THE DISCRETE ELEMENT METHOD
SHORTLY MENTIONED IN LECTURE 01:  
THE DISCRETE ELEMENT METHOD

Definition: a numerical method belongs to DEM if
⇐ it consists of separate, finite-sized elements and their contacts
⇐ its elements have independent degrees of freedom, with large displacements
⇐ contact separation and sliding considered; new contacts can be born

Main steps:
→ define the elements (geometry);
  automatically recognize their contacts
→ specify the material parameters (elements; contacts)
→ loading history: in small steps;
  stepwise: upgrade geometry & topology & material

Detailed introduction to DEM: today in Lecture 05
THIS LECTURE:

What is DEM?

The Geometry
Mechanical Properties
Calculation of the Displacements

DEM models

UDEC/3DEC
Discontinuous Deformation Analysis
Contact Dynamics

Questions
WHAT IS DEM?

The aim: to model materials or structures having discrete internal built-up

„what does it do if loads are put on it?”

The components of the model:

- separate elements + their contacts

- mechanical models for the material of the elements:
  - rigid
  - deformable

- contacts → recognition
  - mechanical models for the contacts:
    - non-deformable
    - deformable: e.g. point-like, deformable
      - e.g. frictional, e.g. finite size, deformable
      - e.g. cemented
WHAT IS DEM?

History overview

→ end of 1960ies:

Peter A Cundall, Imperial College:

**UDEC**
(„Uniform Distinct Element Code”)

model for fractured rocks

→ 1970ies: Molecular Dynamics methods, physics literature
not really DEM
WHAT IS DEM?

History overview

→ end of 1970ies: Cundall & Strack, 1979: BALL

→ from the 1980ies:
  → several new codes, already in 3D
  → general element shapes
  → different mathematical tools

→ from the 1990ies: practical applications in engineering
EXAMPLE

1. Define the geometry:
   ball id 1 x 0.10  y 0.20  rad 0.10
   ball id 2 x 0.55  y 0.20  rad 0.15
   ball id 3 x 0.30  y 0.40  rad 0.15
   wall id 1 nodes 0.0  0.0  0.7  0.0
   wall id 2 nodes 0.7  0.0  0.7  0.5
   wall id 3 nodes 0.0  0.5  0.0  0.0

2. Specify the material parameters:
   property density 10.0
   property kn 1.e4 ks 0.5e4 friction 0.2
   wall id 1 kn 1.e12 ks 0. friction 0.
   wall id 2 kn 1.e12 ks 0. friction 0.
   wall id 3 kn 1.e12 ks 0. friction 0.

3. Specify the loads:
   set gravity 0.0  -9.81

4. Calculate the displacements  [series of small increments]
WHAT IS DEM?

Main steps of the analysis of an engineering problem:

- the model: collection of separate elements (’discrete elements’)  
  \{1 \text{ body} \leftrightarrow 1 \text{ element}\} \text{ or } \{\text{several bodies} \leftrightarrow \text{few elements}\}
  
  \textbf{Step 1.}: define the initial geometry

- rigid or deformable \textit{elements}; rigid or deformable \textit{contacts}
  
  \textbf{Step 2.}: specify the material characteristics

- the loading process:  
  ( e.g. external forces acting on the elements; e.g. prescribed displacements)

- calculation of the state changing: \textit{series of small increments, based on } \textit{\( f = ma \)}
  
  \textbf{Step 3.}: calculation of the actual displacement increments
THE GEOMETRY

Element shapes:

- polygon, polyhedron
- circle, sphere
- ellipse, ellipsoid
- complex shapes

e.g. Lemos (2007):
  masonry blocks & mortar layer:

   ![Masonry blocks and mortar layer]

  ![Toyoura sand model](a)
  ![Ottawa sand model](b)

  (a) Toyoura sand model
  (b) Ottawa sand model

  e.g. Matsushima (2005):
  irregularly shaped sand particles
THE GEOMETRY

Element shapes:

polygon, polyhedron  circle, sphere  ellipse, ellipsoid  complex shapes

e.g. Psycharis et al (2003):  stone blocks:
e.g. Bui et al (2017):  bricks of a house:
How to get the geometry of a masonry structure:

- original plans (if still exist)
- survey the actual geometry, e.g. laser scanner & CAD:
  e.g. McInerney et al (2012):

**St John’s College, Cambridge, UK**

Difficulty e.g.:
how to survey hidden/covered faces

**King’s College, Cambridge, UK**
THE GEOMETRY

Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

the time consuming part: to check the existence of a contact with exact calculations

Trick #1: avoid checking every element with every other element:

→ „body based search” technique:
   consider only those others which are in the vicinity of the analyzed element; then take the next element to analyze, …

→ „space based search”:
   divide the domain into „windows” (overlapping); collect which elements are in which windows; analyze those pairs only where both elements belong to the same window
THE GEOMETRY

Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

the time consuming part: to check the existence of a contact with exact calculations

Trick #2:

Simple surrounding domains checked first (instead of the elements having complicated shapes)

the idea: „surrounding domain” assigned to each element (simple shape: brick; sphere)

→ Phase 1.: intersection between the surrounding domains? (fast)

→ if necessary: Phase 2.: detailed, exact calculations (slow)
MECHANICAL PROPERTIES

Mechanical behaviour of the elements:

*role: to specify how to calculate the stresses from the deformations of the elements*

→ perfectly rigid elements: deformability concentrated into the contacts

→ deformable elements:
  
  stress-strain-relations have to be specified
  
  \[ E, \mu, \ldots \]

Mechanical behaviour of the contacts:

*role: to specify how to calculate the contact forces from the relative displacements at the contact*

→ usually: „deformable” contacts
  
  (relative displ. at the contact regions)
  
  concentrated ↔ distributed

→ sometimes:
  
  infinitely rigid contacts: no overlap neither any other deformation
CALCULATION OF DISPLACEMENTS

Quasi-static methods

From an initial equilibrium state, the incremental displacements \( u \) are to be determined taking the system to the new equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

\[
K \cdot \Delta u + \Delta f = 0
\]

\( \rightarrow \) Kishino (1988); Bagi-Bojtár (1991)

\( \rightarrow \) Meng et al (2017); Baraldi et al (2018)

Time-stepping methods

"\( M \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t)) \)"

\( \rightarrow \) a process in time is searched for

simulate the motion of the system along small, but finite \( \Delta t \) timesteps

Explicit timestepping methods:

\( \rightarrow \) UDEC \( \leftarrow \) deformable polyhedral elements, deformable contacts

\( \rightarrow \) Munjiza’s FEM/DEM \( \leftarrow \) deformable, breakable elements, deformable contacts

Implicit timestepping methods:

\( \rightarrow \) DDA („Discontinuous Deformation Analysis“) \( \leftarrow \) deformable polyhedral elements

\( \rightarrow \) Contact Dynamics models \( \leftarrow \) rigid elements, non-deformable contacts
SOLUTION OF THE EQUATIONS OF MOTION

Numerical solutions only! \[ M \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t)) \]

The aim:
starting from a known \( \mathbf{u}(t_0) = \mathbf{u}_0 \) and \( \mathbf{v}(t_0) = \mathbf{v}_0 \) state at a \( t_0 \) time instant,
the aim is to determine the approximative solutions \((\mathbf{u}_1, \mathbf{v}_1), (\mathbf{u}_2, \mathbf{v}_2), \ldots, (\mathbf{u}_i, \mathbf{v}_i), (\mathbf{u}_{i+1}, \mathbf{v}_{i+1}), \ldots\) belonging to the \( t_1, t_2, \ldots, t_i, t_{i+1}, \ldots \) time instants.

The two basic approaches:
Explicit vs. implicit time integration methods
SOLUTION OF THE EQUATIONS OF MOTION

Explicit vs. implicit methods:

→ explicit methods:

in the state at $t_i$: $(u_i, v_i, f_i) \Rightarrow$ equations of motion $\Rightarrow$ approximate $(u_{i+1}, v_{i+1}, f_{i+1})$ belonging to the state at $t_{i+1}$

NOT checking whether $(u_{i+1}, v_{i+1}, f_{i+1})$ satisfy the eqs of motion: accept them and use them for the calculations of the next timestep $\Rightarrow$ fast, but less reliable; numerical stability problems!
Explicit vs. implicit methods:

→ implicit methods:

in the state at $t_i$: $(u_i, v_i, f_i) \Rightarrow$ equations of motion $\Rightarrow$
approximate $(u_{i+1}, v_{i+1}, f_{i+1})$ belonging to the state at $t_{i+1}$;
then iterations, to improve this approximation belonging to $t_{i+1}$,
so that the eqs of motion be satisfied at $t_{i+1}$
$\Rightarrow$ slow, but longer timesteps;
more reliable, better numerical stability
THIS LECTURE:

What is DEM?
- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models
- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics

Questions
UDEC / 3DEC

UDEC: „Universal Distinct Element Code”
P.A. Cundall, 1971;
development through decades
Itasca Consulting Group
www.itascacg.com

MOST WIDESPREAD IN CIVIL ENGINEERING
UDEC / 3DEC

Elements: polygons / polyhedra (planar faces!);

- rigid elements

degrees of freedom: translation of and rotation about the centroid

- deformable elements (subdivided into simplex zones)

„uniform strain” tetrahedral zones
((10-node tetrahedra – not reliable))

degrees of freedom: translations of the nodes

Material models for the elements:

(rigid) ↔ deformable:

- „null element” (no material in the element)
- linearly elastic, isotropic  \((e.g. \text{ intact rock}; \text{ metal})\)
- lin. elast., with: Mohr-Coulomb  /  Prager-Drucker failure crit.
  \((e.g. \text{ soils, concrete})  \quad (e.g. \text{ clay})\)
  + tensile strength + cohesion + dilation angle

- ...
Contacts:

consist of small „subcontacts”, over which:
uniformly distributed normal and
shear contact forces are transmitted

Material models for the contacts:

[calculate the increments of distrib. contact forces from the increments of rel. disps]

– if no material in the contacts: $\rightarrow k_n, k_s$: numerical parameters, $\infty$
  or express surface roughness ;
  $\rightarrow$ friction: real value

– if material in the joints: (modelled as length or area, with zero thickness):
  – linear behaviour for compression and shear, Coulomb-friction,
    + cohesion and tensile strength
  – linear behaviour for compression and shear, Coulomb-friction,
    + cohesion & tensile strength + softening + dilation angle
     $\Delta U_n(dil) = \Delta U_s \tan \psi$
  – others …
UDEC / 3DEC

Calculation of nodal displacements

Newton II.: „\( ma = f \)”

- mass assigned to the node:

Voronoi-cell

- force on the node: resultant of the forces acting on the Voronoi-cell of the node

  \[ \boldsymbol{V} \leftarrow \text{from the neighbouring element} \]
  \[ \leftarrow \text{from external forces (e.g. self weight, drag force)} \]
  \[ \leftarrow \text{from the stresses inside the simplexes} \]

- force from the stress within a simplex:
  
  \[ \text{--- nodal translations } \Rightarrow \text{simplex strain } \checkmark \]
  
  \[ \text{--- from this and material characteristics } \Rightarrow \text{uniform stress in the simplex } \checkmark \]
  
  \[ \text{--- stress vector acting on the face of the cell: } \boldsymbol{\sigma}_j \mathbf{n}_j = p_i \ ; \text{resultant } \checkmark \]
UDEC / 3DEC

Calculation of nodal displacements

Newton II.: \( ma = f \)

– discretized form of the eqs of motion:

\[
m \frac{v(t_i + \Delta t/2) - v(t_i - \Delta t/2)}{\Delta t} = f(t_i)
\]

or:

\[
v(t_i + \Delta t/2) = v(t_i - \Delta t/2) + \frac{f(t_i)}{m} \Delta t
\]

– at \( t_i \): the \textit{positions of the nodes} and the \textit{forces and stresses} are known;

at \( t_i - \Delta t/2 \): the \textit{nodal velocities} are known;

determine the \textit{nodal velocities} at \( t_{i+1/2} = t_i + \Delta t/2 \)

and the \textit{positions of the nodes} at \( t_{i+1} = t_i + \Delta t \)
Calculation of nodal displacements

- series of small finite time steps:

- explicit time integration; no stiffness matrix!!!

  ⇒ numerical instabilities, convergence problems

- to help numerical stability:
  1. estimate the longest allowed $\Delta t$
  2. artificial damping is introduced [different types can be used]

MAIN DISADVANTAGE:

  strong oscillations around the exact solution

  ⇒ may give unrealistic results [e.g. in case of history dependence]

  ⇒ numerical instabilities may occur
UDEC / 3DEC

Applications for masonry structures:

Quasi-static problems:

- e.g. Sao Vicente de Fora Monastery, Portugal: Giordano et al, 2002

**UDEC advantages:** works well for large displs; realistic crack pattern

- e.g. oval dome statics: Simon & Bagi, 2016

Dynamic problems (use with caution!):

- convergence of the solution with respect to $\Delta t$ should be ensured
- damping type and damping parameters should carefully be selected & calibrated
THIS LECTURE:

What is DEM?

The Geometry
Mechanical Properties
Calculation of the Displacements

DEM models

UDEC/3DEC
Discontinuous Deformation Analysis
Contact Dynamics

Questions
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Gen-Hua Shi (1988), Berkeley
then many others applied or developed research software!!!

The elements: polyhedral; with a reference point (e.g. centroid)
[ Deformable without subdivision ]
„displacement vector” of the \( p \)-th element: \( \textbf{u}^p \)
„reduced load” belonging to the \( p \)-th element: \( \textbf{f}^p \)

The degrees of freedom:
rigid-body translation and rotation of the reference point;
+ the uniform strain of the element

\[
\begin{bmatrix}
\textbf{f}^p \\
\epsilon^p_x \\
\epsilon^p_y \\
\epsilon^p_z \\
\gamma^p_{yz} \\
\gamma^p_{zx} \\
\gamma^p_{xy}
\end{bmatrix} =
\begin{bmatrix}
\textbf{u}^p_x \\
\textbf{u}^p_y \\
\textbf{u}^p_z \\
\phi^p_x \\
\phi^p_y \\
\phi^p_z \\
\varphi^p_{yz} \\
\varphi^p_{zx} \\
\varphi^p_{xy}
\end{bmatrix}
\]

28 / 53
The contacts:  

in 2D: Node – to – Edge contacts

in 3D: Node – to – Face contacts:

Edge – to – Edge contacts:

→ „first entrance position”
⇒ contact deformation: \( \Delta u_N; \Delta u_T \)
  normal & tangential (perhaps sliding)
⇒ direction of the contact:
  the normal vector of the face

Mechanical model:

→ originally: infinitely rigid contacts, Coulomb-friction

→ recent codes: deformable contacts included
  + other friction conditions, cohesion etc.

Remark: infinitely rigid contact: „penalty function”:
\[ F_N = k_N \Delta u_N; \quad dF_T = k_T d(\Delta u_T) \]

\( \equiv \) linearly elastic in normal and in tangential directions
The equations of motion: „Potential energy” stationarity principle

„Potential” of the system:

\[
\Pi = \Pi^{\text{blocks}} + \Pi^{\text{contacts}}
\]

\[
\frac{\partial \Pi}{\partial u_i^p} = 0 \quad \text{for all } p, i
\]

\[
M \cdot a(t) + C \cdot v(t) + K \cdot u(t) = f^{\text{ext}}(t, v(t), u(t))
\]

More exactly: „Hamilton principle”
DDA

Numerical solution of the equations of motion:

\[(t_i, t_{i+1}) \text{ time interval:}\]

at \( t_i \) : known \( u_i, v_i, f(t_i, u_i, v_i) \); satisfy the eqs. of motion

Find \( u_{i+1}, v_{i+1}, a_{i+1} \) so that the eqs of motion would be satisfied at \( t_{i+1} \)

\[
\mathbf{r}(t_{i+1}, u_{i+1}, v_{i+1}) = f(t_{i+1}, u_{i+1}, v_{i+1}) - M \cdot a_{i+1} = 0
\]

Remember: Newmark’s \( \beta \)-method:

\[
\begin{align*}
\mathbf{u}_{i+1} &= \mathbf{u}_i + \Delta t \cdot \mathbf{v}_i + \frac{\Delta t^2}{2} \left[(1 - 2\beta)\mathbf{a}_i + 2\beta \cdot \mathbf{a}_{i+1}\right] \\
\mathbf{v}_{i+1} &= \mathbf{v}_i + (1 - \gamma) \cdot \Delta t \cdot \mathbf{a}_i + \gamma \cdot \Delta t \cdot \mathbf{a}_{i+1}
\end{align*}
\]

[stability: \( 2\beta \geq \gamma \geq \frac{1}{2} \)]

DDA: Newmark’s \( \beta \)-method, with \( \beta = 1/2; \gamma = 1 \):

\[
\begin{align*}
\Delta \mathbf{u}_{i+1} &= \mathbf{u}_{i+1} - \mathbf{u}_i \\
\mathbf{a}_{i+1} &= \frac{1}{\Delta t^2 / 2} (\Delta \mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) \\
\mathbf{v}_{i+1} &= \mathbf{v}_i + \Delta t \cdot \mathbf{a}_{i+1} = \mathbf{v}_i + \frac{2}{\Delta t} (\Delta \mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) = \frac{2}{\Delta t} \Delta \mathbf{u}_{i+1} - \mathbf{v}_i
\end{align*}
\]
DDA

Numerical solution of the equations of motion:

\[ M \cdot a(t) = f(t, u(t), v(t)) \Rightarrow 0 = f(t_{i+1}, u_{i+1}(\Delta u_{i+1}), v_{i+1}(\Delta u_{i+1})) - M \cdot a_{i+1}(\Delta u_{i+1}) \]

Determine \( \Delta u_{i+1} \), so that the residual

\[ r(t_{i+1}, \Delta u_{i+1}) = f(t_{i+1}, u_{i+1}(\Delta u_{i+1}), v_{i+1}(\Delta u_{i+1})) - M \cdot a_{i+1}(\Delta u_{i+1}) \]

would be sufficiently close to zero!

Newton-Raphson:

the Jacobian of the residual: \( \mathcal{K}(t, \Delta u) = \frac{dr(t, \Delta u)}{d\Delta u} \)

this matrix can be compiled from elementary calculations at \( t_i \):

\( \leftarrow \) contains the stiffness matrix
\( \leftarrow \) contains the inertia, contact forces, geometric characteristics etc.

the residual can also be compiled from elementary calculations at \( t_i \):

\( \leftarrow \) contains the external forces, inertia effects, prescribed displacements, damping etc.
DDA

Numerical solution of the equations of motion:

\[
M \cdot a(t) = f(t, u(t), v(t)) \quad \Rightarrow \quad 0 = f(t_{i+1}, u_{i+1}, \Delta u_{i+1}, v_{i+1}(\Delta u_{i+1})) - M \cdot a_{i+1}(\Delta u_{i+1})
\]

\[
\mathbf{r}(t_{i+1}, \Delta u_{i+1}) = f(t_{i+1}, u_{i+1}, \Delta u_{i+1}, v_{i+1}(\Delta u_{i+1})) - M \cdot a_{i+1}(\Delta u_{i+1})
\]

\[
\mathcal{K}(t, \Delta u) = \frac{d\mathbf{r}(t, \Delta u)}{d\Delta u}
\]

Analysis of a time interval:

- initial estimation for \( \Delta u_{i+1} \): \( \Delta u^{(0)}_{i+1} := 0 \)
- \( k+1 \)-th estimation for \( \Delta u_{i+1} \): \( \Delta u^{(k)}_{i+1} := \Delta u^{(k-1)}_{i+1} - \mathcal{K}(t_{i+1}, \Delta u^{(k)}_{i+1})^{-1} \cdot \mathbf{r}(t_{i+1}, \Delta u^{(k)}_{i+1}) \)

then continue until \( |\mathbf{r}(t_{i+1}, \Delta u^{(k+1)}_{i+1})| \) becomes sufficiently small

„Open – close iterations“:

- at the end of \( \Delta t \): check the topology and the forces;
- \( \rightarrow \) modify the topology if necessary (e.g. new contacts, sliding, contact loss)
- \( \rightarrow \) with the new topology, repeat: Newton-Raphson to find another \( \Delta u_{i+1} \)

if acceptable topology not found: decrease timestep \( \Delta t \) to 1/3 of its previous length

CONVERGENCE WITHIN A TIME STEP ????
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Comparison to UDEC:

Main differences from UDEC:

→ basic unknowns: also the components of $\varepsilon$;
→ uniform stress and strain field inside the elements;
→ numerical integration: implicit
→ stiffness matrix included $\Rightarrow$ artificial damping not necessary

– advantages to UDEC: $\rightarrow$ implicit $\Rightarrow$ numerical stability;
   fast convergence if topology does not change
   no artificial damping required

– disadvantages: no commercial software $\Rightarrow$ inconvenient
   (several research codes; e.g. ask from Gen-Hua Shi)
   too simple mechanics of the elements and of the contacts
   large storage requirements & longer computations
   open-close iterations: convergence is not ensured if topology changes
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Comparison to UDEC:
M.S. Kahn (2010)

NOT EFFICIENT IN CASES IF SIGNIFICANT TOPOLOGY MODIFICATIONS OCCUR !!!
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Applications:

- e.g. Rizzi et al (2014): collapse modes of arches
  
  - $\varphi = 0^\circ$ ($\mu = 0$)
  - $\varphi = 10^\circ$ ($\mu = 0.176$)
  - $\varphi = 20^\circ$ ($\mu = 0.364$)
  - $\varphi = 30^\circ$ ($\mu = 0.577$)

- e.g. Kamai and Hatzor (2005): back analysis of seismic events
THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics

Questions
CONTACT DYNAMICS

Jean & Moreau (1992): (2D, 3D) [mostly in physics]


Software:

(1) LMG91 (Dubois & Jean, 2006): OPEN!
   rigid/deformable; spherical/polyhedral elements

(2) SOLFEC (Koziara & Bicanic, 2008):
   rigid/deformable; polyhedral elements

The elements:

ORIGINALLY: rigid, spherical elements
for masonry structures:
   deformable or rigid polyhedral elements
CONTACT DYNAMICS

Contacts of polyhedral elements:

“common plane concept”

Mechanical conditions for the contact forces:

\[ |T_{pq}| \leq -\mathbf{v} \cdot N_{pq} \]

[ the same ]
CONTACT DYNAMICS

Contacts of polyhedral elements:

Rigid polyhedral elements:

   Degrees of freedom: translations & rotations of the reference points

Deformable polyhedral elements:

   constant strain → unfavourable experiences
   uniform-strain tetrahedral subdivision

The point of action of the contact force:

   • : middle point of the face
   „approximated contact point”

   contact: if • touches another face

Masses: distributed to the nodes

Equations of motion: for every node [no rotations considered];

   Degrees of freedom: nodal translations [similar to 3DEC def]
CONTACT DYNAMICS

How to find the solution at the end of a given time step:

implicit solution:

the positions and velocities are repeatedly (iteratively) adjusted,
until the equations of motion AND the contact conditions are satisfied
with the required accuracy at the end of the time step

[≈ Cross method for frames, but randomly sweeping along the pairs of elements]

history dependence! [order of sweeping along contacts makes difference in the results]

⇒ engineers have doubts

Main advantage: extremely fast for dynamic phenomena
CONTACT DYNAMICS

Civil engineering applications

e.g. Rafiee et al (2008):

CD numerical model with deformable elements:

*Arles, aqueduct*

Earthquake simulations:

❓ Experimental verification?
CONTACT DYNAMICS

Civil engineering applications

e.g. Gelo & Mestrovic (2016):
  dome of St Jacob Cathedral, Sibenik, Croatia

Earthquake simulations:

🔍 Experimental verification?

croatiatraveller.com/Heritage_Sites/CathedralSibenik.htm
CONTACT DYNAMICS

Civil engineering applications

e.g. Clementini et al (2018):
   San Benedetto Church, Ferrara
   aim: analyse seismic behaviour

Model assumptions:
   rigid blocks
   Coulomb-frictional contacts
   perfectly plastic impact (no bouncing)

Load: basement oscillations \[ v(t) = C \sin (2\pi f t) \]
   \( \equiv \) earthquake simulations

Model validation: compare first frequency to reality

Outcome: vulnerable regions of the structure
THIS LECTURE:

What is DEM?

   The Geometry
   Mechanical Properties
   Calculation of the Displacements

DEM models

   UDEC/3DEC
   Discontinuous Deformation Analysis
   Contact Dynamics

Questions
QUESTIONS

1. Under what conditions can a numerical technique be classified as a discrete element model? What are the main steps of the discrete element modelling of an engineering problem?

2. What is the difference between quasi-static and time-stepping calculation methods of the displacement increments?

3. What is the difference between explicit and implicit time integration techniques?

4. What are the degrees of freedom in UDEC/3DEC, in DDA, and in Contact Dynamics? What kind of time integration technique is applied in these models?

5. What are the main advantages and disadvantages of UDEC/3DEC, DDA, and Contact Dynamics in comparison to each other?
THIS LECTURE:

What is DEM?
- The Geometry
- Mechanical Properties
- Calculation of the Displacements

DEM models
- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics
- Munjiza’s FEM/DEM

ADDITIONAL TOPIC OF INTEREST

Questions
MUNJIZA’S FEM/DEM METHOD

Ante Munjiza (1999), (2004), …: (2D, 3D)

→ to simulate fracture and fragmentation of discrete elements

Recent years:

→ further development of several algorithmic details
→ applications to historic masonry

Main features:

→ deformable, polyhedral discrete elements; deformable contacts between them

→ discrete elements are subdivided into:
  uniform-strain FEM tetrahedra
→ „joint elements”:
  inside the discrete elements,
  between the FEM tetrahedra:
  able to soften and open up
THE ELEMENTS

Degrees of freedom:
translations of the nodes
→ like in 3DEC def.

Strain in the finite element tetrahedra:
different possibilities available:
small strain tensor; right or left Cauchy-Green strain tensor;

Stress options: Cauchy stress tensor; Ist or IInd Piola-Kirchhoff stress tensor
→ more options than in 3DEC

Constitutive model of the elements:
Hooke law, no plasticity of the finite elements [ very simple ]
→ in 3DEC: plastic yield and user-defined constitutive relations can be used

masses in eqs of motion: masses of the Voronoi cells of the nodes → like in 3DEC
stress field inside the tetrahedra: reduced to the nodes → like in 3DEC
Time integration: central difference method → like in 3DEC
CONTACT INTERACTION ALGORITHM

Advantageous features:

→ distributed contact forces: no unrealistic stress concentrations
→ complicated contact behaviour (sliding, plasticity, cohesion etc): easy to incorporate
→ energy conservation satisfied!
→ computationally relatively efficient

Case of two overlapping discrete elements:

$P$ scans over the total overlap

\[
\begin{align*}
\{ \partial_f \} &= \int \left[ \nabla \varphi_1(P) - \nabla \varphi_2(P) \right] \, dA \\
\Rightarrow \text{distributed force along the overlap:} & \quad \text{then reduced to the nodes}
\end{align*}
\]
FRACTURE & FRAGMENTATION ALGORITHM

aims:
→ to define crack initiation
→ to describe how cracks propagate,
→ to replace the released internal forces with new contact forces

„joint elements”: the surface between FE-s in the interior of DE-s!
THE JOINT ELEMENTS

Mechanical behaviour of joints:

Disadvantage:

simulated fracture behaviour is very sensitive to mesh density & orientation

⇒ very dense subdivision of the DE-s is needed
APPLICATIONS

e.g. Rougier et al (2014):
Seismic analysis of the Dome of the Santa Maria del Fiore cathedral

stress wave propagation  cracked final state

e.g. Zivaljic et al (2014):
Impact loading of a concrete beam

unreinforced  reinforced