

# MASONRY DOMES







### **Citation:**

K. Bagi (2024): Mechanics of Masonry Structures. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

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### **This lecture**

#### Earliest domes

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses criticism: a masonry dome is not in membrane state typical crack pattern; usual protections against cracking tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul Brunelleschi's dome in Florence

Questions

Heyman (1995): "a rounded vault forming a roof" [ no generally accepted definition]

Mezhyrich / Межиріч, Ukraine: remains of four mammoth bone huts ≈ 15.000-14.000 BC; diameters 4-6 m



A Gregorovich, http://209.82.14.226/history/inventions/



http://images.fineartamerica.com/imag es-medium-large/mammoth-bone-hutexcavation-ukraine-ria-novosti.jpg

#### similar other sites Dobranichivka, Gontsy, Ioudinovo ? Cracow, Poland ?

Maltar Corbel domes from Neolith from ≈3600 BC e.g. Hagar Qim: bending (instead of ≈ pure compression)



#### Malta: Corbel domes from Neolith from ≈3600 BC





*Reconstruction of the north temple roof of megalithic temple complex Hagar Qim (3600–3200 BC) Malta. https://commons.wikimedia.org/wiki/File:Couverture\_du\_temple\_Hagar\_Qim.jpg?uselang=fr* 

#### Malta: Corbel domes from Neolith from ≈3600 BC







Barratt (2022) (hypothesis): sometimes stone roofing, sometimes wood, sometimes combined





Everywhere in the Mediterranean:

Corbel dome huts, from Neolith till today

Martynenko, 2017:





Greece: Tholos, Thessaly



Croatia: Kazun, Istria



Italy: Casella, Apula 7/46

Everywhere in the Mediterranean:

Corbel dome huts, from Neolith:

Martynenko, 2017:











### **CORBEL DOMES**

#### Static analysis:

Line of thrust Profile of dome Profile of corbel domes? given: density; height

[thrust line is a very wrong approximation]

1. "Classic Corbelling Theory" (CT) Benvenuto & Corradi, 1987

- 2. "Modified Corbelling Theory" (MCT) Rovero & Tonietti, 2014
- 3. "New Formulation of MCT" (NFMCT) *Foti et al, 2016*
- 4. "Further Refinement of the Corbelling Theory" *Fraddosio et al, 2019*



### **CORBEL DOMES**

Profile of corbel domes? Static analysis: given: density; height

- 1. "Classic Corbelling Theory" (*CT*) Benvenuto & Corradi, 1987
  - $\rightarrow$  separate lunes, no hoop forces
  - $\rightarrow$  the lune consists of horizontal layers
  - $\rightarrow$  rigid blocks
  - $\rightarrow$  no sliding

Outcome: Differential eqs for  $r_{int}$  and  $r_{ext}$ 

2. "Modified Corbelling Theory" (MCT) Rovero & Tonietti, 2014

 $\rightarrow$  Classic Corbelling Theory + hoop forces (,, $\phi$ '')





4. "Further Refinement of the Corbelling Theory" *Fraddosio et al, 2019*→ upfill also taken into account

### **CORBEL DOMES**

Static analysis: Profile of corbel domes? given: density; height

4. "Further Refinement of the Corbelling Theory"  $z = \frac{1}{2} \int z Fraddosio \ et \ al, \ 2019$  $\rightarrow$  upfill also taken into account













### **TRUE DOME SHAPES**

Heyman (1995): "a rounded vault forming a roof" [ no generally accepted definition]

Shapes: huge variety









**O**val



Hemispherical Segmental

Faceted

Pointed

Bulbous etc.

How to support it:



### **This lecture**

Earliest domes

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses criticism: a masonry dome is not in membrane state typical crack pattern; usual protections against cracking tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul Brunelleschi's dome in Florence

Questions

## **Repetition:** Membrane solution for spherical domes $\sqrt[q]{}^q$

 $\sigma_m$ 

 $\sigma_{\scriptscriptstyle h}$ 

*m*: meridional direction*h*: hoop direction

Predicted crack pattern:

lateral thrust! (



## **Repetition:** Membrane solution for spherical domes $\int_{a}^{q} \checkmark$

*m*: meridional direction*h*: hoop direction

**Importance of boundaries:** 



membrane state



 $\sigma_m$ 

 $\sigma_{\scriptscriptstyle h}$ 

membrane state



### **Criticism:** Not a membrane state

The ideal membrane dome shape for pure selfweight & uniform thickness:

$$y = \frac{1}{2} x_0 \left[ \int_0^{x/x_0} e^{t^2} dt - \int_0^{x/x_0} e^{-t^2} dt \right]$$

$$y = (0.7206) [x^{3} + (0.3338) x^{7} + (0.0496) x^{11} + (0.0041) x^{15} + (0.0002) x^{19} + \cdots]$$
(12)

#### Consequence:

- $\Rightarrow$  In real domes the thrust does
  - not run along the middle surface
  - $\Rightarrow$  membrane solution:

only a poor first approximation!



![](_page_17_Figure_10.jpeg)

![](_page_18_Picture_0.jpeg)

#### (Stephen Ressler, West Point)

### **Typical crack pattern under selfweight**

![](_page_19_Picture_1.jpeg)

#### (Stephen Ressler, West Point)

 $https://www.youtube.com/watch?v=cgzh0YfESbA\&list=PLKd0u75kvQExZb0jrI-lr_CpBQegfPo-m&index=1Q_{9/46}$ 

### **Typical crack pattern under selfweight**

![](_page_20_Picture_1.jpeg)

![](_page_20_Picture_2.jpeg)

Atamturktur et al (2012)

#### $\Rightarrow$ crack pattern depends on the stiffness of bottom support

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### **Remark: Direction of the reactions**

also depends on the **stiffness of bottom support**: Chen&Bagi, 2023:

![](_page_21_Figure_2.jpeg)

e.g. under the ground: resistance of neighbouring earth mass

![](_page_22_Picture_2.jpeg)

#### $\rightarrow$ Atreus treasury, BC 1250 :

![](_page_22_Picture_4.jpeg)

#### e.g. iron rings or chains:

![](_page_23_Picture_2.jpeg)

![](_page_23_Picture_3.jpeg)

#### St Peter's Basilica, Rome, strengthened by Poleni

![](_page_23_Picture_5.jpeg)

*Aoki et al (2004)* 

![](_page_23_Picture_7.jpeg)

![](_page_23_Picture_8.jpeg)

Blasi et al (2014)

#### e.g. iron rings or chains:

#### Chen&Bagi, 2023:

![](_page_24_Figure_3.jpeg)

#### e.g. stabilizing effect of a tiburium:

![](_page_25_Picture_2.jpeg)

![](_page_25_Picture_3.jpeg)

Beatini et al (2018)

![](_page_25_Picture_5.jpeg)

e.g. more sophisticated structural solutions:

Hagia Sofia:

![](_page_26_Picture_3.jpeg)

![](_page_26_Picture_4.jpeg)

![](_page_26_Picture_5.jpeg)

![](_page_26_Picture_6.jpeg)

![](_page_26_Picture_7.jpeg)

![](_page_26_Picture_8.jpeg)

### **Protection against typical cracking: Tensile** resistance due to crosswise compression

![](_page_27_Figure_1.jpeg)

### **Protection against typical cracking: Tensile resistance due to crosswise compression**

![](_page_28_Picture_1.jpeg)

![](_page_28_Picture_2.jpeg)

Chen & Bagi (2020): Hemisphere: brick  $\approx 1:4 \dots 1:5 \Rightarrow$  no cracking Simon & Bagi (2016): smaller value found for bricks in bond:  $t_{\min} = 0,037 \cdot R_{middle}$ Beatini et al (2018):  $\tau \leq \mu \cdot (-\sigma_{merid})$   $2 \cdot (\frac{b}{2}\tau) = h \cdot 2\sigma_{hoop}$   $h \cdot 2\sigma_{hoop} \leq b \cdot \mu \cdot (-\sigma_{merid})$   $\sigma_{hoop} \leq \frac{b}{2h} \cdot \mu \cdot (-\sigma_{merid})$  $\sigma_{hoop} \leq \frac{b}{2h} \cdot \mu \cdot (-\sigma_{merid})$ 

### **This lecture**

What is a masonry dome?

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses typical crack pattern; usual protection against cracking criticism: a masonry dome is not in membrane state tension resistance due to crosswise compression

Oval domes

Most famous masonry domes: Hagia Sofia, Istanbul Brunelleschi's dome in Florence

## ← widely applied in the Renaissance & Baroque

#### "Oval":

- $\rightarrow$  closed, convex, smooth curve having
  - two axes of symmetry
- $\rightarrow \approx$  an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

![](_page_30_Picture_7.jpeg)

"Oval": Method 1: compose it from circular arcs!

![](_page_30_Figure_9.jpeg)

![](_page_30_Figure_10.jpeg)

,,eccentricity angle",  $\beta$ : deviation from the circle:

→ b = 0 or  $\beta = 0$ : no deviation from the circle →  $b = \sqrt{3}a$  or  $\beta = 60^\circ$ : largest possible deviation

"Oval":

 → closed, convex, smooth curve having two axes of symmetry
 → ≈ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

"Oval": Method 2: elongate a circle!

![](_page_31_Picture_5.jpeg)

Leonardo da Vinci, Codex Atlanticus, ≈1510

![](_page_31_Picture_7.jpeg)

![](_page_31_Figure_8.jpeg)

Albrecht Dürer, 1525

[ $\Rightarrow$  ellipse received; not used at that time for construction]

The middle surface of oval domes:

<u>Type 1:</u> "flat domes" *Rotate the groundplan about the longer axis!* 

![](_page_32_Figure_3.jpeg)

<u>Type 2:</u> "high domes" *Rotate the groundplan about the shorter axis!* 

![](_page_33_Picture_1.jpeg)

The middle surface of oval domes:

Type 1: "flat domes"

![](_page_33_Picture_4.jpeg)

Rotate the groundplan about the longer axis!

<u>Type 2:</u> "high domes" *Rotate the groundplan about the shorter axis!*  Simon & Bagi (2016)

![](_page_33_Figure_8.jpeg)

Some of the conclusions:

 $\rightarrow$  Both types may require smaller thickness than a semispherical dome.

 $\rightarrow$  Type 2 (,,high") domes are stronger than Type 1 (,,flat") domes.

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#### Failure modes:

![](_page_34_Picture_2.jpeg)

<u>Type 1:</u> "flat domes" *Rotate the groundplan about the longer axis!* 

![](_page_34_Figure_4.jpeg)

(a) Type 1 dome

![](_page_34_Picture_6.jpeg)

Type 2: "high domes"Rotate the groundplanabout the shorter axis!

![](_page_34_Figure_8.jpeg)

![](_page_34_Figure_9.jpeg)

failure for too small t

failure for too low fric

### **This lecture**

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### Main Dome of Hagia Sofia, Istanbul

![](_page_36_Picture_1.jpeg)

![](_page_36_Picture_2.jpeg)

http://www.istanbulturkeybook.com

Cathedral originally built: 537 537 – 1453 Christian church 1453 – 1935 Moslim mosque 1935 – museum

The main dome: 558: previous dome collapsed in earthquake 562: the recent dome (made much higher) is ready survived several earthquakes

[ overview of changes since 537: https://www.youtube.com/watch?v=rFOlOZzO3jY ]

Replacement of the main dome:

![](_page_36_Picture_8.jpeg)

![](_page_36_Picture_9.jpeg)

![](_page_36_Picture_10.jpeg)

### **Dome of Hagia Sofia, Istanbul**

Material: brick and stone

Span: 31 m (largest masonry dome until Brunelleschi's dome in Florence)

<u>Support system:</u> innovation: ,,pendentives" [ sections from a larger sphere ] ; complex system of structural units carry the lateral thrust

![](_page_37_Picture_4.jpeg)

https://www.youtube.com/watch?v=uEKtWii7Vns

![](_page_37_Picture_6.jpeg)

![](_page_37_Picture_7.jpeg)

https://www.youtube.com/watch?v=XfpusWEd2jE

### **Dome of Hagia Sofia, Istanbul**

<u>To prevent cracking due to hoop stress:</u> → 40 brick ribs forming the dome → between them: arched windows

![](_page_38_Picture_2.jpeg)

"suspended from heaven"

![](_page_38_Picture_4.jpeg)

https://www.youtube.com/watch?v=XfpusWEd2jE

https://www.youtube.com/watch?v=XfpusWEd2jE

#### Suggested videos:

https://www.youtube.com/watch?v=5DTh1c-f1uc (long, history & structural) https://www.youtube.com/watch?v=XfpusWEd2jE (cooperating struct. units) https://www.youtube.com/watch?v=S90SMOKeVpA (short, supporting) https://www.youtube.com/watch?v=uEKtWii7Vns (short, structural system)38/46

Beginning of Renaissance, flourishing & competing cities:
Florence cathedral planned still in 14th century, then it was nearly ready but missed a dome <sup>(2)</sup>
44 m span [nearly the span of the Pantheon]

![](_page_39_Picture_2.jpeg)

![](_page_39_Picture_3.jpeg)

https://www.youtube.com/watch?v= \_IOPlGPQPuM&feature=youtu.be

#### Main challenges: HUGE size

- → no external buttressing allowed [ no space around ]
- → no scaffolding allowed [ not enough trees ]

#### aislesalvotimeingh

![](_page_39_Picture_9.jpeg)

Can it be solved at all? public competition launched, 1419 won: Filippo Brunelleschi, a goldsmith; childhood: mathematics and arts; stubborn, self-confident; kept his ideas top secret

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https://www.youtube.com/watch?v=\_IOPlGPQPuM&t=11s

Brunelleschi's answers:

- $\rightarrow$  to decrease the lateral thrust:
  - a) make the dome *pointed*
  - b) make it *light*:

build two domes instead of a single one

- a thick inner, sandstone and marble dome
- a thin outer, brick dome interconnected:

![](_page_40_Picture_9.jpeg)

![](_page_40_Picture_10.jpeg)

http://florencedome.com/blog.html

![](_page_40_Picture_12.jpeg)

http://www.yousubtitles.com/13-Three-Great-Domes-Rome-tothe-Renaissance-id-1373387

![](_page_40_Picture_14.jpeg)

#### Brunelleschi's answers:

- $\rightarrow$  to decrease hoop stress & strain:
  - 3 stone & 1 wooden "chains"

![](_page_41_Picture_4.jpeg)

![](_page_41_Picture_5.jpeg)

http://www.yousubtitles.com/13 -Three-Great-Domes-Rome-tothe-Renaissance-id-1373387

![](_page_41_Picture_7.jpeg)

![](_page_41_Figure_8.jpeg)

#### www.teggelaar.com

http://www.digitalmediaworld.tv/indepth/226-brunelleschi-s-dome

e.g. the lowest stone ring:

![](_page_41_Figure_13.jpeg)

http://florencedome.com/blog.html

Brunelleschi's answers:

 $\rightarrow$  to build the dome without scaffolding:

a self-supporting construction method is needed!

![](_page_42_Picture_4.jpeg)

![](_page_42_Picture_5.jpeg)

https://www.youtube.com/watch?v=kkBaxFuh40E

[ herringbone pattern is found in the dome ]

www.teggelaar.com

Brunelleschi's answers:

 $\rightarrow$  to build the dome without scaffolding:

a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN  $\Rightarrow$  dome successfully built between 1420-1436

![](_page_43_Picture_5.jpeg)

ytaba36.wordpress.com

Experiment by Jones, Sereni & Massimo Ricci (2010):

![](_page_43_Picture_8.jpeg)

https://www.youtube.com/watch?v=kkBaxFuh40E

![](_page_43_Picture_10.jpeg)

#### [ Ricci's dome is unfinished ]

Brunelleschi's answers:

 $\rightarrow$  to build the dome without scaffolding:

a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN

How the idea came?

- $\rightarrow$  may origin from seljukids (moslim architecture)
- $\rightarrow$  Venice: applied too, from XIth century, but for very small niches

![](_page_44_Picture_8.jpeg)

Askarov(2004)

Brunelleschi may had proved that the idea works: [small theatre dome nearby, found in 2012]

![](_page_44_Picture_10.jpeg)

https://www.youtube.com/watch?v=kkBaxFuh40E 44/46

Suggested videos:

https://www.khanacademy.org/humanities/renaissance-reformation/earlyrenaissance1/sculpture-architecture-florence/v/brunelleschi-dome-of-thecathedral-of-florence-1420-36 short

https://www.youtube.com/watch?v=\_IOPIGPQPuM&feature=youtu.be ③ short https://www.youtube.com/watch?v=QWz90KdrDBs short, on Ricci's results

Home study:

on technical details:

https://www.youtube.com/watch?v=RUBnNDloGHg,

51:55 [entitled "Great Cathedral Mystery" ©]

on the nearby small "test dome":

https://www.youtube.com/watch?v=RUBnNDlo GHg

![](_page_45_Picture_10.jpeg)

https://www.youtube.com/watch?v=ex rD5RwjlIo&feature=youtu.be&t=93

### Questions

- 1. Recognize from a figure: *hemispheral / oval / pointed dome*. Explain the following terms: *pendentive*; *drum*; *tiburium*; *corbel dome*; *herringbone pattern*.
- 2. What is a *corbel dome*? What theorical predictions exist about the shape of their intrados and extrados?
- 3. Introduce the typical crack pattern of a hemispherical masonry dome without or with a tension ring at its bottom. How can you *protect* a spherical dome against its typical cracking modes?
- 4. How to calculate the *tensile resistance* in horizontal direction in a dry masonry wall with simple running bond pattern, caused by the vertical compression stress and friction resistance in the horizontal contacts?
- 5. What are the *two main geometrical types* of oval domes? Explain the meaning of the diagram that shows the relation between *minimally necessary wall thickness* and the *eccentricity angle*.