The horizontal coordinate systems

1. Introduction

In surveying calculations, most of the time when we talk about an object's or a point's position, we mean its two or three dimensional coordinates in some coordinate or reference system. There are numerous reference systems used throughout the world, with most countries or regions having their own. The most notable three dimensional reference system is the World Geodetic System 1984 (WGS84), which serves as the reference frame for the Global Positioning System (GPS).

Whenever we carry out surveying work in a region, we have to be aware of the specifics of the reference system used in that particular territory, especially if our work includes dealing with legalities as well.

In this lesson we cover the basics of horizontal (two dimensional) coordinate systems, the conversion between the different coordinates.

2. The Cartesian coordinate system: rectangular and polar coordinates

Typically used in mathematics, the Cartesian coordinate system is a coordinate system that uses a pair of numbers (a coordinate pair) to represent any point on the plane. The two values used to describe the position of a point are the *x* and the *y* coordinates (see Figure 1). Each of these values are measured from the origin, the (0, 0) point, along the x-axis and the y-axis respectively. If we have a point P with the coordinates (5, 2), it means that we have to go to 5 steps on the x-axis and 2 steps on the y-axis to get to point P. These are also called the point's rectangular coordinates.

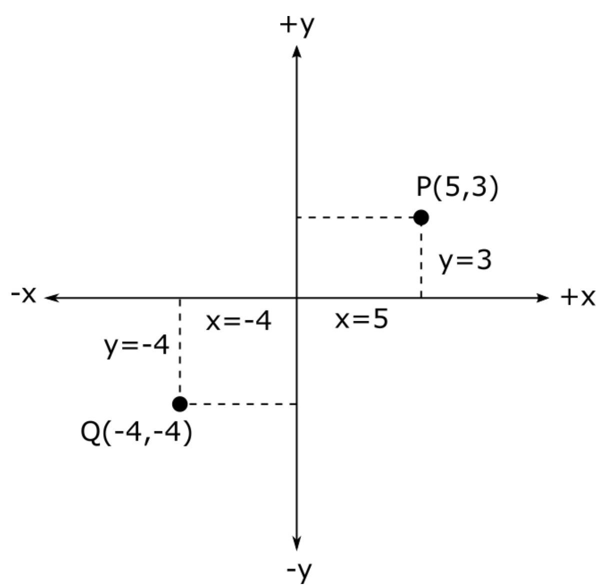

Figure 1. The Cartesian coordinate system with rectangular coordinates.

Another way to specify a point's position in the Cartesian coordinate system is the use of the so called polar coordinates. In this case, we describe each point using two quantities, the horizontal distance of the point measured from the origin of the coordinate system and the angle between the +x-axis and the line connecting the origin and the point (see Figure 2). The first part of our coordinate pair for any point is the horizontal distance (usually denoted *r*), while the second part is the angle (denoted Θ), which is measured between +180°and –180° (or $+\pi$ and $-\pi$ in radians). If the value of Θ is positive, it means that the angle is measured from the +x-axis counterclockwise, while if it's negative, the angle is measured clockwise from the +x-axis (see Figure 2).

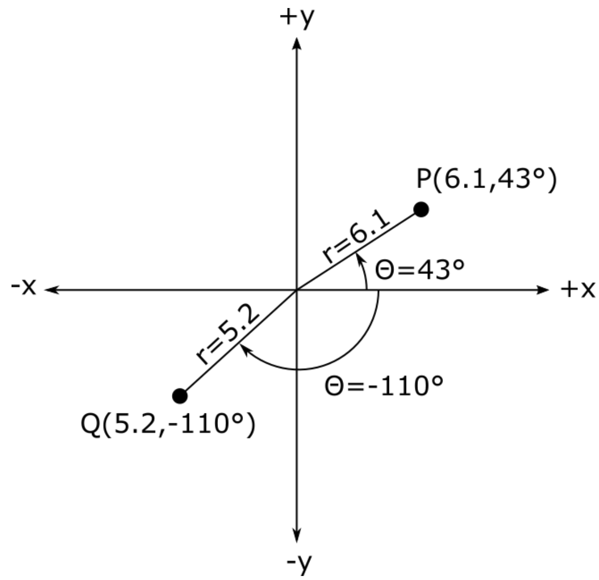

Figure 2. The Cartesian coordinate system with polar coordinates.

3. Conversion between the rectangular and the polar coordinates

3.1. Conversion from rectangular to polar

If we have a rectangular coordinate pair (for example, the coordinates of point P1 on Figure 3Figure 1), we can convert them to polar coordinates in the following way:

1. We calculate the point's distance from the origin:

$$r = \sqrt{x^2 + y^2} \ .$$

2. We calculate the angle between the +x-axis and the line connecting the point and the origin:

$$\Theta = \tan^{-1}\left(\frac{y}{x}\right) \ .$$

If we take a look at point P2 in Figure 3 and compute the Θ angle using the formula given above, we will not arrive at the correct angle. The reason for that is the range of the inverse tangent function which is between +90° and –90°. As the Θ we use is measured between +180° and –180°, we will get the wrong answer using the formula above whenever we have a Θ that is greater than +90° or lower than –90°. We can counter this problem in two ways. The first one is by checking the signs of the coordinates of the point and deciding in which quadrant our point is located prior to calculating the Θ value. We can use the following decision matrix:

Table 1. Decision matrix for calculating the value of Θ.

Signs		Quadrant	Θ
x	y		
+	+	1.	$\Theta = \tan^{-1}\left(\frac{y}{x}\right)$
–	+	2.	$\Theta = \tan^{-1}\left(\frac{y}{x}\right) + 180°$
–	–	3.	$\Theta = \tan^{-1}\left(\frac{y}{x}\right) - 180°$
+	–	4.	$\Theta = \tan^{-1}\left(\frac{y}{x}\right)$

The second way to counter the problem is to use the built in conversion function found in most scientific calculators. In order to convert from rectangular coordinates to polar ones, we have to look for the polar function on our calculator, which in most cases is denoted by "POL(" (on most Casio type calculators) or by "$\rightarrow r\Theta$" (on Sharp type calculators). After pressing the button, we first have to specify the x coordinate of the point, then type a comma (","), specify the y coordinate of the point, type a close bracket (")", this might not be necessary on certain types of calculators) and press the "=" button. If we use this conversion, we don't have to worry about the decision matrix mentioned above as the calculator will decide for us and give back the correct value of Θ. The calculator saves the values of r and Θ in two memory slots (typically X-Y, A-B or E-F) so we have to recall them from memory if we need to use them further (for example, to change the degree value into DMS).

Couple of examples:

- P(3,4) → r = 5, Θ = 53°07'48"
- Q(-5, -2) → r = 5.385 Θ = –158°11'55"

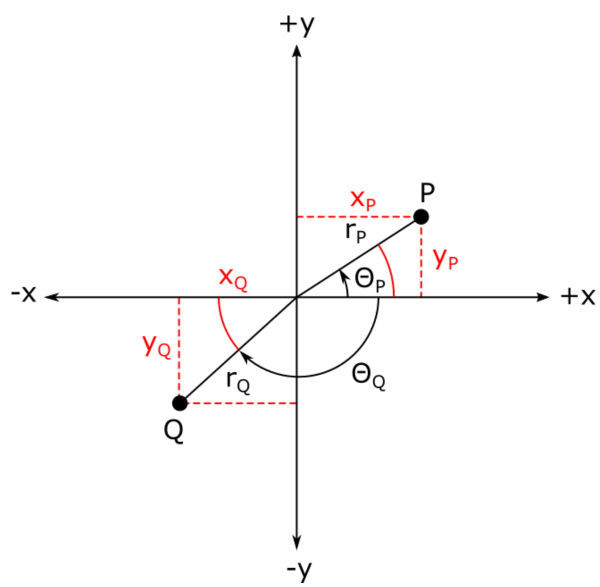

Figure 3. Conversion between rectangular and polar coordinates.

3.2. Conversion from polar to rectangular

Conversion in the opposite direction is a bit simpler. If we consult Figure 3 again, we can see the relationship between the two polar coordinates (r, Θ) and the two rectangular coordinates (x, y). To compute (x, y) from (r, Θ), we can use the following formulae:

$$x = r \cdot \cos(\Theta)$$
$$y = r \cdot \sin(\Theta)$$

Just like in section 3.1, we can also use the built-in function of the calculator to convert from the polar coordinates to the rectangular ones. In this case, we have to find the "REC(" or "$\rightarrow xy$" buttons, press them, then first specify the value of r, then – separated by a comma – the value of Θ. The calculator saves the value of the calculated x and y coordinates into the memory, from where we recall them.

Examples:

- R(6.2, 127°52'30") → x = –3.806, y = 4.894
- T(14.5, –36°42'10") → x = 11.625, y = –8.666