MASONRY DOMES
This lecture

What is a masonry dome?

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses

criticism: a masonry dome is not in membrane state

typical crack pattern; usual protection against cracking

tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul

Brunelleschi’s dome in Florence
WHAT IS A MASONRY DOME?

Heyman (1995): „a rounded vault forming a roof”

[ no generally accepted definition]

Shapes: huge variety

Hemispherical  Segmental  Faceted  Pointed  Oval  Bulbous  etc.

How to support it: e.g. dome rests on a drum  e.g. dome rests on pendentives

Beatini et al (2018)

http://sridhistory.blogspot.com/
This lecture

What is a masonry dome?

**Spherical domes**

repetition of membrane solution:

distribution of meridian and hoop stresses

typical crack pattern; usual protection against cracking

criticism: a masonry dome is not in membrane state

tension resistance due to crosswise compression

Oval domes

**Most famous masonry domes:**

Hagia Sofia, Istanbul

Brunelleschi’s dome in Florence
**Repetition: Membrane solution for spherical domes**

- $m$: meridional direction
- $h$: hoop direction

Predicted crack pattern:

- lateral thrust!

\[ \sigma_m = -\frac{q}{t} R \frac{1}{1 + \cos \varphi} \]
\[ \sigma_h = \frac{q}{t} R \left( \cos \varphi - \frac{1}{1 + \cos \varphi} \right) \]
Repetition: Membrane solution for spherical domes

\( m: \) meridional direction
\( h: \) hoop direction

**Importance of boundaries:**

\[
s_{m} = -\frac{q}{t} \frac{1}{R (1 + \cos \varphi)}
\]

\[
s_{h} = \frac{q}{t} R \left( \cos \varphi - \frac{1}{(1 + \cos \varphi)} \right)
\]
Criticism: Not a membrane state

The ideal dome shape for pure selfweight & uniform thickness:

\[ y = \frac{1}{2} x_0 \left[ \int_0^{x_0} e^2 \, dt - \int_0^{x_0} e^{-2} \, dt \right] \]

\[ y = (0.7206) [x^3 + (0.3338) x^7 + (0.0496) x^{11} + (0.0041) x^{15} + (0.0002) x^{19} + \ldots] \]  

Consequence:

⇒ In real domes the thrust does not run along the middle surface

⇒ membrane solution: only a poor first approximation!

Heyman (1998)

e.g Beatini et al (2018)
Typical crack pattern under weight

(Stephen Ressler, West Point)

Typical crack pattern under weight

(Stephen Ressler, West Point)

Typical crack pattern under weight

Ottoni (2014):
Santa Maria del Fiore, Florence

Garro (1962):
Sanctuary of Vicoforte

Blasi et al (2014)

Atamurktur et al (2012)
Protection against typical cracking

e.g. resistance of the dome neighbourhood:

works even for corbel domes:
→ Atreus treasury, BC 1250:

[simply the neighbouring earth mass takes the lateral forces]

→ „Cardenha” in Northern Portugal, used for shelter since neolith:

[heavy, thick dome & support]

Martynenko (2017)
Protection against typical cracking

e.g. iron rings or chains:

St Peter’s Basilica, Rome, strengthened by Poleni


Blasi et al (2014)
Protection against typical cracking

e.g. stabilizing effect of a tiburium:

Beatini et al (2018)
Protection against typical cracking

e.g. more sophisticated structural solutions:

Hagia Sofia:
Tension resistance due to crosswise compression

Heyman (1967):

minimally necessary wall thickness for a hemispherical dome: \( t_{\text{min}} = 0.042 \cdot R \)

Simon & Bagi (2016): smaller value found for bricks in bond:

\( t_{\text{min}} = 0.037 \cdot R_{\text{middle}} \)

Beatini et al (2018): for simple running bond pattern:
Tension resistance due to crosswise compression

Simon & Bagi (2016): smaller value found for bricks in bond:
\[ t_{\text{min}} = 0.037 \cdot R_{\text{middle}} \]

Beatini et al (2018):
\[ \tau \leq \mu \cdot (-\sigma_{\text{merid}}) \]
\[ 2 \cdot \left( \frac{b}{2} \cdot \tau \right) = h \cdot 2\sigma_{\text{hoop}} \]
\[ h \cdot 2\sigma_{\text{hoop}} \leq b \cdot \mu \cdot (-\sigma_{\text{merid}}) \]
\[ \sigma_{\text{hoop}} \leq \frac{b}{2h} \cdot \mu \cdot (-\sigma_{\text{merid}}) \]
This lecture

What is a masonry dome?

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses

typical crack pattern; usual protection against cracking

criticism: a masonry dome is not in membrane state

tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul

Brunelleschi’s dome in Florence
OVAL DOMES

„Oval“:
→ closed, convex, smooth curve having two axes of symmetry
→ ≈ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

Method 1: compose it from circular arcs!

„eccentricity angle“, $\beta$:
development from the circle:

→ $b = 0$ or $\beta = 0$: no deviation from the circle
→ $b = \sqrt{3}a$ or $\beta = 60^\circ$: largest possible deviation

← widely applied in the Renaissance & Baroque
OVAL DOMES

„Oval”:
→ closed, convex, smooth curve having
two axes of symmetry
→ ≈ an elongated circle, approximate ellipse

Groundplan: (Huerta, 2007)

Method 2: elongate a circle!

[ ⇒ ellipse received;
  not known at that time ]

Method 2 was not used in building practice;
Method 1 could provide an excellent approximation for ellipse
OVAL DOMES

The middle surface of oval domes:

**Type 1**: „flat domes”

*Rotate the groundplan about the longer axis!*

**Type 2**: „high domes”

*Rotate the groundplan about the shorter axis!*
OVAL DOMES

The middle surface of oval domes:

**Type 1:** „flat domes”  
*Rotate the groundplan about the longer axis!*

**Type 2:** „high domes”  
*Rotate the groundplan about the shorter axis!*

Some of the conclusions:

→ Both types may require smaller thickness than a semispherical dome.  
→ Type 2 („high”) domes are stronger than Type 1 („flat”) domes.
This lecture

What is a masonry dome?

Spherical domes

repetition of membrane solution:

distribution of meridian and hoop stresses

typical crack pattern; usual protection against cracking

criticism: a masonry dome is not in membrane state

tension resistance due to crosswise compression

Oval domes

Most famous masonry domes:

Hagia Sofia, Istanbul

Brunelleschi’s dome in Florence
Main Dome of Hagia Sofia, Istanbul

Cathedral originally built: 537
537 – 1453 Christian church
1453 – 1935 Moslim mosque
1935 – museum

http://www.istanbulturkeybook.com

The main dome:  558: previous dome collapsed in earthquake
562: the recent dome (made much higher) is ready
survived several earthquakes
[ overview of changes since 537: https://www.youtube.com/watch?v=rFOIOZzO3jY ]

Replacement of the main dome:
Dome of Hagia Sofia, Istanbul

**Material:** brick and stone

**Span:** 31 m (largest masonry dome until Brunelleschi’s dome in Florence)

**Support system:** innovation: „pendentives” [sections from a larger sphere]; complex system of structural units carry the lateral thrust

https://www.youtube.com/watch?v=uEKtWii7Vns

https://www.youtube.com/watch?v=XfpusWEd2jE
Dome of Hagia Sofia, Istanbul

To prevent cracking due to hoop stress:

→ 40 brick ribs forming the dome
→ between them: arched windows

„suspended from heaven”

Suggested videos:

https://www.youtube.com/watch?v=5DTh1c-f1uc (long, history & structural)
https://www.youtube.com/watch?v=XfpusWEd2jE (cooperating struct. units)
https://www.youtube.com/watch?v=S90SMOKeVpA (short, supporting)
https://www.youtube.com/watch?v=uEKtWii7Vns (short, structural system)
Dome of Santa Maria del Fiore, Florence

Beginning of Renaissance, flourishing & competing cities:
- Florence cathedral planned still in 14th century,
- then it was nearly ready but missed a dome 😊
- 44 m span [ nearly the span of the Pantheon ]

Main challenges: HUGE size
- → no external buttressing allowed [ no space around ]
- → no scaffolding allowed [ not enough trees ]

Can it be solved at all? public competition launched, 1419
- won: Filippo Brunelleschi, a goldsmith;
  childhood: mathematics and arts;
  stubborn, self-confident; kept his ideas top secret

https://www.youtube.com/watch?v= _IOPlGPQPuM&feature=youtu.be
Dome of Santa Maria del Fiore, Florence

https://www.youtube.com/watch?v=_IOPlGPQPuM&t=11s

Brunelleschi’s answers:

→ to decrease the lateral thrust:

a) make the dome pointed

b) make it light:

build two domes instead of a single one

• a thick inner, sandstone and marble dome
• a thin outer, brick dome

interconnected:
Dome of Santa Maria del Fiore, Florence

Brunelleschi’s answers:
→ to decrease hoop stress & strain:
  3 stone & 1 wooden „chains“

www.teggelaar.com

http://www.digitalmediaworld.tv/in-depth/226-brunelleschi-s-dome

e.g. the lowest stone ring:

Why stone?
⇐ shortage in iron

http://florencecdome.com/blog.html
Dome of Santa Maria del Fiore, Florence

Brunelleschi’s answers:

→ to build the dome without scaffolding:
  a self-supporting construction method is needed!

https://www.youtube.com/watch?v=kkBaxFu40E

[ herringbone pattern is found in the dome ]

www.teggelaar.com
Dome of Santa Maria del Fiore, Florence

Brunelleschi’s answers:
→ to build the dome without scaffolding:
    a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN
⇒ dome successfully built between 1420-1436

Experiment by Jones, Sereni & Massimo Ricci (2010):

[ Ricci’s dome is not ready yet! ]

https://www.youtube.com/watch?v=kkBaxFuh40E
Dome of Santa Maria del Fiore, Florence

Brunelleschi’s answers:
→ to build the dome without scaffolding:
  a self-supporting construction method is needed!

the huge trick: to use HERRINGBONE PATTERN

How the idea came?
→ may origin from seljukids (moslim architecture)
→ Venice: applied too, from XIth century, but for very small niches

Brunelleschi may had proved
  that the idea works:
  [small theatre dome found nearby]

https://www.youtube.com/watch?v=kkBaxFu40E
Dome of Santa Maria del Fiore, Florence

Suggested videos:
https://www.youtube.com/watch?v=_IOPlGPQPuM&feature=youtu.be 😊 short
https://www.youtube.com/watch?v=QWz90KdrDBs short, on Ricci’s results

Homework:
on technical details:
https://www.youtube.com/watch?v=kkBaxFuh40E 51:55
[entitled „Great Cathedral Mystery” 😊]

https://www.youtube.com/watch?v=exrD5Rwjllo&feature=youtu.be&t=93
Questions

1. Recognize from a figure: **hemispherical / oval / pointed dome**. Explain the following terms: **pendentive; drum; tiburium; corbel dome; herringbone pattern**.

2. Introduce the typical crack pattern of a hemispherical masonry dome without or with a tension ring at its bottom. How can you **protect** a spherical dome against its typical cracking modes?

3. How to calculate the **tension resistance** in horizontal direction in a dry masonry wall with simple running bond pattern, caused by the vertical compression stress and friction resistance in the horizontal contacts?

4. What are the **two main geometrical types** of oval domes? Explain the meaning of the diagram that shows the relation between **minimally necessary wall thickness** and the **eccentricity angle**.