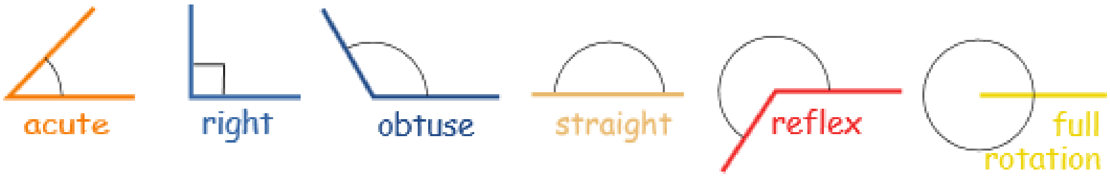
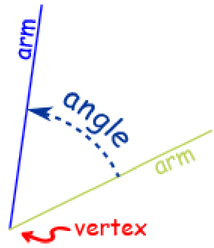
1. Types of angles



Acute Angle	is less than 90°
Right Angle	is 90° exactly
Obtuse Angle	is greater than 90° but less than 180°
Straight Angle	is 180° exactly
Reflex Angle	is greater than 180°
Full Rotation	is 360° exactly



Parts of angles:

The corner point of an angle is called the vertex

And the two straight sides are called arms

The angle is the amount of turn between each arm.

2. Practice of addition, subtraction and average of angles using DMS values

Additon of angles: $\gamma = \alpha + \beta$ If γ > 360° then subtract 360°

incorrect		correct	
α =	86-59-42.1	α =	86-59-42.1
+β =	33-45-37.3	+β =	33-45-37.3
γ =	**119-104-79.4**	γ =	**120-45-19.4**

incorrect		correct	
α =	217-34-19	α =	217-34-19
+β =	186-29-57	+β =	186-29-57
γ =	**403-63-76**	γ =	**44-04-16**

Practice:

α = 214-21-54
+β = 135-44-12
γ = 350-06-06

α = 314-24-41
+β = 222-11-42
γ = **536-36-23 (-360°) = 176-36-23**

α = 180-00-01
+β = 180-00-00
γ = **360-00-01 (-360°) = 0-00-01**

α = 145-25-45
+β = 122-57-54
γ = **268-23-39**

Calculate the angle between two directions (subtraction of angles)

$\beta = l_R - l_L$ (right side - left side)   If $\beta<0°$ (negative) then add 360°

				341-76-79
l_R =	342-17-19		l_R =	342-17-19
$-l_L$ =	141-27-41		$-l_L$ =	141-27-41
β =	**200-49-38**		β =	**200-49-38**

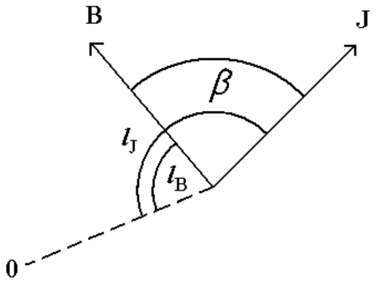
Practice:

l_R = 214-21-54
$-l_L$ = 135-44-12
β = **78-37-42**

l_R = 0-00-00 **(+360°)**
$-l_L$ = 184-54-11
β = **175-05-49**

l_R = 331-43-18 **(+360°)**
$-l_L$ = 331-43-19
β = **359-59-59**

l_R = 98-22-32
$-l_L$ = 211-55-49
β =**-(113-33-17)+360°=246-26-43**



Calculate the average (arithmetic mean) of 2 angles (using banker's rounding)

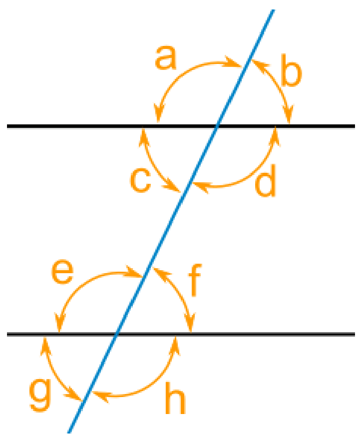
α_1 = 352-51-27
α_2 = 352-51-21
α = **352-51-24**

α_1 = 78-17-49
α_2 = 78-17-32
α = **78-17-40**

α_1 = 3-19-58
α_2 = 3-20-04
α = **3-20-01**

α_1 = 246-59-54
α_2 = 247-00-06
α = **247-00-00**

3. Pairs of angles



Classification of angles using the figure above

- Opposite angles: a-d, b-c, e-h, g-f (equal)
- Corresponding angles: a-e, b-f, c-g, h-d (equal)
- Alternate interior angles: c-f, d-e (equal)
- Alternate exterior angles: a-h, b-g (equal)
- Consecutive interior angles: c-e, d-f (c + e = 180°)

- Two angles are Supplementary when they add up to 180 degrees: a-b, b-d, d-c, c-a, e-f, f-h, h-g, g-e (Notice that together they make a straight angle. These two (60° and 120°) are supplementary because 60° + 120° = 180°)
- Two angles are Complementary when they add up to 90 degrees (Notice that together they make a right angle. These two (27° and 63°) are complementary because 27° + 63° = 90°)
- +1: angles with perpendicular arms (equal)

4. Types of triangles

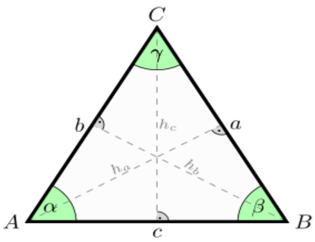
Triangles Contain 180°. In a triangle, the three interior angles always add to 180°: A + B + C = 180°

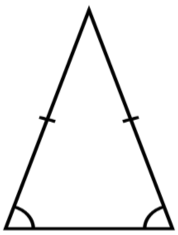
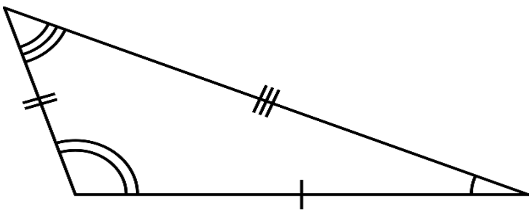
Classification by angles

- Obtuse: has one angle which is > 90°
- Acute: all the angles are < 90°
- Right: one angle is 90°

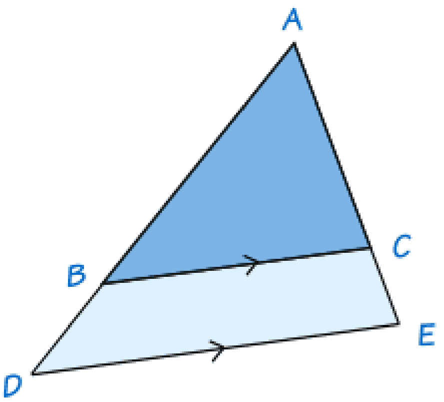
Classification by angles and sides

- Equilateral: equal sides, equal angles (left figure)
- Isosceles: two equal sides, two equal angles (center figure)
- Scalene: no equal sides, no equal angles (right figure)



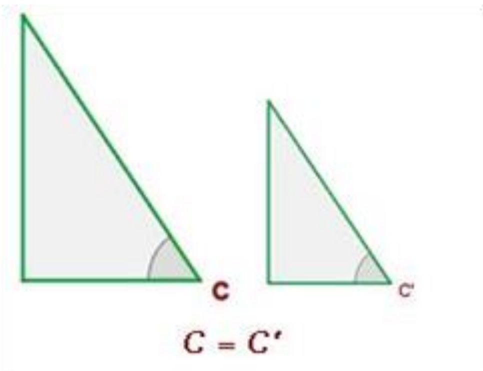
5. Similar and congruent triangles

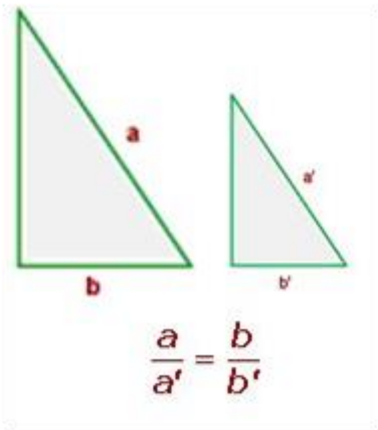


The two triangles has the same angles, the line length are in proportion.

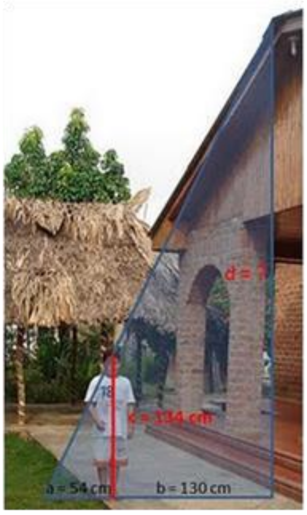
When two triangles are congruent they will have exactly the same three sides and exactly the same three angles.

If ADE is any triangle and BC is drawn parallel to DE, then $\frac{AB}{BD} = \frac{AC}{CE}$.





Problem: Calculate the height of Pepito's house considering the given distances and Pepito's height in the picture:

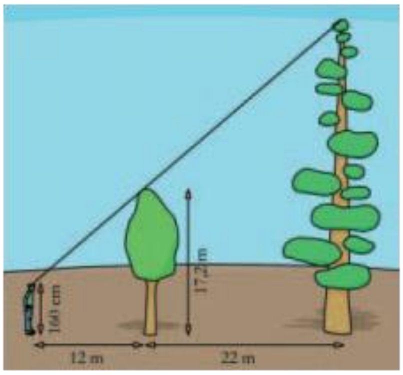


The triangles are similar because they share an acute angle

$$\frac{54}{184} = \frac{134}{d} \Rightarrow d = \frac{184 \cdot 134}{54} = 456{,}6$$

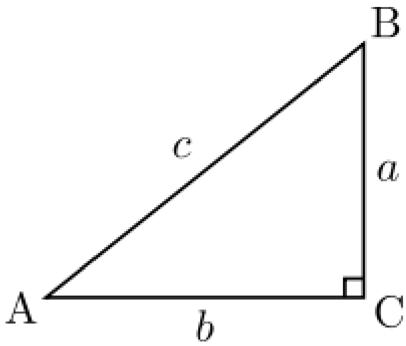
The height of the house is 4,57 m

Calculate the height of the tallest tree:



Solutions: 45.8 m

6. The right-angled triangle (right triangle)



In the right triangle *a,b* are called legs (or arms) of the triangle, c is the hypotenuse.

In a right angled triangle, the two non-right angles are complementary, because in a triangle the three angles add to 180°, and 90° has already been taken by the right angle.

The most important theorem concerning the right triangle is Pythagoras's theorem.

6.1. Pythagoras' theorem

The square of the hypotenuse is equal to the sum of squares of the other two sides.

$$c^2 = a^2 + b^2$$

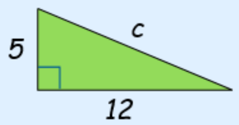
Pythagorean triples: 3, 4, 5. They solve the Pythagorean equation.

How can we tell if a triangle is a right triangle? Example: a = 28.2 m, b = 34.4 m, c = 44.5 m.

Practice (check the correct precision!)

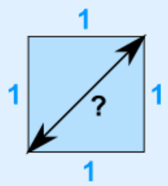
a) a=16.12 m b=12.34 m
b) a=16.120 m b=12.340 m
Tell correct length of c! (In mathematics: 20.3 m or 20.3009852 m is also a good answer, in geodesy, the correct answer for a) is 20.30 m, for b) is 20.301 m.

Example: Solve this triangle



Start with:	$a^2 + b^2 = c^2$
Put in what we know:	$5^2 + 12^2 = c^2$
Calculate squares:	$25 + 144 = c^2$
25+144=169:	$169 = c^2$
Swap sides:	$c^2 = 169$
Square root of both sides:	$c = \sqrt{169}$
Calculate:	$\mathbf{c = 13}$

Example: What is the diagonal distance across a square of size 1?



Start with:	$a^2 + b^2 = c^2$
Put in what we know:	$1^2 + 1^2 = c^2$
Calculate squares:	$1 + 1 = c^2$
1+1=2:	$2 = c^2$
Swap sides:	$c^2 = 2$
Square root of both sides:	$\mathbf{c = \sqrt{2}}$
Which is about:	$\mathbf{c = 1.4142...}$

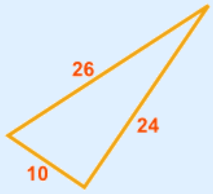
Example: Does an 8, 15, 16 triangle have a Right Angle?

Does $8^2 + 15^2 = 16^2$?

- $8^2 + 15^2 = 64 + 225 =$ **289**,
- but $16^2 =$ **256**

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

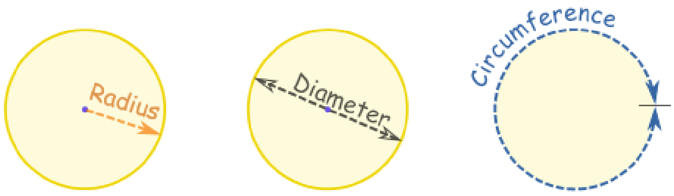
- $a^2 + b^2 = 10^2 + 24^2 = 100 + 576 =$ **676**
- $c^2 = 26^2 =$ **676**

They are equal, so ...

Yes, it does have a Right Angle!

7. Circle

All points are the same distance from the center. The **Radius** is the distance from the center outwards. The **Diameter** goes straight across the circle, through the center. The **Circumference** is the distance once around the circle.

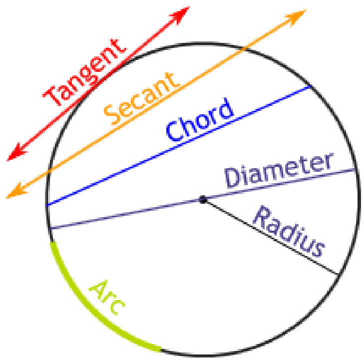


$$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159...$$

When we divide the circumference by the diameter we get 3.141592654... which is the number π (Pi).

The area of a circle is π times the radius squared, which is written: $A = \pi r^2$. The circumference is $2 \pi r$.

Special lines and parts of a circle



Lines:

A line that "just touches" the circle as it passes by is called a **Tangent**. It always forms a right angle with the circle's radius.

A line that cuts the circle at two points is called a **Secant**.

A line segment that goes from one point to another on the circle's circumference is called a **Chord**.

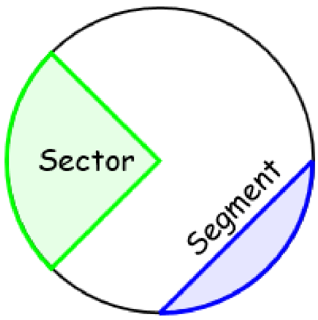
If it passes through the center it is called a **Diameter**.

And a part of the circumference is called an **Arc**.

Slices:

The "pizza" slice is called a **Sector**.

And the slice made by a chord is called a **Segment**.

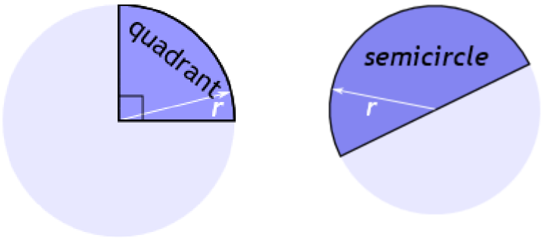


Common Sectors:

The Quadrant and Semicircle are two special types of Sector:

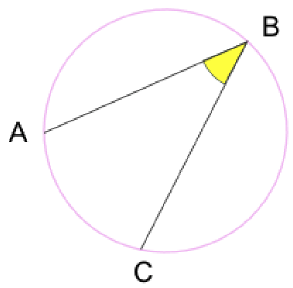
Quarter of a circle is called a Quadrant.

Half a circle is called a Semicircle.



8. Circle Theorems

Inscribed Angle



Inscribed Angle: an angle made from points sitting on the circle's circumference. A and C are "end points" B is the "apex point"

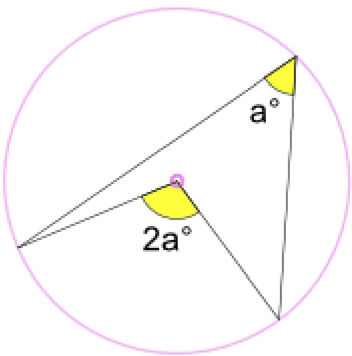
Equivalently, an inscribed angle is defined by two chords of the circle sharing an endpoint.

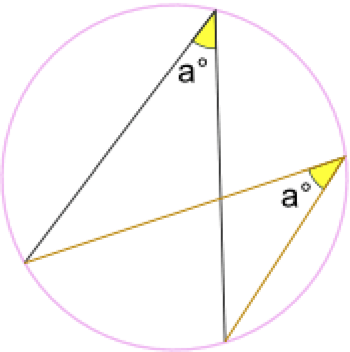
The inscribed angle theorem relates the measure of an inscribed angle to that of the central angle subtending the same arc.

Inscribed Angle Theorems

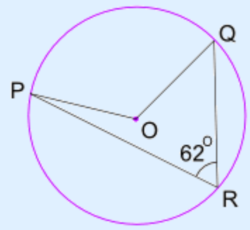
The inscribed angle theorem (or Angle at the Center Theorem) states that an angle θ inscribed in a circle is half of the central angle 2θ that subtends the same arc on the circle. Therefore, the angle does not change as its vertex is moved to different positions on the circle.

Angle at the Center Theorem: An inscribed angle a° is half of the central angle 2a° and (keeping the endpoints fixed) the angle a° is always the same, no matter where it is on the circumference:



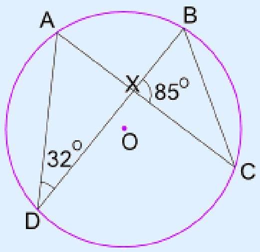


Example: What is the size of Angle POQ? (O is circle's center)



Angle POQ = 2 × Angle PRQ = 2 × 62° = **124°**

Example: What is the size of Angle CBX?



Angle ADB = 32° also equals Angle ACB.

And Angle ACB also equals Angle XCB.

So in triangle BXC we know Angle BXC = 85°, and Angle XCB = 32°

Now use angles of a triangle add to 180° :

Angle CBX + Angle BXC + Angle XCB = 180°

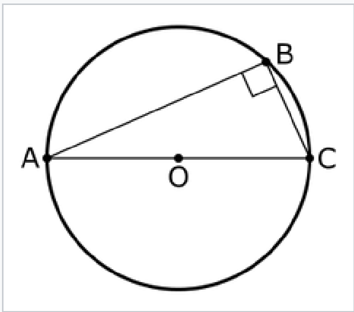
Angle CBX + 85° + 32° = 180°

Angle CBX = 63°

Thales' Theorem (or Angle in a Semicircle)

The diameter of a circle always subtends a right angle to any point on the circle. (An angle **inscribed** in a **semicircle** is always a right angle.)

Put another way: If a triangle has, as one side, the diameter of a circle, and the third vertex of the triangle is any point on the circumference of the circle, then the triangle will always be a right triangle.

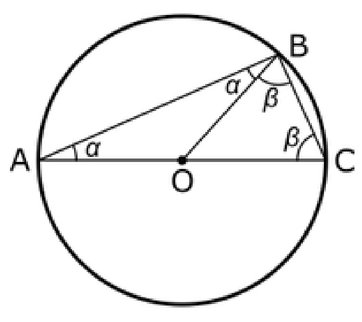


Thales's theorem: if $\overline{AC}$ is a diameter, then the angle at B is a right angle.

Proof

Since OA = OB = OC, ΔOBA and ΔOBC are isosceles triangles, and by the equality of the base angles of an isosceles triangle, ∠OBC = ∠OCB and ∠BAO = ∠ABO.

Let α = ∠BAO and β = ∠OBC. The three internal angles of the ΔABC triangle are α, (α + β), and β. Since the sum of the angles of a triangle is equal to 180°, we have



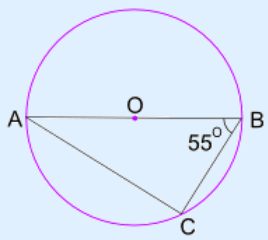
$\alpha + (\alpha + \beta) + \beta = 180°$

$2\alpha + 2\beta = 180°$

$2(\alpha + \beta) = 180°$

$\alpha + \beta = 90°$

Example: What is the size of Angle BAC?



The Angle in the Semicircle Theorem tells us that Angle ACB = 90°

Now use angles of a triangle add to 180° to find Angle BAC:

Angle BAC + 55° + 90° = 180°

Angle BAC = 35°