ARCHES

This lecture

The first arches

Basic mechanics of arches

- → How to construct an arch?
- → Reactions for selfweight
- → The Couplet-Heyman problem
- → Live loads
- → How to resist the lateral thrust?
- → Crack pattern under selfweight; How to protect the arch

Most important arch types

- → Romanesque vs Gothic arch
- → Flat arches
- → Flying buttresses

Multispan arch bridges

THE FIRST ARCHES

The aim: to span gaps so that the loads from above would be carried mainly by compression, and to lead the forces downwards to the sides

Sumerian invention:

earliest remaining arch found:

Mesopotamia, Ur, ≈ 2100 BC

[mostly made of sun-dried mud brick]

moulds for arch voussoirs: ≈ 3000 BC

Edublalmahr Temple 2100 BC. Ancient Ur, Iraq https://traveltoeat.com/the-arch-in-architecture-and-history/

Romans: revolutionarized architecture!

extensive use of arches (vaults, domes):

aqueducts, bridges, baths, churches, public buildings, …

THE FIRST ARCHES

The aim: to span gaps so that the loads from above would be carried mainly by compression, and to lead the forces downwards to the sides

Roman aqueduct, Segovia, Spain, 100 AD; travelguide.michelin.com

Roman bath house, Kaiserthermen, Germany, it.wikipedia.org

Pont St Martin, Italy https://www2.uned.es/geo-1-historia-antigua-universal

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BASIC MECHANICS OF ARCHES

<u>How to construct an arch?</u>








pictures:
Ressler
(2011)

BASIC MECHANICS OF ARCHES

How to construct an arch?

How to build a brick archway, https://www.youtube.com/watch?v=-9RPeneyIMI

https://www.khanacademy.org/humanities/renaissance-reformation/early-renaissance1/sculpture-architecture-florence/v/brunelleschi-dome-of-the-cathedral-of-florence-1420-36

→ supports and centring placed first

→ wedge-shaped blocks, proceeding upwards

→ keystone located

→ centring can be removed

BASIC MECHANICS OF ARCHES

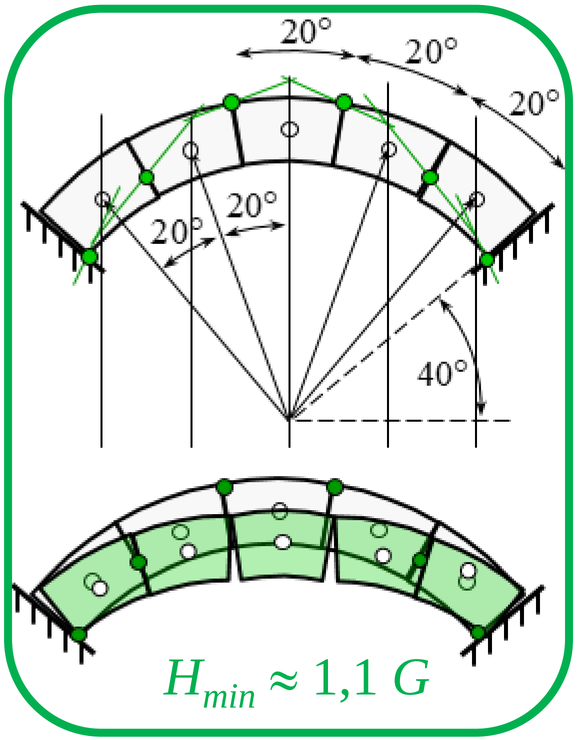
Importance of lateral supports:

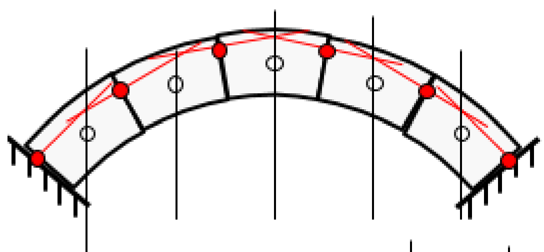
pictures: Ressler (2011)

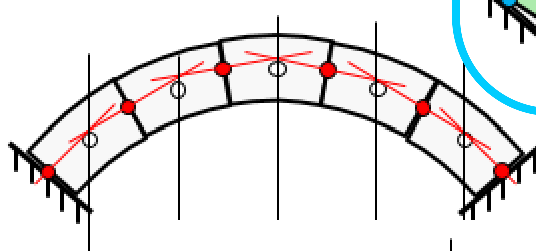
BASIC MECHANICS OF ARCHES

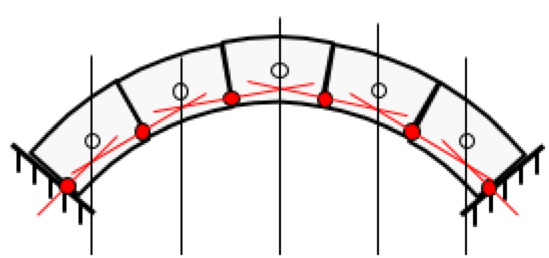
Reactions for selfweight:

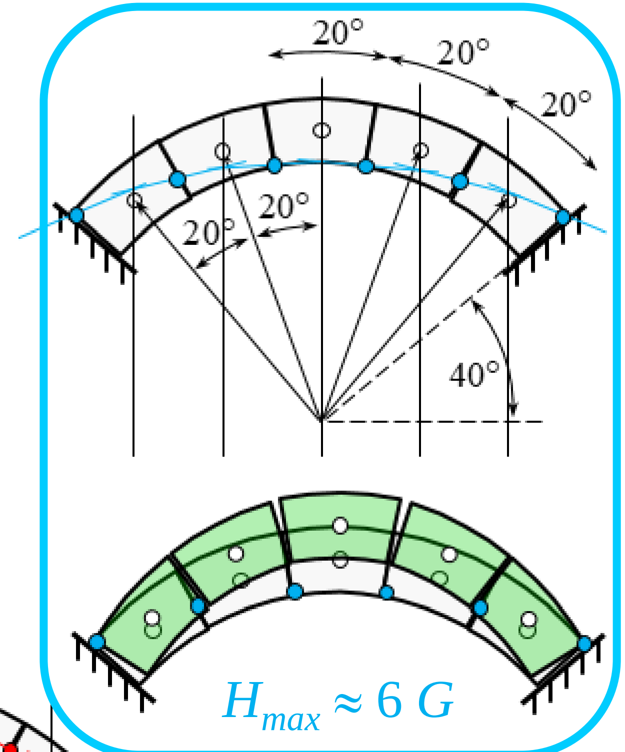
Remember from Lecture 02:











⇒ not „***THE*** solution”, but a

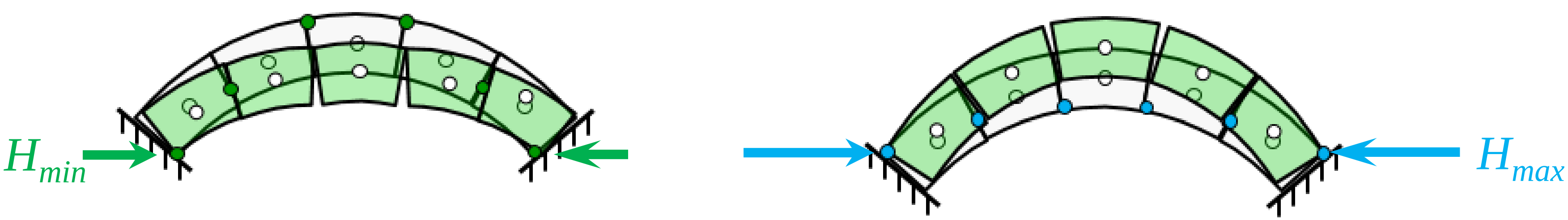
RANGE OF SOLUTIONS exist!

BASIC MECHANICS OF ARCHES

Reactions for selfweight:

Heyman (1966):

statical indeterminacy ⇒ multiple solutions [if thick enough]



if „the actual” state is attempted to be calculated with e.g. elastic analysis or FEM:

⇒ the results are very sensitive to slight support displacements or inaccuracies in geometric data or small modifications of geometry

⇒ „the actual” state is not reasonable to search for;
instead: can the structure be in equilibrium at all?
≡ is there a non-empty range for H ?

BASIC MECHANICS OF ARCHES

The Couplet-Heyman problem:

→ ***circular*** arch with ***uniform thickness***; infinitely dense ***radial*** contacts;

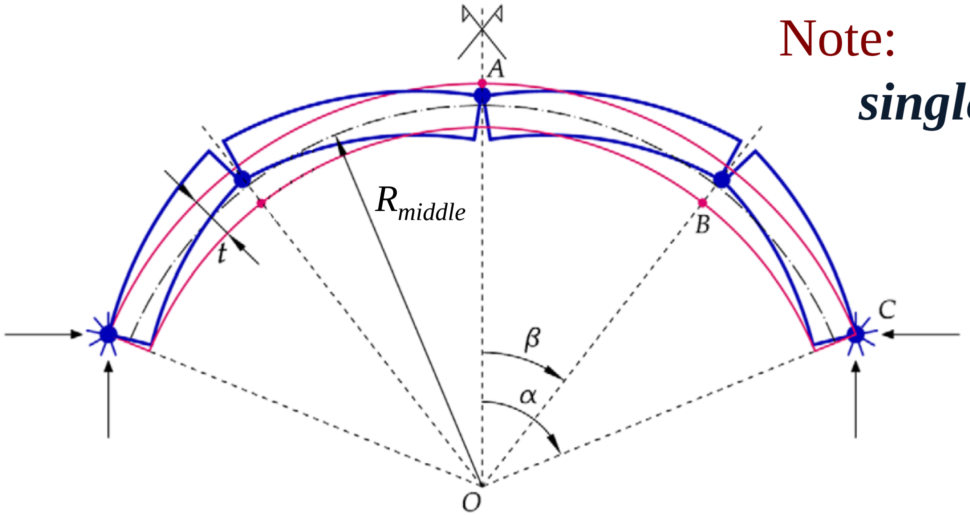
→ sliding and material crushing excluded: arch can ***fail by hinging only***;

→ what is the ***minimally necessary thickness*** to carry its selfweight?

($t_{min}(\alpha) = ?$)

→ having this, what will be the ***collapse mechanism*** for this thickness?

($\beta(\alpha) = ?$)



Note:

single thrust line exists!

BASIC MECHANICS OF ARCHES

Solution given by Couplet (1730):

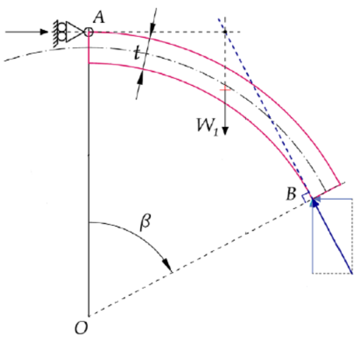
assumed : $\beta(90°) = 45°$; then elementary statics for the just failing arch

$$\Rightarrow t_{min}(90°) \approx 0,101 \cdot R_{middle}$$

Solution given by Heyman (1977): *details missing; see Cochetti et al (2011)*

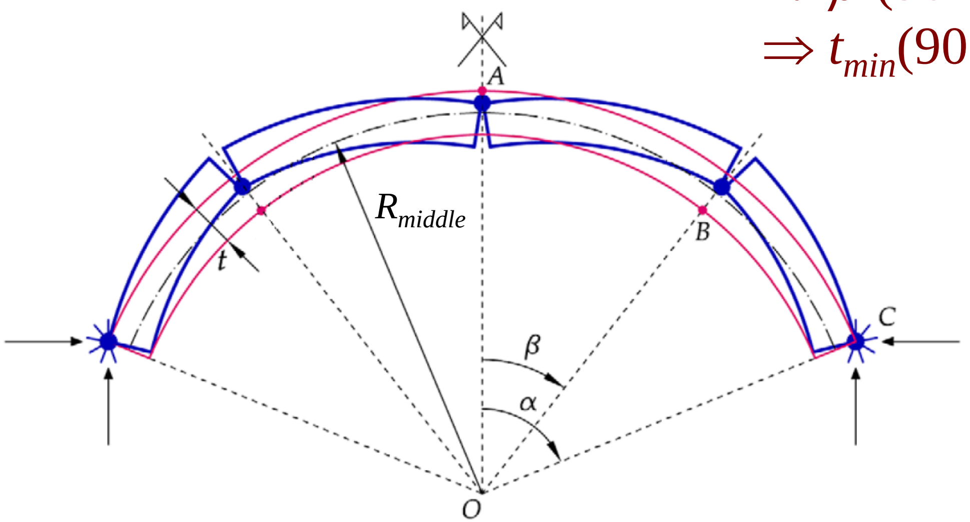
find unique equilibrium force system while minimizing the thickness;

$\Rightarrow$ two unsafe approximations (*wedge centroid*; *tangent force*)



$$\Rightarrow \beta(90°) \approx 58,8°$$

$$\Rightarrow t_{min}(90°) \approx 0,106 \cdot R_{middle}$$



BASIC MECHANICS OF ARCHES

Solution given by Milankovitch (1904; 1907):

forgotten; re-discovered by Foce (2007)

implicitly applied the statical theorem:

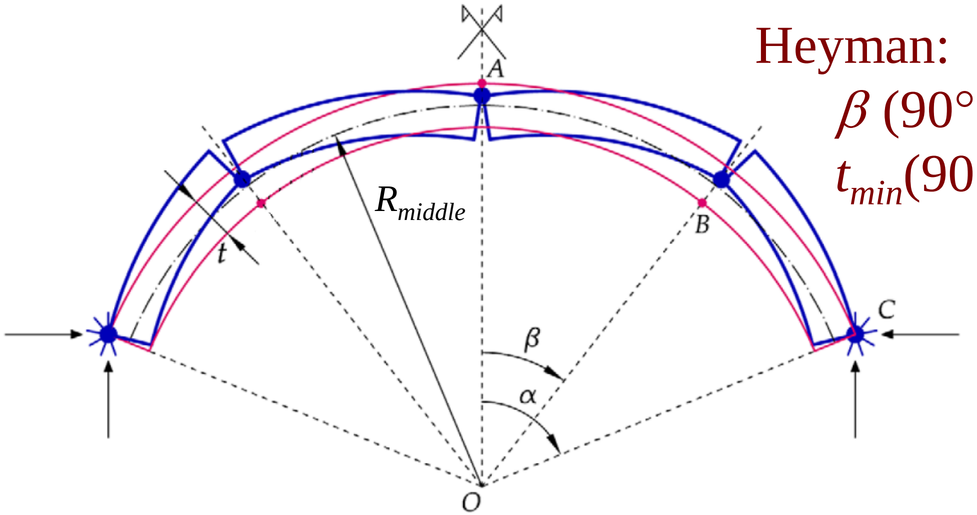
$\Rightarrow \beta\,(90°) = 54{,}5°$;

$\Rightarrow t_{min}(90°) \approx 0{,}1075 \cdot R_{middle}$

Solution given by Cochetti et al (2011): *later confirmed by DDA simulations*

$\Rightarrow \beta\,(90°) = 54{,}5°$;

$\Rightarrow t_{min}(90°) \approx 0{,}1074 \cdot R_{middle}$



Heyman:

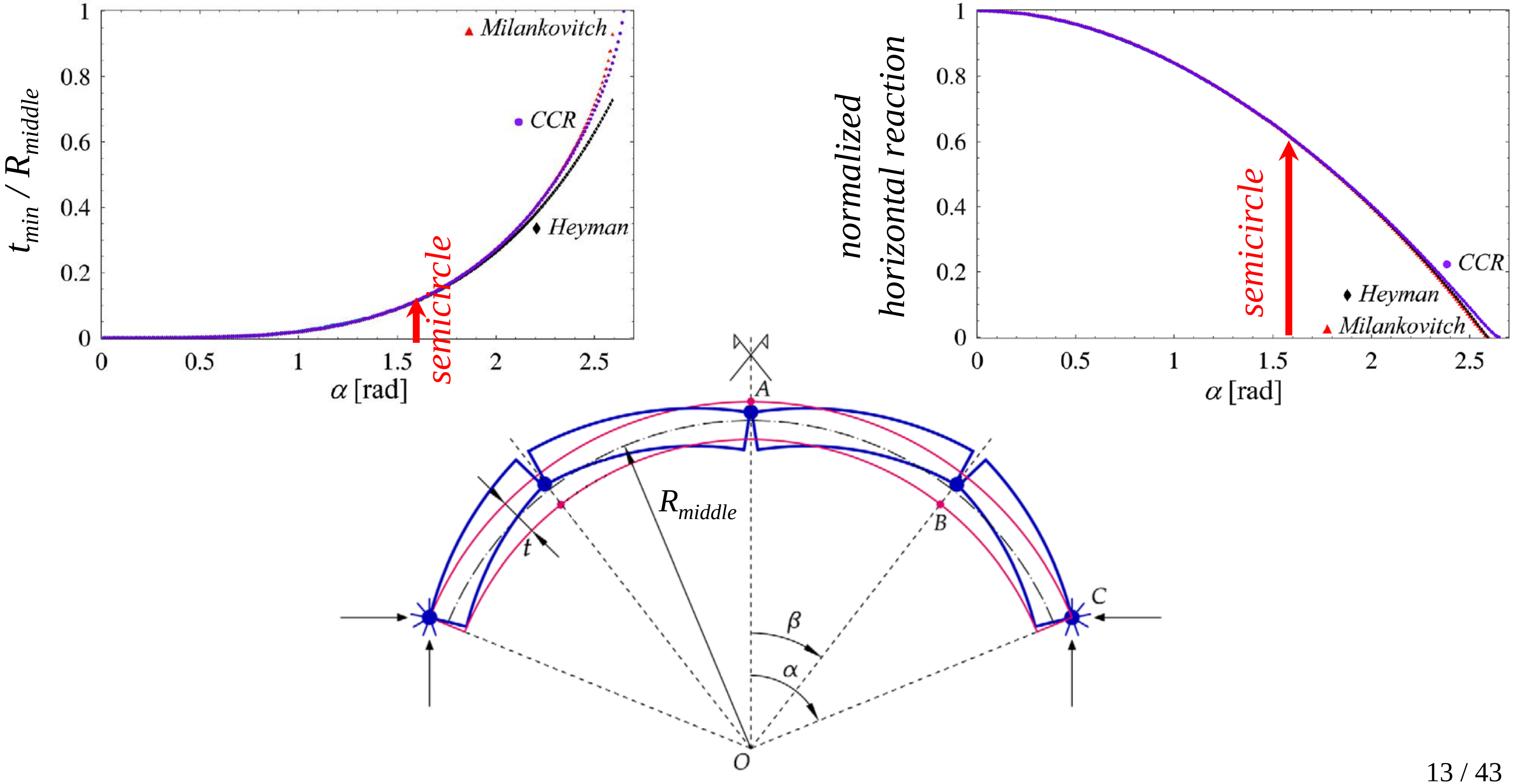
$\beta\,(90°) \approx 58{,}8°$

$t_{min}(90°) \approx 0{,}106 \cdot R_{middle}$

BASIC MECHANICS OF ARCHES

CONCLUSION:

$$\begin{cases} \beta\,(90°) \approx 54{,}5° \\ t_{min}(90°) \approx 0{,}1074 \cdot R_{middle} \end{cases}$$

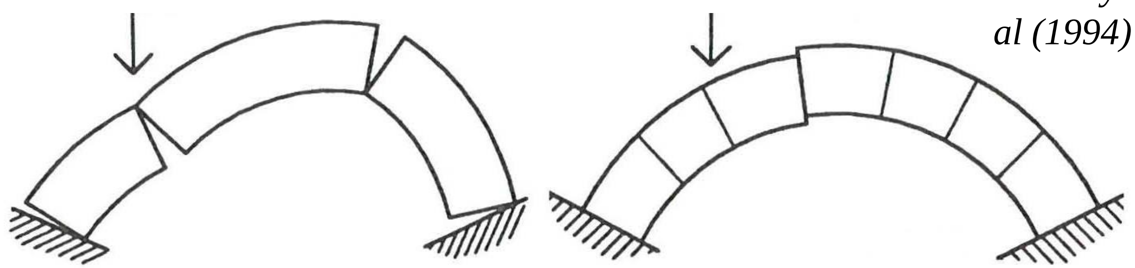


BASIC MECHANICS OF ARCHES

Failure modes of arches under live loads:

Boothby et al (1994)

Without material failure:



hinging mechanism ↔ **sliding** mechanism

[several combined types of mechanisms exist]

→ Can the arch carry the given live load?

→ What is the admissible max load?

Practice today:

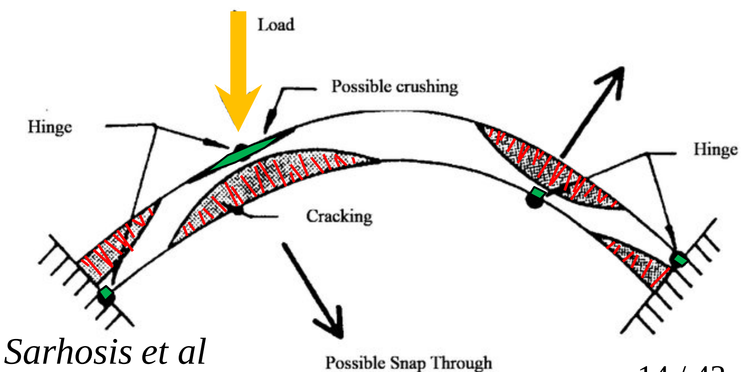
→ graphostatics: find a thrust line

→ limit state analysis codes

With material failure:

→ sharp corner points: stress peaks

→ mortar cracking for tension



Sarhosis et al (2016)

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Multispan arch bridges

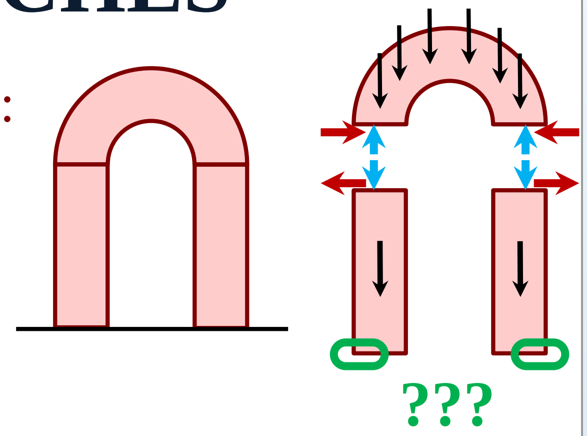
BASIC MECHANICS OF ARCHES

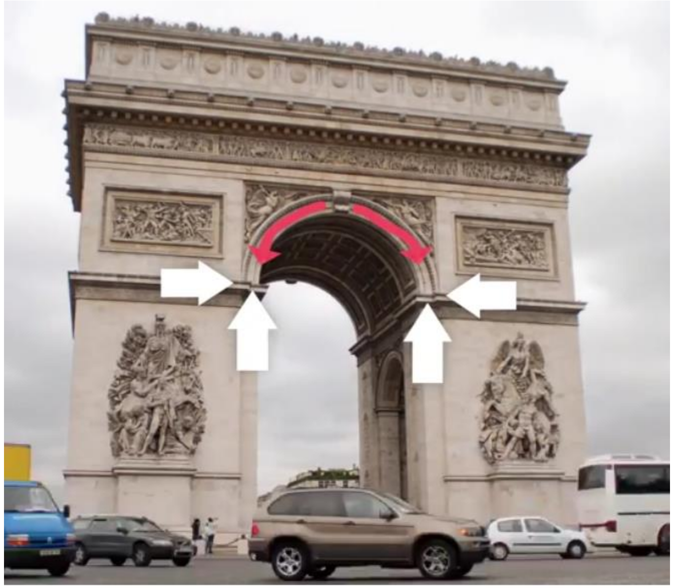
Effect of the arch on its supporting structural members:

THE LATERAL THRUST

How to resist it?

1. ***Heavy***, ***wide***, ***downloaded*** lateral neighbours





Arc de Triomphe, Paris, 1836; https://www.youtube.com/watch?v=qL0w_rhMH3o

BASIC MECHANICS OF ARCHES

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How to resist it?

1. ***Heavy***, ***wide***, ***downloaded*** lateral neighbours

Triumphal Arch, Glanum, France, 1st century; www.lonelyplanet.com

Arc de Triomf, Paseo de Lluís Companys, Barcelona, 1888; http://barcelona-home.com

BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:

THE LATERAL THRUST

How to resist it?

2. Solid rock ***walls*** of the valley

3. Neighbouring arches
⇒ ***arcade*** is formed

Pont StMartin, 1st century BC, Italy; Flickr.com, copyright BeNowMeHere

Colosseum, Rome, 80 AD; www.enca.com/life/travel/

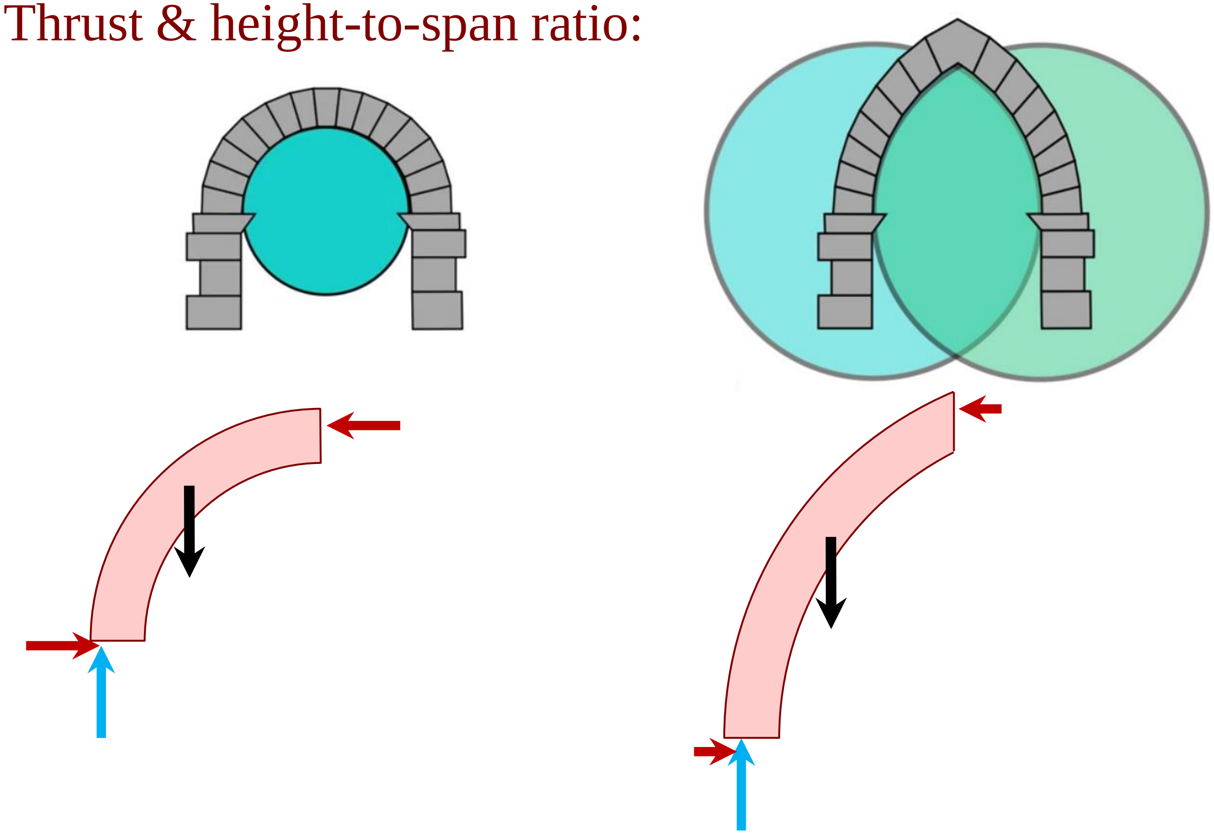
San Angelo, Rome, 2nd century, https://bridgevalleyroad.wordpress.com/stone-bridges/

photo: Ressler (2011)

BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:

Remark: Thrust & height-to-span ratio:



⇒ ***pointed*** arches require ***thinner***, ***higher*** columns & buttresses

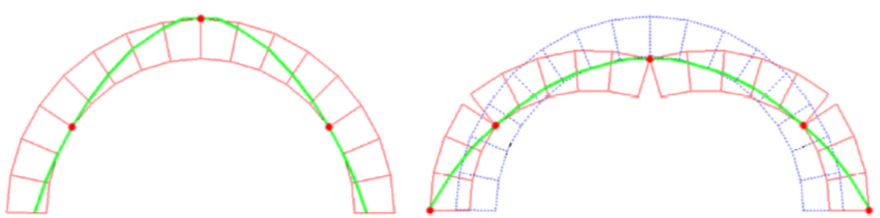
BUT: increasing height: increasing danger of ***sliding failure*** at top

BASIC MECHANICS OF ARCHES

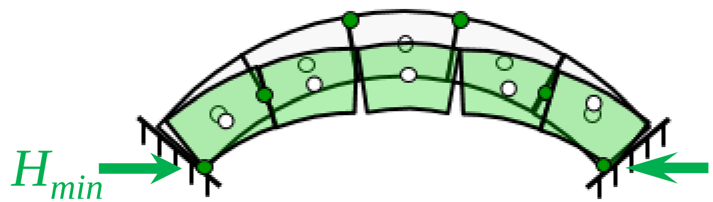
Typical crack pattern of a single arch under selfweight:

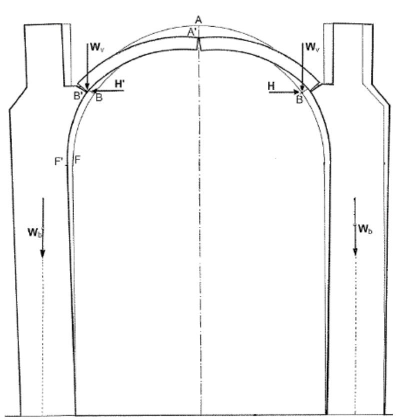
the arch presses the supports ***outwards***

$\Rightarrow$ shifts towards the H_{min} case :



Coccia et al, 2015





Huerta, 2010

Protection against these cracks:

$\rightarrow$ ***fix*** abutments as much as possible

$\rightarrow$ apply ***buttresses***

$\rightarrow$ apply ***tension rods***

$\rightarrow$ strengthening ***strips*** on the surface

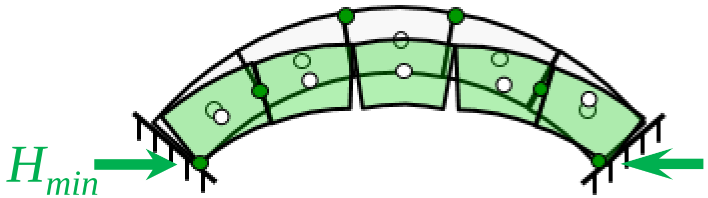
$\rightarrow$ …

BASIC MECHANICS OF ARCHES

Apply buttresses:

https://www.slideshare.net/apehuva/ romanesque-and-gothic-55265863

imagedatabase.st-andrews.ac.uk/images



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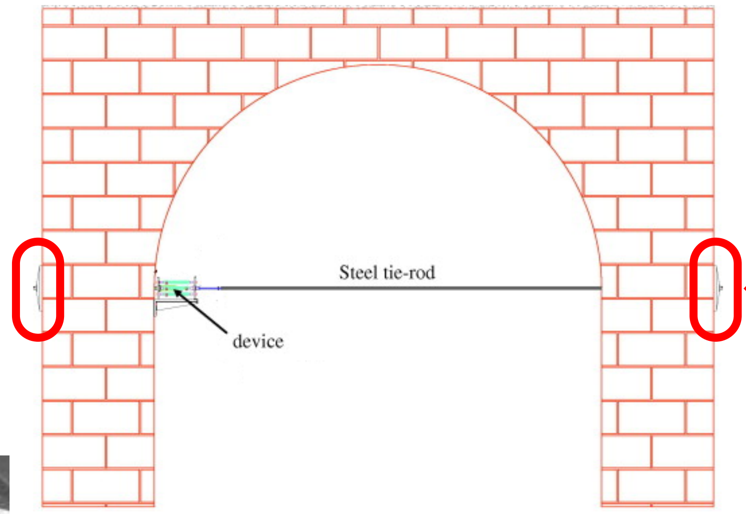
- → …

BASIC MECHANICS OF ARCHES

Apply tension rods:

CAREFULLY !!!



Cardone & Gesualdi (2014)

yourmodelrailway.net

Gesualdo (Campania, Italy), Chiesa del Santissimo Rosario; De Guglielmo, F (2015)

Protection against these cracks:

→ ***fix*** abutments as much as possible

→ apply ***buttresses***

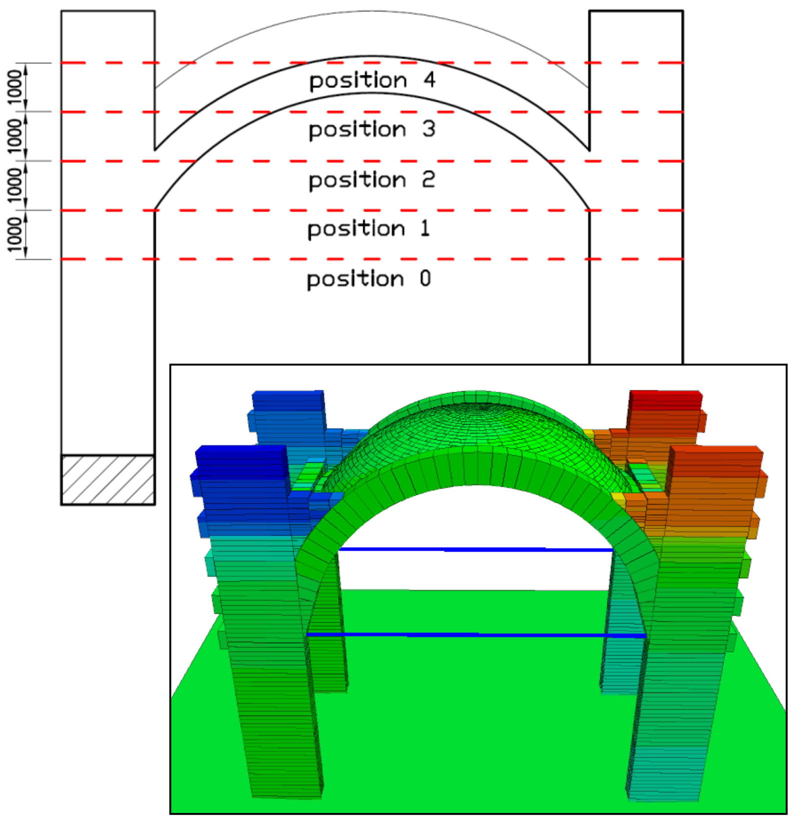
→ apply ***tension rods***

→ strengthening ***strips*** on the surface

→ ...

BASIC MECHANICS OF ARCHES

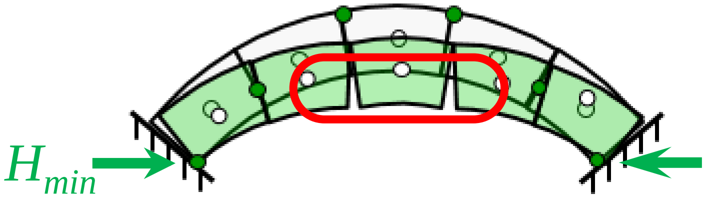
Apply tension rods: *Orosz, A., 2014, 3DEC*



BASIC MECHANICS OF ARCHES

Strengthening strips on the surfaces:

FRP strips, Oliveira et al (2010)



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BASIC MECHANICS OF ARCHES

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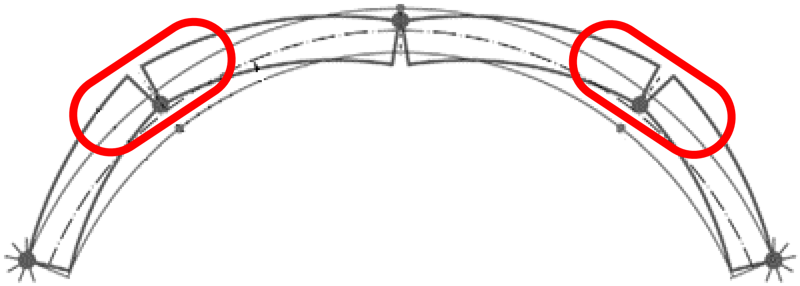

Protection against these cracks:

→ ***fix*** abutments as much as possible

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→ apply ***tension rods***

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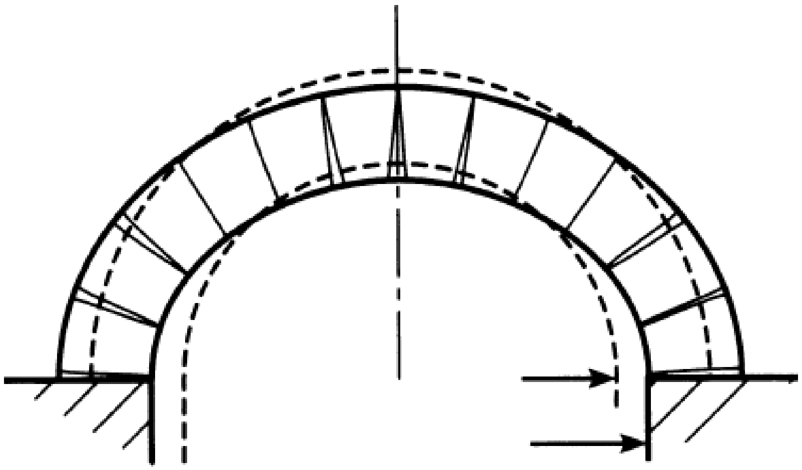
→ …

BASIC MECHANICS OF ARCHES

Until now:

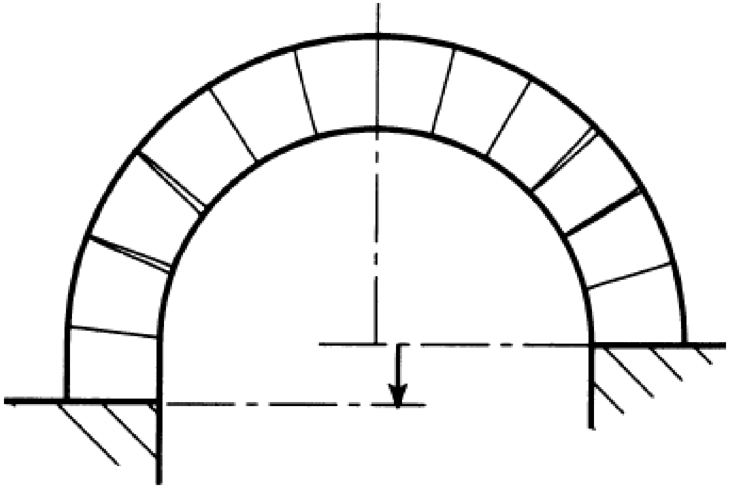
Outwards support displacement:



The other main reason for cracking:

Uneven downwards support displacement

→ can be recognized from
non-symmetric cracks location



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Most important arch types

- → Corbel vs true arch; Romanesque vs Gothic arch
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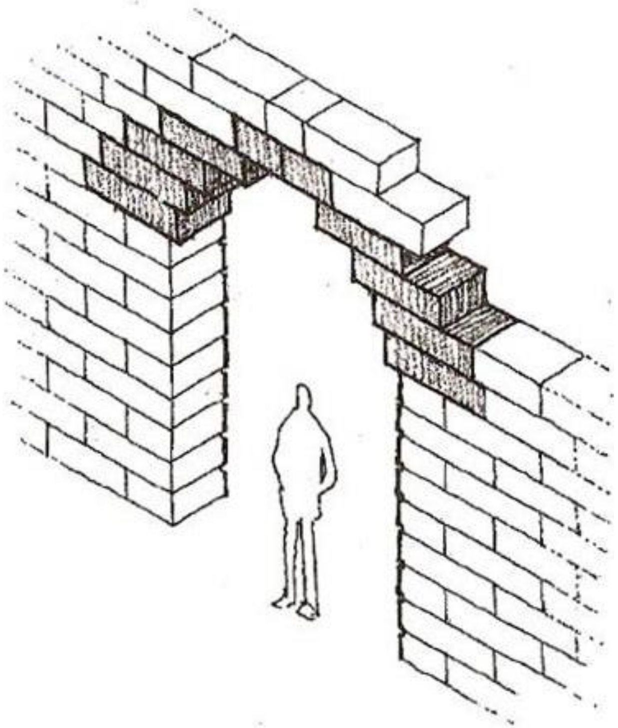
- → Flying buttresses

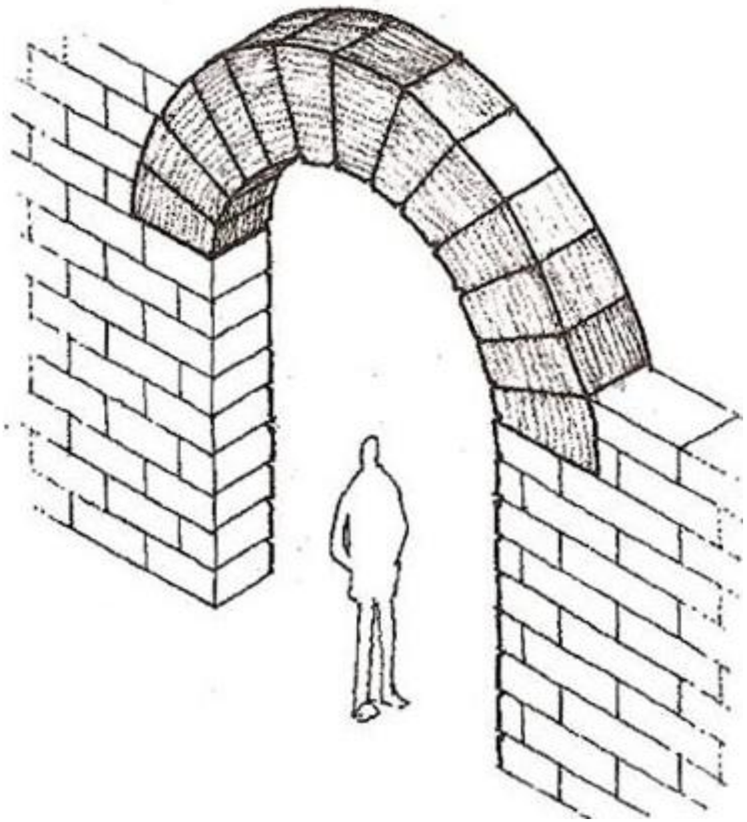
Multispan arch bridges

MOST IMPORTANT ARCH TYPES

According to its mechanics:



Corbel arch („false arch”):
cantilever

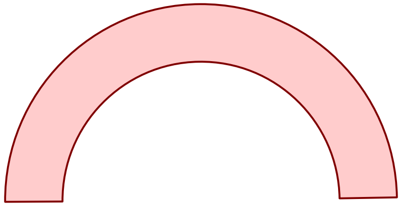


True arch:
compression

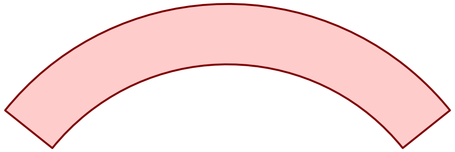
MOST IMPORTANT ARCH TYPES

True arch types according to middle line geometry:

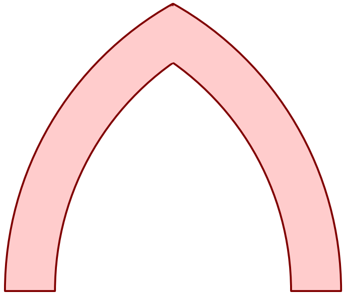
Semicircular („Roman”, „Romanesque”) arch:



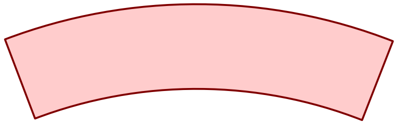
Segmental arch:



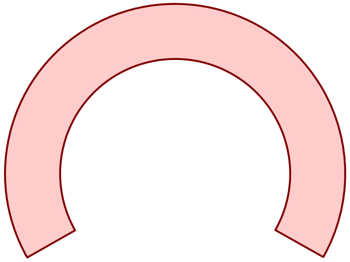
Pointed („Gothic”) arch:



Flat arch:



Moslim („horseshoe”) arch:



MOST IMPORTANT ARCH TYPES

e.g. in Toledo, Spain:

https://www.youtube.com/watch?v=EU4Fx5R0Ows

MOST IMPORTANT ARCH TYPES

Romanesque vs Gothic arch:

Aquitaine, France;
in metmuseum.org

pinterest.com, 300 Archiecture
Travel Inspiration Pictures

Romanesque	Gothic
massive, ***thick***, ***semicircular*** arch	***pointed***, ***thin*** arch
strong piers	light piers up to the sky
thick walls, heavy pillars inside	thin walls, flying buttresses outside
small windows; darkness	***large*** stained glass ***windows***; light

MOST IMPORTANT ARCH TYPES

Romanesque vs Gothic arch:

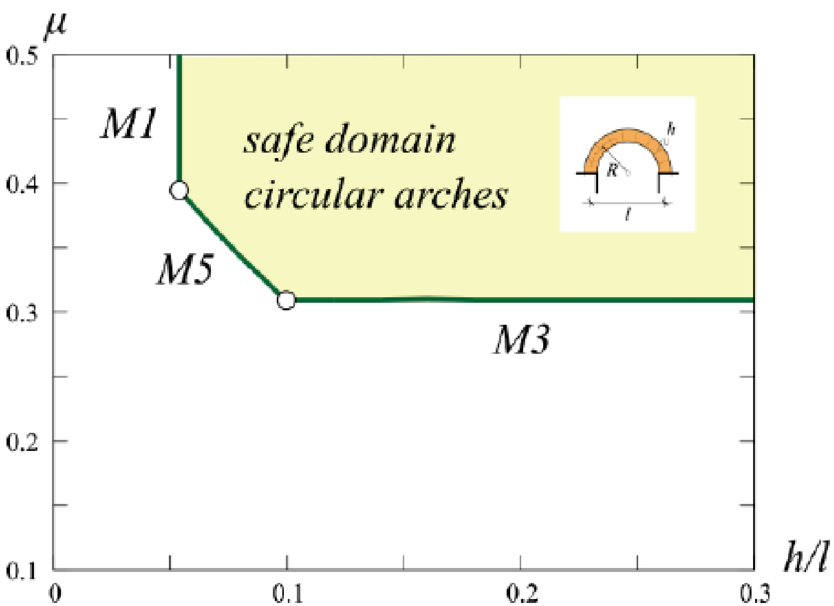
<u>Remember the Durand-Claye method:</u>

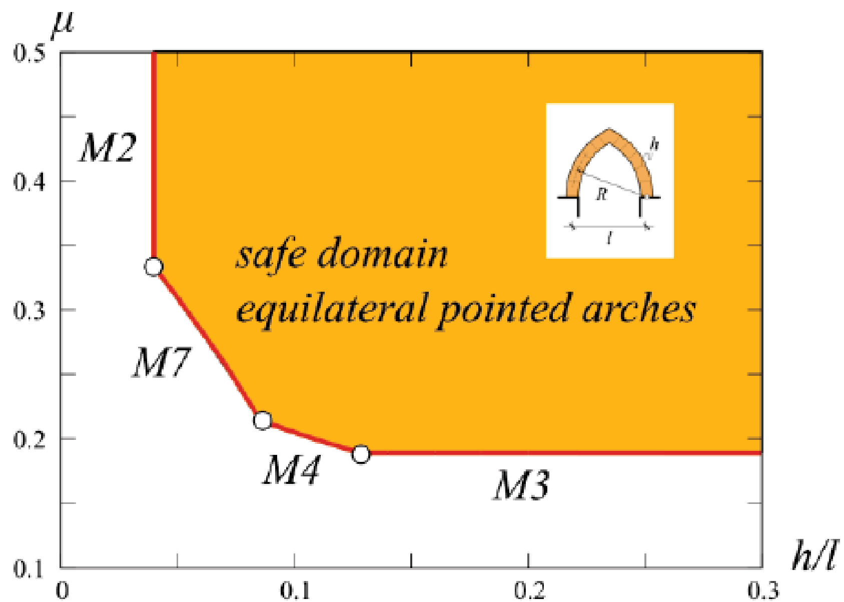
Barsotti et al (2017):

comparison of different arch types and their possible collapse modes

μ : friction coefficient

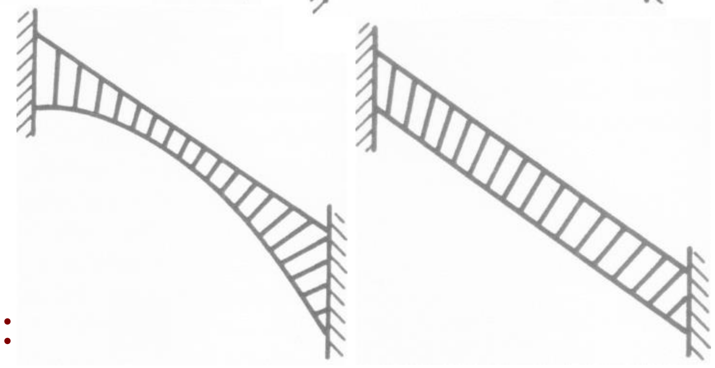
h : arch thickness





MOST IMPORTANT ARCH TYPES

Flat arches:



Definition by Heyman (1982):

for arbitrary downwards loads on them:

statically admissible force system can always be found;

in other words: no hinging mechanism exist

$\Rightarrow$ they cannot fail with any Heymanian collapse modes

$\rightarrow$ failure can happen due to material crushing

$\rightarrow$ failure can happen due to contact sliding

<u>Note:</u> a small sliding is usually no problem!

Kamai & Hatzor, 2005

MOST IMPORTANT ARCH TYPES

Flat arches:

Pithole, Venango, Pennsylvania; https://hu.pinterest.com

https://www.livingstonemasons.com/glossary.html

https://www.locallocalhistory.co.uk/studies/lintels/index-m.htm

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FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Ressler, 2011:

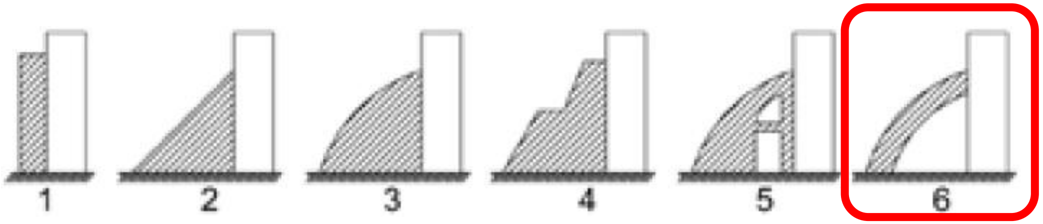
FAILS

STANDS

FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

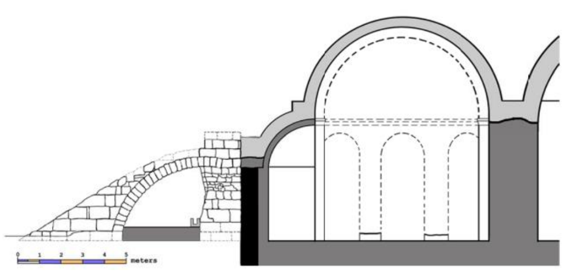
Evolution: fundamental in Gothic architecture; may be of Islamic origin



Karimi et al (2016)

Probably the oldest flying buttress in Europe:
Grand Baths at Salamis-Constantia, Cyprus, 3rd-7th century

Stewart (2013)



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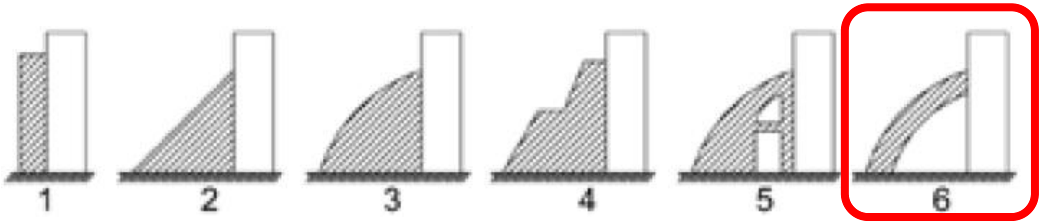
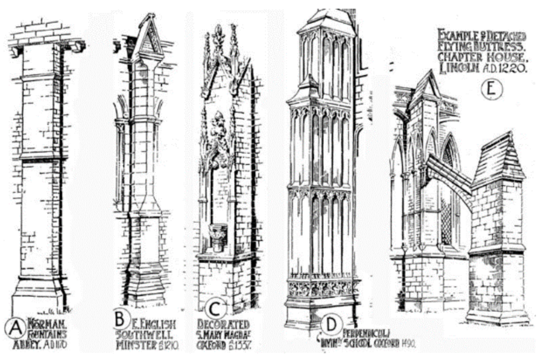


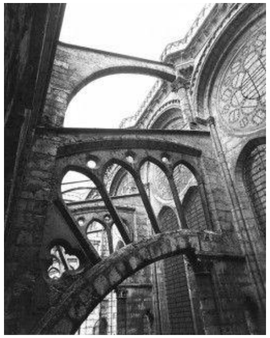
Karimi et al (2016)



Fletcher & Fletcher (1905)

Wide variety of shapes:






https://www.pinterest.co.uk/ckefn/flying-buttress

FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Mechanics:

receives vertical & horizontal thrust from the supported vault;

loads are dominant over selfweight;

works simply as a masonry arch $\Rightarrow$ stability (and not strength) problem

www.pinterest.co.uk

Stability analysis:

$\rightarrow$ admissible range of thrust in the supported vault?

$\rightarrow$ admissible range of thrust in the flying buttress?

$\rightarrow$ is the first range included in the second range?

$\rightarrow$ often: the flying buttress is a „flat arch"

[OPEN ISSUES]

This lecture

The first arches

Basic mechanics of arches

→ How to construct an arch?

→ Reactions for selfweight

→ The Couplet-Heyman problem

→ Live loads

→ How to resist the lateral thrust?

→ Crack pattern under selfweight; How to protect the arch

Most important arch types

→ Romanesque vs Gothic arch

→ Flat arches

→ Flying buttresses

Multispan arch bridges

MULTISPAN ARCH BRIDGES

Neighbouring arches support each other:

Skopje Aqueduct, Macedonia, 1st century; exploringmacedonia.com

Railway viaduct in Switzerland, http://crea.bunshun.jp/articles/-/6770

Gatehampton Railway Bridge, http://thames.me.uk/s01240.htm

Practical analysis:

→ rules based on experience; (?) MEXE

→ limit state codes

→ nonlinear FEM

→ …

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? how the neighbours act on each other ?

? slender / stocky pillars ?

? …

[OPEN ISSUES]

SUGGESTED VIDEOS

https://www.youtube.com/watch?v=Rkxlxm26G_s
(Ressler (2016): Seeing Structure in the Great Architecture of Western Civilization; long, but excellent and easy to understand)

https://www.youtube.com/watch?v=EU4Fx5R0Ows (short; basic arch types)
„Know before you go": Identifying Arch Types → excellent!

https://www.youtube.com/watch?v=mstZhReh31k
World's largest brick bridge – just nice to see

https://www.youtube.com/watch?v=qL0w_rhMH3o
(Arches, Domes, Vaults: short, elementary intro)

https://www.youtube.com/watch?v=awTkIr1TIoY
(examples for nonlinear FEM modelling of single-span arch)

QUESTIONS

1. What is the ***Couplet-Heyman*** problem? How much is the ***minimally necessary thickness*** for a semicircular arch?

2. Introduce three techniques to ***resist the lateral thrust*** expressed by an arch. Introduce four techniques to ***protect the arch itself against cracking***.

3. Why is it advantageous to have a ***pointed*** arch instead of a ***semicircular*** arch on the same span?

4. Recognize from a picture: ***corbel*** arch, ***Roman*** arch, ***segmental*** arch, ***flat*** arch, ***pointed*** arch, ***horseshoe*** arch. What is the difference between segmental arch and flat arch?