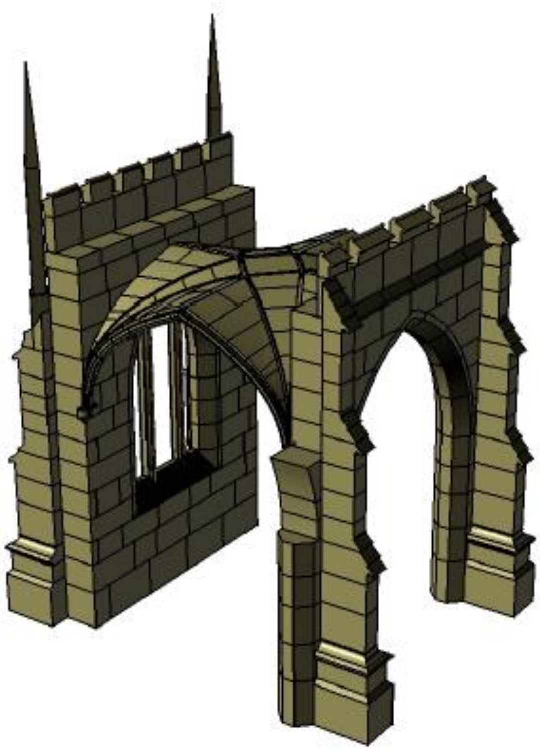
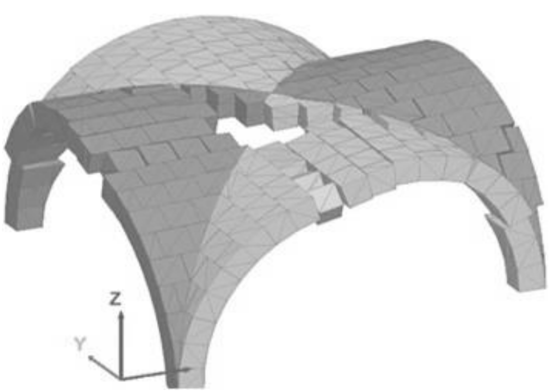
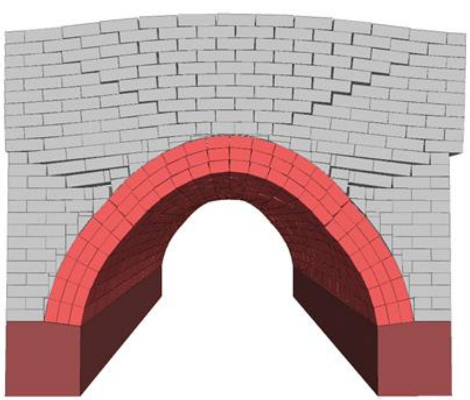


THE DISCRETE ELEMENT METHOD





Citation:

K. Bagi (2024): Mechanics of Masonry Structures. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

In case of any question or problem, do not hesitate to contact Prof. K. Bagi, kbagi.bme@gmail.com .

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

Most important DEM techniques

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics
- Munjiza's FEM/DEM

Questions

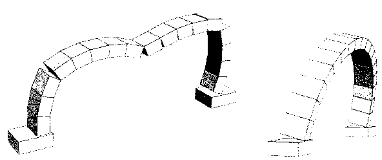
WHAT IS DEM?

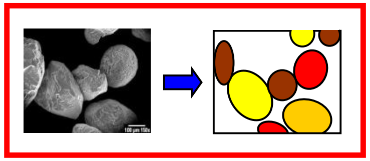
The aim: to model materials or structures having discrete internal builtup

„what does it do if loads are put on it?"

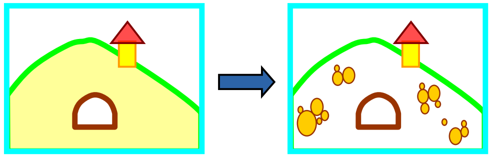
The components of the model:

separate elements + their contacts





or



Definition: a numerical method belongs to DEM if

← it consists of separate, ***finite-sized*** solid bodies and their contacts

← its elements have ***independent*** degrees of frredom, with ***large displs***

← contact separation and sliding considered; ***new contacts*** can be born

Main steps: → define the ***elements*** (geometry); automatically recognize their ***contacts***

→ specify the ***material parameters*** (elements; contacts)

→ loading history: movements in small ***incremental steps***;

stepwise: ***upgrade*** geometry & topology & material

Remark: Why is DEM needed at all?

Continuum shell theories; FEM: Why not enough?

<u>Common misbelief:</u>

„If the number of stones forming the structure tends to infinity,
the discrete structure tends to a continuum."

NO !

countably infinite — uncountably infinite

Domokos & Holmes (2003; …): „Ghost solutions" for boundary value problems:

if a continuous domain is discretized (for the aim of solving it numerically),
ghost solutions appear: they are not solutions of the continuum problem,
their existence is due to the discretization only

Remark: Why is DEM needed at all?

Continuum shell theories; FEM: Why not enough?

Domokos & Holmes (2003; …): „Ghost solutions"

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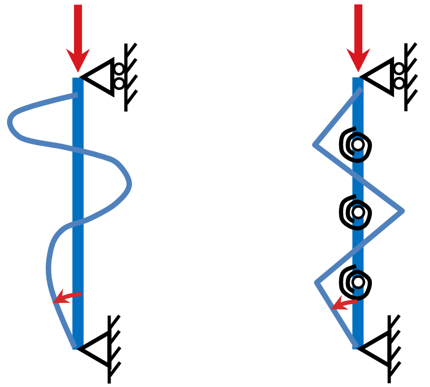
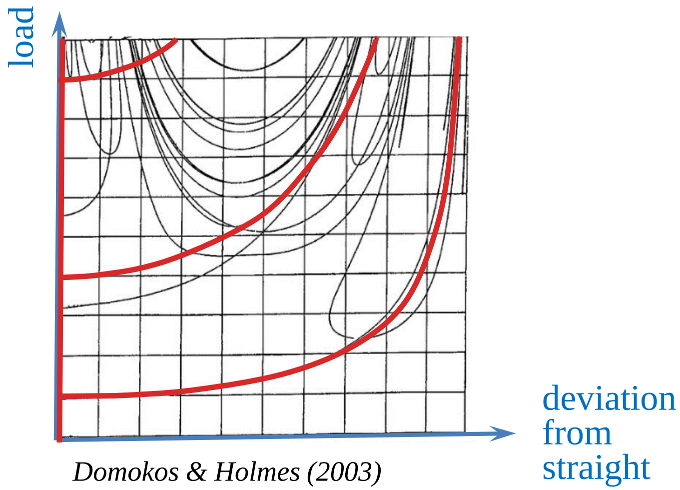
ghost solutions appear: they are not solutions of the continuum problem,

their existence is due to the discretization only

Example: Bar under compression:





Domokos & Holmes (2003)

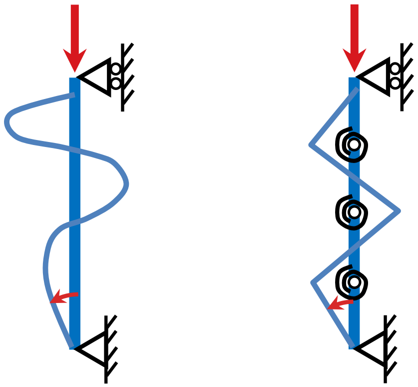
Remark: Why is DEM needed at all?

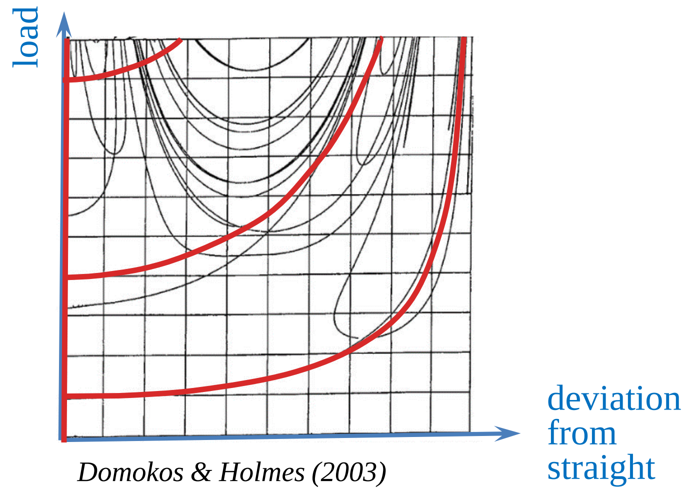
Continuum shell theories; FEM: Why not enough?

Domokos & Holmes (2003; …): „Ghost solutions"

Meaning in the context of masonry mechanics: [think of it backwards]

MAIN MESSAGE: the discrete reality is much more „rich" than the continuous models





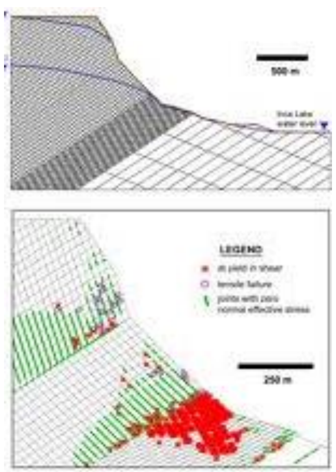
Domokos & Holmes (2003)

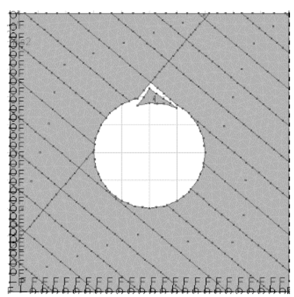
WHAT IS DEM?

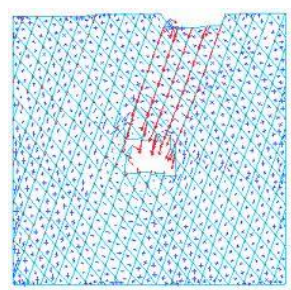
<u>History overview</u>

→ end of 1960ies:

Peter A Cundall,
Imperial College:
UDEC
(„Uniform Distinct Element Code”)

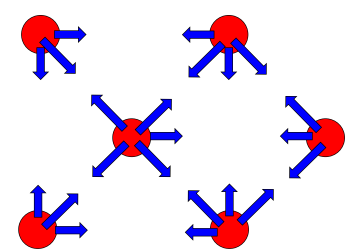




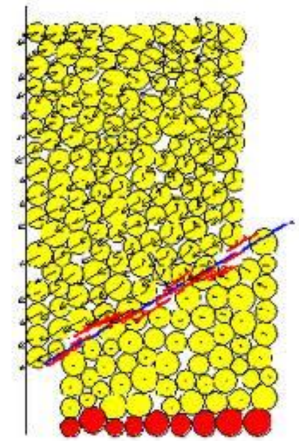


model for fractured rocks

→ 1970ies: Molecular Dynamics methods, physics literature
not really DEM



WHAT IS DEM?

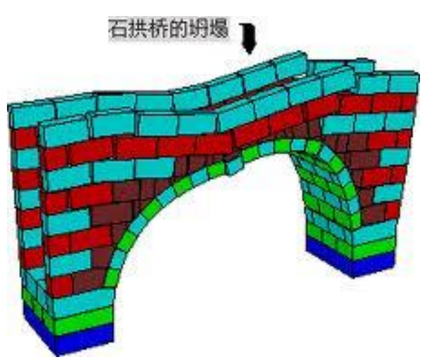


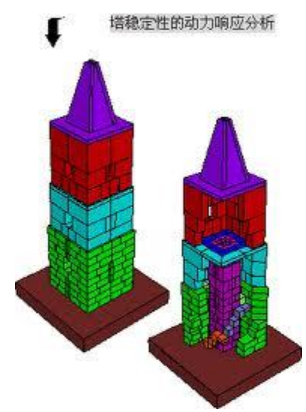
History overview

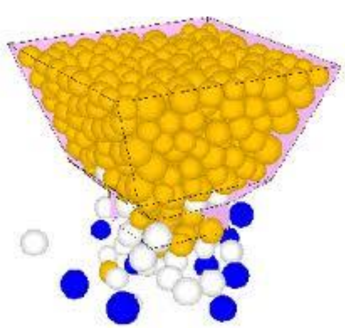
→ end of 1970ies: Cundall & Strack, 1979:
BALL

→ from the 1980ies: USA; Japan; …
→ several new codes, already in 3D
→ general element shapes
→ different mathematical tools

→ from the 1990ies: practical applications in engineering



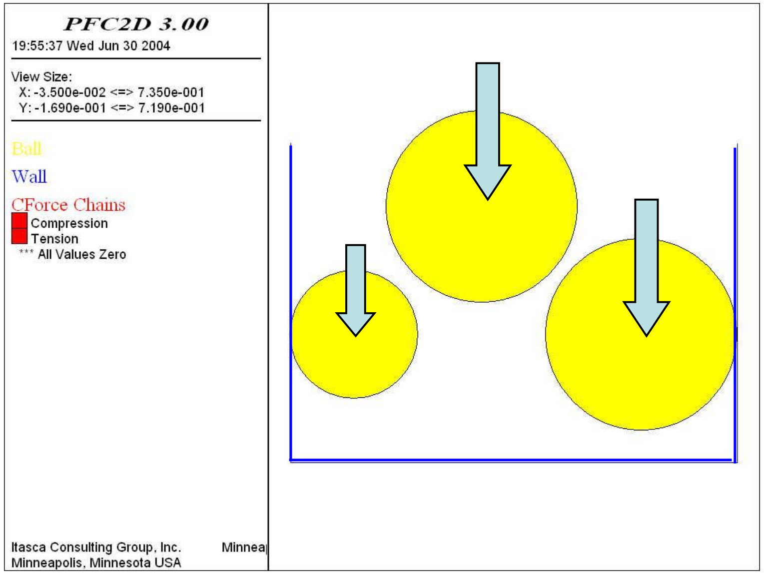




EXAMPLE

1. Define the geometry:

ball id 1 x 0.10 y 0.20 rad 0.10
ball id 2 x 0.55 y 0.20 rad 0.15
ball id 3 x 0.30 y 0.40 rad 0.15
wall id 1 nodes 0.0 0.0 0.7 0.0
wall id 2 nodes 0.7 0.0 0.7 0.5
wall id 3 nodes 0.0 0.5 0.0 0.0



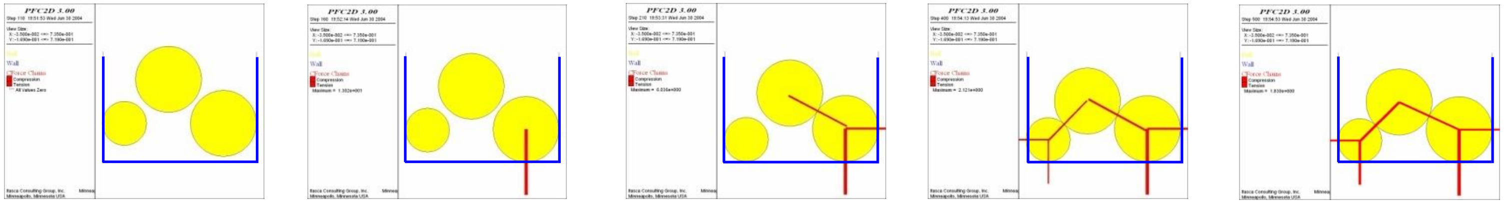
2. Specify the material parameters:

property density 10.0
property kn 1.e4 ks 0.5e4 friction 0.2
wall id 1 kn 1.e12 ks 0. friction 0.
wall id 2 kn 1.e12 ks 0. friction 0.
wall id 3 kn 1.e12 ks 0. friction 0.

3. Specify the loads:

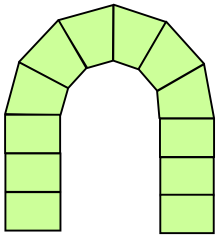
set gravity 0.0 -9.81

4. Calculate the displacements [series of small increments]



WHAT IS DEM?

Main steps of the analysis of an engineering problem:



- the model: collection of separate elements (’**discrete elements**’)

  {1 body ↔ 1 element} or {several bodies ↔ few elements}

  Step 1.: define the initial geometry

- rigid or deformable ***elements***; rigid or deformable ***contacts***

  Step 2.: specify the material characteristics

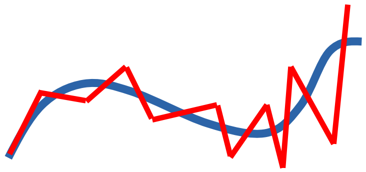
  (maybe time dependent, stress dependent, …)

- the loading process:

  (e.g. external forces acting on the elements; e.g. prescribed displacements)

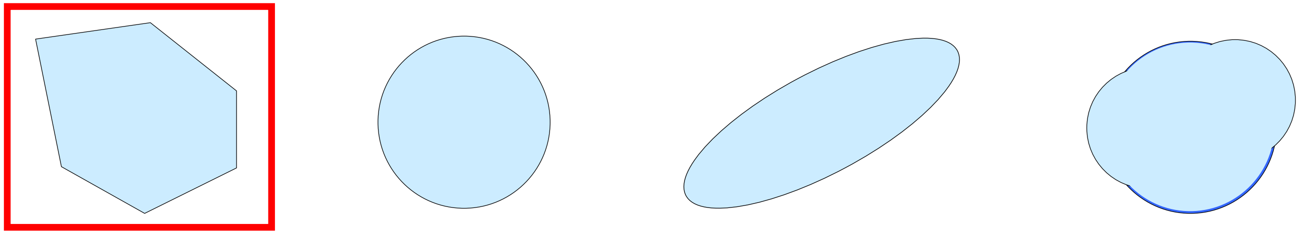
- calculation of the state changing: ***series of small increments,*** „**f** = m**a**” or „**f** = **Ku**”

  Step 3.: calculation of the actual displacement increments



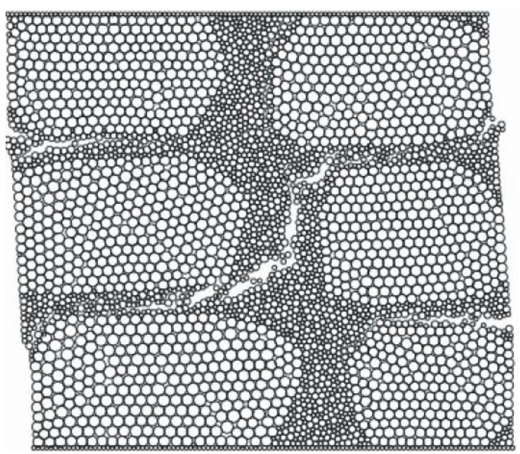
THE GEOMETRY

Element shapes:

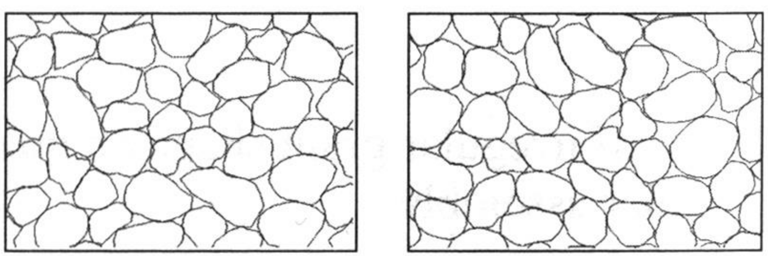


polygon, polyhedron   circle, sphere   ellipse, ellipsoid   complex shapes

e.g. Lemos (2007):
masonry blocks & mortar layer:



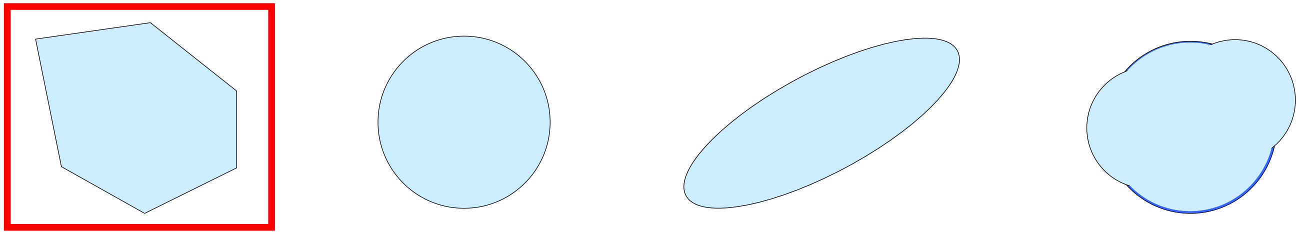
e.g. Matsushima (2005):
irregularly shaped particles for upfill



(a) Toyoura sand model   (b) Ottawa sand model

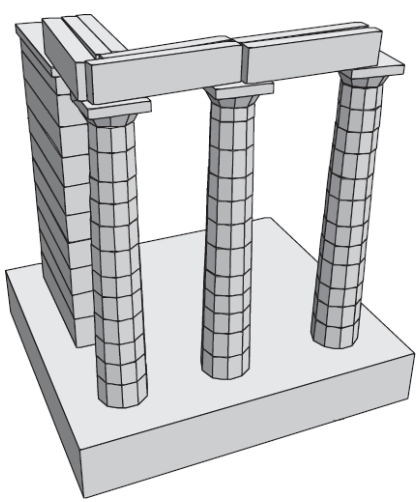
THE GEOMETRY

Element shapes:

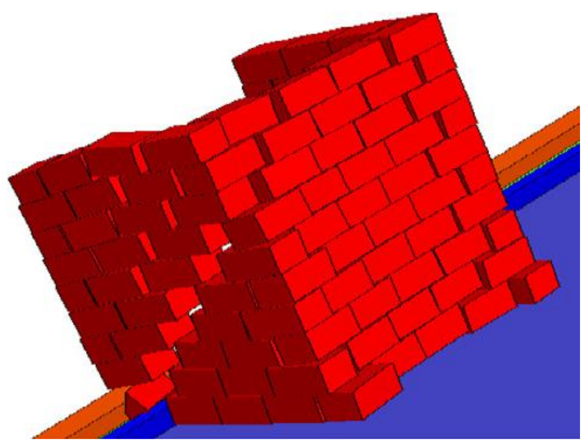


polygon, polyhedron   circle, sphere   ellipse, ellipsoid   complex shapes

e.g. Psycharis et al (2003):
stone blocks:



e.g. Bui et al (2017):
bricks of a house:



THE GEOMETRY

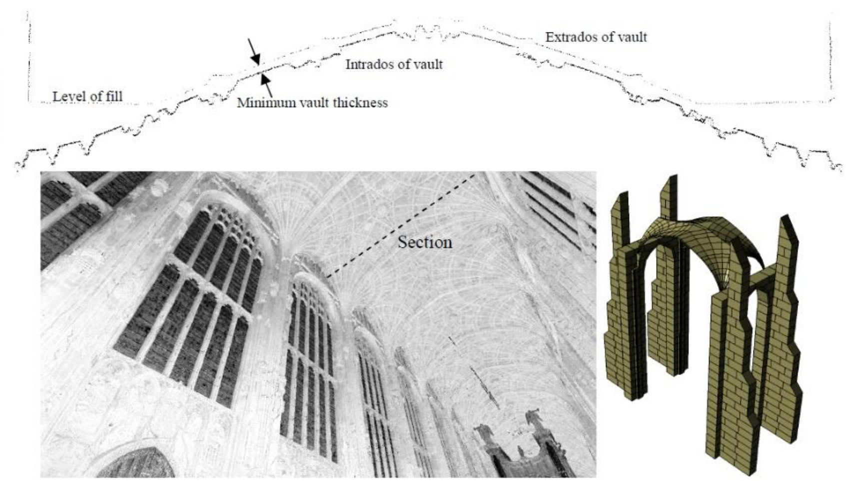
How to get the geometry of a masonry structure:

→ original plans (if still exist)

→ survey the actual geometry, e.g. laser scanner & CAD:

e.g. McInerney et al (2012):

St John's College, Cambridge, UK



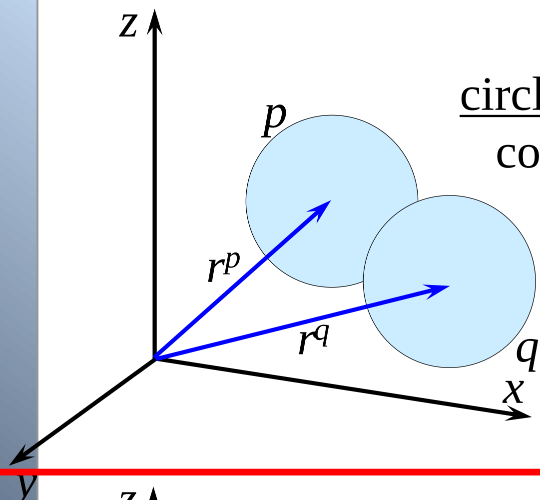
Difficulty e.g.:

how to survey hidden/covered faces

King's College, Cambridge, UK

THE GEOMETRY

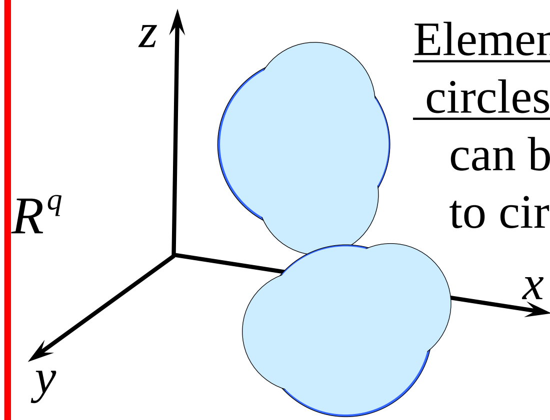
<u>Contact:</u> ≡ any point of an element gets into the interior / on surface of another element



<u>circles/spheres:</u>

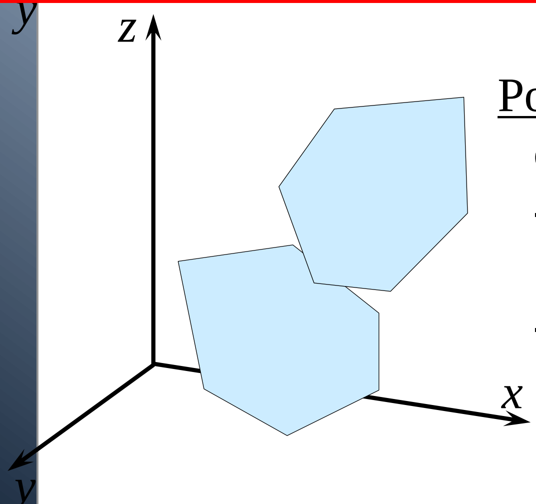
contact if:

$$\left|\mathbf{r}^q - \mathbf{r}^p\right| \le R^p + R^q$$



<u>Elements composed of circles/spheres:</u>

can be treated similarly to circles/spheres

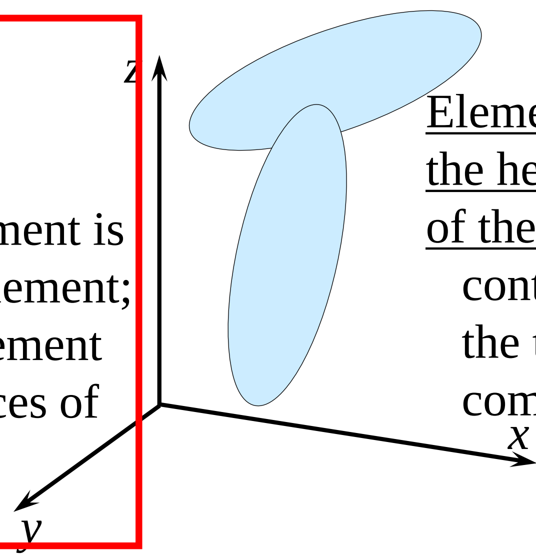


<u>Polygons/polyhedra:</u>

contact if :

-- a node of an element is inside another element;

-- an edge of an element intersects the faces of another element



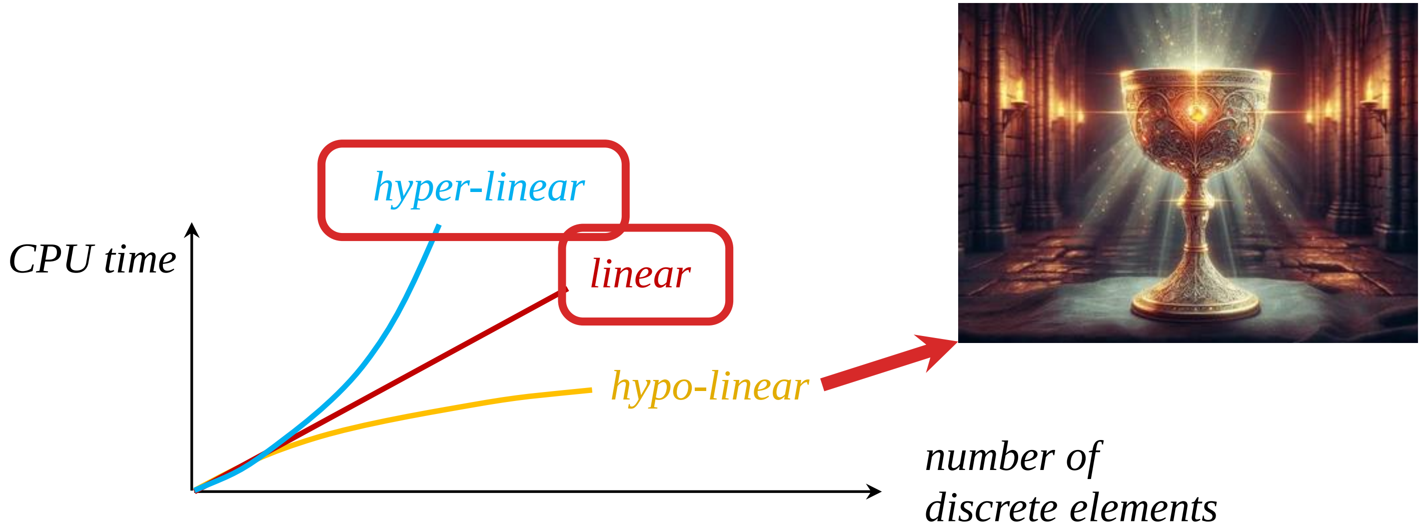
<u>Elements defined with the help of the equation of their surface:</u>

contact if:

the two equations have common solutions

THE GEOMETRY

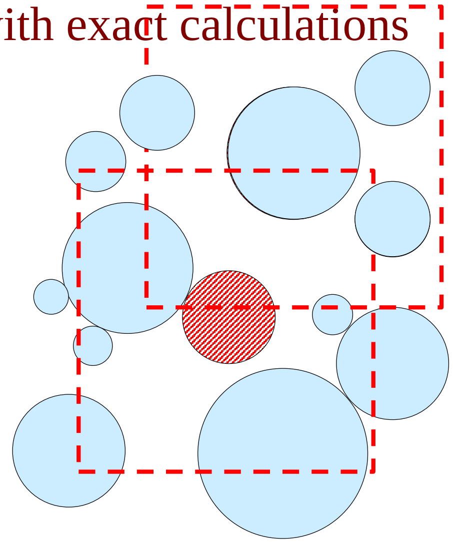
Contact recognition: several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!



THE GEOMETRY

<u>Contact recognition:</u> several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

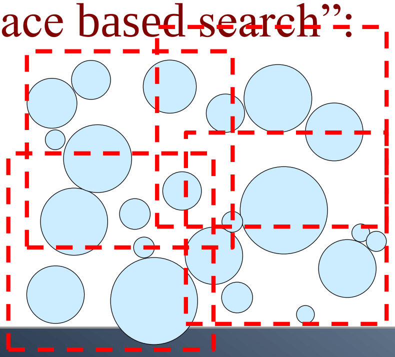
<u>the time consuming part:</u> to check the existence of a contact with exact calculations



Trick #1:
avoid checking <u>every</u> element with <u>every other</u> element:

→ „body based search" technique:
consider only those others which are in the vicinity of the analyzed element;
then take the next element to analyze, …

→ „space based search":

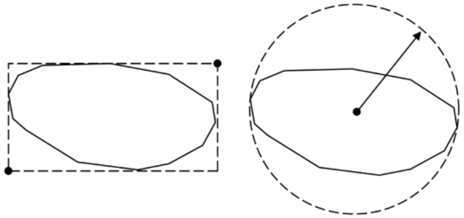


→ divide the domain into „windows" (overlapping);
→ collect which elements are in which windows;
→ analyze those pairs only where both elements belong to the same window

THE GEOMETRY

<u>Contact recognition:</u> several different algorithms exist; its speed basically determines the computational efficiency of the whole DEM code!

<u>the time consuming part:</u> to check the existence of a contact with exact calculations



Trick #2:
avoid majority of the <u>analysis with exact shapes</u>
(useful for elements having complicated shapes)

the idea: „surrounding domain” assigned to each element
(simple shape: brick; sphere)

→ Phase 1.: intersection beween the surrounding domains? (fast)

→ if necessary: Phase 2.: detailed, exact calculations (slow)

MECHANICAL PROPERTIES

Mechanical behaviour of the elements:

role: to specify how to calculate the stresses from the deformations of the elements

→ perfectly rigid elements: deformability concentrated into the contacts

→ deformable elements:
stress-strain-relations have to be specified
[e.g. E, μ, …]

Mechanical behaviour of the contacts:

MECHANICAL PROPERTIES

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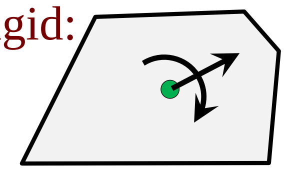
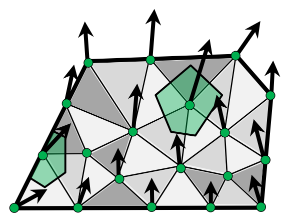
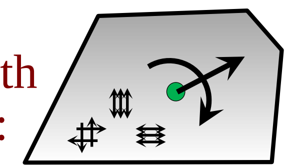
rigid:



deformable
with inner FEM:



deformable with
uniform strain:



MECHANICAL PROPERTIES

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MECHANICAL PROPERTIES

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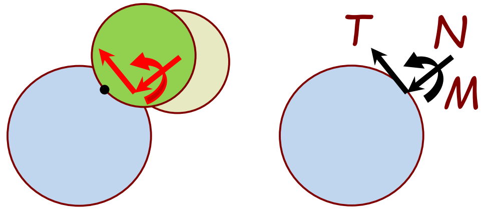
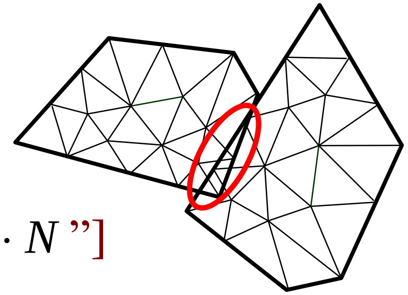


Mechanical behaviour of the contacts:

role: to specify how to calculate the contact forces from the relative displacements at the contact



→ usually: „deformable" contacts
(relative displ. at the contact regions)
concentrated ↔ distributed
[e.g. „ $\Delta N = k_N \cdot \Delta u_N$; $\Delta T = k_T \cdot \Delta u_T$ but $T \leq -f \cdot N$ "]

→ sometimes:
infinitely rigid contacts: no overlap neither any other deformation

CALCULATION OF DISPLACEMENTS

Quasi-static methods ← *an equilibrium state is searched for*

From an initial equilibrium state, the incremental displacements **u** are to be determined taking the system to the new equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\Delta\mathbf{f}=\mathbf{0}"$$

→ Kishino (1988); Bagi-Bojtár (1991) { *circular, perfectly rigid elemets, deformable contacts*

→ Meng et al (2017); Baraldi et al (2018) { *not really DEM yet: small displs; no new contacts;*

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← *a process in time is searched for*

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ UDEC ← *deformable polyhedral elements, deformable contacts*

→ Munjiza's FEM/DEM ← *deformable, breakable elements, deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis") ← *deformable polyhedral elements*

→ Contact Dynamics models ← *rigid elements, non-deformable contacts*

SOLUTION OF THE EQUATIONS OF MOTION

Numerical solutions only!

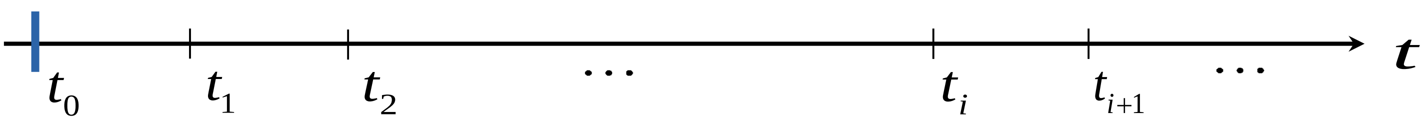
$$\mathbf{M} \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))$$

The aim:

starting from a known $\mathbf{u}(t_0) = \mathbf{u}_0$ and $\mathbf{v}(t_0) = \mathbf{v}_0$ state at a t_0 time instant,

the aim is to determine the approximative solutions $(\mathbf{u}_1, \mathbf{v}_1)$, $(\mathbf{u}_2, \mathbf{v}_2)$, …,

$(\mathbf{u}_i, \mathbf{v}_i)$, $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1})$, … belonging to the $t_1, t_2, \ldots, t_i, t_{i+1}, \ldots$ time instants.

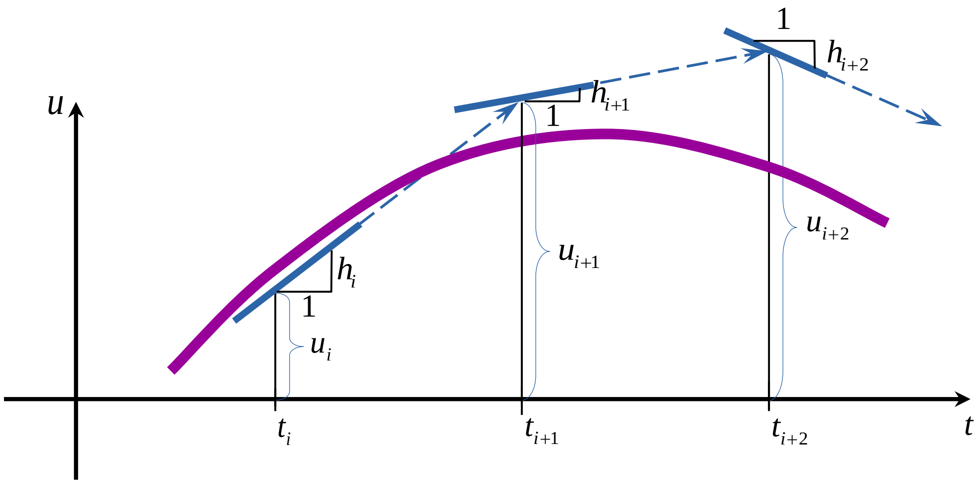


The two basic approaches:

Explicit vs. implicit time integration methods

SOLUTION OF THE EQUATIONS OF MOTION

Explicit vs. implicit methods:



→ explicit methods:

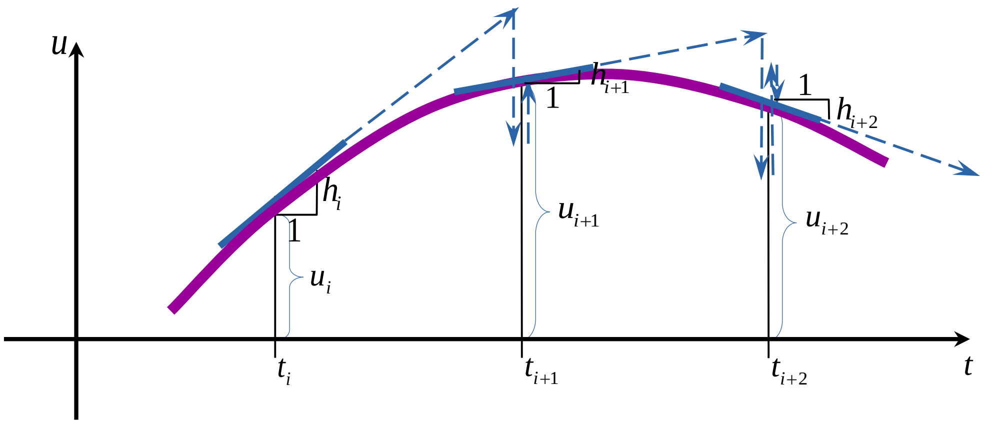
in the state at t_i: ($\mathbf{u}_i$, $\mathbf{v}_i$, $\mathbf{f}_i$) ⇒ equations of motion ⇒ approximate ($\mathbf{u}_{i+1}$, $\mathbf{v}_{i+1}$, $\mathbf{f}_{i+1}$) belonging to the state at t_{i+1}

NOT checking whether ($\mathbf{u}_{i+1}$, $\mathbf{v}_{i+1}$, $\mathbf{f}_{i+1}$) satisfy the eqs of motion: accept them and use them for the calculations of the next timestep

⇒ fast, but less reliable; numerical stability problems!

SOLUTION OF THE EQUATIONS OF MOTION

Explicit vs. implicit methods:



→ implicit methods:

in the state at t_i: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{f}_i) \Rightarrow$ equations of motion $\Rightarrow$

approximate $(\mathbf{u}_{i+1}, \mathbf{v}_{i+1}, \mathbf{f}_{i+1})$ belonging to the state at t_{i+1};

then iterations, to improve this approximation belonging to t_{i+1},

so that the eqs of motion be satisfied at t_{i+1}

$\Rightarrow$ slow, but longer timesteps;

more reliable, better numerical stability

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

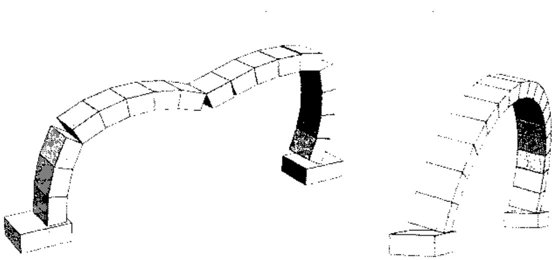
Most important DEM techniques

- UDEC/3DEC
- Discontinuous Deformation Analysis
- Contact Dynamics
- Munjiza's FEM/DEM

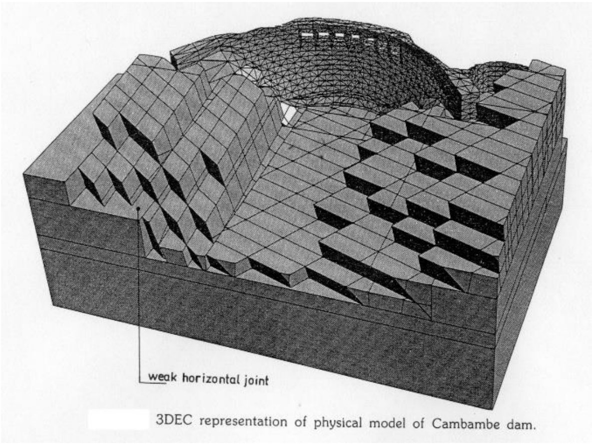
Questions

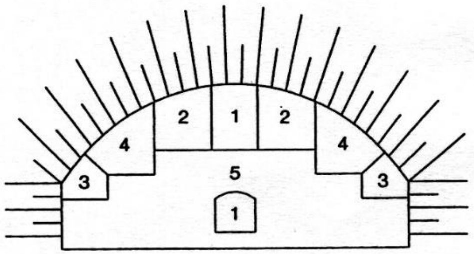
UDEC / 3DEC

UDEC: „Universal Distinct Element Code”
P.A. Cundall, 1971;
development through decades
Itasca Consulting Group
www.itascacg.com



MOST WIDESPREAD IN CIVIL ENGINEERING

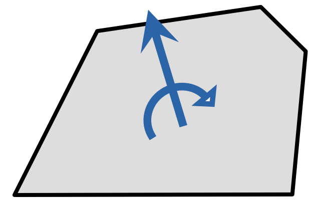




UDEC / 3DEC

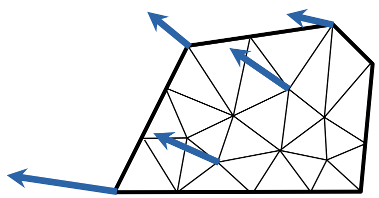
Elements: polygons / polyhedra (planar faces!);

– rigid elements



degrees of freedom:
translation of and rotation about the centroid

– deformable elements (subdivided into simplex zones)



„uniform strain” tetrahedral zones
((*10-node tetrahedra – not reliable*))

degrees of freedom: translations of the nodes

Material models for the elements:

(rigid) ↔ deformable with an inner FEM inside the elements:

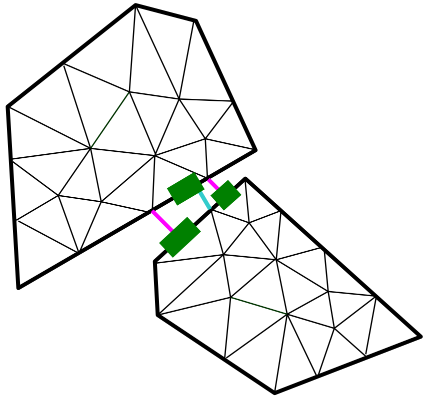
- „null element” (no material in the element)
- linearly elastic, isotropic *(e.g. intact rock; metal)*
- lin. elast., with: Mohr-Coulomb / Prager-Drucker failure crit.
  (e.g. soils, concrete) *(e.g. clay)*
  + tensile strengh + cohesion + dilation angle
- ...

UDEC / 3DEC

<u>Contacts:</u> „common plane" recognition

consist of small „subcontacts", over which: uniformly distributed normal and shear contact forces are transmitted

$$\Delta\sigma = k_N \Delta u_N$$
$$\Delta\tau = k_T \Delta u_T$$



<u>Material models for the contacts:</u>

[calculate the increments of distrib. contact forces from the increments of rel. disps]

– if no material in the contacts: → k_n, k_s: numerical parameters, ∞
or express surface roughness ;
→ friction: real value

– if material in the joints: (modelled as length or area, with zero thickness):

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion and tensile strength

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion & tensile strength + softening + dilation angle

$$\Delta U_n(dil) = \Delta U_s \tan\psi$$

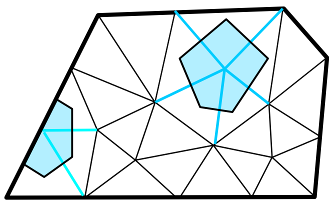
– others …

UDEC / 3DEC

<u>Calculation of nodal displacements</u>

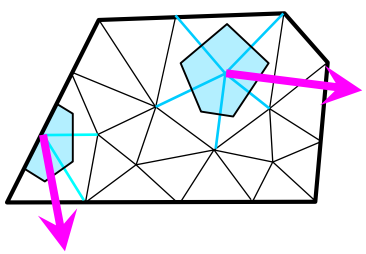
Newton II.: „ $m\,a = f$ ”

– mass assigned to the node:



Voronoi-cell

– force on the node: resultant of the forces acting on the Voronoi-cell of the node



← from the neighbouring element

← from external forces (e.g. self weight, drag force)

← from the stresses inside the simplexes

- force from the stress within a simplex:
  - --- nodal translations ⇒ simplex strain ✓
  - --- from this and material characteristics ⇒ uniform stress in the simplex ✓
  - --- stress vector acting on the face of the cell: $\sigma_{ij} n_j = p_i$; resultant ✓

UDEC / 3DEC

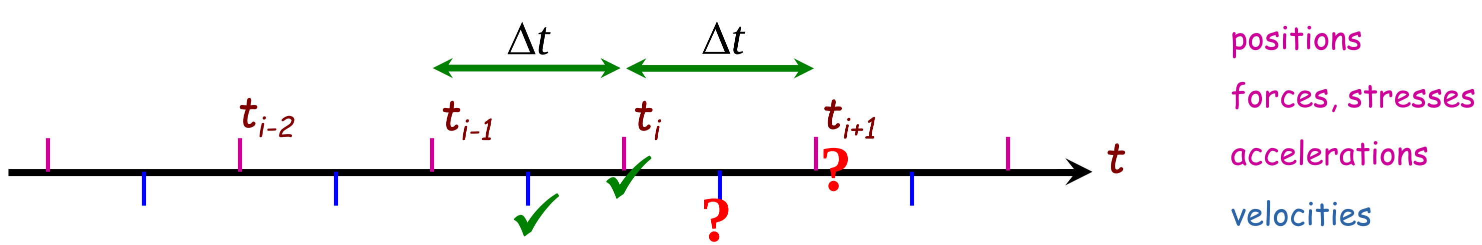
Calculation of nodal displacements

Newton II.: „ $m\,a = f$ ”

– discretized form of the eqs of motion: $$m\frac{\mathbf{v}(t_i + \Delta t/2) - \mathbf{v}(t_i - \Delta t/2)}{\Delta t} = \mathbf{f}(t_i)$$

or: $$\mathbf{v}(t_i + \Delta t/2) = \mathbf{v}(t_i - \Delta t/2) + \frac{\mathbf{f}(t_i)}{m}\Delta t$$

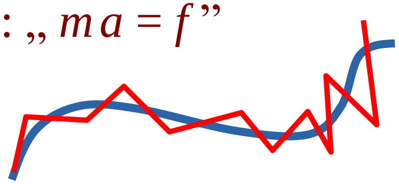
– at t_i : the ***positions of the nodes*** and the ***forces and stresses*** are known;
at $t_i - \Delta t/2$: the ***nodal velocities*** are known;
determine the ***nodal velocities*** at $t_{i+1/2} = t_i + \Delta t/2$
and the ***positions of the nodes*** at $t_{i+1} = t_i + \Delta t$



UDEC / 3DEC

Calculation of nodal displacements

Newton II.: „ $m\,a = f$ ”



– series of small finite time steps:

– explicit time integration; no stiffness matrix!!!

$\Rightarrow$ numerical instabilities, convergence problems

– to help numerical stability:

1. estimate the longest allowed Δt
2. artifical damping is introduced [different types can be used]

MAIN DISADVANTAGE:

strong oscillations around the exact solution

$\Rightarrow$ may give unrealistic results [e.g. in case of history dependence]

$\Rightarrow$ numerical instabilities may occur

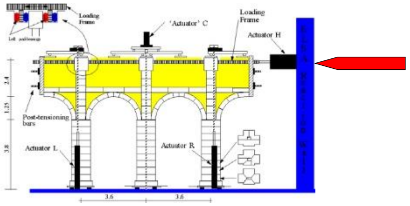
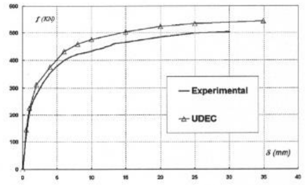
UDEC / 3DEC

<u>Applications for masonry structures:</u>

Quasi-static problems:

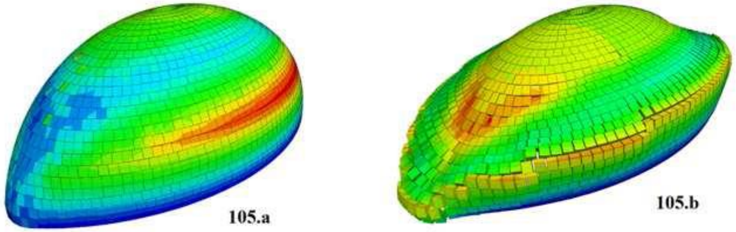
e.g. Sao Vicente de Fora Monastery, Portugal: Giordano et al, 2002

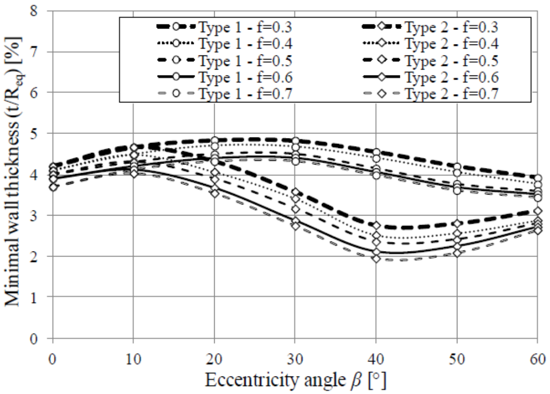


<u>UDEC advantages:</u> works well for ***large displs***; realistic ***crack pattern***

e.g. oval dome statics: Simon & Bagi, 2016





Dynamic problems (use with caution!):

→ convergence of the solution with respect to Δt should be ensured

→ damping type and damping parameters should carefully be selected & calibrated

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

Most important DEM techniques

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Questions

DDA: „DISCONTINUOUS DEFORMATION ANALYSIS"

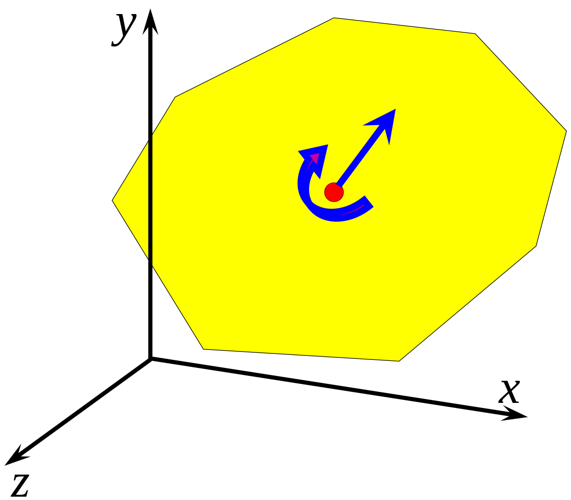
Gen-Hua Shi (1988), Berkeley
then many others applied or developed
research software!!!

<u>The elements:</u> polyhedral; with a reference point (e.g. centroid)
[Deformable without subdivision]

„displacement vector" of the *p*-th element: $\mathbf{u}^p$

„reduced load" belonging to the *p*-th element: $\mathbf{f}^p$

$$\mathbf{u}^p = \begin{bmatrix} u_x^p \\ u_y^p \\ u_z^p \\ \varphi_x^p \\ \varphi_y^p \\ \varphi_z^p \\ \varepsilon_x^p \\ \varepsilon_y^p \\ \varepsilon_z^p \\ \gamma_{yz}^p \\ \gamma_{zx}^p \\ \gamma_{xy}^p \end{bmatrix} \qquad \mathbf{f}^p = \begin{bmatrix} f_x^p \\ f_y^p \\ f_z^p \\ m_x^p \\ m_y^p \\ m_z^p \\ V^p \sigma_x^p \\ V^p \sigma_y^p \\ V^p \sigma_z^p \\ V^p \tau_{yz}^p \\ V^p \tau_{zx}^p \\ V^p \tau_{xy}^p \end{bmatrix}$$



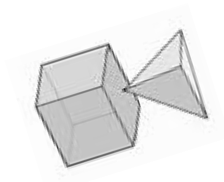
The degrees of freedom:
rigid-body translation and rotation
of the reference point;
+ the ***uniform*** strain of the element

DDA

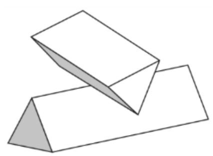
The contacts: (material point) with (material point)

in 2D: Node – to – Edge contacts

in 3D: Node – to – Face contacts:



Edge – to – Edge contacts:



→ „first entrance position"

⇒ contact deformation: $\Delta u_N ; \Delta u_T$

normal & tangential (perhaps sliding)

→ direction of the contact:

the normal vector of the face

???? for edge-to-edge contact

Mechanical model:

→ originally: infinitely rigid contacts, Coulomb-friction

→ recent codes: deformable contacts included

+ other friction conditions, cohesion etc.

Remark: infinitely rigid contact: „penalty function": $F_N = k_N \Delta u_N ; \; dF_T = k_T d(\Delta u_T)$

≡ linearly elastic in normal and in tangential directions

DDA

↓ rather: „ Veubecke-Hu-Washizu principle ”

The equations of motion: „Potential energy” stationarity principle

„Potential” of the system: $\Pi = \Pi^{blocks} + \Pi^{contacts}$

- $\Pi^{contacts}$: deformed springs
- Π^{blocks}:
  - external pot.
  - strain energy
  - inertial forces
  - velocity-proportional damping
  - initial stress
  - prescribed displacement history

$$\frac{\partial \Pi}{\partial u_i^p} = 0 \quad \text{for all } p, i$$

$$\mathbf{M} \cdot \mathbf{a}(t) + \mathbf{C} \cdot \mathbf{v}(t) + \mathbf{K} \cdot \mathbf{u}(t) = \mathbf{f}^{ext}(t, \mathbf{v}(t), \mathbf{u}(t))$$

- $\mathbf{u}$: generalized displacement increment
- $\mathbf{K}$: stiffness matrix + etc
- $\mathbf{C}$: damping + etc
- $\mathbf{M}$: inertia

or: $\mathbf{M} \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))$

DDA

Numerical solution of the equations of motion:

(t_i, t_{i+1}) time interval:

at t_i : known $\mathbf{u}_i$, $\mathbf{v}_i$, $\mathbf{f}(t_i, \mathbf{u}_i, \mathbf{v}_i)$; satisfy the eqs. of motion

Find $\mathbf{u}_{i+1}$, $\mathbf{v}_{i+1}$, $\mathbf{a}_{i+1}$ so that the eqs of motion would be satisfied at t_{i+1}

$$\mathbf{r}(t_{i+1}, \mathbf{u}_{i+1}, \mathbf{v}_{i+1}) = \mathbf{f}(t_{i+1}, \mathbf{u}_{i+1}, \mathbf{v}_{i+1}) - \mathbf{M} \cdot \mathbf{a}_{i+1} = 0$$

Remember: Newmark's β–method:

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t \cdot \mathbf{v}_i + \frac{\Delta t^2}{2}\left[(1-2\beta)\mathbf{a}_i + 2\beta \cdot \mathbf{a}_{i+1}\right]$$

$$\mathbf{v}_{i+1} := \mathbf{v}_i + (1-\gamma) \cdot \Delta t \cdot \mathbf{a}_i + \gamma \cdot \Delta t \cdot \mathbf{a}_{i+1}$$

[stability: $2\beta \geq \gamma \geq ½$]

DDA: Newmark's β–method, with $\beta = 1/2$; $\gamma = 1$:

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t \cdot \mathbf{v}_i + \frac{\Delta t^2}{2}\mathbf{a}_{i+1}$$

$$\mathbf{v}_{i+1} := \mathbf{v}_i + \Delta t \cdot \mathbf{a}_{i+1}$$

let

$$\begin{cases} \Delta\mathbf{u}_{i+1} = \mathbf{u}_{i+1} - \mathbf{u}_i \\ \mathbf{a}_{i+1} = \dfrac{1}{\Delta t^2/2}(\Delta\mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) \\ \mathbf{v}_{i+1} = \mathbf{v}_i + \Delta t \cdot \mathbf{a}_{i+1} = \mathbf{v}_i + \dfrac{2}{\Delta t}(\Delta\mathbf{u}_{i+1} - \Delta t \cdot \mathbf{v}_i) = \dfrac{2}{\Delta t}\Delta\mathbf{u}_{i+1} - \mathbf{v}_i \end{cases}$$

DDA

Numerical solution of the equations of motion :

$$\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t)) \quad \Rightarrow \quad \mathbf{0}=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1},\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

Determine $\Delta\mathbf{u}_{i+1}$, so that the residual

$$\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1})=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1}),\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

would be sufficiently close to zero!

Newton-Raphson:

the Jacobian of the residual: $\mathcal{K}(t,\Delta\mathbf{u})=\dfrac{d\mathbf{r}(t,\Delta\mathbf{u})}{d\Delta\mathbf{u}}$

this matrix can be compiled from elementary calculations at t_i:
← contains the stiffness matrix
← contains the inertia, contact forces, geometric characteristics etc.

the residual can also be compiled from elementary calculations at t_i:
← contains the external forces, inertia effects, prescribed displacements, damping etc.

DDA

Numerical solution of the equations of motion :

$$\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t)) \quad\Rightarrow\quad \mathbf{0}=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1},\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})$$

$$\boxed{\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1})=\mathbf{f}(t_{i+1},\mathbf{u}_{i+1}(\Delta\mathbf{u}_{i+1}),\mathbf{v}_{i+1}(\Delta\mathbf{u}_{i+1}))-\mathbf{M}\cdot\mathbf{a}_{i+1}(\Delta\mathbf{u}_{i+1})}$$

$$\mathcal{K}(t,\Delta\mathbf{u})=\frac{d\mathbf{r}(t,\Delta\mathbf{u})}{d\Delta\mathbf{u}}$$

Analysis of a time interval:

initial estimation for $\Delta\mathbf{u}_{i+1}$: $\Delta\mathbf{u}_{i+1}^{(0)}:=\mathbf{0}$

k+1-th estimation for $\Delta\mathbf{u}_{i+1}$: $\Delta\mathbf{u}_{i+1}^{(k+1)}:=\Delta\mathbf{u}_{i+1}^{(k)}-\mathcal{K}(t_{i+1},\Delta\mathbf{u}_{i+1}^{(k)})^{-1}\cdot\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1}^{(k)})$

then continue until $\left|\mathbf{r}(t_{i+1},\Delta\mathbf{u}_{i+1}^{k+1})\right|$ becomes sufficiently small

„Open – close iterations”: at the end of Δt: **check** the topology and the forces;

→ **modify the topology** if necessary (e.g. new contacts, sliding, contact loss)

→ with the new topology, **repeat:** Newton-Raphson to find another $\Delta\mathbf{u}_{i+1}$

if acceptable topology not found: **decrease timestep** Δt to 1/3 of its previous length

CONVERGENCE WITHIN A TIME STEP ???

DDA: „DISCONTINUOUS DEFORMATION ANALYSIS"

Comparison to UDEC/3DEC:

Main differences from UDEC/3DEC:

→ basic unknowns: also the components of **ε** ;

→ uniform stress and strain field inside the elements;

→ numerical integration: implicit

→ stiffness matrix included ⇒ artificial damping not necessary

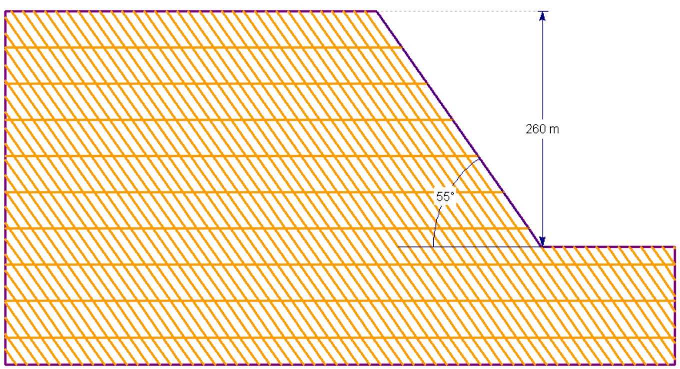
– advantages to UDEC/3DEC: implicit ⇒ numerical stability;
fast convergence if topology does not change
no artificial damping required

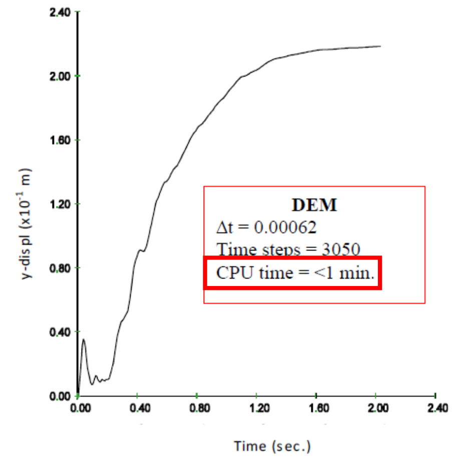
– disadvantages: no commercial software ⇒ inconvenient
(several research codes; e.g. ask from Gen-Hua Shi)
too simple mechanics of the elements and of the contacts
large storage requirements & longer computations
open-close iterations: convergence is not ensured if topology changes

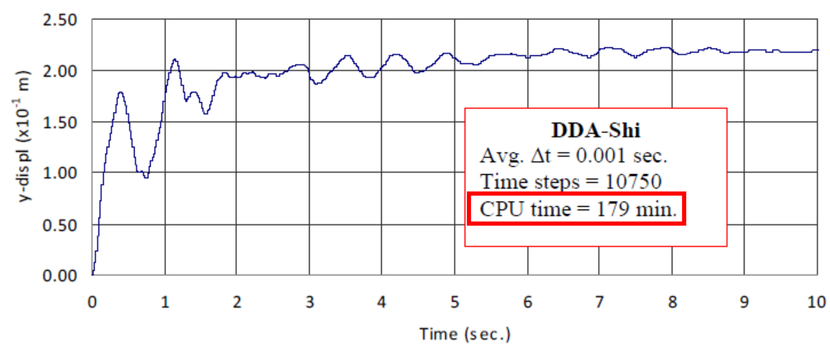
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Comparison to UDEC:

M.S. Kahn (2010)





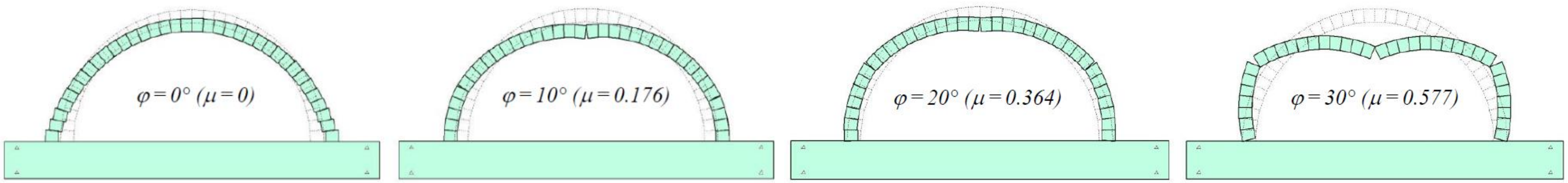


NOT EFFICIENT IN CASES IF SIGNIFICANT TOPOLOGY MODIFICTIONS OCCUR !!!

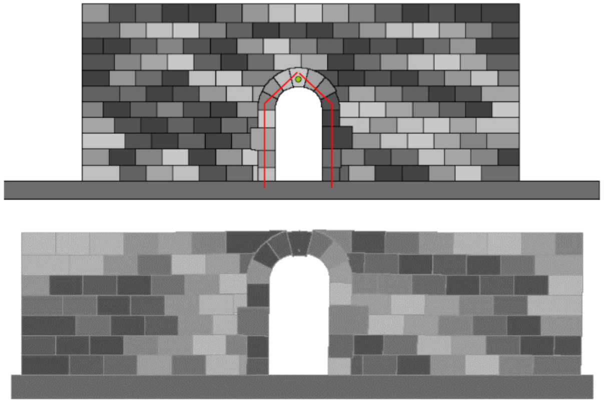
DDA: „DISCONTINUOUS DEFORMATION ANALYSIS”

Applications:

e.g. Rizzi et al (2014): collapse modes of arches



e.g. Kamai and Hatzor (2005): back analysis of seismic events



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Questions

CONTACT DYNAMICS

Jean & Moreau (1992): (2D, 3D) [mostly in physics]

Unger, T. – Kertész, J. (2003): The contact dynamics method for granular media. In: Modeling of Complex Systems, Melville, New York, American Institute of Physics, pp. 116-138

Software:

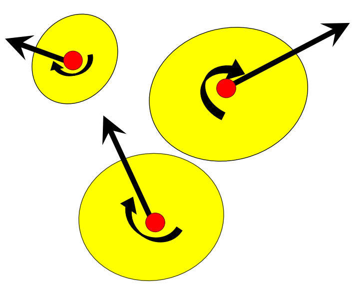
(1) LMGC91 (Dubois & Jean, 2006): **OPEN!**

rigid/deformable; spherical/polyhedral elements

(2) SOLFEC (Koziara & Bicanic, 2008):

rigid/deformable; polyhedral elements

The elements:



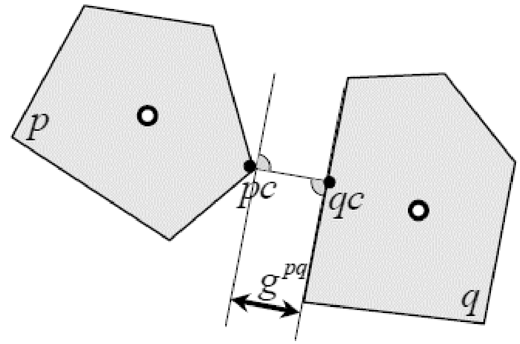
ORIGINALLY: rigid, spherical elements

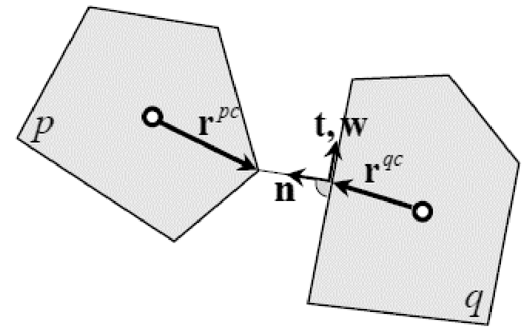
for masonry structures:

deformable or rigid polyhedral elements

CONTACT DYNAMICS

Contacts of polyhedral elements:

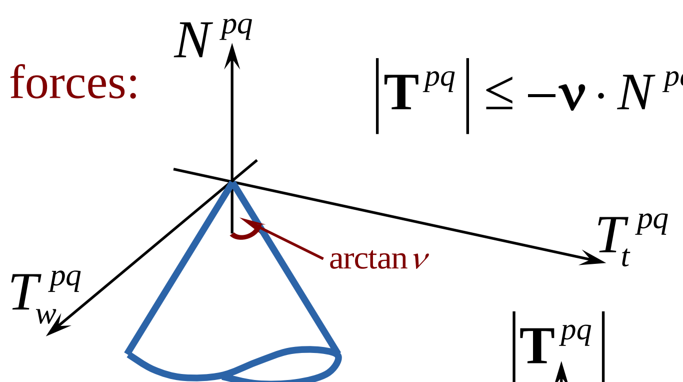




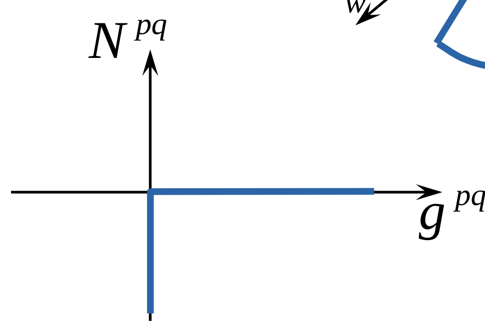
„common plane concept”

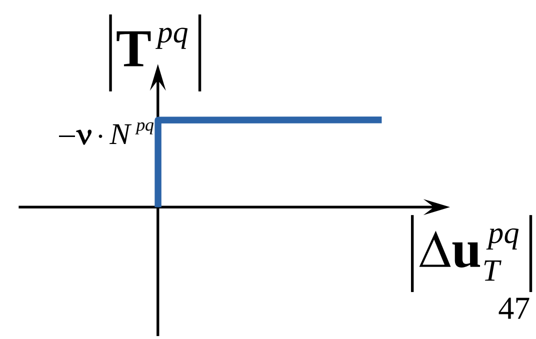
Mechanical conditions for the contact forces:

$$\left|\mathbf{T}^{pq}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}$$



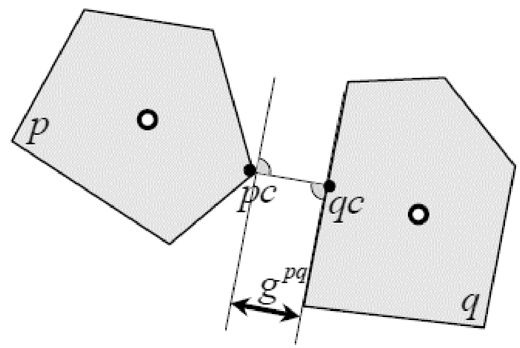
[the same]

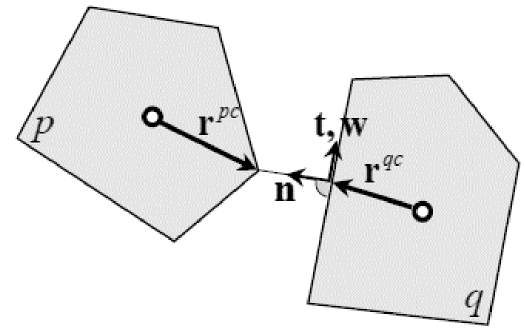




CONTACT DYNAMICS

<u>Contacts of polyhedral elements:</u>





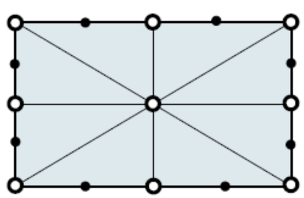
Rigid polyhedral elements:

Degrees of freedom: translations & rotations of the reference points

Deformable polyhedral elements: ~~constant strain~~ → unfavourable experiences

uniform-strain tetrahedral subdivision

<u>The point of action of the contact force:</u>



• : middle point of the face

„**approximated contact point**"

contact: if • touches another face

<u>Masses:</u> distributed to the **nodes**

<u>Equations of motion:</u> for every **node** [no rotations considered];

Degrees of freedom: nodal translations [similar to 3DEC def]

CONTACT DYNAMICS

How to find the solution at the end of a given time step:

implicit solution:

the positions and velocities are repeatedly (iteratively) adjusted,
until the equations of motion AND the contact conditions are satisfied
with the required accuracy at the end of the time step

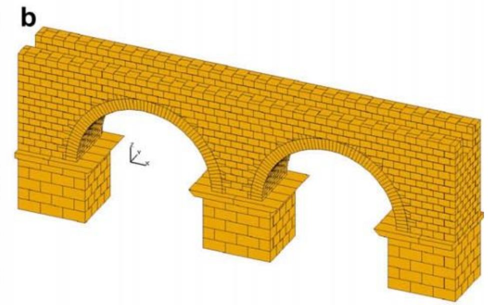
[≈ Cross method for frames, but randomly sweeping along the pairs of elements]

history dependence! [order of sweeping along contacts makes difference in the results]

⇒ engineers have serious doubts

Main advantage: extremely fast for dynamic phenomena

CONTACT DYNAMICS



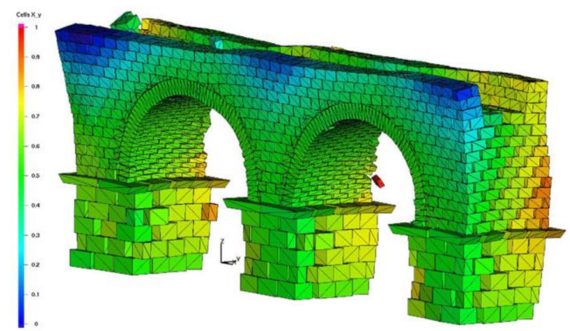
<u>Civil engineering applications</u>

e.g. Rafiee et al (2008):

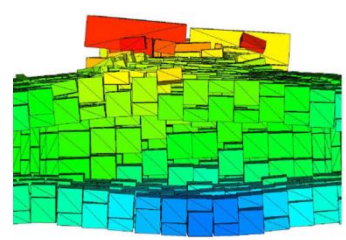
CD numerical model with deformable elements:

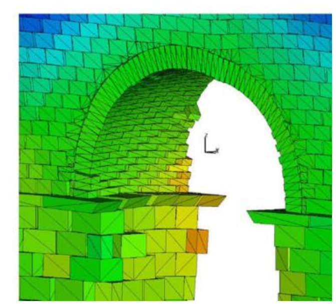
Arles, aqueduct

Earthquake simulations:



☹ Experimental verification?





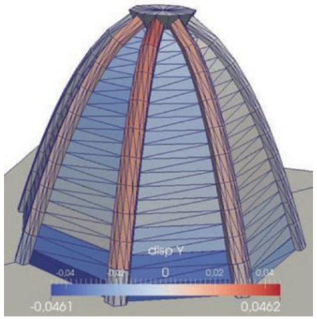
CONTACT DYNAMICS

croatiatraveller.com/Heritage_Sites/CathedralSibenik.htm

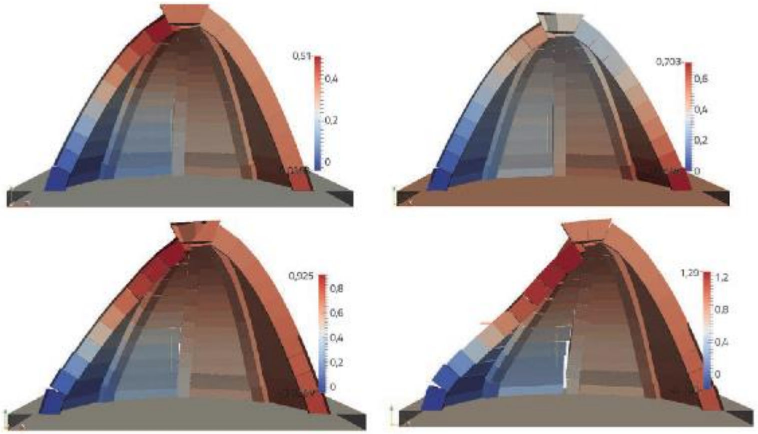
Civil engineering applications

e.g. Gelo & Mestrovic (2016):

dome of St Jacob Cathedral, Sibenik, Croatia

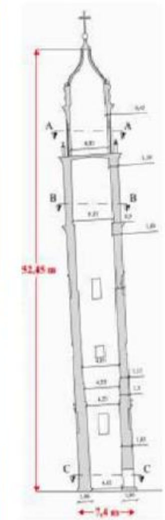


Earthquake simulations:



☹ Experimental verification?

CONTACT DYNAMICS

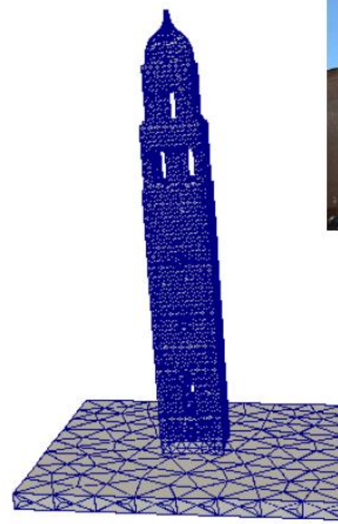


Civil engineering applications

e.g. Clementini et al (2018):

San Benedetto Church, Ferrara

aim: analyse seismic behaviour



Model assumptions:

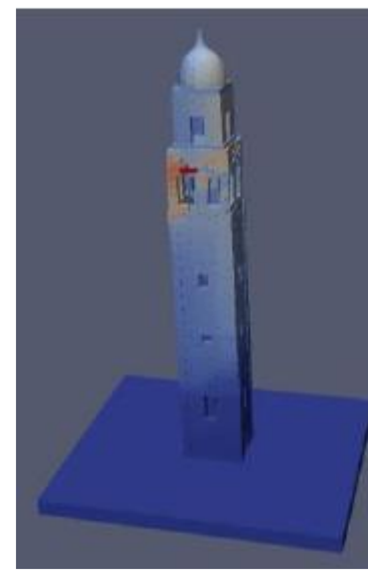
rigid blocks

Coulomb-frictional contacts

perfectly plastic impact (no bouncing)

Load: basement oscillations $v(t) = C \sin(2\pi \cdot f \cdot t)$

$\equiv$ earthquake simulations



Model validation: compare first frequency to reality

Outcome: vulnerable regions of the structure

THIS LECTURE:

What is DEM?

- The Geometry
- Mechanical Properties
- Calculation of the Displacements

Most important DEM techniques

- UDEC/3DEC
- Discotninuous Deformation Analysis
- Contact Dynamics
- Munjiza’s FEM/DEM

Questions

MUNJIZA'S FEM/DEM METHOD

Ante Munjiza (1999), (2004), …: (2D, 3D)

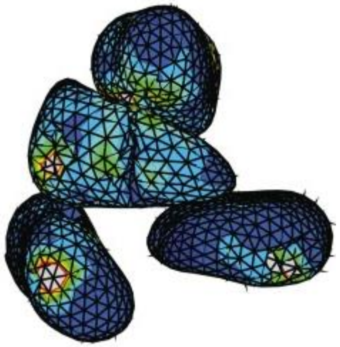
→ to simulate fracture and fragmentation of discrete elements

Recent years:

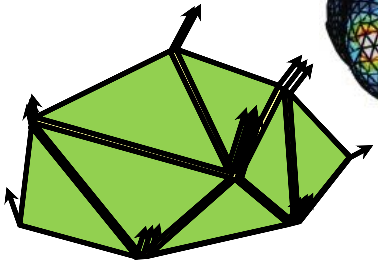
→ further development of several algorithmic details

→ applications to historic masonry

Main features:

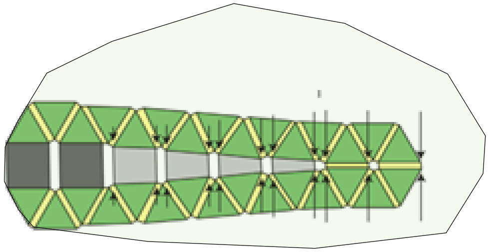


→ deformable, polyhedral discrete elements ;
deformable contacts between them

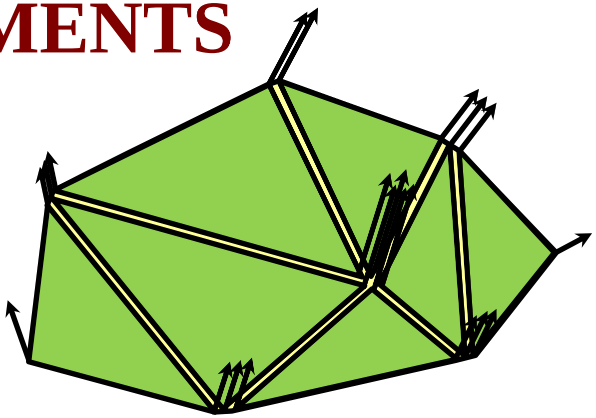


→ discrete elements are subdivided into:
uniform-strain FEM tetrahedra

→ „joint elements":
inside the discrete elements,
between the FEM tetrahedra:
able to soften and open up



MUNJIZA'S: THE ELEMENTS



Degrees of freedom:

translations of the nodes

→ like in 3DEC def.

Strain in the finite element tetrahedra:

different possibilities available:

small strain tensor; right or left Cauchy-Green strain tensor;

Stress options: Cauchy stress tensor; Ist or IInd Piola-Kirchhoff stress tensor

→ more options than in 3DEC

Constitutive model of the elements:

Hooke law, no plasticity of the finite elements [very simple]

→ in 3DEC: plastic yield and user-defined constitutive relations can be used

masses in eqs of motion: masses of the Voronoi cells of the nodes → like in 3DEC

stress field inside the tetrahedra: reduced to the nodes → like in 3DEC

Time integration: central difference method → like in 3DEC

CONTACT INTERACTION ALGORITHM

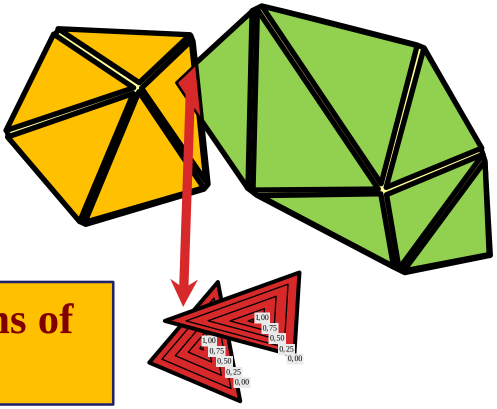
<u>Advantageous features:</u>

→ distributed contact forces: no unrealistic stress concentrations

→ complicated contact behaviour (sliding, plasticity, cohesion etc): easy to incorporate

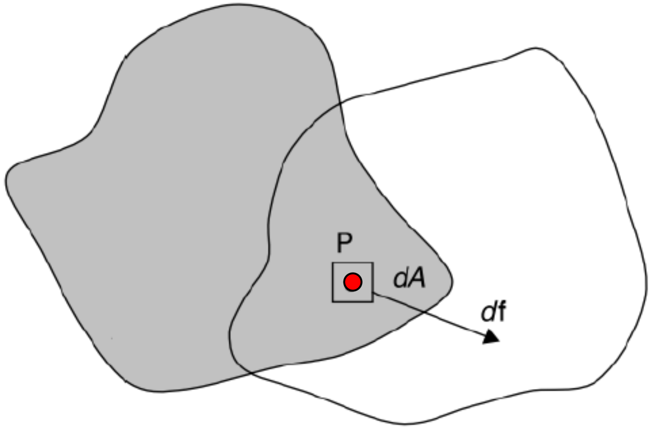
→ energy conservation satisfied!

→ computationally relatively efficient



Case of two overlapping discrete elements:

P scans over the total overlap



Potential functions of the two FE-s

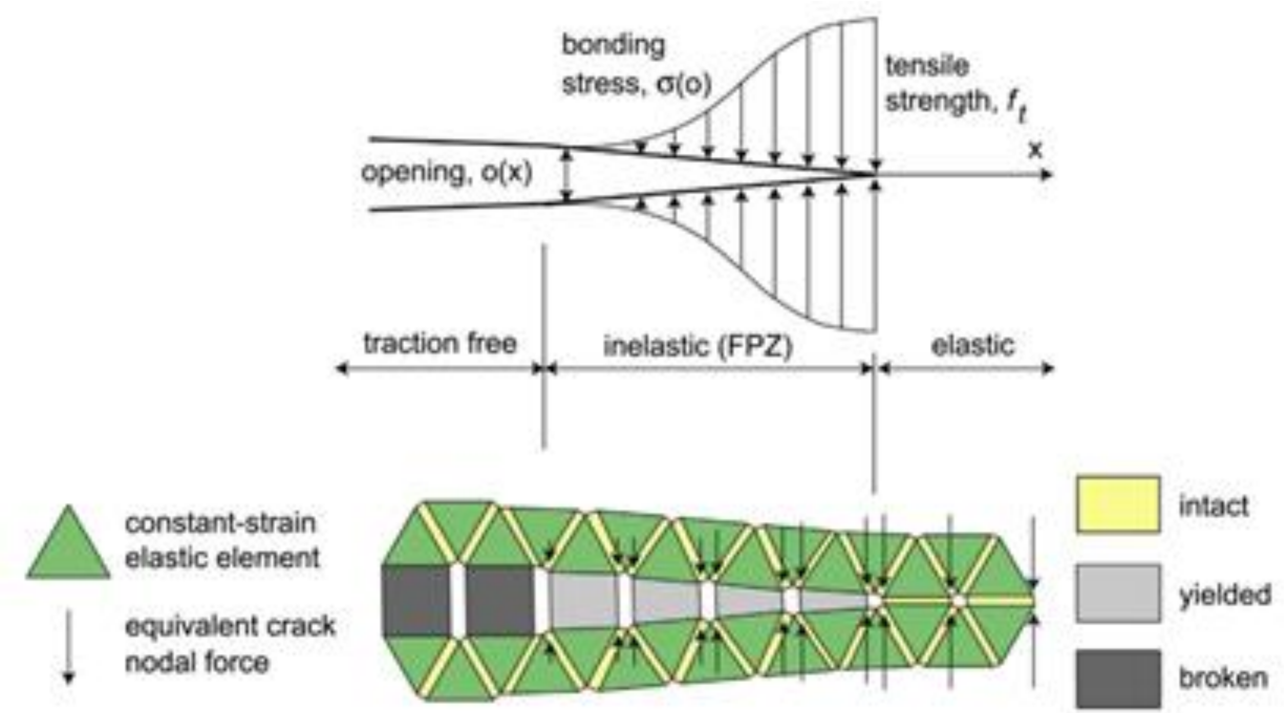
$$\mathrm{d}\boldsymbol{f} = \left[\mathrm{grad}\,\varphi_1(P) - \mathrm{grad}\,\varphi_2(P)\right]\mathrm{d}A$$

⇒ distributed force along the overlap: then reduced to the nodes

FRACTURE & FRAGMENTATION ALGORITHM

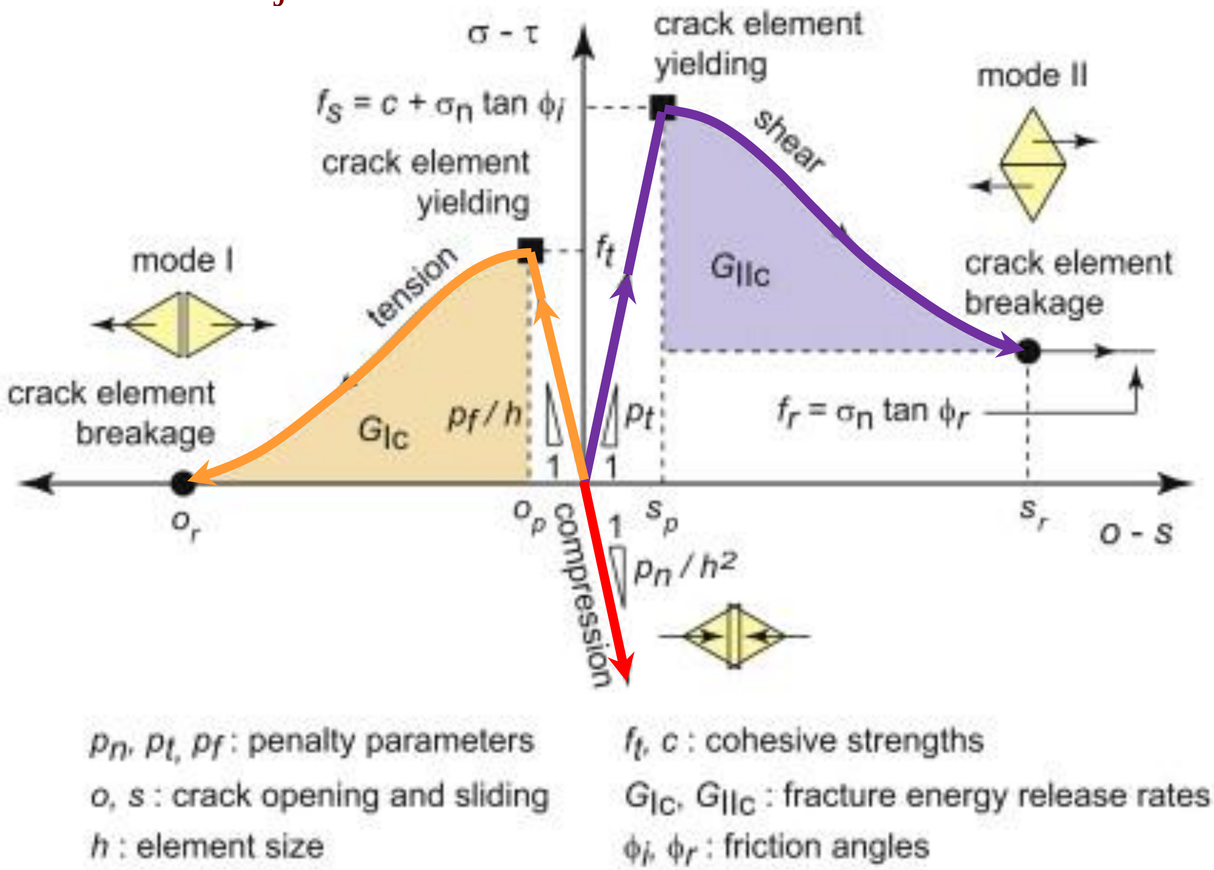
aims: → to define crack initiation
→ to describe how cracks propagate,
→ to replace the released internal forces with new contact forces

„joint elements”: the surface between FE-s ! in the interior of DE-s !



MUNJIZA’S: THE JOINT ELEMENTS

Mechanical behaviour of joints:



Disadvantage:

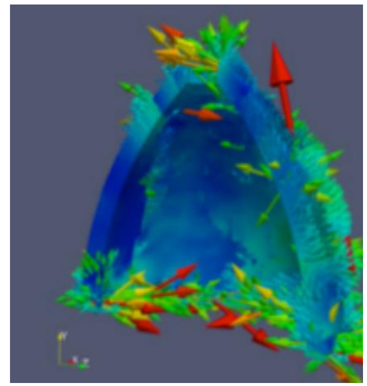
simulated fracture behaviour is very sensitive to mesh density & orientation

⇒ very dense subdivision of the DE-s is needed

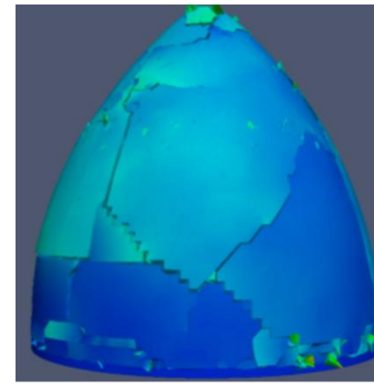
MUNJIZA’S: APPLICATIONS

e.g. Rougier et al (2014):

Seismic analysis of the Dome of the Santa Maria del Fiore cathedral



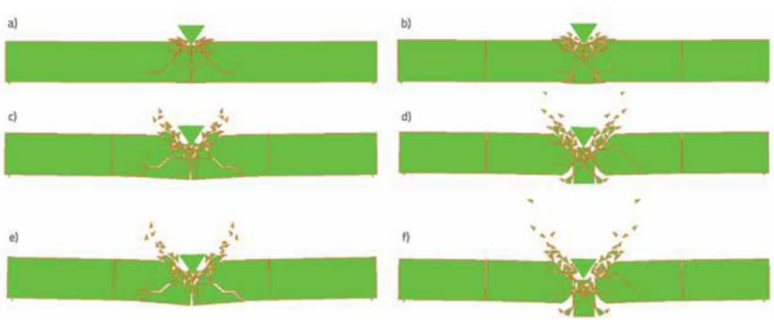
stress vawe propagation



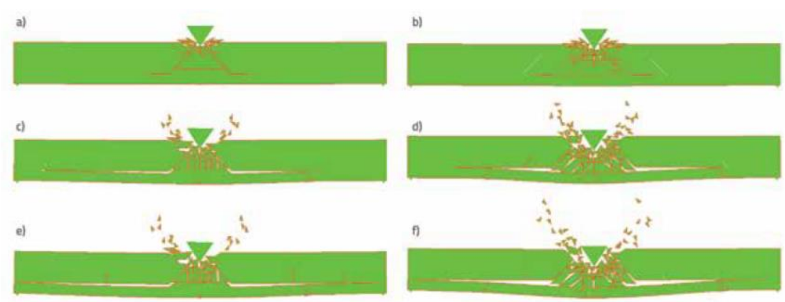
cracked final state

e.g. Zivaljic et al (2014):

Impact loading of a concrete beam



unreinforced



reinforced

Additional remarks

Remarks about other codes:

YADE: (free, open source code; rather an international community)

– : contact model for polyhedra: too simple, damping cannot be calibrated

Further info: **https://yade-dem.org**

PFC: („Particle Flow Code”, Cundall, 1979)

polyhedral elements: recently included

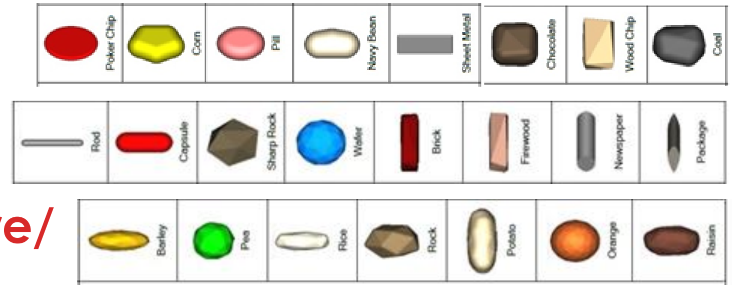
application in the past: spheres glued together to form voussoirs

Further info: **www.itascacg.com**

Rocky-DEM:

wide variety of element shapes; breakable

Further info: **https://rocky.esss.co/software/**

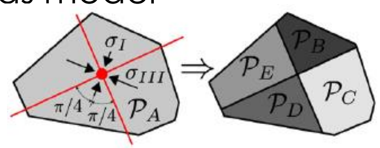


Additional remarks

Remark: How to consider element breakage:

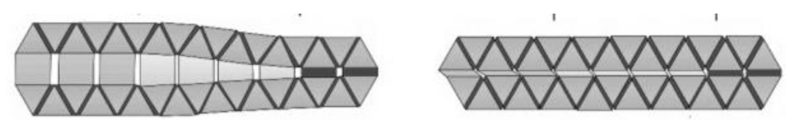
→ If a breakage criterion is met,
replace the discrete element with several smaller discrete elements

- YADE: the Elias model



Eliáš (2014)

- Munjiza:
  inside the discrete elements:
  „joints" between FEM-tetrahedra can break



Deng et al (2021)

- Rocky-DEM: Tavares
  based on (accumulated) collision energy

https://rocky.esss.co/blog

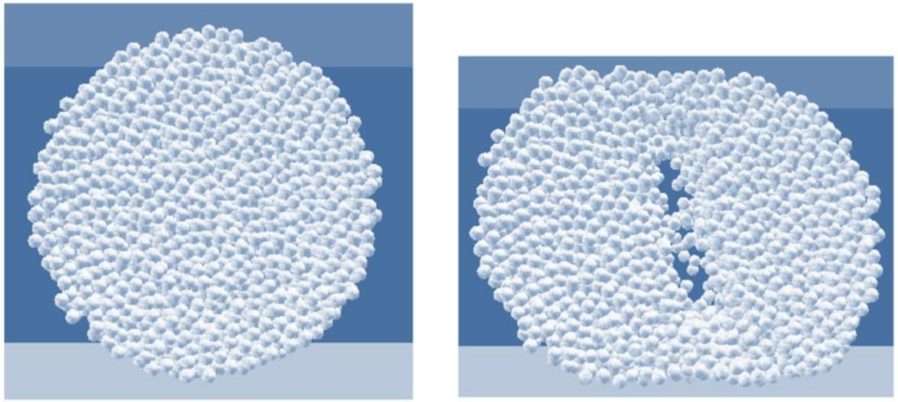
- PFC: recent attempts, reseaerches just going on

Additional remarks

Remark: How to consider element breakage:

→ The alternative:

compose your masonry stone from many small discrete elements; contacts break



Gupta et al (2017)

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Questions

QUESTIONS

1. Under what conditions can a numerical technique be classified as a discrete element model? What are the main steps of the discrete element modelling of an engineering problem?

2. What is the difference between quasi-static and time-stepping calculation methods of the displacement increments?

3. What is the difference between explicit and implicit time integration techniques?

4. What are the degrees of freedom in UDEC/3DEC, in DDA, and in Contact Dynamics? What kind of time integration technique is applied in these models?

5. What are the main advantages and disadvantages of UDEC/3DEC, DDA, and Contact Dynamics in comparison to each other?