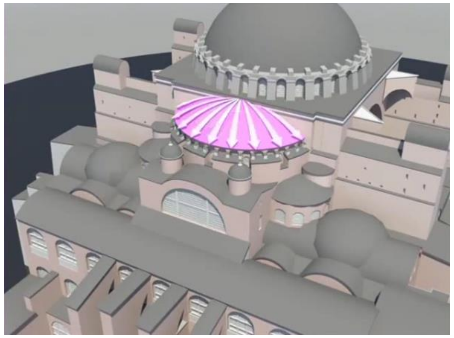
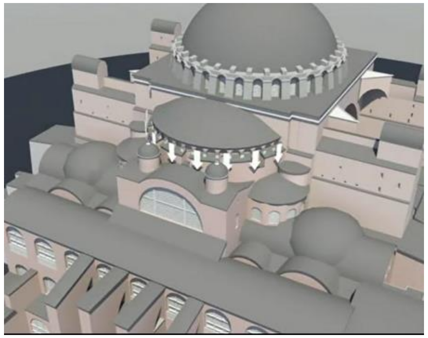
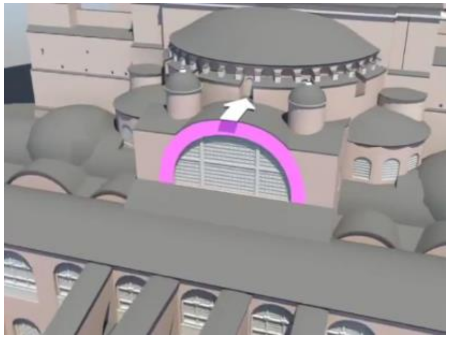
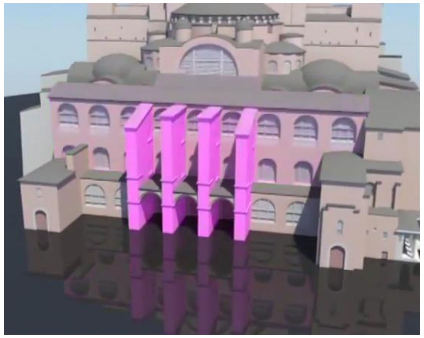


Beatini et al (2018)

Protection against typical cracking

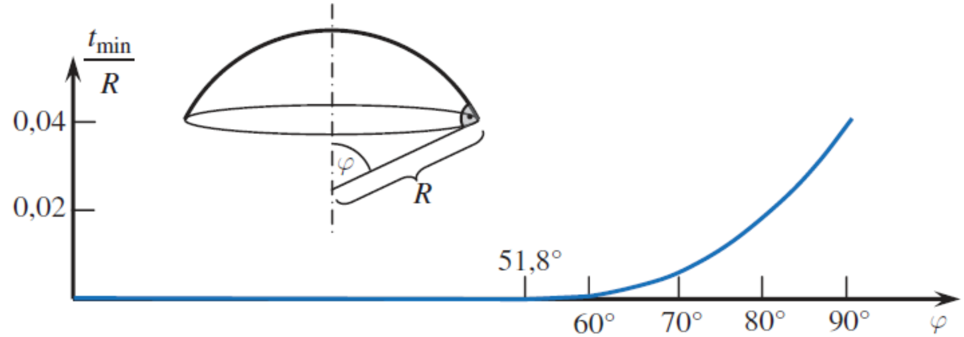
e.g. more sophisticated structural solutions:

Hagia Sofia:

Tension resistance due to crosswise compression

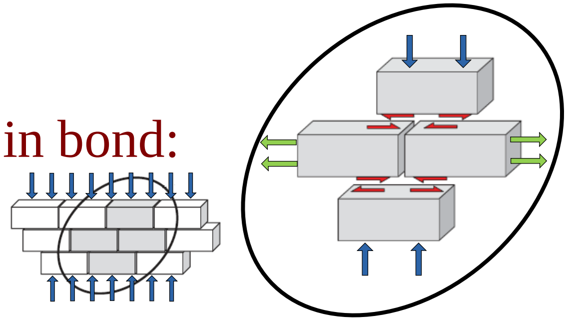
Heyman (1967):



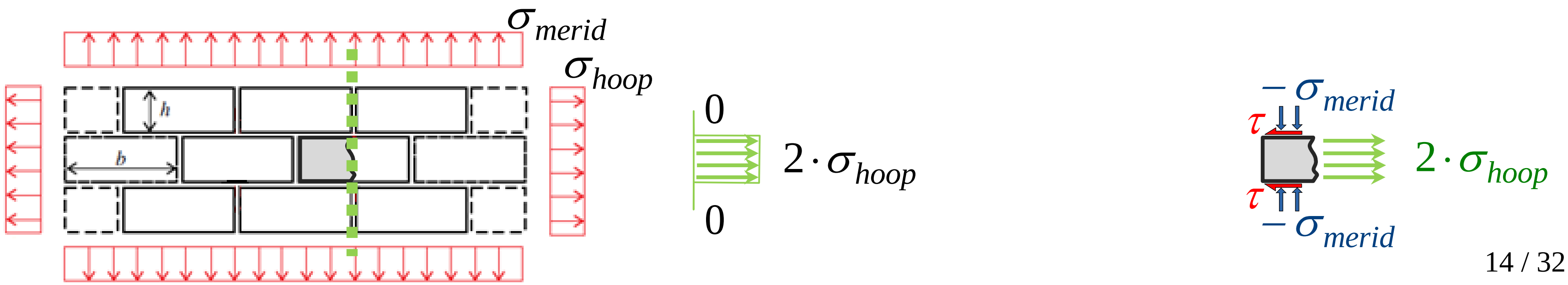
minimally necessary wall thickness for a hemispheral dome: $t_{min} = 0,042 \cdot R$

Simon & Bagi (2016): smaller value found for bricks in bond:

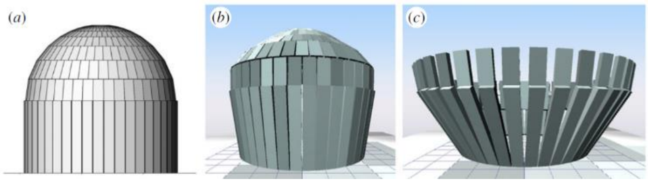
$$t_{min} = 0,037 \cdot R_{middle}$$

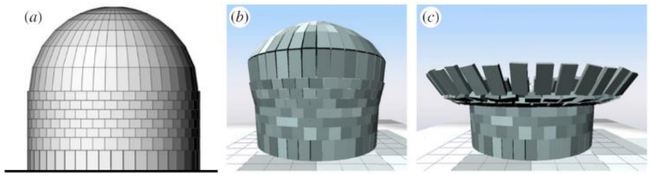


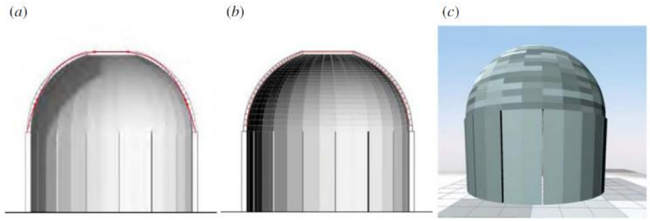
Beatini et al (2018): for simple running bond pattern:



Tension resistance due to crosswise compression







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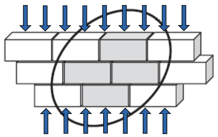
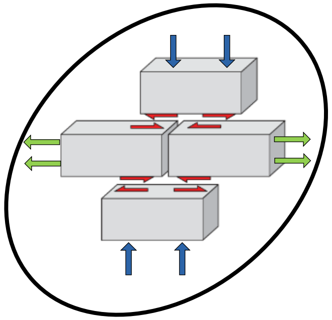
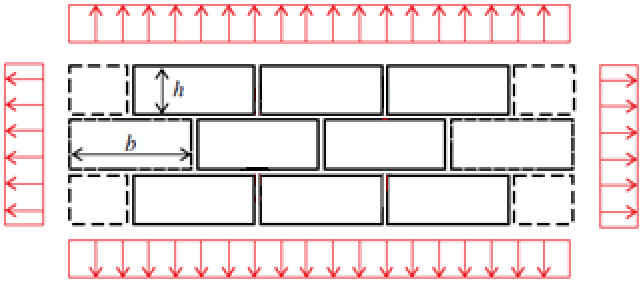




Beatini et al (2018):

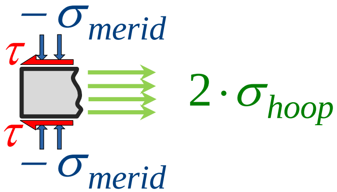


$$\tau \le \mu \cdot (-\sigma_{merid})$$

$$2 \cdot \left(\frac{b}{2} \cdot \tau\right) = h \cdot 2\sigma_{hoop}$$

$$h \cdot 2\sigma_{hoop} \le b \cdot \mu \cdot (-\sigma_{merid})$$

$$\sigma_{hoop} \le \frac{b}{2h} \cdot \mu \cdot (-\sigma_{merid})$$



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OVAL DOMES

← widely applied in the Renaissance & Baroque

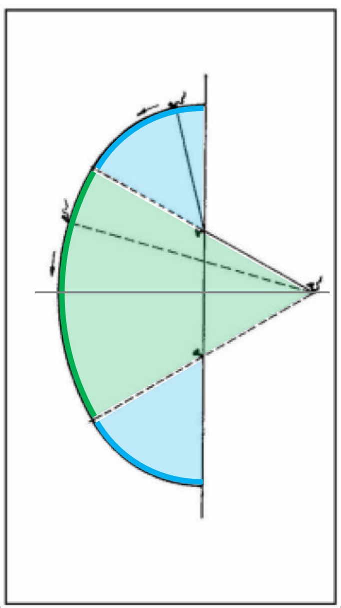
„Oval”:

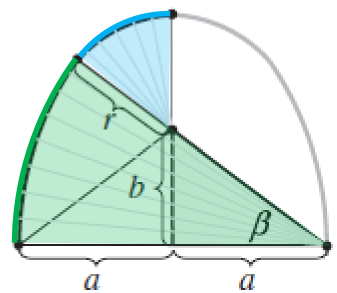
→ closed, convex, smooth curve having two axes of symmetry

→ ≈ an elongated circle, approximate ellipse

<u>Groundplan:</u> (Huerta, 2007)

Method 1: compose it from circular arcs!





„eccentricity angle”, β : deviation from the circle:

→ $b = 0$ or $\beta = 0$: no deviation from the circle

→ $b = \sqrt{3}a$ or $\beta = 60°$: largest possible deviation

OVAL DOMES

„Oval”:

→ closed, convex, smooth curve having

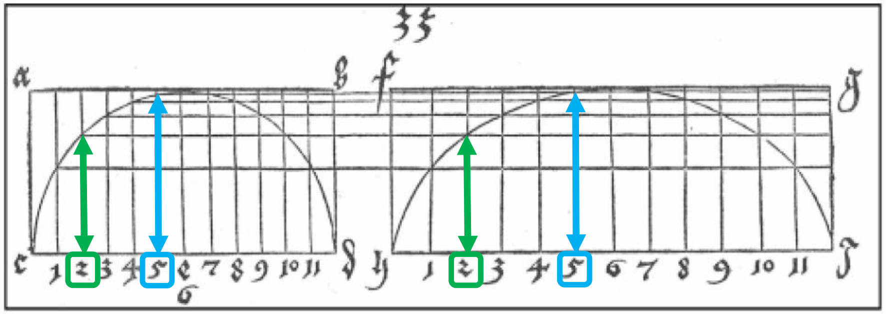
two axes of symmetry

→ ≈ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

Method 2: elongate a circle!

[⇒ ellipse received;
not known at that time]



Method 2 was not used in building practice;

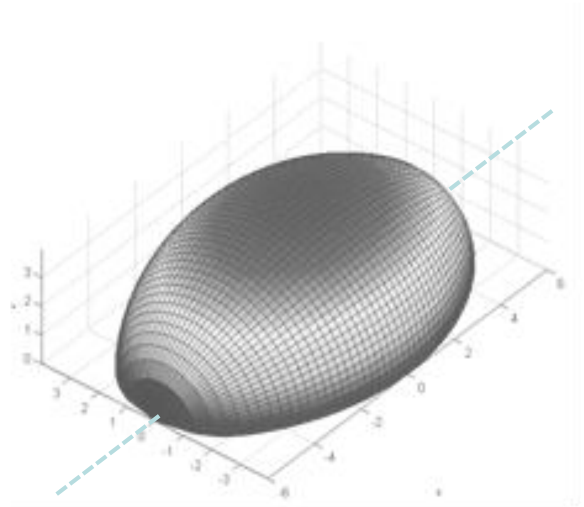
Method 1 could provide an excellent approximation for ellipse

OVAL DOMES

The middle surface of oval domes:

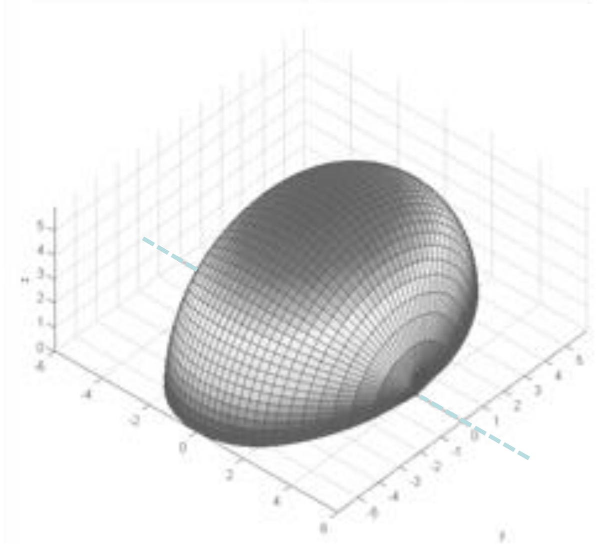
Type 1: „flat domes"

Rotate the groundplan about the longer axis!



Type 2: „high domes"

Rotate the groundplan about the shorter axis!



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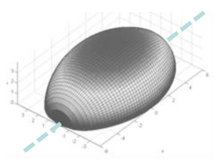
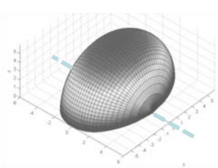


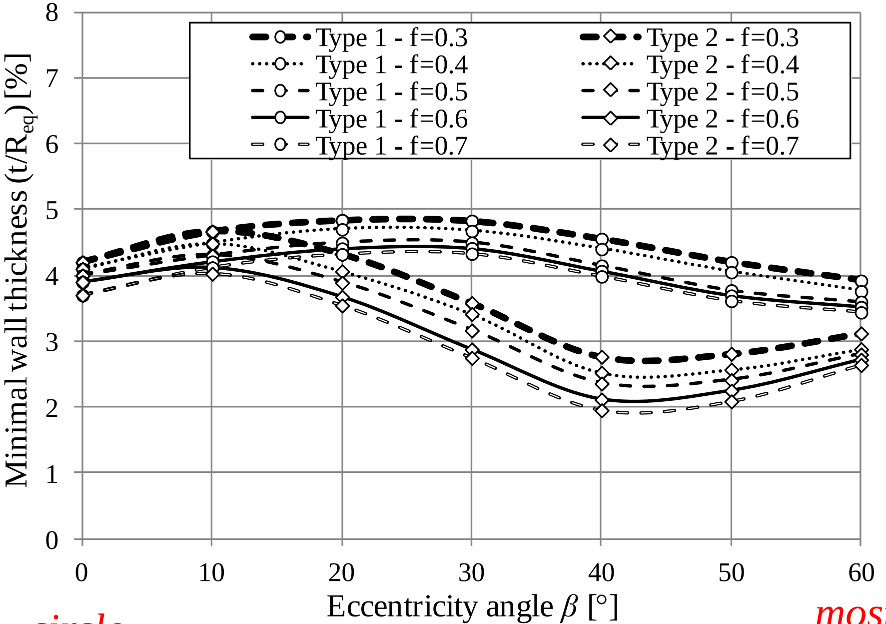
Rotate the groundplan about the longer axis!

Type 2: „high domes"



Rotate the groundplan about the shorter axis!

Simon & Bagi (2016)



Some of the conclusions:

→ Both types may require smaller thickness than a semispherical dome.

→ Type 2 („high") domes are stronger than Type 1 („flat") domes.

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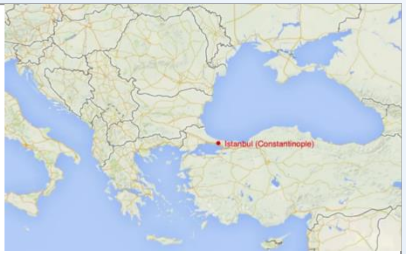
Main Dome of Hagia Sofia, Istanbul



http://www.istanbulturkeybook.com

Cathedral originally built: 537

537 – 1453 Christian church

1453 – 1935 Moslim mosque

1935 – museum

The main dome: 558: previous dome collapsed in earthquake

562: the recent dome (made much higher) is ready

survived several earthquakes

[overview of changes since 537: https://www.youtube.com/watch?v=rFOlOZzO3jY]

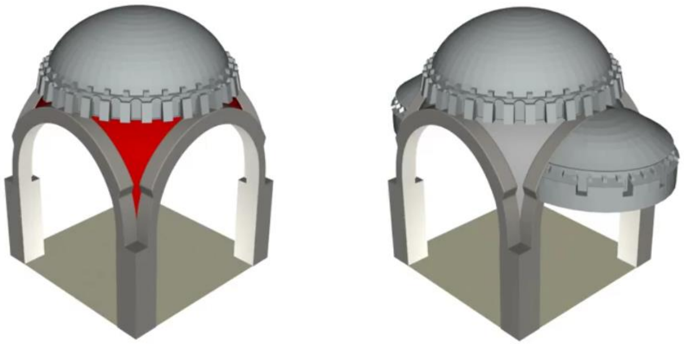
Replacement of the main dome:

Dome of Hagia Sofia, Istanbul

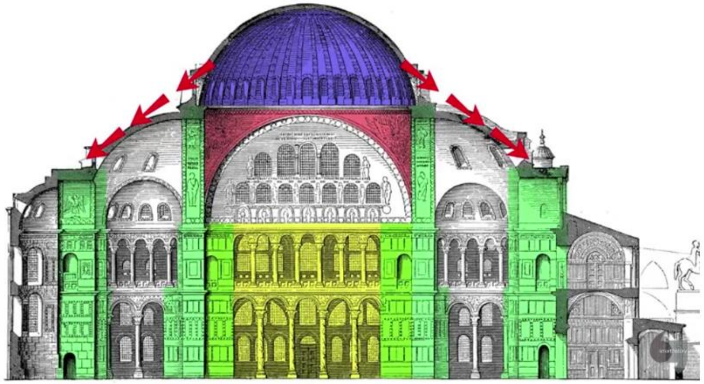
Material: brick and stone

Span: 31 m (largest masonry dome until Brunelleschi's dome in Florence)

Support system: innovation: „pendentives" [sections from a larger sphere] ; complex system of structural units carry the lateral thrust

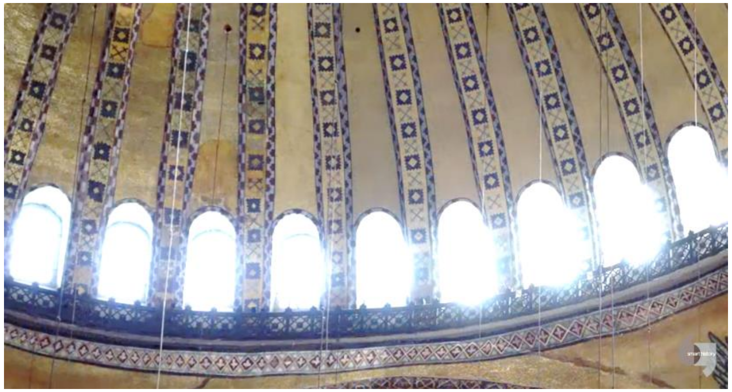


https://www.youtube.com/watch?v=uEKtWii7Vns



https://www.youtube.com/watch?v=XfpusWEd2jE

Dome of Hagia Sofia, Istanbul



https://www.youtube.com/watch?v=XfpusWEd2jE

To prevent cracking due to hoop stress:

→ 40 brick ribs forming the dome

→ between them: arched windows

„suspended from heaven"

https://www.youtube.com/watch?v=XfpusWEd2jE

Suggested videos:

https://www.youtube.com/watch?v=5DTh1c-f1uc (long, history & structural)
https://www.youtube.com/watch?v=XfpusWEd2jE (cooperating struct. units)
https://www.youtube.com/watch?v=S90SMOKeVpA (short, supporting)
https://www.youtube.com/watch?v=uEKtWii7Vns (short, structural system)

Dome of Santa Maria del Fiore, Florence

Beginning of Renaissance, flourishing & competing cities:

Florence cathedral planned still in 14th century,
then it was nearly ready but missed a dome ☺

44 m span [nearly the span of the Pantheon]

https://www.youtube.com/watch?v=_IOPlGPQPuM&feature=youtu.be

Main challenges: HUGE size

→ no external buttressing allowed [no space around]

→ no scaffolding allowed [not enough trees]

aislesalvotimeingh

Can it be solved at all? public competition launched, 1419

won: Filippo Brunelleschi, a goldsmith;
childhood: mathematics and arts;
stubborn, self-confident; kept his ideas top secret

Dome of Santa Maria del Fiore, Florence

https://www.youtube.com/watch?v=_IOPlGPQPuM&t=11s

Brunelleschi's answers:

→ to decrease the lateral thrust:

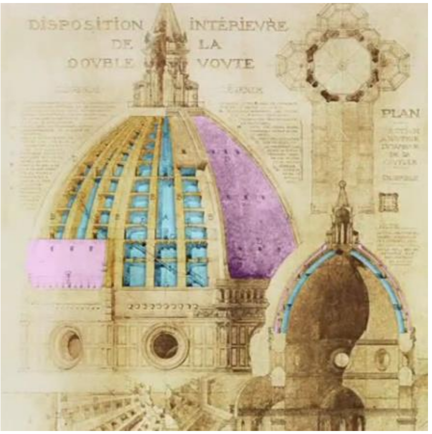
a) make the dome ***pointed***

b) make it ***light***:

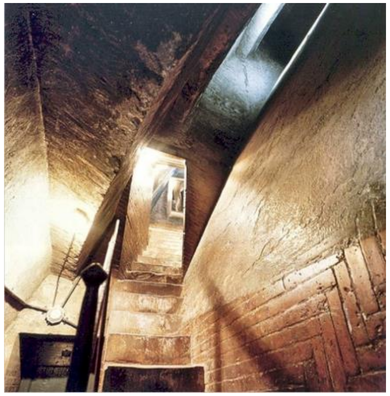
build two domes instead of a single one

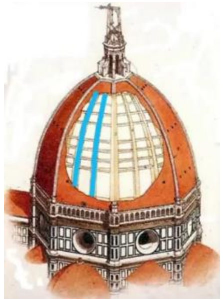
- a thick inner, sandstone and marble dome
- a thin outer, brick dome

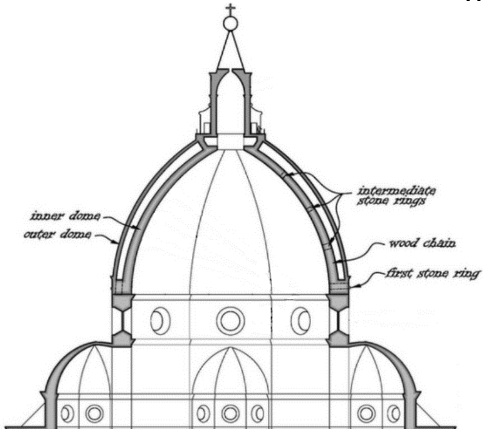
interconnected:



http://www.yousubtitles.com/13-Three-Great-Domes-Rome-to-the-Renaissance-id-1373387







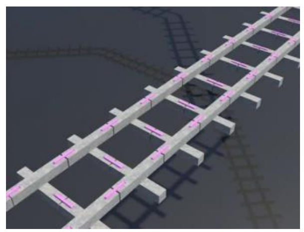
http://florencedome.com/blog.html

Dome of Santa Maria del Fiore, Florence

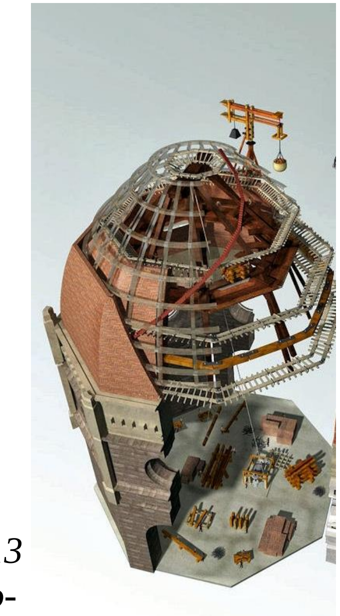
Brunelleschi's answers:

→ to decrease hoop stress & strain:

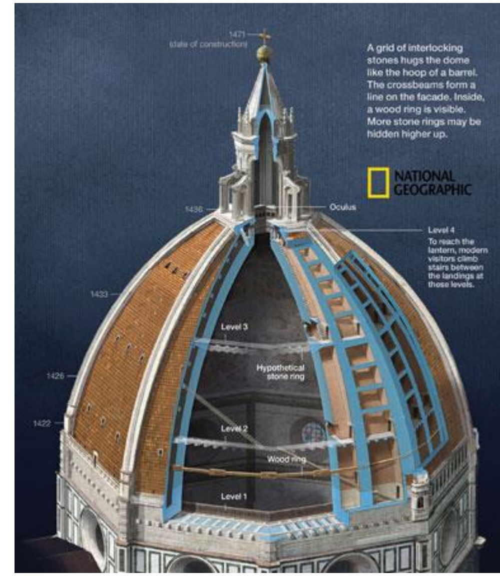
3 stone & 1 wooden „chains"



http://www.yousubtitles.com/13-Three-Great-Domes-Rome-to-the-Renaissance-id-1373387

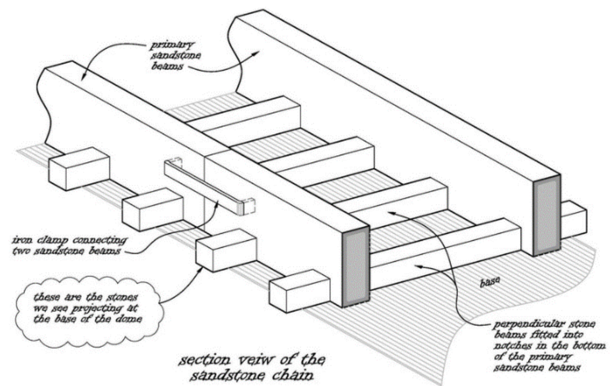


www.teggelaar.com



http://www.digitalmediaworld.tv/in-depth/226-brunelleschi-s-dome

e.g. the lowest stone ring:



http://florencedome.com/blog.html

Why stone?

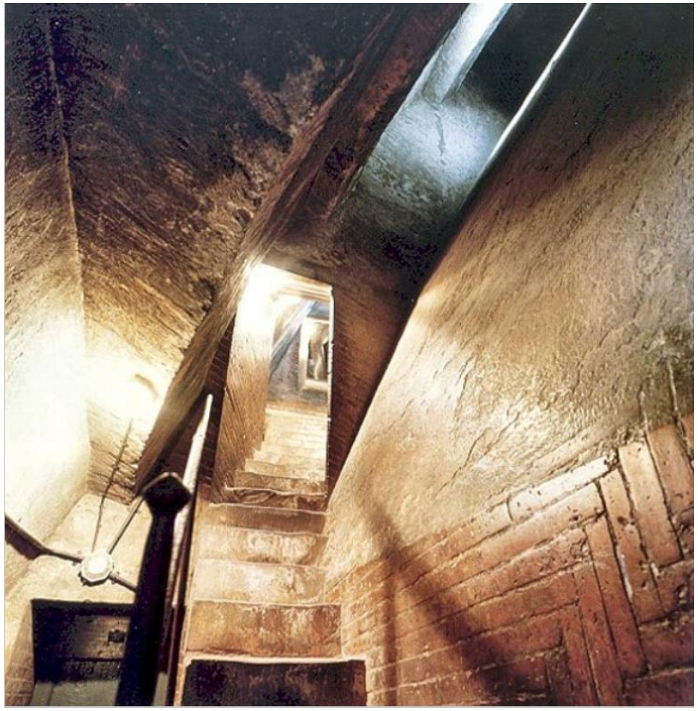
⇐ shortage in iron

Dome of Santa Maria del Fiore, Florence

Brunelleschi's answers:

$\rightarrow$ to build the dome without scaffolding:

a self-supporting construction method is needed!



www.teggelaar.com

https://www.youtube.com/watch?v=kkBaxFuh40E

[herringbone pattern is found in the dome]

Dome of Santa Maria del Fiore, Florence

Brunelleschi's answers:

→ to build the dome without scaffolding:

a self-supporting construction method is needed!

ytaba36.wordpress.com

the huge trick: to use HERRINGBONE PATTERN

⇒ dome successfully built between 1420-1436

Experiment by Jones, Sereni & Massimo Ricci (2010):

https://www.youtube.com/watch?v=kkBaxFuh40E

[Ricci's dome is not ready yet !]

Dome of Santa Maria del Fiore, Florence

Brunelleschi’s answers:

→ to build the dome without scaffolding:

a self-supporting construction method is needed!

Askarov(2004)

the huge trick: to use HERRINGBONE PATTERN

How the idea came?

→ may origin from seljukids (moslim architecture)

→ Venice: applied too, from XIth century, but for very small niches

Brunelleschi may had proved

that the idea works:

[small theatre dome found nearby]

https://www.youtube.com/watch?v=kkBaxFuh40E

Dome of Santa Maria del Fiore, Florence

Suggested videos:

https://www.khanacademy.org/humanities/renaissance-reformation/early-renaissance1/sculpture-architecture-florence/v/brunelleschi-dome-of-the-cathedral-of-florence-1420-36 short

https://www.youtube.com/watch?v=_IOPlGPQPuM&feature=youtu.be ☺ short

https://www.youtube.com/watch?v=QWz90KdrDBs short, on Ricci's results

Homework:

on technical details:
https://www.youtube.com/watch?v=kkBaxFuh40E 51:55
[entitled „Great Cathedral Mystery" ☺]

https://www.youtube.com/watch?v=exrD5RwjlIo&feature=youtu.be&t=93

Questions

1. Recognize from a figure: ***hemispheral*** / ***oval*** / ***pointed dome***. Explain the following terms: ***pendentive***; ***drum***; ***tiburium***; ***corbel dome***; ***herringbone pattern***.

2. Introduce the typical crack pattern of a hemispherical masonry dome without or with a tension ring at its bottom. How can you ***protect*** a spherical dome against its typical cracking modes?

3. How to calculate the ***tension resistance*** in horizontal direction in a dry masonry wall with simple running bond pattern, caused by the vertical compression stress and friction resistance in the horizontal contacts?

4. What are the ***two main geometrical types*** of oval domes? Explain the meaning of the diagram that shows the relation between ***minimally necessary wall thickness*** and the ***eccentricity angle***.