Plane surveying: fundamental tasks of plane surveying

1. Introduction

In plane surveying calculations, we can talk about a subset of tasks that are repeatedly carried out. These rudimentary computational steps concern the calculation of a point's coordinates using a known point (a control point) and the measured distance and whole circle bearing. The inverse of this computation, when we calculate the distance and the whole circle bearing using the coordinates of two control points is also essential. These two computations are therefore called the fundamental tasks of plane surveying.

Whenever we survey new points or compute the position of an object, we very rarely use absolute measurements, that is, measure the position or orientation of the point directly. In most cases, we use some sort of control network that consists of control points, points with known coordinates, and measure the relative position to these points. This is mostly due to the fact that direct measurements would be difficult or even impossible to carry out in many cases, but it can also have beneficial effects by reducing certain errors in the measurement.

2. First fundamental task of surveying

Our task is to find the coordinates of the unknown point P using control point A with known coordinates, the whole circle bearing WCB_{AP} and the distance d_{AP} measured at the control point (see Figure 1). Control points on the following figures are denoted by a double circle, while unknown points are denoted by a single circle.

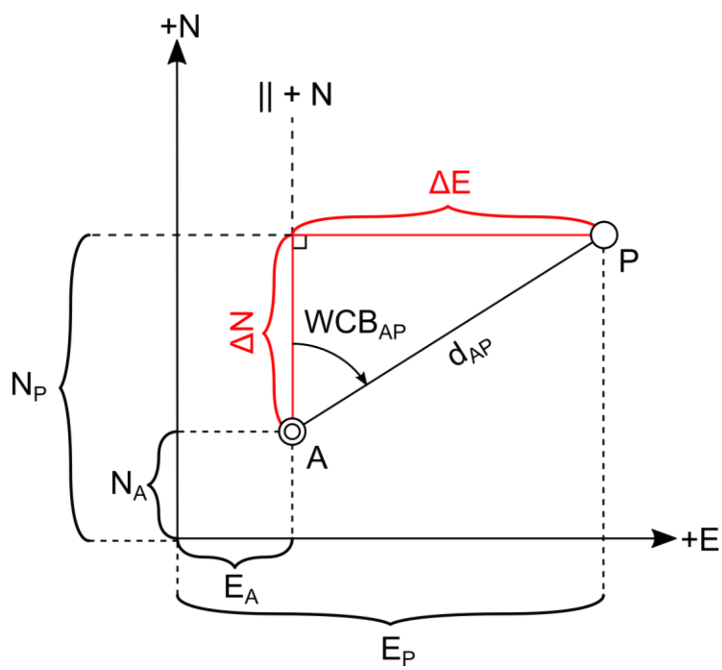

Figure 1. The first fundamental task of surveying.

We can calculate the coordinate difference between the control point and the unknown point using the right triangle in the figure:

$$\Delta E_{AP} = d_{AP} \cdot \sin(\text{WCB}_{AP})$$

$$\Delta N_{AP} = d_{AP} \cdot \cos(\text{WCB}_{AP})$$

Now, we just simply have to add the coordinate differences to the coordinates of the control point to arrive at the coordinates of point P:

$$E_P = E_A + \Delta E_{AP}$$

$$N_P = N_A + \Delta N_{AP}$$

Examples:

1. Calculate the coordinates of point B using the following data:

   $E_A = 1045.634$ m

   $N_A = 2931.587$ m

   $\text{WCB}_{AB} = 156°35'14"$

   $d_{AB} = 699.321$ m

   The coordinate differences:

   $\Delta E_{AB} = 699.321 \cdot \sin(156°35'14") = 277.877$ m

   $\Delta N_{AB} = 699.321 \cdot \cos(156°35'14") = -641.743$ m

   The coordinates of point B:

   $E_B = 1045.634 + 277.877 = 1323.511$ m

   $N_B = 2931.587 + (-641.743) = 2289.844$ m

2. Calculate the coordinates of point C using the following data:

   $E_A = -6773.596$ m

   $N_A = -4826.167$ m

   $\text{WCB}_{AC} = 299°10'58"$

   $d_{AC} = 1556.148$ m

   The coordinate differences:

   $\Delta E_{AC} = 1556.148 \cdot \sin(299°10'58") = -1358.624$ m

   $\Delta N_{AC} = 1556.148 \cdot \cos(299°10'58") = 758.773$ m

   The coordinates of point B:

   $E_C = -6773.596 + (-1358.624) = -8132.220$ m

   $N_C = -4826.167 + 758.773 = -4067.394$ m

3. Second fundamental task of surveying

The second fundamental task is considered the inverse of the first task. Using the coordinates of two control points, our task is to compute the distance between the two tasks and the whole circle bearing of the direction between them.

Using Figure 2, we can derive the formula for the computation. We first have to calculate the coordinate differences between the two points. The order of the points in the formula is essential as the signs of the coordinate differences directly affect the value of the whole circle bearing. The rule is that if we are computing the whole circle bearing of the direction from point A to point B, then we subtract the coordinates of point A from point B (starting point from end point):

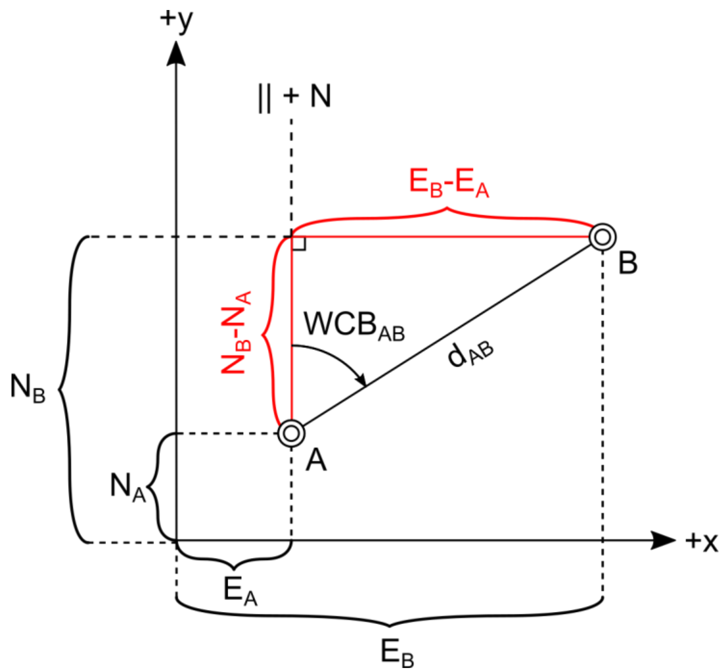

Figure 2. The second fundamental task of surveying.

$$\Delta E_{AB} = E_B - E_A$$

$$\Delta N_{AB} = N_B - N_A$$

Next, we can compute the distance between the two points using the Pythagoras' theorem:

$$d_{AB} = \sqrt{\Delta E_{AB}^2 + \Delta N_{AB}^2}$$

Finally, we compute the whole circle bearing going from point A to point B. As the range of the inverse tangent function is between –90° and +90°, we have use a two-step approach. We first calculate the value of an auxiliary angle:

$$\alpha = \tan^{-1}\left(\frac{\Delta E_{AB}}{\Delta N_{AB}}\right)$$

We then use a decision matrix that gives us the correct formula for the WCB depending on the sign of the coordinate differences:

Table 1. Decision matrix for calculating the WCB.

Signs		Quadrant	Θ
E	N		
+	+	1.	$\mathrm{WCB} = \alpha$
+	–	2.	$\mathrm{WCB} = \alpha + 180°$
–	–	3.	$\mathrm{WCB} = \alpha + 180°$
–	+	4.	$\mathrm{WCB} = \alpha + 360°$

We may of course utilize the calculator's polar function to compute the WCB and the distance all at once. In the most common calculator types, either the $POL($ or the $\rightarrow r\Theta$ button have to be used. In the case of geodetic coordinates, we first have to specify the northing (N) then the easting (E) coordinate differences. If the polar function returns a negative value for the WCB, we have to add 360° to it.

Examples:

1. Calculate the distance and the whole circle bearing between from point A to point B using the following data:

Point ID	Easting [m]	Northing [m]
A	648.634	798.491
B	–489.169	–621.498

The coordinate differences:

$$\Delta E_{AB} = -489.169 - 648.634 = -1137.803 \text{ m}$$
$$\Delta N_{AB} = -621.498 - 798.491 = -1419.989 \text{ m}$$

The distance between the two points:

$$d_{AB} = \sqrt{(-1137.803)^2 + (-1419.989)^2} = 1819.606 \text{ m}$$

Looking at the signs of the coordinate differences, we can see that the WCB is in 3rd quadrant, therefore:

$$\mathrm{WCB}_{AB} = \tan^{-1}\left(\frac{-1137.803}{-1419.989}\right) + 180° = 218°42'16''$$

Using the calculator's polar function, we get the same results:

$$POL(-1419.989, -1137.803) \rightarrow r = 1819.606, \Theta = -141°17'44''(+360°) = 218°42'16''$$

2. Calculate the distance and the whole circle bearing between from point C to point D using the following data:

Point ID	Easting [m]	Northing [m]
C	4169.563	–1563.561
D	–3597.491	6189.135

The coordinate differences:

$$\Delta E_{CD} = -3597.491 - 4169.563 = -7767.054\text{ m}$$
$$\Delta N_{CD} = 6189.135 - (-1563.561) = 7752.696\text{ m}$$

The distance between the two points:

$$d_{AB} = \sqrt{(-7767.054)^2 + (7752.696)^2} = 10974.125\text{ m}$$

Looking at the signs of the coordinate differences, we can see that the WCB is in 3rd quadrant, therefore:

$$\text{WCB}_{AB} = \tan^{-1}\left(\frac{-7767.054}{7752.696}\right) + 360° = 314°56'49"$$

Using the calculator's polar function, we get the same results:

$$POL(7752.696\,, -7767.054) \rightarrow r = 10974.125, \Theta = -45°03'11"(+360°) = 314°56'49"$$

4. Whole circle bearing of a reverse direction

Let us assume that we know the WCB of the direction AB and we want to find the WCB of the reverse direction BA. According to Figure 3, we have change the already known WCB by 180°:

$$\text{WCB}_{BA} = \text{WCB}_{AB} \pm 180°$$

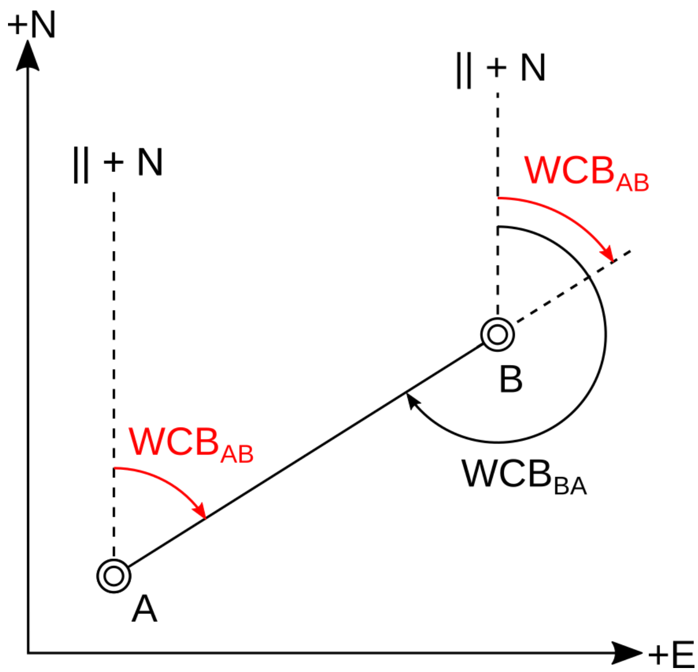

Figure 3. Whole circle bearing of a reverse direction.