ARCHES
This lecture

The first arches

Basic mechanics of arches

→ How to construct an arch?
→ Reactions for selfweight
→ The Couplet-Heyman problem
→ Live loads
→ How to resist the lateral thrust?
→ Crack pattern under selfweight; How to protect the arch

Most important arch types

→ Romanesque vs Gothic arch
→ Flat arches
→ Flying buttresses

Multispan arch bridges
THE FIRST ARCHES

The aim: to span gaps so that the loads from above would be carried mainly by compression, and to lead the forces downwards to the sides

Sumerian invention:
- earliest remaining arch found: Mesopotamia, Ur, ≈ 2100 BC
- [mostly made of sun-dried mud brick]
- moulds for arch voussoirs: ≈ 3000 BC

Romans: revolutionarized architecture!
- extensive use of arches (vaults, domes):
  - aqueducts, bridges, baths, churches, public buildings, …
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BASIC MECHANICS OF ARCHES

How to construct an arch?

pictures: Ressler (2011)
BASIC MECHANICS OF ARCHES

How to construct an arch?

→ supports and centring placed first
→ wedge-shaped blocks, proceeding upwards
→ keystone located
→ centring can be removed

How to build a brick archway,
https://www.youtube.com/watch?v=-9RPeneyIMI

BASIC MECHANICS OF ARCHES

Importance of lateral supports:

pictures: Ressler (2011)
Reactions for selfweight:

Remember from Lecture 02:

⇒ not „THE solution”, but a 
RANGE OF SOLUTIONS exist!

\[ H_{\text{min}} \approx 1.1 \ G \]

\[ H_{\text{max}} \approx 6 \ G \]
Reactions for selfweight:

Heyman (1966):

statical indeterminacy $\Rightarrow$ multiple solutions [if thick enough]

if „the actual” state is attempted to be calculated with e.g. elastic analysis or FEM:

$\Rightarrow$ the results are very sensitive to slight support displacements or inaccuracies in geometric data or small modifications of geometry

$\Rightarrow$ „the actual” state is not reasonable to search for;

instead: can the structure be in equilibrium at all?

$\equiv$ is there a non-empty range for $H$?
BASIC MECHANICS OF ARCHES

The Couplet-Heyman problem:

→ circular arch with uniform thickness; infinitely dense radial contacts;
→ sliding and material crushing excluded: arch can \textit{fail by hinging only};
→ what is the \textit{minimally necessary thickness} to carry its selfweight?
  \( t_{\text{min}}(\alpha) = ? \)
→ having this, what will be the \textit{collapse mechanism} for this thickness?
  \( \beta(\alpha) = ? \)

Note: \textit{single} thrust line exists!
BASIC MECHANICS OF ARCHES

Solution given by Couplet (1730):
assumed: $\beta (90^\circ) = 45^\circ$; then elementary statics for the just failing arch
$\Rightarrow t_{min}(90^\circ) \approx 0,101 \cdot R_{middle}$

Solution given by Heyman (1977): details missing; see Cochetti et al (2011)
find unique equilibrium force system while minimizing the thickness;
$\Rightarrow$ two unsafe approximations (wedge centroid; tangent force)

$\Rightarrow \beta (90^\circ) \approx 58,8^\circ$
$\Rightarrow t_{min}(90^\circ) \approx 0,106 \cdot R_{middle}$
Solution given by Milankovitch (1904; 1907): forgotten; re-discovered by Foce (2007)

implicitly applied the statical theorem:

\[ \Rightarrow \beta (90^\circ) = 54,5^\circ ; \]
\[ \Rightarrow t_{\text{min}} (90^\circ) \approx 0,1075 \cdot R_{\text{middle}} \]

Solution given by Cochetti et al (2011): later confirmed by DDA simulations

\[ \Rightarrow \beta (90^\circ) = 54,5^\circ ; \]
\[ \Rightarrow t_{\text{min}} (90^\circ) \approx 0,1074 \cdot R_{\text{middle}} \]

Heyman:
\[ \beta (90^\circ) \approx 58,8^\circ \]
\[ t_{\text{min}} (90^\circ) \approx 0,106 \cdot R_{\text{middle}} \]
CONCLUSION:

\[
\beta \ (90^\circ) \approx 54.5^\circ \\
t_{\min} \ (90^\circ) \approx 0.1074 \cdot R_{\text{middle}}
\]
BASIC MECHANICS OF ARCHES

Failure modes of arches under live loads:

Without material failure:

- **hinging** mechanism ↔ **sliding** mechanism
  - [several combined types of mechanisms exist]
  - Can the arch carry the given live load?
  - What is the admissible max load?

Practice today:
- → graphostatics: find a thrust line
- → limit state analysis codes

With material failure:
- → sharp corner points: stress peaks
- → mortar cracking for tension

Boothby et al (1994)

Sarhosis et al (2016)
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BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:
THE LATERAL THRUST

How to resist it?

1. Heavy, wide, downloaded lateral neighbours

Arc de Triomphe, Paris, 1836; https://www.youtube.com/watch?v=qL0w_rHMH3o
BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:

THE LATERAL THRUST

How to resist it?

1. Heavy, wide, downloaded lateral neighbours

Triumphal Arch, Glanum, France, 1st century; www.lonelyplanet.com

Arc de Triomf, Paseo de Lluís Companys, Barcelona, 1888; http://barcelona-home.com
BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:

THE LATERAL THRUST

How to resist it?

2. Solid rock *walls* of the valley

3. Neighbouring arches
   \[\Rightarrow \text{arcade is formed}\]
BASIC MECHANICS OF ARCHES

Effect of the arch on its supporting structural members:
Remark: Thrust & height-to-span ratio:

⇒ **pointed** arches require **thinner, higher** columns & buttresses

**BUT:** increasing height: increasing danger of **sliding failure** at top
BASIC MECHANICS OF ARCHES

Typical crack pattern of a single arch under selfweight:

the arch presses the supports \textit{outwards}

⇒ shifts towards the $H_{\text{min}}$ case:

Protection against these cracks:

→ \textit{fix} abutments as much as possible
→ apply \textit{buttresses}
→ apply \textit{tension rods}
→ strengthening \textit{strips} on the surface
→ …
BASIC MECHANICS OF ARCHES

Apply buttresses:

Protection against these cracks:

→ **fix** abutments as much as possible
→ apply **buttresses**
→ apply **tension rods**
→ strengthening **strips** on the surface
→ ...

https://www.slideshare.net/apehuva/romanesque-and-gothic-55265863

imagedatabase.st-andrews.ac.uk/images
BASIC MECHANICS OF ARCHES

Apply tension rods:

CAREFULLY !!!

Protection against these cracks:
- fix abutments as much as possible
- apply buttresses
- apply tension rods
- strengthening strips on the surface
- ...

Cardone & Gesualdi (2014)

Gesualdo (Campania, Italy), Chiesa del Santissimo Rosario; De Guglielmo, F (2015)
BASIC MECHANICS OF ARCHES

Apply tension rods:  

Orosz, A., 2014, 3DEC
BASIC MECHANICS OF ARCHES

Strengthening strips on the surfaces:

Protection against these cracks:

→ fix abutments as much as possible
→ apply buttresses
→ apply tension rods
→ strengthening strips on the surfaces
→ …
**BASIC MECHANICS OF ARCHES**

Strengthening strips:

- FRP strips, Oliveira et al (2010)

Protection against these cracks:

- *fix* abutments as much as possible
- apply *buttresses*
- apply *tension rods*
- strengthening *strips* on the surface
- …
Until now:

Outwards support displacement:

The other main reason for cracking:

Uneven downwards support displacement

→ can be recognized from non-symmetric cracks location
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→ Corbel vs true arch; Romanesque vs Gothic arch
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Multispan arch bridges
MOST IMPORTANT ARCH TYPES

According to its mechanics:

Corbel arch („false arch”):
- cantilever

True arch:
- compression
MOST IMPORTANT ARCH TYPES

True arch types according to middle line geometry:

Semicircular („Roman”, „Romanesque”) arch:

Segmental arch:

Pointed („Gothic”) arch:

Flat arch:

Moslim („horseshoe”) arch:
MOST IMPORTANT ARCH TYPES

e.g. in Toledo, Spain:

https://www.youtube.com/watch?v=EU4Fx5R0Ows
MOST IMPORTANT ARCH TYPES

Romanesque vs Gothic arch:

- **Massive, thick, semicircular arch**
  - Strong piers
  - Thick walls, heavy pillars inside
  - Small windows; darkness

- **Pointed, thin arch**
  - Light piers up to the sky
  - Thin walls, flying buttresses outside
  - Large stained glass windows; light

*Images from:*
- Aquitaine, France; in metmuseum.org
- Pinterest.com, 300 Architecture Travel Inspiration Pictures
MOST IMPORTANT ARCH TYPES

Romanesque vs Gothic arch:

Remember the Durand-Claye method:

Barsotti et al (2017):
comparison of different arch types and their possible collapse modes

\[ \mu : \text{friction coefficient} \]

\[ h : \text{arch thickness} \]
MOST IMPORTANT ARCH TYPES

Flat arches:

Definition by Heyman (1982):
for arbitrary downwards loads on them:

- statically admissible force system can always be found;
- in other words: no hinging mechanism exist

⇒ they cannot fail with any Heymanian collapse modes

→ failure can happen due to material crushing
→ failure can happen due to contact sliding

Note: a small sliding is usually no problem!

Kamai & Hatzor, 2005
MOST IMPORTANT ARCH TYPES

Flat arches:

⇒ they cannot fail with any Heymanian collapse modes
  → failure can happen due to material crushing
  → failure can happen due to contact sliding

Note: a small sliding is usually no problem!
FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Ressler, 2011:

FAILS

STANDS
FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Evolution: fundamental in Gothic architecture; may be of Islamic origin

Probably the oldest flying buttress in Europe:
Grand Baths at Salamis-Constantia,
Cyprus, 3rd-7th century
FLYING BUTTRESSSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Evolution: fundamental in Gothic architecture; may be of Islamic origin

Wide variety of shapes:

https://www.pinterest.co.uk/ckefn/flying-buttress
FLYING BUTTRESSES

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Mechanics:

receives vertical & horizontal thrust from the supported vault;
loads are dominant over selfweight;
works simply as a masonry arch ⇒ stability (and not strength) problem

Stability analysis:

→ admissible range of thrust in the supported vault?
→ admissible range of thrust in the flying buttress?
→ is the first range included in the second range?
→ often: the flying buttress is a „flat arch”
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MULTISPAN ARCH BRIDGES

Neighbouring arches support each other:

- Skopje Aqueduct, Macedonia, 1st century; exploringmacedonia.com
- Railway viaduct in Switzerland, http://crea.bunshun.jp/articles/-/6770
- Gatehampton Railway Bridge, http://thames.me.uk/s01240.htm

Practical analysis:

→ rules based on experience; (?) MEXE
→ limit state codes
→ nonlinear FEM
→ …
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? how the neighbours act on each other?
? slender / stocky pillars?
? …

[ OPEN ISSUES ]
SUGGESTED VIDEOS

https://www.youtube.com/watch?v=Rkxlxm26G_s
   (Ressler (2016): Seeing Structure in the Great Architecture of Western Civilization; long, but excellent and easy to understand)

https://www.youtube.com/watch?v=EU4Fx5R0Ows (short; basic arch types)
   „Know before you go”: Identifying Arch Types → excellent!

https://www.youtube.com/watch?v=mstZhReh31k
   World’s largest brick bridge – just nice to see

https://www.youtube.com/watch?v=qL0w_rhMH3o
   (Arches, Domes, Vaults: short, elementary intro)

https://www.youtube.com/watch?v=awTkIr1TioY
   (examples for nonlinear FEM modelling of single-span arch)
QUESTIONS

1. What is the **Couplet-Heyman** problem? How much is the minimally necessary thickness for a semicircular arch?

2. Introduce three techniques to **resist the lateral thrust** expressed by an arch. Introduce four techniques to **protect the arch itself against cracking**.

3. Why is it advantageous to have a **pointed** arch instead of a **semicircular** arch on the same span?

4. Recognize from a picture: **corbel** arch, **Roman** arch, **segmental** arch, **flat** arch, **pointed** arch, **horseshoe** arch. What is the difference between segmental arch and flat arch?