



# ARCHES





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## **This lecture**

The first arches

Basic mechanics of arches

- $\rightarrow$  How to construct an arch?
- $\rightarrow$  Reactions for selfweight
- $\rightarrow$  The Couplet-Heyman problem
- $\rightarrow$  Live loads
- $\rightarrow$  How to resist the lateral thrust?
- $\rightarrow$  Crack pattern under selfweight; How to protect the arch

Most important arch types

- $\rightarrow$  Romanesque vs Gothic arch
- $\rightarrow$  Flat arches
- $\rightarrow$  Flying buttresses

Multispan arch bridges

## THE FIRST ARCHES

<u>The aim</u>: to span gaps so that the loads from above would be carried mainly by compression, and to lead the forces downwards to the sides

## Sumerian invention:

earliest arches (?): Mesopotamia, ≈ 3500 BC [ mostly made of sun-dried mud brick ]



Edublalmahr Temple 2100 BC. Ancient Ur, Iraq https://traveltoeat.com/the-arch-in-architectureand-history/

<u>Romans:</u> revolutionarized architecture! and extensive use of arches (vaults, domes):

aqueducts, bridges, baths, churches, public buildings, ...

# THE FIRST ARCHES

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Roman aqueduct, Segovia, Spain, 100 AD; travelguide.michelin.com



Roman bath house, Kaiserthermen, Germany, it.wikipedia.org



Pont St Martin, Italy https://www2.uned.es/geo-1historia-antigua-universal

 Romans:
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## How to construct an arch?









pictures: Ressler (2011)





## How to construct an arch?



*How to build a brick archway, https://www.youtube.com/watch?v=-9RPeneyIMI* 



https://www.khanacademy.org/humanities/renais sance-reformation/early-renaissance1/sculpturearchitecture-florence/v/brunelleschi-dome-ofthe-cathedral-of-florence-1420-36

- $\rightarrow$  supports and centring placed first
- $\rightarrow$  wedge-shaped blocks, proceeding upwards
- $\rightarrow$  keystone located
- $\rightarrow$  centring can be removed

## Importance of lateral supports:









pictures: Ressler (2011)

## **BASIC MECHANICS OF ARCHES** 20° Reactions for selfweight: 20° 20° Remember from Lecture 02: 20° 20° 20° 40 20° 40° $H_{max} \approx 6 G$ $H_{min} \approx 1,1 G$ $\Rightarrow$ not ,,*THE* solution", but a **RANGE OF SOLUTIONS exist!** 9/52

## **Reactions for selfweight:**

Heyman (1966):

statical indeterminacy  $\Rightarrow$  multiple solutions [if thick enough]



if "the actual" state is attempted to be calculated with e.g. elastic analysis or FEM:

⇒ the results are very sensitive to slight support displacements or inaccuracies in geometric data or small modifications of geometry

⇒ ,,the actual" state is not reasonable to search for;
instead: can the structure be in equilibrium at all?
≡ is there a non-empty range for H ?

The Couplet-Heyman problem:

- → *circular* arch with *uniform thickness*; infinitely dense *radial* contacts;
- → sliding and material crushing excluded: arch can *fail by hinging only*;
- $\rightarrow$  what is the *minimally necessary thickness* to carry its selfweight?  $t_{min}(\alpha) = ?$
- $\rightarrow$  what is the corresponding *collapse mechanism* for this thickness?  $\beta(\alpha) = ?$





Solution given by Heyman (1977): *details missing, but: Cochetti et al (2011)* find unique equilibrium force system while minimizing the thickness;  $\Rightarrow$  two unsafe approximations Two approximations: (*wedge centroid*; *tangent force*) [turned out unsafe] 1) (I):  $\Sigma M_i^{(B)} = 0$ ; ( $\Sigma$ ):  $\Sigma M_i^{(C)} = 0$ ; tangent at B  $\equiv$ [rectangle]  $t_{min}(\alpha), H/W(\alpha), \beta(\alpha)$  received R<sub>middle</sub> Heyman:  $\Rightarrow \beta (90^\circ) \approx 58.8^\circ$  $\Rightarrow t_{min}(90^\circ) \approx 0,106 \cdot R_{middle}$ 13 / 52

Solution given by Milankovitch (1904; 1907):





Failure modes of arches under live loads:

Without material failure:



hinging mechanism ↔ sliding mechanism
[ several combined types of mechanisms exist ]

- $\rightarrow$  Can the arch carry the given live load?
- $\rightarrow$  What is the admissible max load?

Practice today:

- $\rightarrow$  graphostatics: find a thrust line
- $\rightarrow$  limit state analysis codes

With material failure:

- $\rightarrow$  sharp corner points: stress peaks
- $\rightarrow$  mortar cracking for tension



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Multispan arch bridges

Effect of the arch on its supporting structural members: THE LATERAL THRUST

How to resist it?

1. Heavy, wide, downloaded lateral neighbours







Arc de Triomphe, Paris, 1836; https://www.youtube.com/watch?v=qL0w\_rhMH3o

Effect of the arch on its supporting structural members: THE LATERAL THRUST

How to resist it?

1. Heavy, wide, downloaded lateral neighbours



Triumphal Arch, Glanum, France, 1st century; www.lonelyplanet.com



Arc de Triomf, Paseo de Lluís Companys, Barcelona, 1888; http://barcelona-home.com

Effect of the arch on its supporting structural members:

THE LATERAL THRUST

How to resist it?

- 2. Solid rock *walls* of the valley
- 3. Neighbouring arches  $\Rightarrow$  *arcade* is formed



Colosseum, Rome, 80 AD; www.enca.com/life/travel/



San Angelo, Rome, 2nd century, https://bridgevalleyroad.wordpress. com/stone-bridges/



Pont StMartin, 1st century BC, Italy; Flickr.com, copyright BeNowMeHere



photo: Ressler (2011)

20 / 52

Effect of the arch on its supporting structural members:

Remark: Thrust & height-to-span ratio:



21 / 52

Typical crack pattern of a single arch under selfweight:

the arch presses the supports *outwards* 

 $\Rightarrow$  shifts towards the  $H_{min}$  case :







Coccia et al, 2015



Protection against these cracks:

- $\rightarrow$  *fix* abutments as much as possible
- $\rightarrow$  apply *buttresses*
- $\rightarrow$  apply *tension rods*
- $\rightarrow$  strengthening *strips* on the surface

Huerta, 2010

## Apply buttresses:



https://www.slideshare.net/apehuva/ romanesque-and-gothic-55265863



imagedatabase.st-andrews.ac.uk/images



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Cardone & Gesualdi (2014)

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Gesualdo (Campania, Italy), Chiesa del Santissimo Rosario; De Guglielmo, F (2015)

Apply tension rods: Orosz, A., 2014, 3DEC





Strengthening strips on the surfaces:





FRP strips, Oliveira et al (2010)



Protection against these cracks:

- $\rightarrow$  *fix* abutments as much as possible
- $\rightarrow$  apply *buttresses*
- $\rightarrow$  apply *tension rods* 
  - $\rightarrow$  strengthening *strips* on the surfaces

## Strengthening strips:



FRP strips, Oliveira et al (2010)



Protection against these cracks:

- $\rightarrow$  *fix* abutments as much as possible
- $\rightarrow$  apply *buttresses*
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Until now:

Outwards support displacement:



→ can be recognized from non-symmetric cracks location





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## According to its mechanics:



Corbel arch ("false arch"): cantilever



True arch: compression

True arch types according to middle line geometry:

Semicircular (,,Roman", ,,Romanesque") arch:



Segmental arch:



Pointed (,,Gothic") arch:

Flat arch:

Moslim ("horseshoe") arch:





# Flat arches: Image: Control of the second secon

Definition by Heyman (1982):

for arbitrary downwards loads on top:

statically admissible force system can always be found;

in other words: no hinging mechanism exist

- $\Rightarrow$  they cannot fail with any Heymanian collapse modes
  - $\rightarrow$  failure can happen due to material crushing
  - $\rightarrow$  failure can happen due to contact sliding

Note: a small sliding is usually no problem!



Kamai & Hatzor, 2005

## Flat arches:



Pithole, Venango, Pennsylvania; https://hu.pinterest.com



https://www.livingstonemasons .com/glossary.html



https://www.locallocalhistory.co. uk/studies/lintels/index-m.htm

⇒ they cannot fail with any Heymanian collapse modes
→ failure can happen due to material crushing
→ failure can happen due to contact sliding

Note: a small sliding is usually no problem!

# Flat arch: Image: Control of the second se

Definition:

*M. Herczeg* (2024)

From any point of the extrados, the supports are visible.

Meaning:

For any downwards load acting on the extrados, an equilibrium force system can be drawn.

 $\Rightarrow$  cannot fail with Heymanian collapse mode



Codex Atlanticus:



## M. Herczeg (2024):





*optimal shape* of the voussoirs ? [ limestone; 3D-printed concrete ]



## Static load:





### worst location of the load:



Static load:



Load bearing capacity depending on s / k:



Outwards displacements:



39 / 52

## How to continue:

- $\rightarrow$  Experimental confirmation
- $\rightarrow$  Optimize vertically; size effect
- $\rightarrow$  Imperfectnesses



## e.g. in Toledo, Spain:



https://www.youtube.com/watch?v=EU4Fx5R0Ows



## Romanesque vs Gothic arch:



Aquitaine, France; in metmuseum.org

pinterest.com, 300 Archiecture Travel Inspiration Pictures

massive, *thick*, *semicircular* arch strong piers thick walls, heavy pillars inside *small windows*; darkness

*pointed*, *thin* arch light piers up to the sky thin walls, flying buttresses outside *large* stained glass *windows*; light

Romanesque vs Gothic arch:

<u>Remember the Durand-Claye method:</u> Barsotti et al (2017):

comparison of different arch types (*semicircular*; *equilateral pointed*) and their possible collapse modes



Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Ressler, 2011:





FAILS

<u>Definition:</u> A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Evolution: fundamental in Gothic architecture; may be of Islamic origin



Probably the oldest flying buttress in Europe: Grand Baths at Salamis-Constantia, Cyprus, 3rd-7th century







<u>Definition:</u> A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

fundamental in Gothic architecture; may be of Islamic origin





Fletcher & Fletcher (1905)

## Wide variety of shapes:

**Evolution**:



https://www.pinterest.co.uk/ckefn/flying-buttress

Definition: A buttress / flying buttress is a structural unit placed on the outer side of a wall, to support the lateral thrust of an arch or vault inside.

Mechanics:

receives vertical & horizontal thrust from the supported vault; loads are dominant over selfweight; works simply as a masonry arch  $\Rightarrow$  stability (and not strength) problem



Stability analysis:

- $\rightarrow$  admissible range of thrust in the supported vault?
- $\rightarrow$  admissible range of thrust in the flying buttress?
- $\rightarrow$  is the first range included in the second range?
- $\rightarrow$  often: the flying buttress is a "flat arch"

www.pinterest.co.uk

## [ OPEN ISSUES ]

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# **MULTISPAN ARCH BRIDGES**

## Neighbouring arches support each other:



Skopje Aqueduct, Macedonia, 1st century; exploringmacedonia.com



Railway viaduct in Switzerland, http://crea.bunshun.jp/articles/-/6770



Gatehampton Railway Bridge, http://thames.me.uk/s01240.htm

## Practical analysis:

- $\rightarrow$  rules based on experience; (?) MEXE
- $\rightarrow$  limit state codes
- $\rightarrow$  nonlinear FEM

 $\rightarrow \dots$ 

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? how the neighbours act on each other ?? slender / stocky pillars ?

? ...

## [ OPEN ISSUES ]

## **SUGGESTED VIDEOS**

https://www.youtube.com/watch?v=Rkxlxm26G\_s

(Ressler (2016): Seeing Structure in the Great Architecture of Western Civilization; long, but excellent and easy to understand)

https://www.youtube.com/watch?v=EU4Fx5R0Ows (short; basic arch types) "Know before you go": Identifying Arch Types → excellent!

https://www.youtube.com/watch?v=mstZhReh31k World's largest brick bridge – just nice to see

https://www.youtube.com/watch?v=qL0w\_rhMH3o (Arches, Domes, Vaults: short, elementary intro)

https://www.youtube.com/watch?v=awTkIr1TIoY (examples for nonlinear FEM modelling of single-span arch)

## QUESTIONS

1. What is the *Couplet-Heyman* problem? How much is the *minimally necessary thickness* for a semicircular arch, according to Couplet; Heyman; Milankovitch; Cochetti et al?

2. Why is it advantageous to have a *pointed* arch instead of a *semicircular* arch above the same span?

3. Recognize from a picture: *corbel* arch, *Roman* arch, *segmental* arch, *flat* arch, *pointed* arch, *horseshoe* arch; *multispan* arch. What is the difference between a *segmental arch* and a *flat arch*?