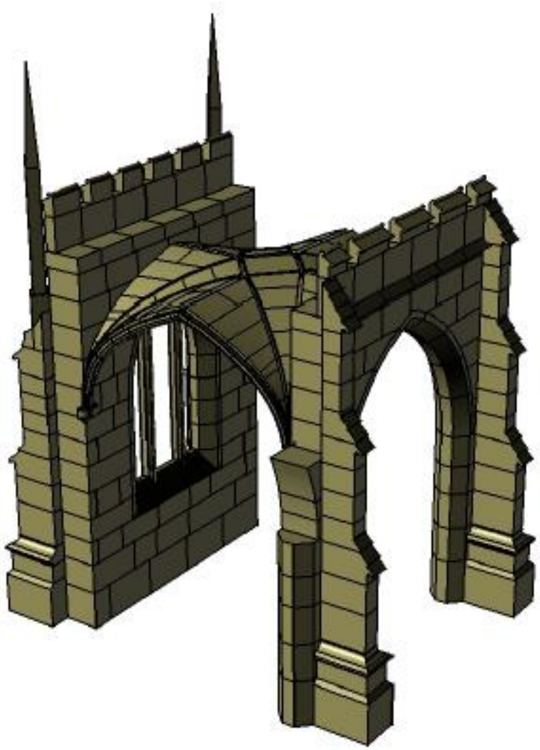
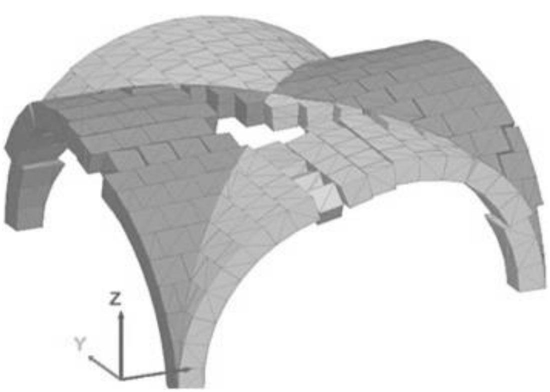
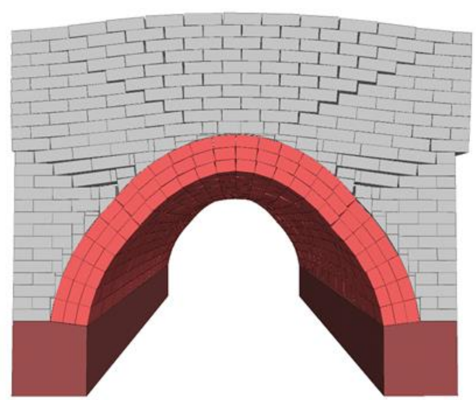


THE DISCRETE ELEMENT METHOD





SHORTLY MENTIONED IN LECTURE 01:
THE DISCRETE ELEMENT METHOD

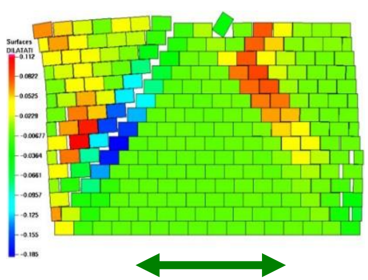
Definition: a numerical method belongs to DEM if

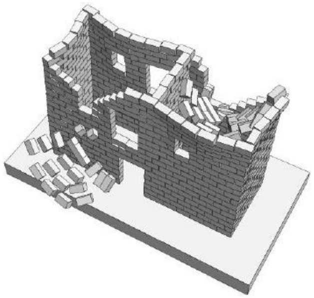
- ← it consists of separate, ***finite-sized*** elements and their contacts
- ← its elements have ***independent*** degrees of frredom, with ***large displs***
- ← contact separation and sliding considered; ***new contacts*** can be born

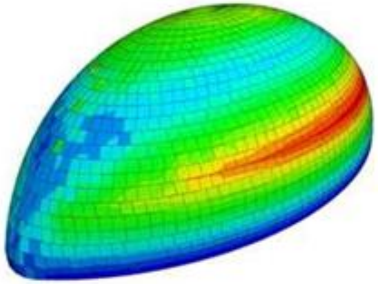
Main steps:

- → define the elements (geometry);
  automatically recognize their contacts
- → specify the material parameters (elements; contacts)
- → loading history: in small steps;
  stepwise: upgrade geometry & topology & material

Detailed introduction to DEM: today in Lecture 05







THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics

Questions

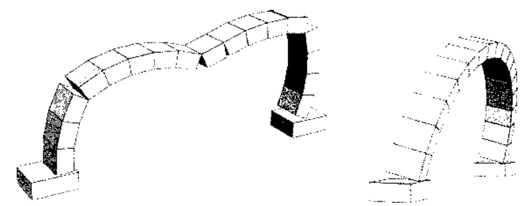
WHAT IS DEM?

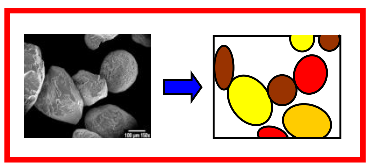
The aim: to model materials or structures having discrete internal builtup

„what does it do if loads are put on it?”

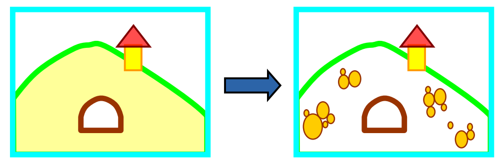
The components of the model:

separate elements + their contacts





or



- mechanical models for the material of the elements: → rigid
  → deformable
- contacts → recognition
  → mechanical models for the contacts:
    → non-deformable
    → deformable: e.g.point-like, deformable; e.g. finite size, deformable } e.g. frictional, e.g. cemented

WHAT IS DEM?

History overview

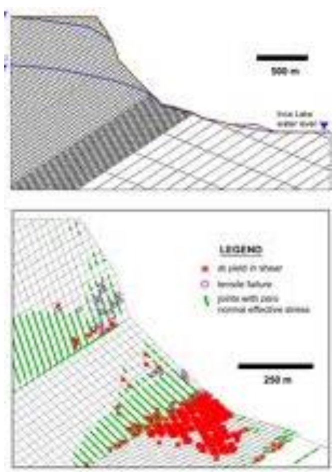
→ end of 1960ies:

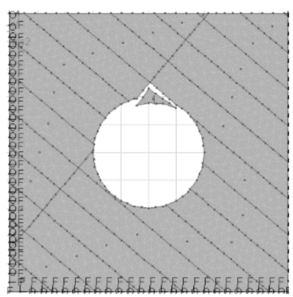
Peter A Cundall,

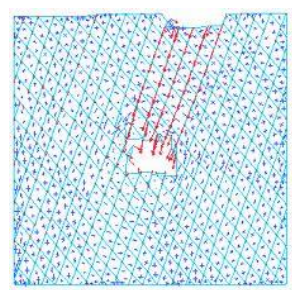
Imperial College:

UDEC

(„Uniform Distinct Element Code")



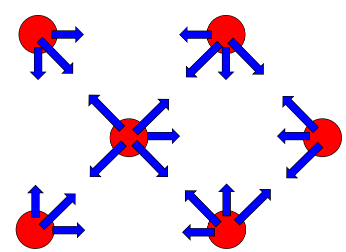




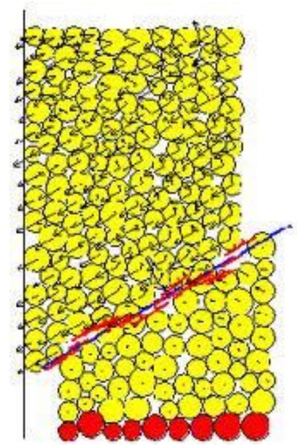
model for fractured rocks

→ 1970ies: Molecular Dynamics methods, physics literature

not really DEM



WHAT IS DEM?



History overview

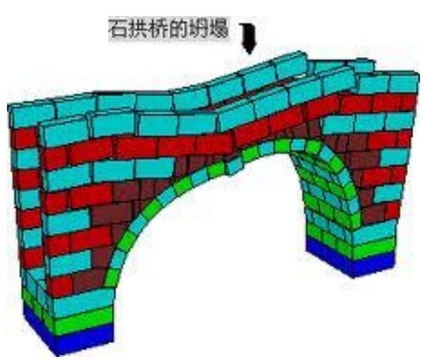
→ end of 1970ies: Cundall & Strack, 1979:

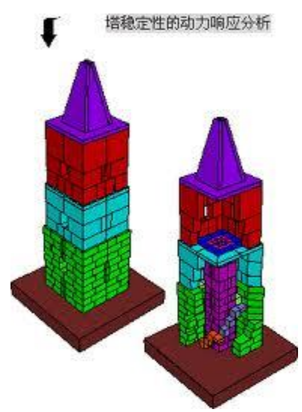
BALL

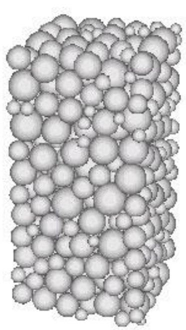
→ from the 1980ies:

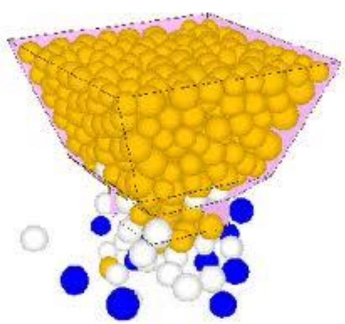
→ several new codes, already in 3D
→ general element shapes
→ different mathematical tools

→ from the 1990ies: practical applications in engineering





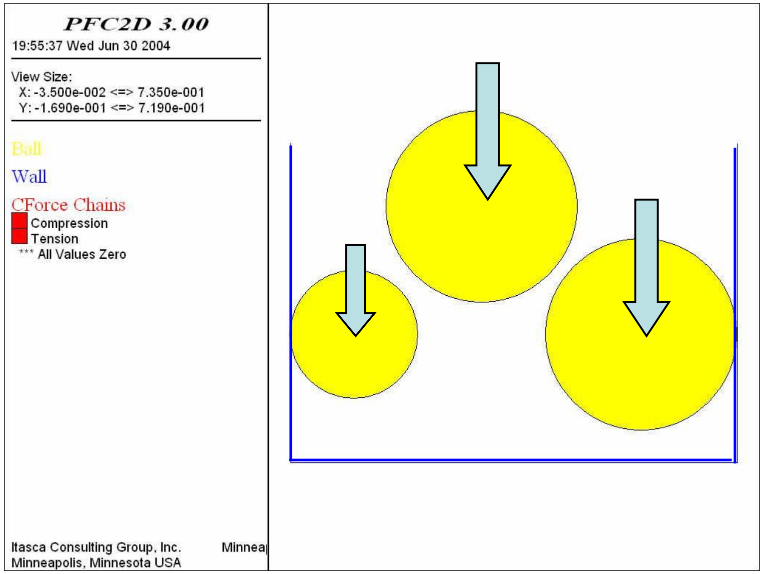




EXAMPLE

1. Define the geometry:

```
ball id 1 x 0.10  y 0.20  rad 0.10
ball id 2 x 0.55  y 0.20  rad 0.15
ball id 3 x 0.30  y 0.40  rad 0.15
wall id 1 nodes 0.0  0.0  0.7  0.0
wall id 2 nodes 0.7  0.0  0.7  0.5
wall id 3 nodes 0.0  0.5  0.0  0.0
```



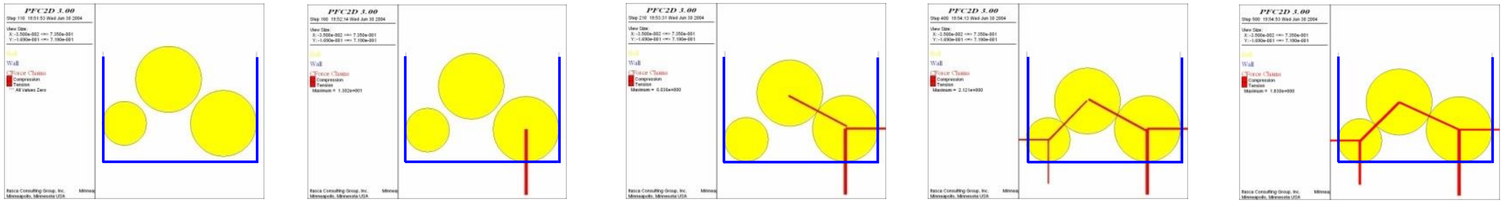
2. Specify the material parameters:

```
property density 10.0
property kn 1.e4 ks 0.5e4 friction 0.2
wall id 1 kn 1.e12 ks 0. friction 0.
wall id 2 kn 1.e12 ks 0. friction 0.
wall id 3 kn 1.e12 ks 0. friction 0.
```

3. Specify the loads:

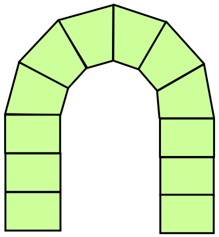
```
set gravity 0.0  -9.81
```

4. Calculate the displacements [series of small increments]



WHAT IS DEM?

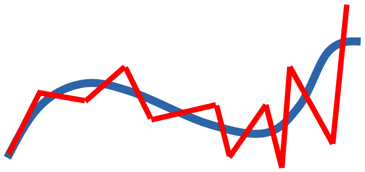
<u>Main steps of the analysis of an engineering problem:</u>



- the model: collection of separate elements (’**discrete elements**’)
  {1 body ↔ 1 element} or {several bodies ↔ few elements}
  <u>Step 1.:</u> define the initial geometry

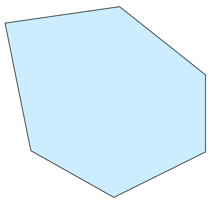
- rigid or deformable ***elements***; rigid or deformable ***contacts***
  <u>Step 2.:</u> specify the material characteristics

- the loading process:
  (e.g. external forces acting on the elements; e.g. prescribed displacements)
- calculation of the state changing: ***series of small increments, based on*** „ **f** = m**a** ”
  <u>Step 3.:</u> calculation of the actual displacement increments

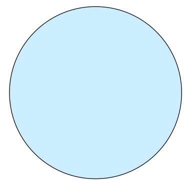


THE GEOMETRY

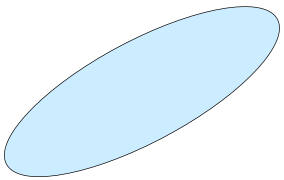
Element shapes:



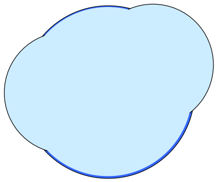
polygon, polyhedron



circle, sphere

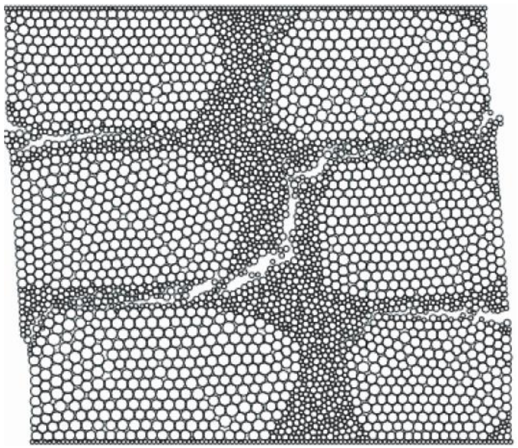


ellipse, ellipsoid

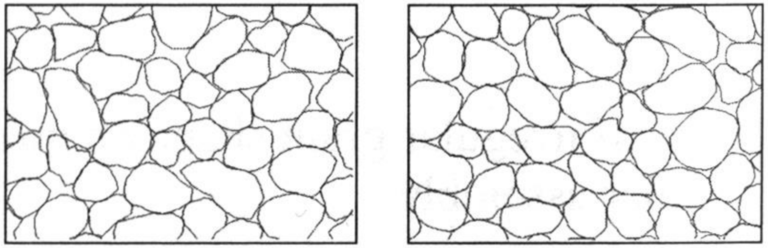


complex shapes

e.g. Lemos (2007):
masonry blocks & mortar layer:



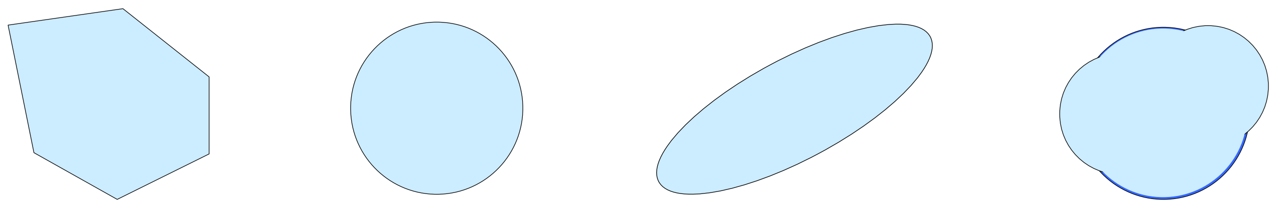
e.g. Matsushima (2005):
irregularly shaped sand particles



(a) Toyoura sand model (b) Ottawa sand model

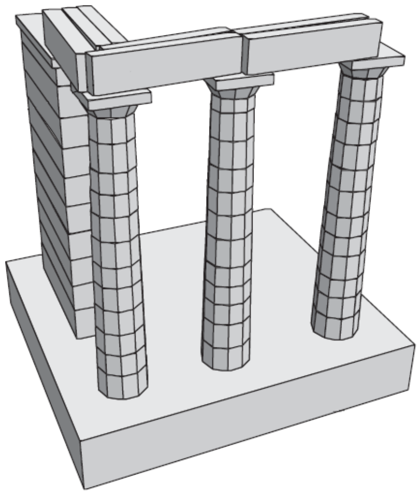
THE GEOMETRY

Element shapes:

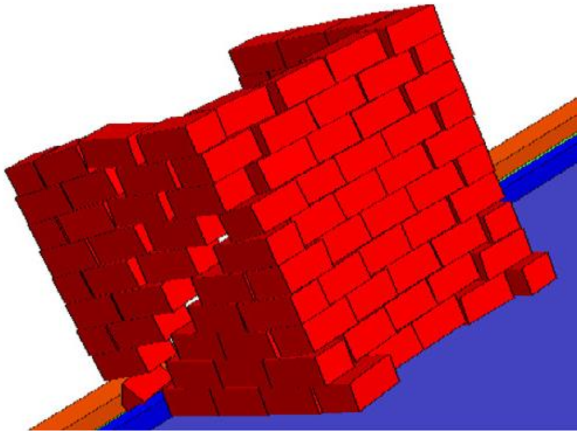


polygon, polyhedron   circle, sphere   ellipse, ellipsoid   complex shapes

e.g. Psycharis et al (2003):
stone blocks:



e.g. Bui et al (2017):
bricks of a house:



THE GEOMETRY

How to get the geometry of a masonry structure:

→ original plans (if still exist)

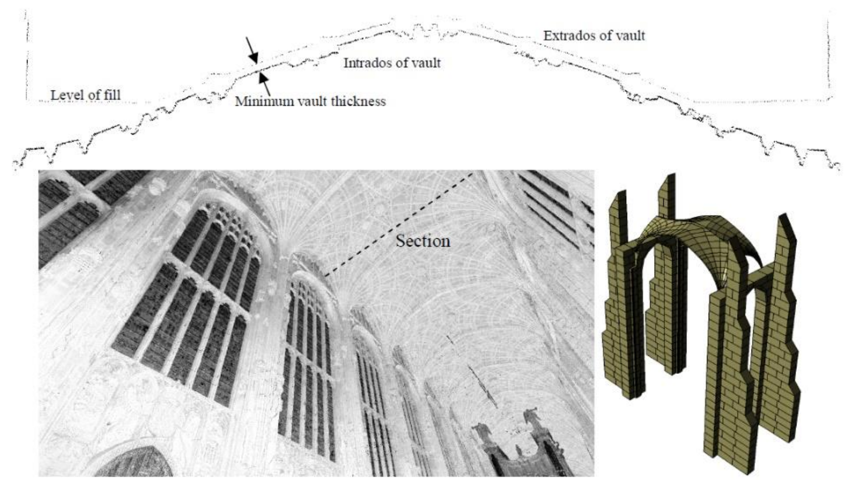
→ survey the actual geometry, e.g. laser scanner & CAD:

e.g. McInerney et al (2012):

St John's College, Cambridge, UK

Difficulty e.g.:

how to survey hidden/covered faces

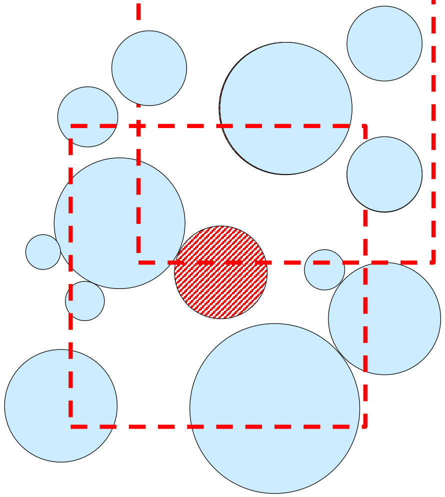


King's College, Cambridge, UK

THE GEOMETRY

<u>Contact recognition:</u> several different algorithms exist;
its speed basically determines the computational efficiency of the whole DEM code!

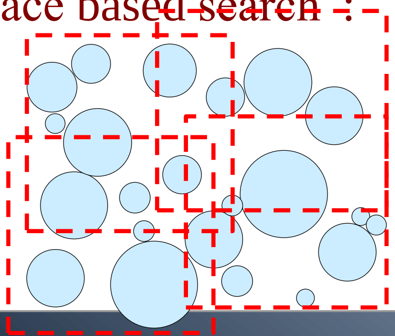
<u>the time consuming part:</u> to check the existence of a contact with exact calculations



Trick #1:
avoid checking <u>every</u> element with <u>every other</u> element:

→ „body based search” technique:
consider only those others which are in the vicinity of the analyzed element;
then take the next element to analyze, …

→ „space based search”:

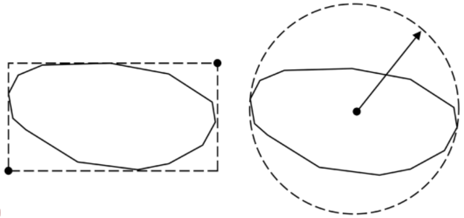


divide the domain into „windows” (overlapping);
collect which elements are in which windows;
analyze those pairs only where both elements belong to the same window

THE GEOMETRY

Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

the time consuming part: to check the existence of a contact with exact calculations



Trick #2:

Simple surrounding domains checked first
(instead of the elements having complicated shapes)

the idea: „surrounding domain” assigned to each element
(simple shape: brick; sphere)

→ Phase 1.: intersection beween the surrounding domains? (fast)

→ if necessary: Phase 2.: detailed, exact calculations (slow)

MECHANICAL PROPERTIES

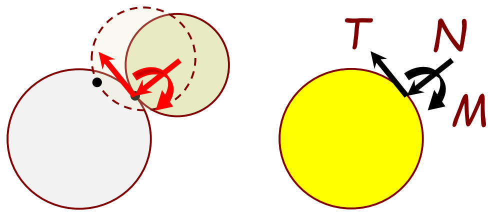
Mechanical behaviour of the elements:

role: to specify how to calculate the stresses from the deformations of the elements

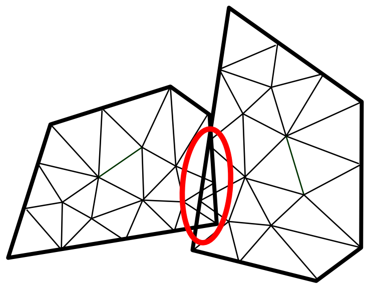
→ perfectly rigid elements: deformability concentrated into the contacts

→ deformable elements:
stress-strain-relations have to be specified
[e.g. E, μ, …]

Mechanical behaviour of the contacts:



role: to specify how to calculate the contact forces from the relative displacements at the contact



→ usually: „deformable" contacts
(relative displ. at the contact regions)
concentrated ↔ distributed

→ sometimes:
infinitely rigid contacts: no overlap neither any other deformation

CALCULATION OF DISPLACEMENTS

Quasi-static methods ← *an equilibrium state is searched for*

From an initial equilibrium state, the incremental displacements **u** are to be determined taking the system to the new equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\Delta\mathbf{f}=\mathbf{0}"$$

→ Kishino (1988); Bagi-Bojtár (1991) { *circular, perfectly rigid elemets, deformable contacts*

→ Meng et al (2017); Baraldi et al (2018) { *not really DEM yet: small displs; no new contacts;*

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← *a process in time is searched for*

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ UDEC ← *deformable polyhedral elements, deformable contacts*

→ Munjiza's FEM/DEM ← *deformable, breakable elements, deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis”) ← *deformable polyhedral elements*

→ Contact Dynamics models ← *rigid elements, non-deformable contacts*

SOLUTION OF THE EQUATIONS OF MOTION

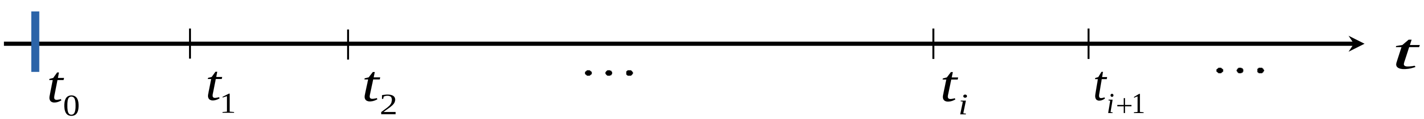
<u>Numerical solutions only!</u>

$$\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}\,(t,\mathbf{u}(t),\mathbf{v}(t))$$

The aim:

starting from a known $\mathbf{u}(t_0) = \mathbf{u}_0$ and $\mathbf{v}(t_0) = \mathbf{v}_0$ state at a t_0 time instant,

the aim is to determine the approximative solutions $(\mathbf{u}_1, \mathbf{v}_1)$, $(\mathbf{u}_2, \mathbf{v}_2)$, …, $(\mathbf{u}_i, \mathbf{v}_i)$, $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1})$, … belonging to the $t_1, t_2, \ldots, t_i, t_{i+1}, \ldots$ time instants.

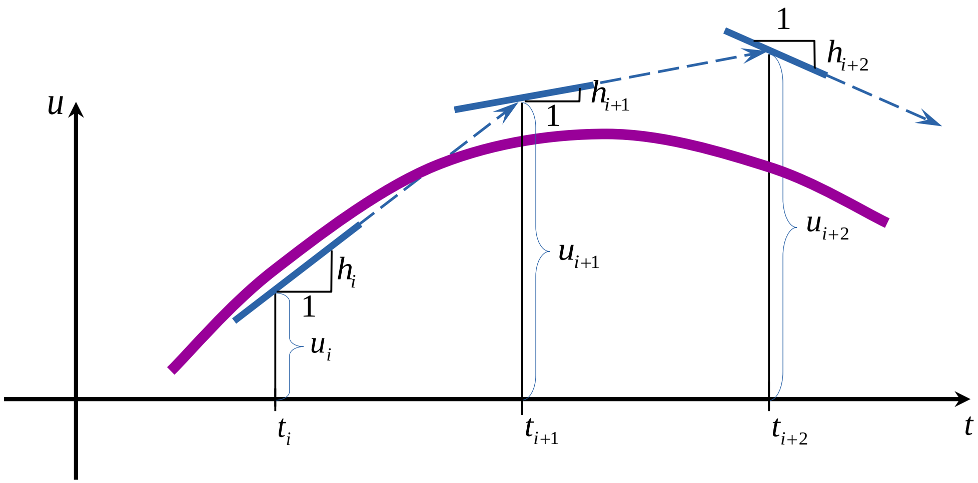


The two basic approaches:

Explicit vs. implicit time integration methods

SOLUTION OF THE EQUATIONS OF MOTION

Explicit vs. implicit methods:



→ explicit methods:

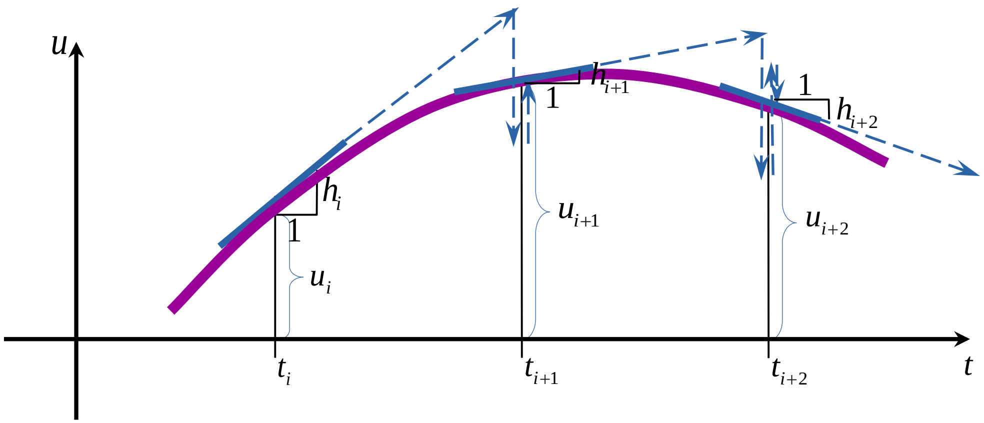
in the state at t_i: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{f}_i)$ $\Rightarrow$ equations of motion $\Rightarrow$ approximate $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1}, \mathbf{f}_{i+1})$ belonging to the state at t_{i+1}

NOT checking whether $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1}, \mathbf{f}_{i+1})$ satisfy the eqs of motion: accept them and use them for the calculations of the next timestep

$\Rightarrow$ fast, but less reliable; numerical stability problems!

SOLUTION OF THE EQUATIONS OF MOTION

Explicit vs. implicit methods:



→ implicit methods:

in the state at t_i: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{f}_i) \Rightarrow$ equations of motion $\Rightarrow$

approximate $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1}, \mathbf{f}_{i+1})$ belonging to the state at t_{i+1};

then iterations, to improve this approximation belonging to t_{i+1},

so that the eqs of motion be satisfied at t_{i+1}

$\Rightarrow$ slow, but longer timesteps;

more reliable, better numerical stability

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics

Questions

UDEC / 3DEC

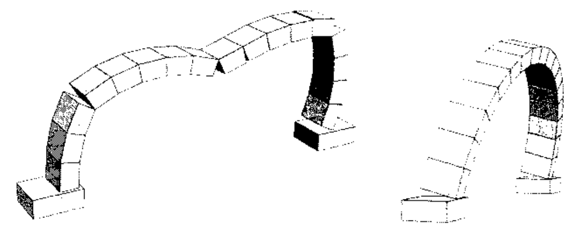
<u>UDEC:</u> „Universal Distinct Element Code”

P.A. Cundall, 1971;

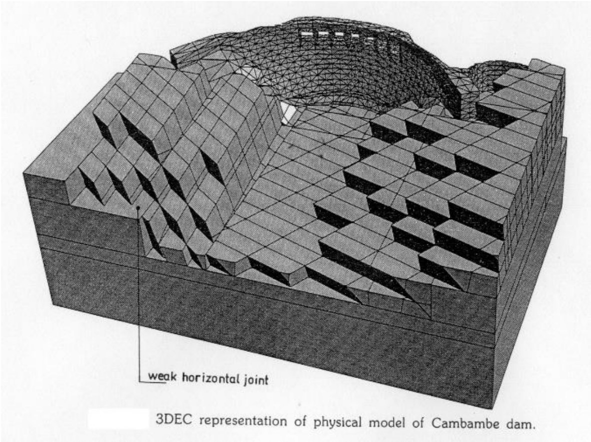
development through decades

Itasca Consulting Group

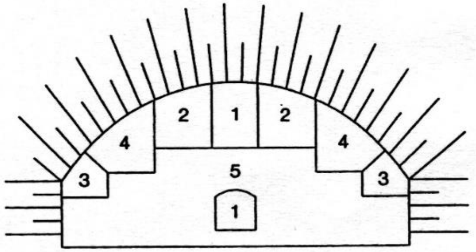
www.itascacg.com



MOST WIDESPREAD IN CIVIL ENGINEERING



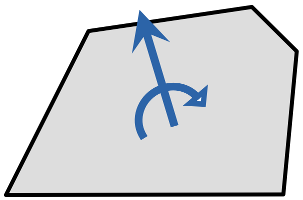
3DEC representation of physical model of Cambambe dam.



UDEC / 3DEC

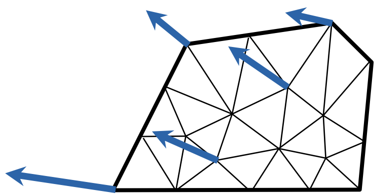
Elements: polygons / polyhedra (planar faces!);

– rigid elements



degrees of freedom:
translation of and rotation about the centroid

– deformable elements (subdivided into simplex zones)



„uniform strain" tetrahedral zones
((*10-node tetrahedra – not reliable*))
degrees of freedom: translations of the nodes

Material models for the elements:

(rigid) ↔ deformable:

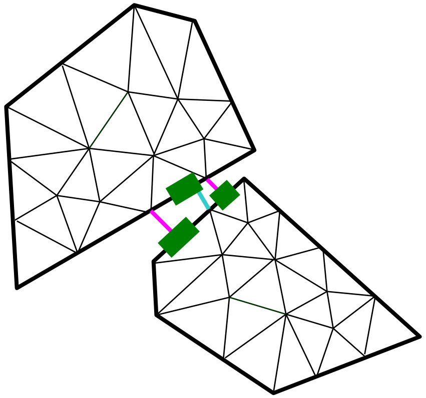
– „null element" (no material in the element)
– linearly elastic, isotropic *(e.g. intact rock; metal)*
– lin. elast., with: Mohr-Coulomb / Prager-Drucker failure crit.
(e.g. soils, concrete) *(e.g. clay)*
+ tensile strengh + cohesion + dilation angle
– ...

UDEC / 3DEC

Contacts:

consist of small „subcontacts", over which:
uniformly distributed normal and
shear contact forces are transmitted

$$\Delta\sigma = k_N \Delta u_N$$
$$\Delta\tau = k_T \Delta u_T$$



Material models for the contacts:

[calculate the increments of distrib. contact forces from the increments of rel. disps]

– if no material in the contacts: $\rightarrow$ k_n, k_s: numerical parameters, ∞
or express surface roughness ;
$\rightarrow$ friction: real value

– if material in the joints: (modelled as length or area, with zero thickness):

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion and tensile strength

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion & tensile strength + softening + dilation angle

$$\Delta U_n(dil) = \Delta U_s \tan\psi$$

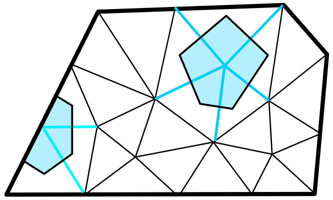
– others …

UDEC / 3DEC

<u>Calculation of nodal displacements</u>

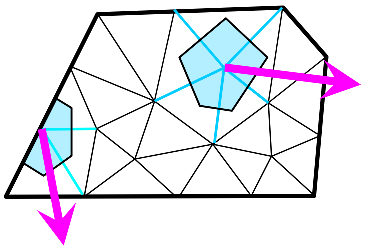
Newton II.: „ $m\,a = f$ ”

– mass assigned to the node:



Voronoi-cell

– force on the node: resultant of the forces acting on the Voronoi-cell of the node



← from the neighbouring element

← from external forces (e.g. self weight, drag force)

← from the stresses inside the simplexes

- force from the stress within a simplex:
    - --- nodal translations ⇒ simplex strain ✓
    - --- from this and material characteristics ⇒ uniform stress in the simplex ✓
    - --- stress vector acting on the face of the cell: $\sigma_{ij} n_j = p_i$; resultant ✓

UDEC / 3DEC

<u>Calculation of nodal displacements</u>

Newton II.: „ $ma = f$ ”

– discretized form of the eqs of motion:

$$m\frac{\mathbf{v}(t_i+\Delta t/2)-\mathbf{v}(t_i-\Delta t/2)}{\Delta t}=\mathbf{f}(t_i)$$

or:

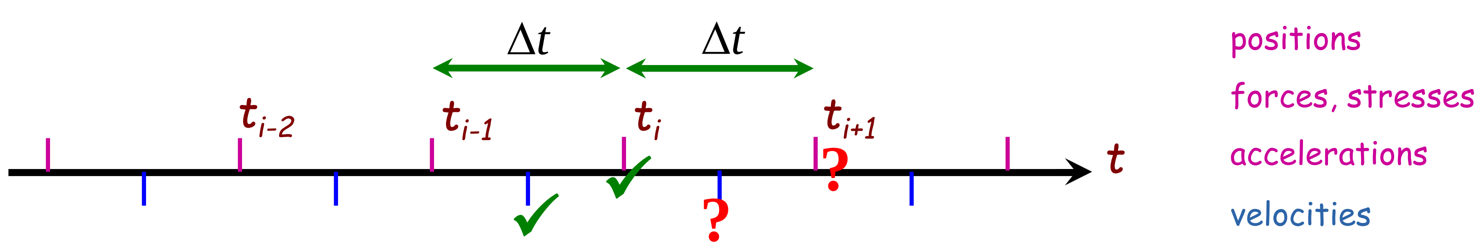
$$\boxed{\mathbf{v}(t_i+\Delta t/2)=\mathbf{v}(t_i-\Delta t/2)+\frac{\mathbf{f}(t_i)}{m}\Delta t}$$

– at t_i : the ***positions of the nodes*** and the ***forces and stresses*** are known;

at $t_i-\Delta t/2$: the ***nodal velocities*** are known;

determine the ***nodal velocities*** at $t_{i+1/2}=t_i+\Delta t/2$

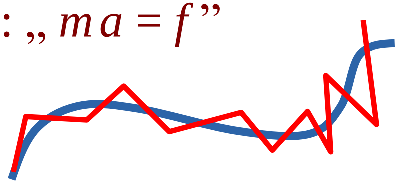
and the ***positions of the nodes*** at $t_{i+1}=t_i+\Delta t$



UDEC / 3DEC

<u>Calculation of nodal displacements</u>

Newton II.: „ $m\,a = f$ ”



– series of small finite time steps:

– explicit time integration; no stiffness matrix!!!

$\Rightarrow$ numerical instabilities, convergence problems

– <u>to help numerical stability:</u>

1. estimate the longest allowed Δt
2. artifical damping is introduced [different types can be used]

MAIN DISADVANTAGE:

strong oscillations around the exact solution

$\Rightarrow$ may give unrealistic results [e.g. in case of history dependence]

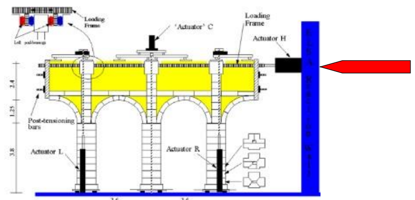
$\Rightarrow$ numerical instabilities may occur

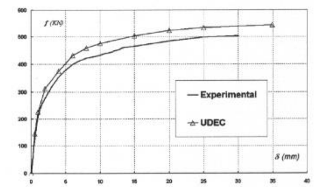
UDEC / 3DEC

<u>Applications for masonry structures:</u>

Quasi-static problems:

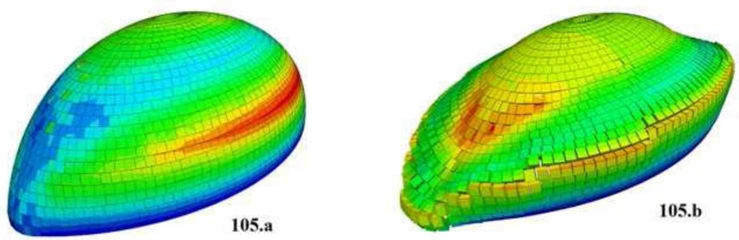
e.g. Sao Vicente de Fora Monastery, Portugal: Giordano et al, 2002

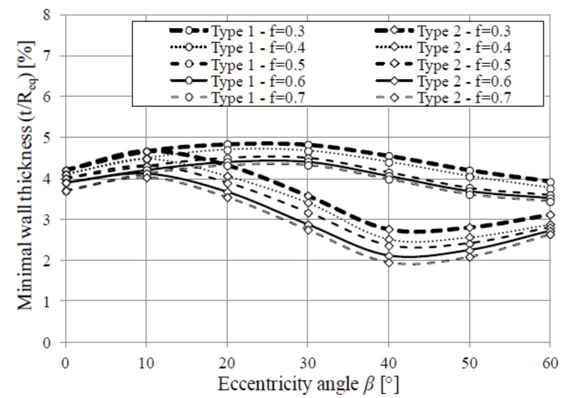




<u>UDEC advantages:</u> works well for ***large displs***; realistic ***crack pattern***

e.g. oval dome statics: Simon & Bagi, 2016





Dynamic problems (use with caution!):

→ convergence of the solution with respect to Δt should be ensured

→ damping type and damping parameters should carefully be selected & calibrated

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics

Questions

DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

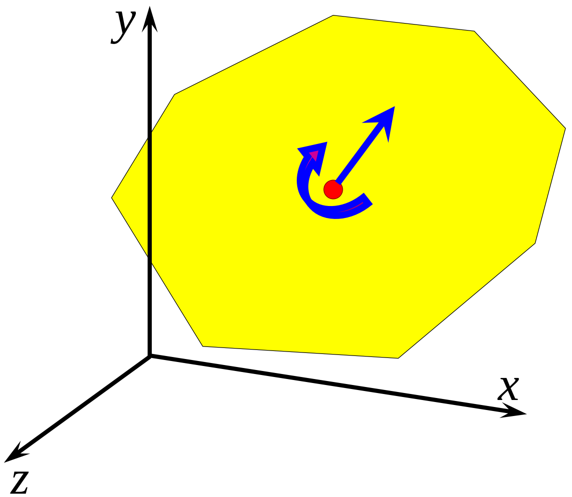
Gen-Hua Shi (1988), Berkeley
then many others applied or developed
research software!!!

<u>The elements:</u> polyhedral; with a reference point (e.g. centroid)
[Deformable without subdivision]

„displacement vector” of the *p*-th element: $\mathbf{u}^p$

„reduced load” belonging to the *p*-th element: $\mathbf{f}^p$

$$\mathbf{u}^p = \begin{bmatrix} u_x^p \\ u_y^p \\ u_z^p \\ \varphi_x^p \\ \varphi_y^p \\ \varphi_z^p \\ \varepsilon_x^p \\ \varepsilon_y^p \\ \varepsilon_z^p \\ \gamma_{yz}^p \\ \gamma_{zx}^p \\ \gamma_{xy}^p \end{bmatrix} \qquad \mathbf{f}^p = \begin{bmatrix} f_x^p \\ f_y^p \\ f_z^p \\ m_x^p \\ m_y^p \\ m_z^p \\ V^p\sigma_x^p \\ V^p\sigma_y^p \\ V^p\sigma_z^p \\ V^p\tau_{yz}^p \\ V^p\tau_{zx}^p \\ V^p\tau_{xy}^p \end{bmatrix}$$



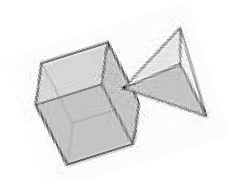
The degrees of freedom:
rigid-body translation and rotation
of the reference point;
+ the ***<u>uniform</u>*** strain of the element

DDA

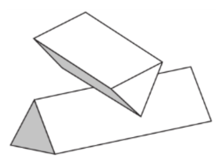
The contacts: (material point) with (material point)

in 2D: Node – to – Edge contacts

in 3D: Node – to – Face contacts:



Edge – to – Edge contacts:



→ „first entrance position”

⇒ contact deformation: $\Delta u_N ; \Delta u_T$

normal & tangential (perhaps sliding)

→ direction of the contact:

the normal vector of the face

???? for edge-to-edge contact

Mechanical model:

→ originally: infinitely rigid contacts, Coulomb-friction

→ recent codes: deformable contacts included

\+ other friction conditions, cohesion etc.

Remark: infinitely rigid contact: „penalty function”: $F_N = k_N \Delta u_N ;\; dF_T = k_T d(\Delta u_T)$

≡ linearly elastic in normal and in tangential directions

DDA

↓ more exactly: „ Hamilton principle ”

The equations of motion: „Potential energy” stationarity principle

„Potential” of the system: $\Pi = \Pi^{blocks} + \Pi^{contacts}$

deformed springs

external pot.

strain energy

inertial forces

velocity-proportional damping

initial stress

prescribed displacement history

$$\boxed{\frac{\partial \Pi}{\partial u_i^p} = 0} \quad \text{for all } p, i$$

$$\mathbf{M}\cdot\mathbf{a}(t) + \mathbf{C}\cdot\mathbf{v}(t) + \mathbf{K}\cdot\mathbf{u}(t) = \mathbf{f}^{ext}(t, \mathbf{v}(t), \mathbf{u}(t))$$

generalized displacement increment

stiffness matrix + etc

damping + etc

inertia

or: $\boxed{\mathbf{M}\cdot\mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))}$

DDA

Numerical solution of the equations of motion:

(t_i, t_{i+1}) time interval:

at t_i : known $\mathbf{u}_i$, $\mathbf{v}_i$, $\mathbf{f}(t_i, \mathbf{u}_i, \mathbf{v}_i)$; satisfy the eqs. of motion

Find $\mathbf{u}_{i+1}$, $\mathbf{v}_{i+1}$, $\mathbf{a}_{i+1}$ so that the eqs of motion would be satisfied at t_{i+1}

$$\boxed{\mathbf{r}(t_{i+1}, \mathbf{u}_{i+1}, \mathbf{v}_{i+1}) = \mathbf{f}(t_{i+1}, \mathbf{u}_{i+1}, \mathbf{v}_{i+1}) - \mathbf{M} \cdot \mathbf{a}_{i+1} = 0}$$

Remember: Newmark's β–method:

$$\boxed{\begin{aligned} \mathbf{u}_{i+1} &= \mathbf{u}_i + \Delta t \cdot \mathbf{v}_i + \frac{\Delta t^2}{2}\left[(1-2\beta)\mathbf{a}_i + 2\beta \cdot \mathbf{a}_{i+1}\right] \\ \mathbf{v}_{i+1} &:= \mathbf{v}_i + (1-\gamma) \cdot \Delta t \cdot \mathbf{a}_i + \gamma \cdot \Delta t \cdot \mathbf{a}_{i+1} \end{aligned}}$$

[stability: $2\beta \geq \gamma \geq ½$]

DDA: Newmark's β–method, with $\beta = 1/2$; $\gamma = 1$:

$$\boxed{\begin{aligned} \mathbf{u}_{i+1} &= \mathbf{u}_i + \Delta t \cdot \mathbf{v}_i + \frac{\Delta t^2}{2}\mathbf{a}_{i+1} \\ \mathbf{v}_{i+1} &:= \mathbf{v}_i + \Delta t \cdot \mathbf{a}_{i+1} \end{aligned}}$$

let
$$\left\{ \begin{aligned} \Delta\mathbf{u}_{i+1} &= \mathbf{u}_{i+1} - \mathbf{u}_i \\ \mathbf{a}_{i+1} &= \frac{1}{\Delta t^2/2}(\Delta\mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) \\ \mathbf{v}_{i+1} &= \mathbf{v}_i + \Delta t \cdot \mathbf{a}_{i+1} = \mathbf{v}_i + \frac{2}{\Delta t}(\Delta\mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) = \frac{2}{\Delta t}\Delta\mathbf{u}_{i+1} - \mathbf{v}_i \end{aligned} \right.$$

DDA

Numerical solution of the equations of motion :

$$\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t)) \quad\Rightarrow\quad \mathbf{0}=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1},\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

Determine $\Delta\mathbf{u}_{i+1}$, so that the residual

$$\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1})=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1}),\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

would be sufficiently close to zero!

Newton-Raphson:

the Jacobian of the residual: $\mathcal{K}(t,\Delta\mathbf{u})=\dfrac{d\mathbf{r}(t,\Delta\mathbf{u})}{d\Delta\mathbf{u}}$

this matrix can be compiled from elementary calculations at t_i:
← contains the stiffness matrix
← contains the inertia, contact forces, geometric characteristics etc.

the residual can also be compiled from elementary calculations at t_i:
← contains the external forces, inertia effects, prescribed displacements, damping etc.

DDA

Numerical solution of the equations of motion :

$$\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t)) \quad\Rightarrow\quad \mathbf{0}=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1},\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

$$\boxed{\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1})=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1}),\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})}$$

$$\mathcal{K}(t,\Delta\mathbf{u})=\frac{d\mathbf{r}(t,\Delta\mathbf{u})}{d\Delta\mathbf{u}}$$

Analysis of a time interval:

initial estimation for $\Delta\mathbf{u}_{i+1}$: $\Delta\mathbf{u}_{i+1}^{(0)}:=\mathbf{0}$

k+1-th estimation for $\Delta\mathbf{u}_{i+1}$: $\Delta\mathbf{u}_{i+1}^{(k+1)}:=\Delta\mathbf{u}_{i+1}^{(k)}-\mathcal{K}(t_{i+1},\Delta\mathbf{u}_{i+1}^{(k)})^{-1}\cdot\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1}^{(k)})$

then continue until $\left|\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1}^{k+1})\right|$ becomes sufficiently small

„Open – close iterations”: at the end of Δt: **check** the topology and the forces;

→ **modify the topology** if necessary (e.g. new contacts, sliding, contact loss)

→ with the new topology, **repeat:** Newton-Raphson to find another $\Delta\mathbf{u}_{i+1}$

if acceptable topology not found: **decrease timestep** Δt to 1/3 of its previous length

CONVERGENCE WITHIN A TIME STEP ???

DDA: „DISCONTINUOUS DEFORMATION ANALYSIS"

Comparison to UDEC:

Main differences from UDEC:

→ basic unknowns: also the components of $\boldsymbol{\varepsilon}$;
→ uniform stress and strain field inside the elements;
→ numerical integration: implicit
→ stiffness matrix included ⇒ artificial damping not necessary

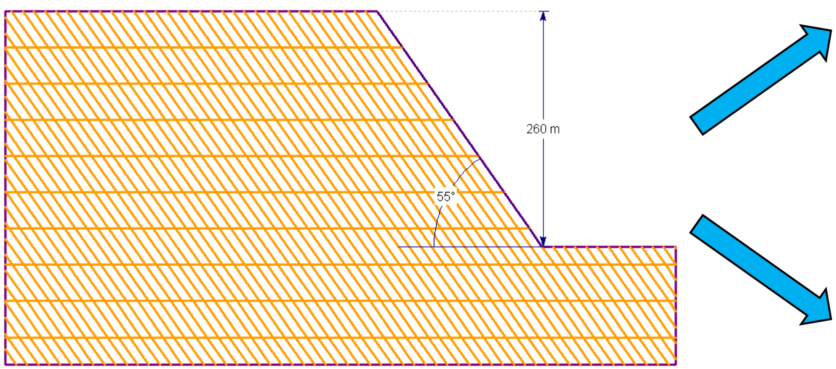
– advantages to UDEC: → implicit ⇒ numerical stability;
fast convergence if topology does not change
no artificial damping required

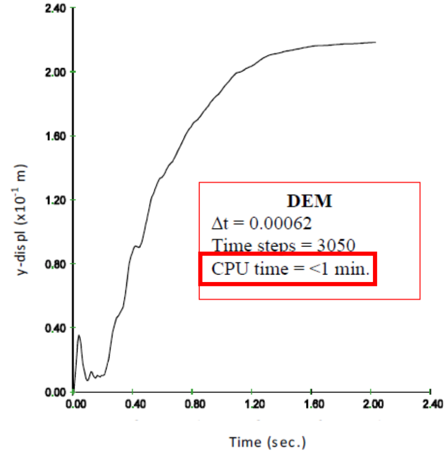
– disadvantages: no commercial software ⇒ inconvenient
(several research codes; e.g. ask from Gen-Hua Shi)
too simple mechanics of the elements and of the contacts
large storage requirements & longer computations
open-close iterations: convergence is not ensured if topology changes

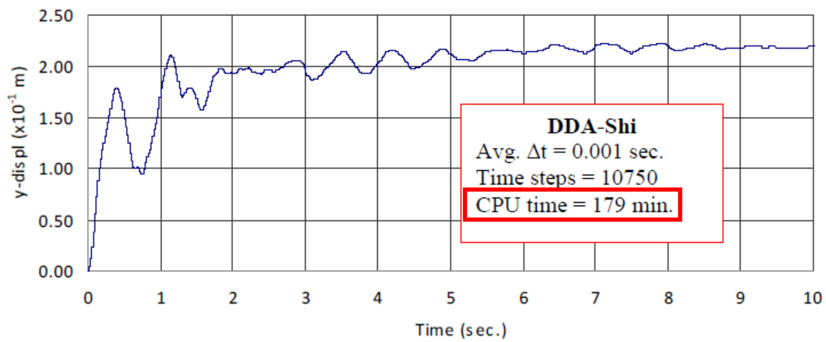
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Comparison to UDEC:

M.S. Kahn (2010)





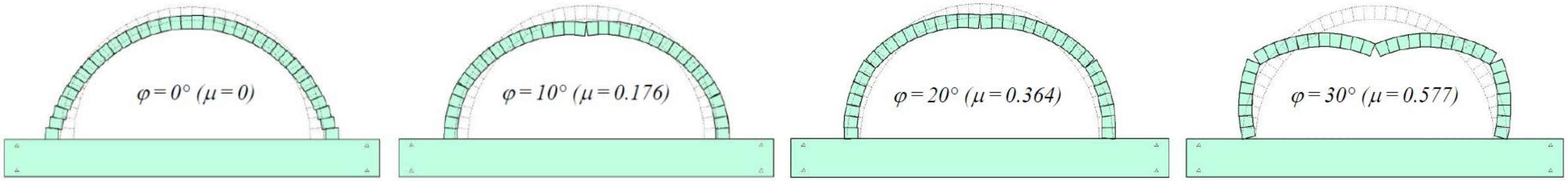


NOT EFFICIENT IN CASES IF SIGNIFICANT TOPOLOGY MODIFICTIONS OCCUR !!!

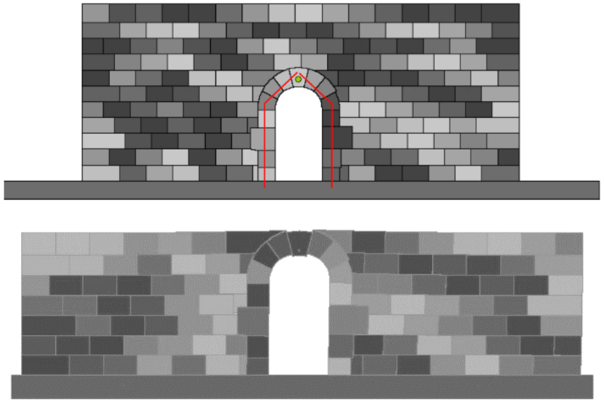
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS"

Applications:

e.g. Rizzi et al (2014): collapse modes of arches



e.g. Kamai and Hatzor (2005): back analysis of seismic events



THIS LECTURE:

What is DEM?

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DEM models

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Questions

CONTACT DYNAMICS

<u>Jean & Moreau (1992):</u> (2D, 3D) [mostly in physics]

Unger, T. – Kertész, J. (2003): The contact dynamics method for granular media. In: Modeling of Complex Systems, Melville, New York, American Institute of Physics, pp. 116-138

Software:

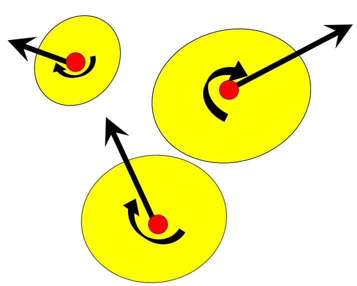
(1) LMGC91 (Dubois & Jean, 2006): **OPEN!**

rigid/deformable; spherical/polyhedral elements

(2) SOLFEC (Koziara & Bicanic, 2008):

rigid/deformable; polyhedral elements

<u>The elements:</u>



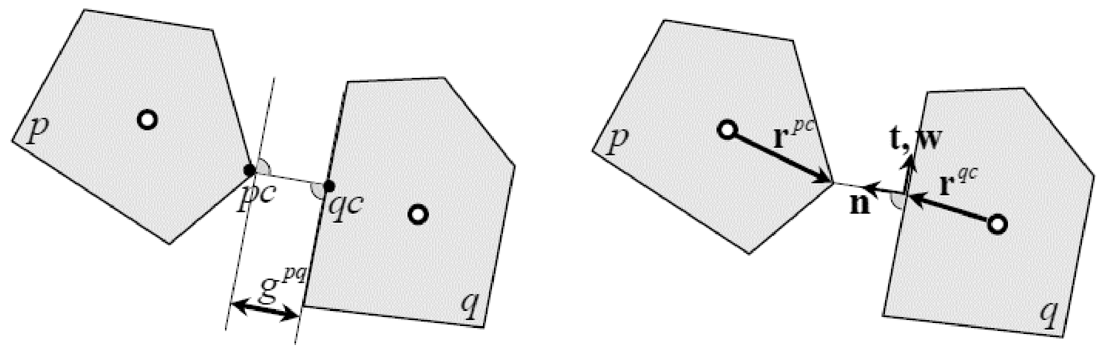
<u>ORIGINALLY:</u> rigid, spherical elements

for masonry structures:

deformable or rigid polyhedral elements

CONTACT DYNAMICS

Contacts of polyhedral elements:

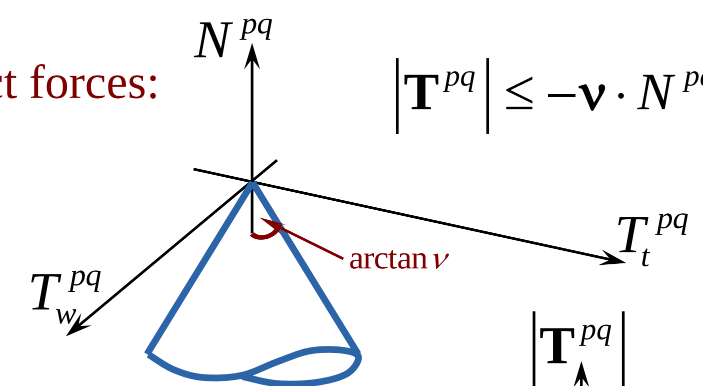


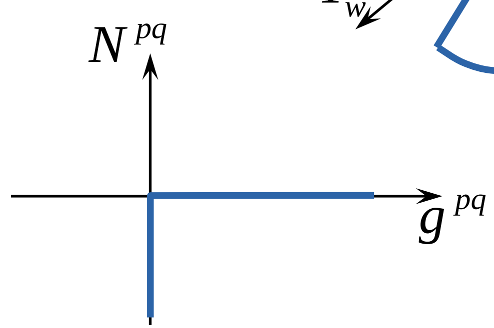
„common plane concept”

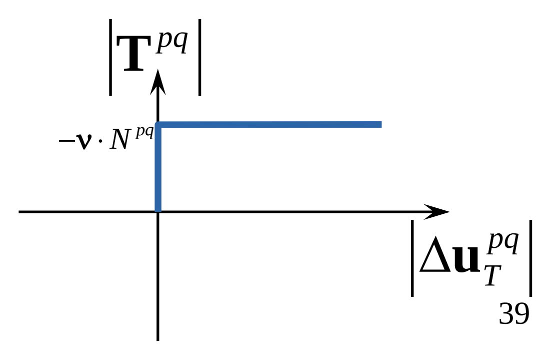
Mechanical conditions for the contact forces:

$$\left|\mathbf{T}^{pq}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}$$

[the same]

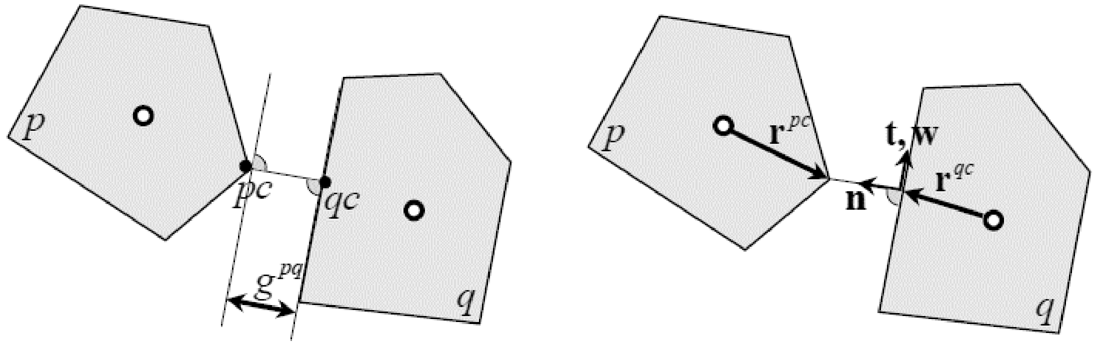






CONTACT DYNAMICS

Contacts of polyhedral elements:



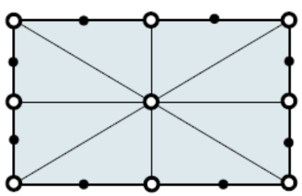
Rigid polyhedral elements:

Degrees of freedom: translations & rotations of the reference points

Deformable polyhedral elements: ~~constant strain~~ → unfavourable experiences

$\boxed{\text{uniform-strain tetrahedral subdivision}}$

The point of action of the contact force:



• : middle point of the face

„**approximated contact point**"

contact: if • touches another face

Masses: distributed to the **nodes**

Equations of motion: for every **node** [no rotations considered];

Degrees of freedom: nodal translations [similar to 3DEC def]

CONTACT DYNAMICS

How to find the solution at the end of a given time step:

implicit solution:

the positions and velocities are repeatedly (iteratively) adjusted,
until the equations of motion AND the contact conditions are satisfied
with the required accuracy at the end of the time step

[≈ Cross method for frames, but randomly sweeping along the pairs of elements]

history dependence! [order of sweeping along contacts makes difference in the results]

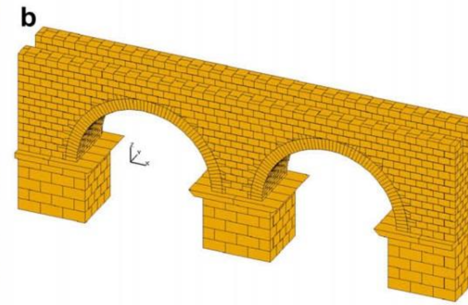
⇒ engineers have doubts

Main advantage: extremely fast for dynamic phenomena

CONTACT DYNAMICS

<u>Civil engineering applications</u>

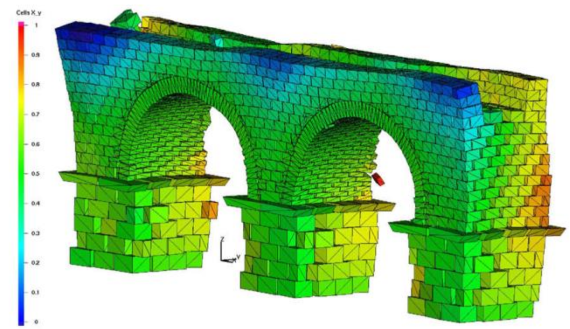
e.g. Rafiee et al (2008):



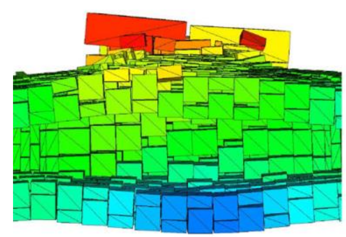
CD numerical model with deformable elements:

Arles, aqueduct

Earthquake simulations:



☹ Experimental verification?

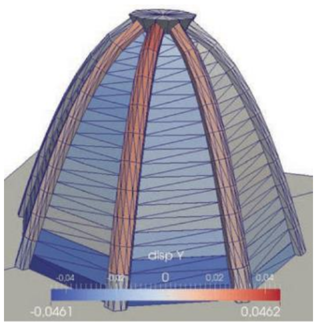


CONTACT DYNAMICS

<u>Civil engineering applications</u>

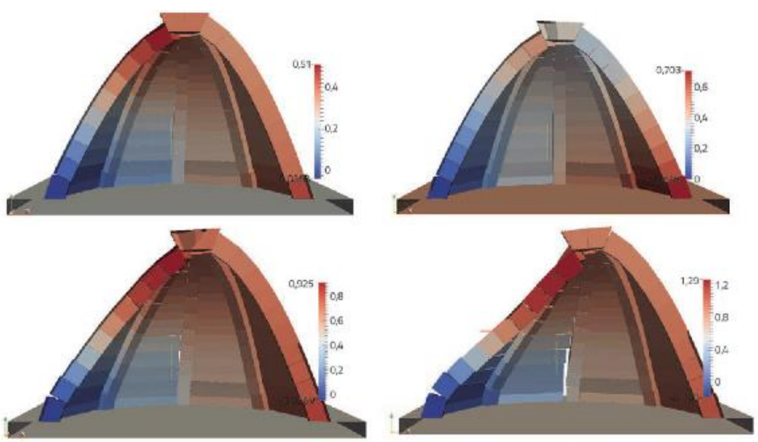
e.g. Gelo & Mestrovic (2016):

dome of St Jacob Cathedral, Sibenik, Croatia



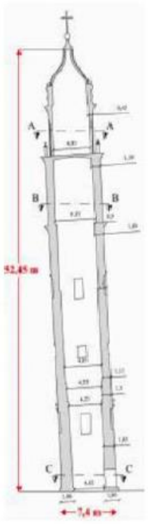
croatiatraveller.com/Heritage_Sites/CathedralSibenik.htm

Earthquake simulations:



☹ Experimental verification?

CONTACT DYNAMICS

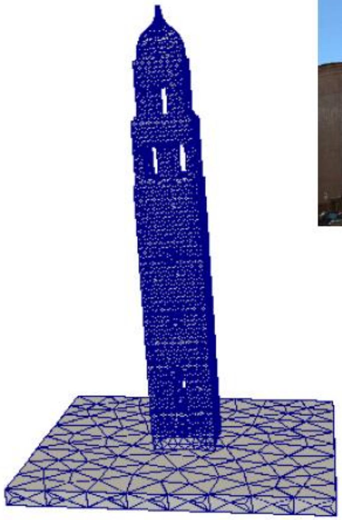


Civil engineering applications

e.g. Clementini et al (2018):

San Benedetto Church, Ferrara

aim: analyse seismic behaviour



Model assumptions:

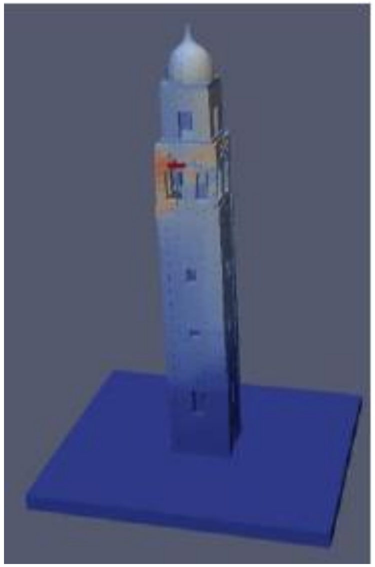
rigid blocks

Coulomb-frictional contacts

perfectly plastic impact (no bouncing)

Load: basement oscillations $v(t) = C \sin(2\pi \cdot f \cdot t)$

$\equiv$ earthquake simulations



Model validation: compare first frequency to reality

Outcome: vulnerable regions of the structure

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Questions

QUESTIONS

1. Under what conditions can a numerical technique be classified as a discrete element model? What are the main steps of the discrete element modelling of an engineering problem?

2. What is the difference between quasi-static and time-stepping calculation methods of the displacement increments?

3. What is the difference between explicit and implicit time integration techniques?

4. What are the degrees of freedom in UDEC/3DEC, in DDA, and in Contact Dynamics? What kind of time integration technique is applied in these models?

5. What are the main advantages and disadvantages of UDEC/3DEC, DDA, and Contact Dynamics in comparison to each other?

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- Munjiza's FEM/DEM   ADDITIONAL TOPIC OF INTEREST

Questions

MUNJIZA'S FEM/DEM METHOD

Ante Munjiza (1999), (2004), …: (2D, 3D)

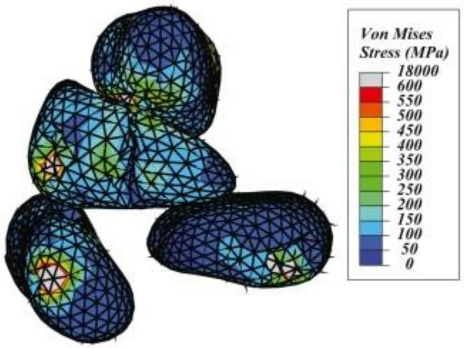
→ to simulate fracture and fragmentation of discrete elements

Recent years:

→ further development of several algorithmic details

→ applications to historic masonry

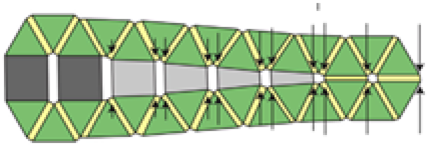
Main features:



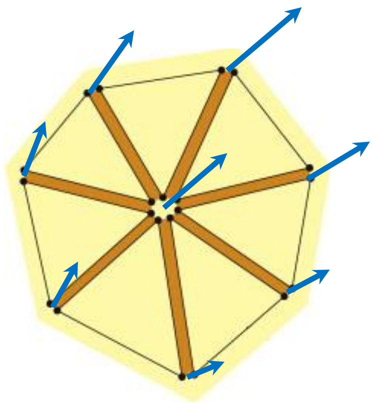
→ deformable, polyhedral discrete elements ;
deformable contacts between them

→ discrete elements are subdivided into:
uniform-strain FEM tetrahedra

→ „joint elements":
inside the discrete elements,
between the FEM tetrahedra:
able to soften and open up



THE ELEMENTS



Degrees of freedom:

translations of the nodes

→ like in 3DEC def.

Strain in the finite element tetrahedra:

different possibilities available:

small strain tensor;   right or left Cauchy-Green strain tensor;

Stress options: Cauchy stress tensor; Ist or IInd Piola-Kirchhoff stress tensor

→ more options than in 3DEC

Constitutive model of the elements:

Hooke law, no plasticity of the finite elements   [very simple]

→ in 3DEC: plastic yield and user-defined constitutive relations can be used

masses in eqs of motion: masses of the Voronoi cells of the nodes   → like in 3DEC

stress field inside the tetrahedra: reduced to the nodes   → like in 3DEC

Time integration: central difference method   → like in 3DEC

CONTACT INTERACTION ALGORITHM

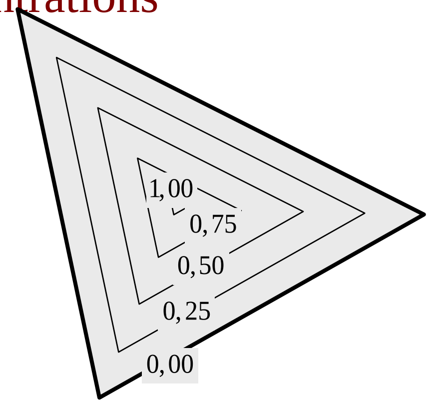
Advantageous features:

→ distributed contact forces: no unrealistic stress concentrations

→ complicated contact behaviour (sliding, plasticity, cohesion etc): easy to incorporate

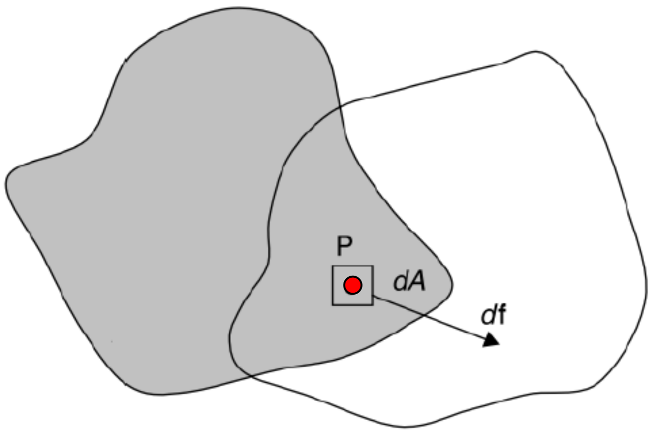
→ energy conservation satisfied!

→ computationally relatively efficient



Case of two overlapping discrete elements:

P scans over the total overlap



Potential functions of the two FE-s

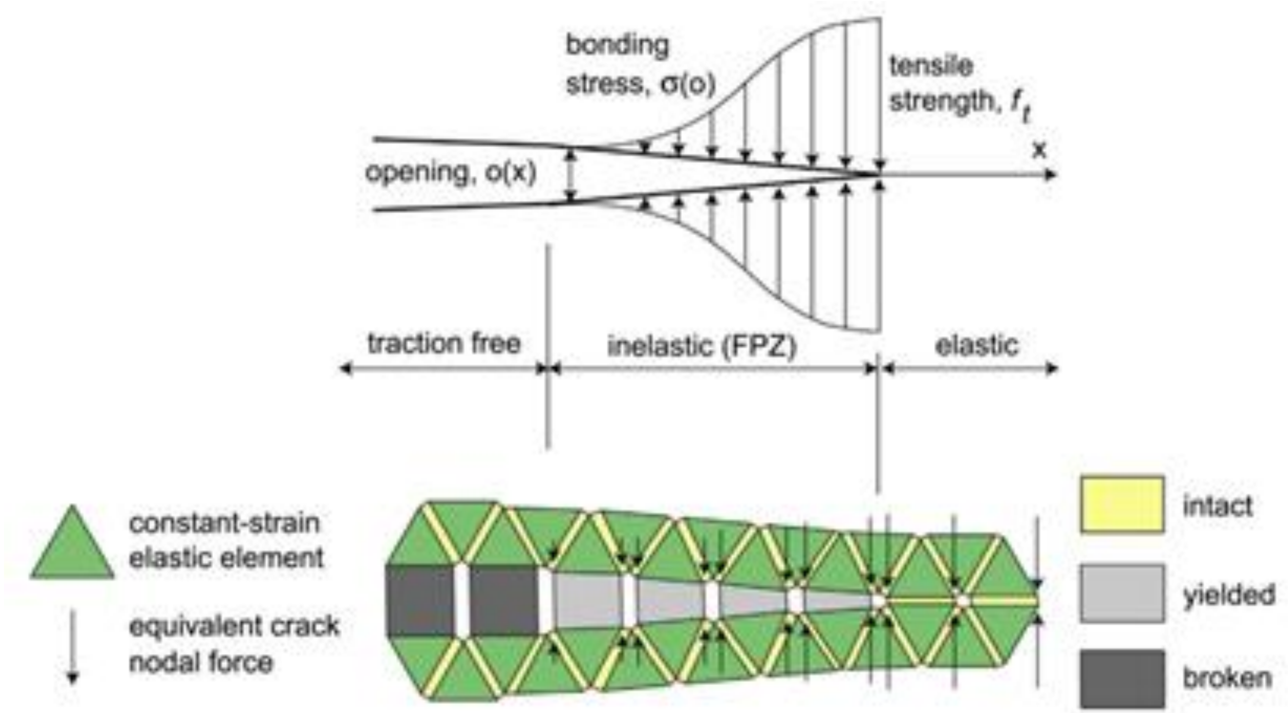
$$\mathrm{d}\boldsymbol{f} = \left[\operatorname{grad}\varphi_1(P) - \operatorname{grad}\varphi_2(P)\right]\mathrm{d}A$$

⇒ distributed force along the overlap: then reduced to the nodes

FRACTURE & FRAGMENTATION ALGORITHM

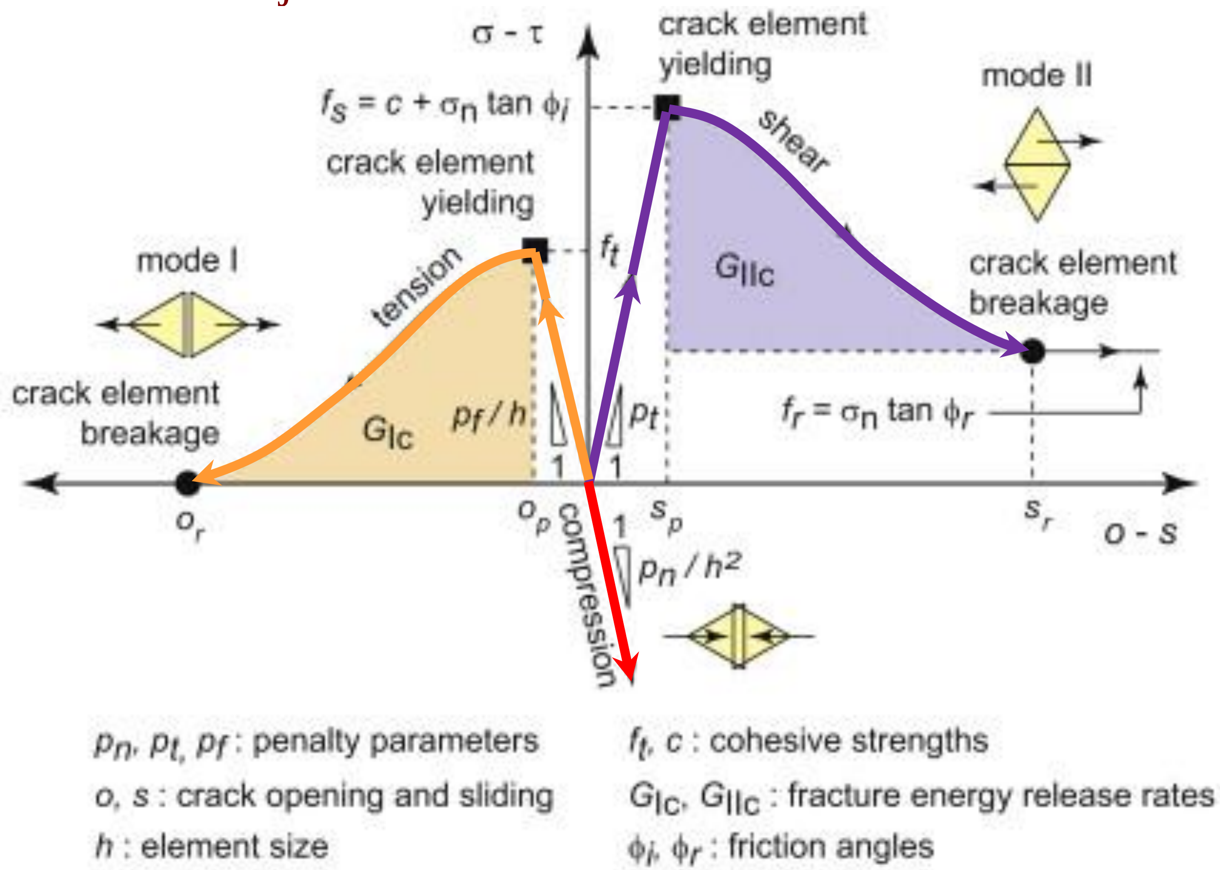
aims: → to define crack initiation
→ to describe how cracks propagate,
→ to replace the released internal forces with new contact forces

„joint elements”: the surface between FE-s ! in the interior of DE-s !



THE JOINT ELEMENTS

Mechanical behaviour of joints:



Disadvantage:

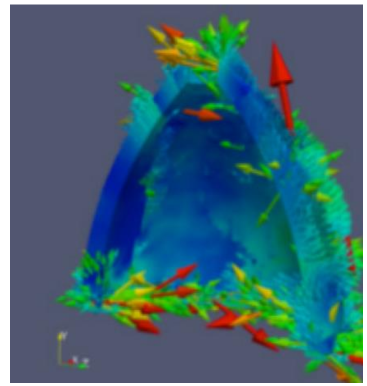
simulated fracture behaviour is very sensitive to mesh density & orientation

⇒ very dense subdivision of the DE-s is needed

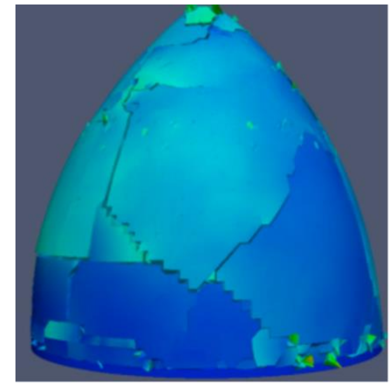
APPLICATIONS

e.g. Rougier et al (2014):

Seismic analysis of the Dome of the Santa Maria del Fiore cathedral



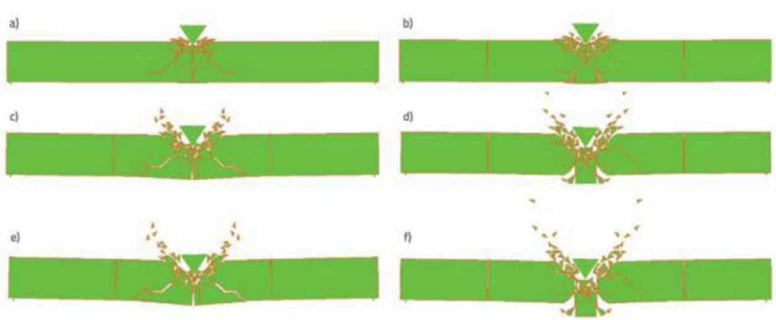
stress vawe propagation



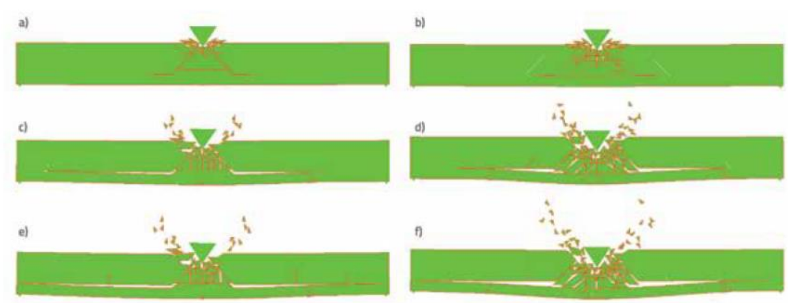
cracked final state

e.g. Zivaljic et al (2014):

Impact loading of a concrete beam



unreinforced



reinforced