Basics of geometrical optics

1. Geometrical optics

1.1. Light rays and light waves

Light, the part of the electromagnetic spectrum that is visible for the human eye (between wavelengths of ~400 nm to ~700 nm), can be modelled in physics as three distinct phenomena: as light particles, as light waves and as light rays. The first modeling approach describes the energy properties of light and is of no interest to us now. In the second modelling approach, we use the so-called wave nature of light and imagine light waves similarly to regular water waves. The waves are represented by the wave front, which is a group of points that connect identical wave displacements, that is, identical points above or below the normal surface of the medium, the waves are travelling in.

Geometrically, a light ray is a line perpendicular to a series of successive wave fronts, specifying the direction of energy flow in the wave. In geometrical optics, simplifying light to light rays gives us much cleaner and easier to understand concepts while the representation still remains accurate enough for our purposes.

Thinking about light as light rays helps us illustrate effects like propagation, reflection and refraction, while using the wave nature of light, we may understand phenomena like interference and diffraction.

1.2. Basics of geometrical optics

There are a couple of basic simplifying assumptions in geometrical optics. These include that

- light rays propagate in a straight line while travelling in a homogeneous medium,
- light rays refract or are reflected at the interface (the surface that separates one optical medium from another) between two media that are not similar,
- light rays travel along a reversible path and
- there is no interference when two or more light rays meet.

1.3. Reflection of light from optical surfaces

When light hits an interface between two transparent optical media (such as between air and glass), four things can happen to the incident light:

- it can partly or totally be reflected at the surface,
- it can be scattered in random directions at the interface,
- it can be partly transmitted via refraction and enter the second medium or
- it can be partly or totally absorbed in either medium.

The law of reflections states that when light reflects from a plane surface, the angle that the reflected light ray makes with the normal of the surface at the point of incidence is always equal to the angle the incident ray makes with the same normal. The incident ray, the reflected ray and the normal always lie in the same plane.

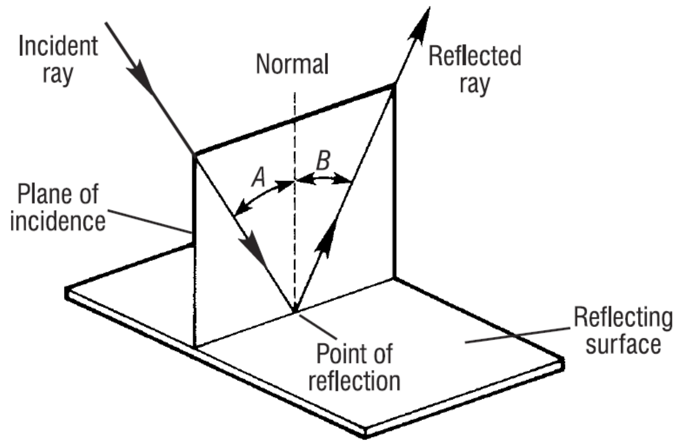


Figure 1. Law of reflection. Angles A and B are equal.

1.4. Refraction of light on optical interfaces

When light is incident at an interface, it will usually be party reflected and partly transmitted. The bending of light rays at an interface between two media is called refraction. Each optical medium has an index of refraction which can be calculated by the formula

$$n = \frac{c}{v}$$

where c is the speed of light in vacuum, v is the speed of light in the medium and n is the index of refraction of the medium. From the formula, we can see that the refraction index is always greater than 1 (vacuum is the "medium" where light travels the fastest) and the higher its value, the slower light travels in that particular medium. The following two figures show two distinct cases, one for light passing from a medium of lower index to higher index and one that is the opposite case. In the first case (lower to higher) the ray bends toward the normal and in the second case, the ray bends away from the normal

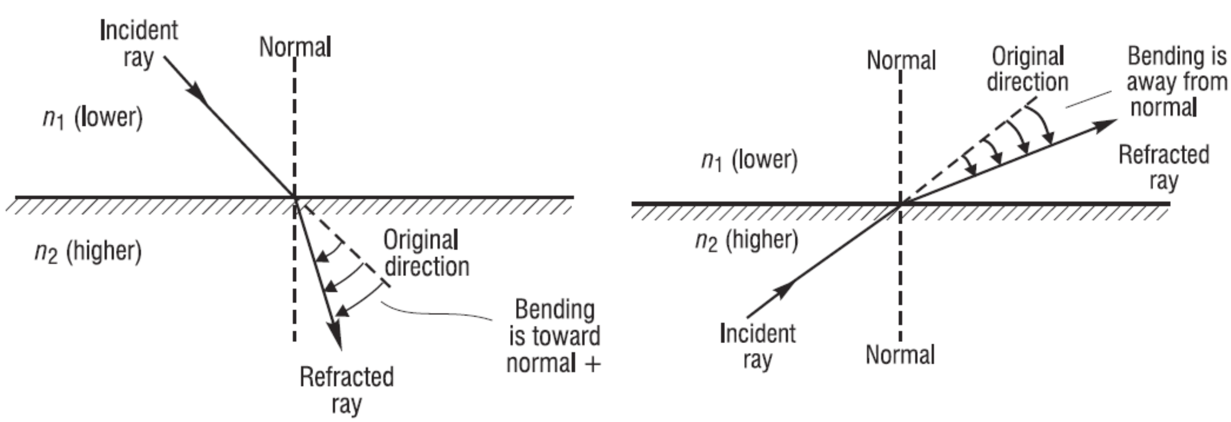


Figure 2. Refraction of light while passing from a medium with lower refraction index to a higher one (left) and the opposite case (right).

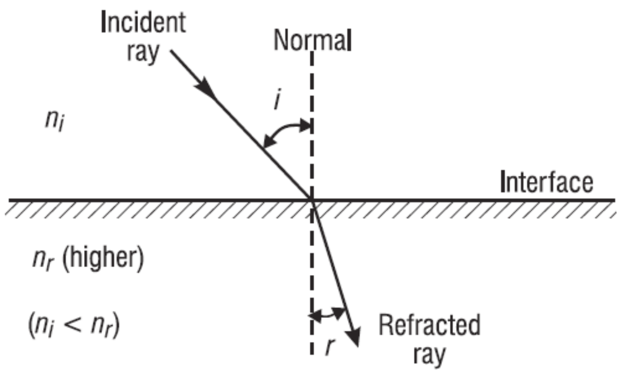
Snell's law of refraction relates the sines of the angles of incidence and refraction to the indices of refraction of the two media. The law lets us calculate the direction of the refracted ray if we know the refractive indices of the two media and the direction of the incident ray:

$$\frac{\sin(i)}{\sin(r)} = \frac{n_r}{n_i}$$

Or, in the form it is usually given in practice:

$$n_i \cdot \sin(i) = n_r \cdot \sin(r)$$

where n_i and n_r are the refractive indices of the incident and the refracting media, i is the angle of incidence and r is the angle of refraction.



Example 1:

In a handheld optical instrument used under water, light is incident from the water onto the plane surface of glass at an angle of incidence of 45°. The index of refraction of water is 1.33 and 1.63 for the glass.

(a) What is the angle of reflection of light off the glass?

The law of reflection states that the angle of reflection is the same as the angle of incidence on a plane surface. Therefore, the angle of reflection is 45°.

(b) Does the refracted ray bend toward or away from the normal?

The light ray travels from a medium with refractive index 1.33 (water) to a medium with refractive index 1.63 (glass). In other words, from a faster medium to a slower one. In such a case, the ray always bends toward the normal.

(c) What is the angle of refraction in the glass?

We can use Snell's law to find the angle of refraction:

$$\frac{\sin(i)}{\sin(r)} = \frac{n_r}{n_i} \rightarrow r = \arcsin\left(\frac{n_i}{n_r} \cdot sin(i)\right) = \arcsin\left(\frac{1.33}{1.63} \cdot sin(45°)\right) \approx 35.24°$$

1.5. Critical angle and total internal reflection

We may encounter some interesting results as light travels from a medium with a higher index of refraction to a medium with a lower one. Looking at the figure below, we see four rays of light originating from point O in the higher-index medium. Ray 1 is incident on the interface at 90°, which means that there is no bending. Ray 2 is incident at angle i and refracts at angle r. Ray 3 is incident at angle i_c, refracts at 90° away from the normal and therefore travels along the interface between the two media (in other words, the ray is trapped in the interface). Ray 4 is incident at an angle which is greater than i_c and is totally reflected back into the same medium from which it arrived at the interface. As ray 4 obeys the law of reflection, its angle of refraction is exactly the same as its angle of incidence.

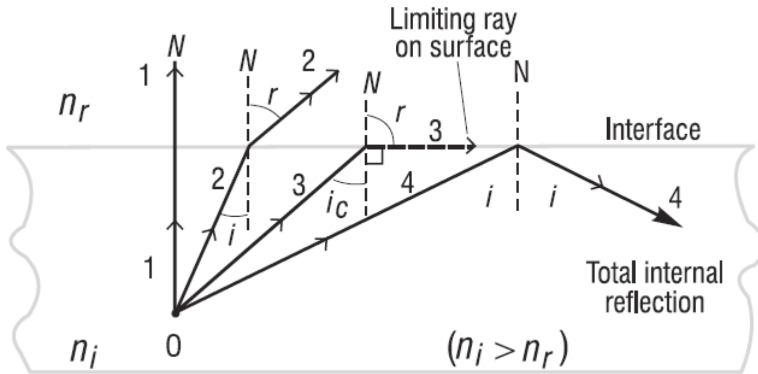


Figure 3. Rays travelling from a medium with high index of refractivity to a medium with a lower index.

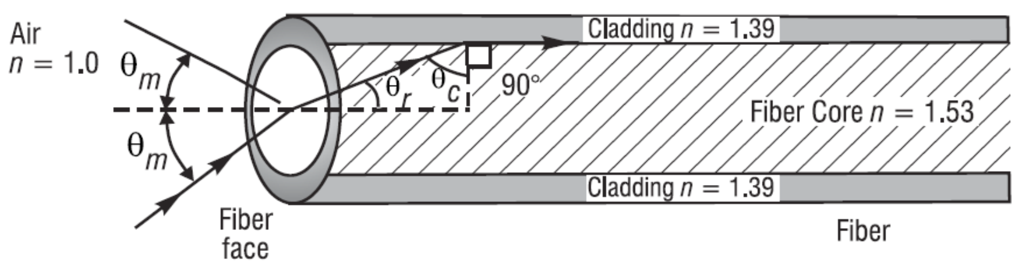
We can calculate this i_c critical incidence angle using Snell's law:

$$n_i \cdot \sin(i) = n_r \cdot \sin(90°)$$

$$i = \arcsin\left(\frac{n_r}{n_i}\right)$$

Example 2:

A fiber with a diameter of 6 mm has a core with refractivity index of 1.53 and a cladding with an index of 1.39. What is the maximum acceptance angle Θ_m for a cone of light rays incident on the face of the fiber such, that the refracted ray in the core of the fiber is incident on the cladding at the critical angle?



As our first step, we find the critical angle Θ_c at the interface between the core and the cladding:

$$\Theta_c = \arcsin\left(\frac{n_{\text{cladding}}}{n_{\text{core}}}\right) = \arcsin\left(\frac{1.39}{1.53}\right) \approx 65.3°$$

Using the right triangle on the figure, we can compute the refraction angle Θ_r at the fiber face:

$$\Theta_r = 90° - \Theta_c = 24.7°$$

Using Snell's law at this interface, we can calculate the Θ_m incidence angle:

$$\Theta_m = \arcsin\left(\frac{n_{\text{core}}}{n_{\text{air}}} \cdot sin(\Theta_r)\right) = \arcsin\left(\frac{1.53}{1} \cdot sin(24.7°)\right) \approx 39.7°$$

The maximum acceptance angle of the cone is twice the Θ_m angle: $2 \cdot \Theta_m = 79.4°$.