Mirrors and lenses

1. Plane mirrors

A plane mirror is a surface that can reflect a ray of light in one direction instead of scattering it or absorbing it. It may be a shiny metal surface or some other sort of reflective material. The figure below shows a point light source O (called the object) at a perpendicular distance p from the plane of the mirror. If we extend the rays of light that are incident on the mirror backward (behind the mirror), we find that the extensions intersect at a point that is a perpendicular distance i from the mirror.

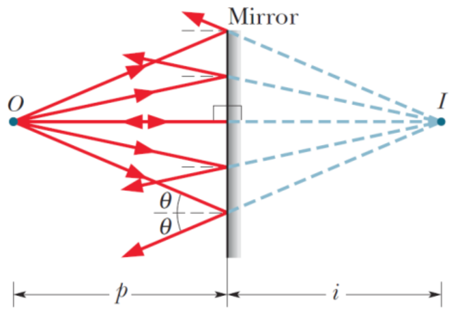

Figure 1. Point source O and its image I.

If our eyes intercept parts of the light that is reflected, we perceive a point source of light that is behind the mirror and located at the intersection of the extensions of the light rays. This perceived point source is the image I of object O. In this case, it is called a point image and it is virtual as light rays do not actually pass through it (only their extensions).

1.1. Ray tracing with plane mirrors

An intuitive way to find the image I of an object O that is in from of the mirror is the so-called ray tracing method. We can choose two arbitrary rays, find the corresponding reflected rays and look for the intersection of the extensions of these reflected beams. The figure below shows an example for this. One of the light rays that comes from O is perpendicular to the surface of the mirror and reaches the mirror at point b. A perpendicular ray is reflected back onto it itself so its extension lies is the same straight line behind the mirror. Another ray reaches that goes in an arbitrary direction reaches the mirror at point a, under an incidence angle of Θ. Its reflection will have the same Θ angle measured from the normal of the surface. The reflected ray can be extended and where it intersects the other extension, we have our image I of object O.

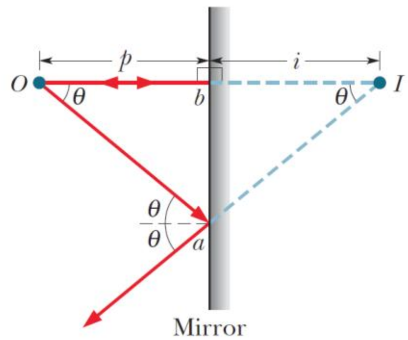

Figure 2. Ray tracing with a plane mirror.

The right triangles on the figure are congruent (similar and equal in size), therefore their horizontal sides have the same length, that is:

$$\overline{Ib} = \overline{Ob}\,.$$

By convention, object distances are taken to be positive and image distances for virtual images are negative. So the above equation can written as

$$p = -i$$

The image of extended objects can be derived in the same way as for a point object, but now each small – point-like – portion of the extended object acts like the point source O in the above example.

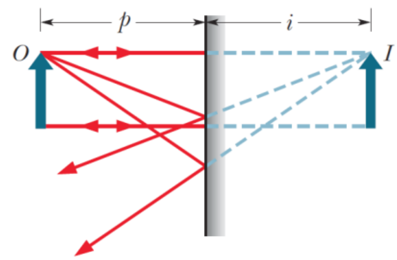

Figure 3. Image of extended object from ray tracing.

2. Spherical mirrors

Spherical mirrors are in the shape of small section of the surface of a sphere. The plane mirror can be considered a special case of the spherical mirror, where the radius of curvature is infinity.

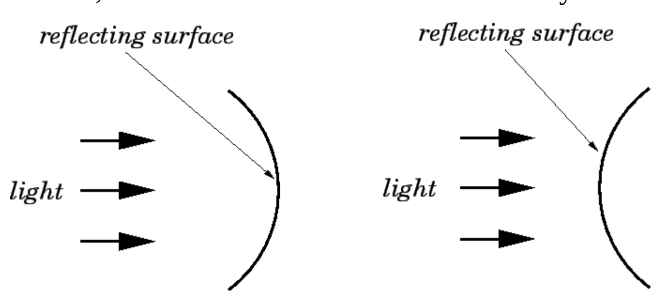

Figure 4. *A concave (left) and a convex (right) mirror*

A concave mirror can be seen on the figure above (left), concave meaning in this case that the surface is “caved in” from the object side, a convex mirror (right) is the opposite of the concave one, its surface is “flexed out” towards the object.

In Figure 5. we can see the basic concepts of image formation. The normal to the center of the mirror is called the *principal axis*. The mirror is *rotationally symmetric* about this axis. The point V where principal axis meets the mirror is called the *vertex*. Point C is equidistant from all points on the reflecting surface, it is called the *centre of curvature* or the *center of the mirror*. The distance between C and V is called the *radius of curvature* and is denoted R. Light-rays striking a concave mirror parallel to its principal axis, and not too far away from this axis, are reflected by the mirror through the same point F, the *focal point*, or *focus*. on the principal axis. The distance along the principal axis from the focus to the vertex is called the *focal length* of the mirror, and is denoted f.

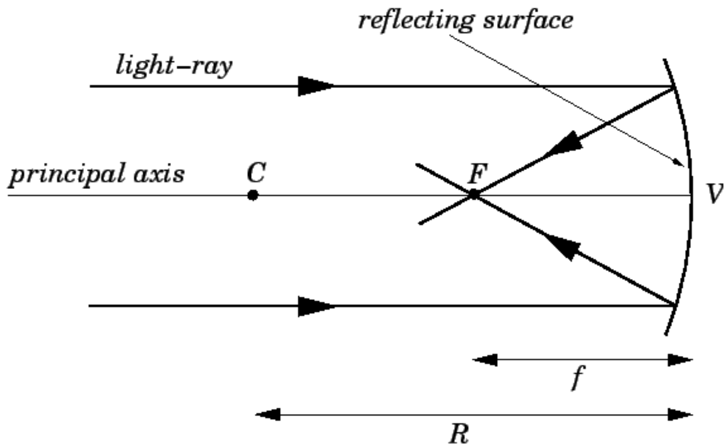

Figure 5. *Image formation by a concave mirror.*

(more details: https://farside.ph.utexas.edu/teaching/316/lectures/node136.html)

2.1. Graphical Image Formation by Concave Mirrors

There are two alternative methods of locating the image formed by a concave mirror. The first is graphical, the second is analytical. The graphical method of locating the image produced by a concave mirror consists of ray tracing using four simple rules (see Figure 6.):

1) A ray that is initially parallel to the central axis will pass through the focus of the mirror after reflection.
2) A ray that passes through the focus will be parallel to the principal axis after reflection.
3) .A ray which passes through the centre of curvature C of the mirror is reflected back along its own path (since it is perpendicular to the mirror).
4) A ray that is incident on mirror in the vertex point V (where the principal axis intersects the surface of the mirror, see below) is reflected symmetrically about the central axis.

The intersection of any two ray will define the image of the corresponding point of the object.

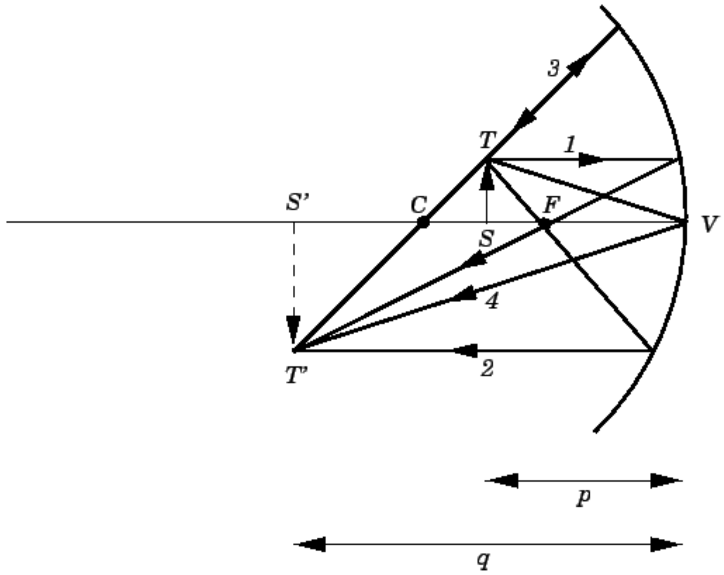

Figure 6. *Formation of a real image by a concave mirror.*

Consider an object *ST* which is placed a distance *p* from a concave spherical mirror, as shown in Fig. 5. Let us suppose that the object distance *p* is greater than the focal length *f* of the mirror. Consider four light-rays emanating from the tip *T* of the object which strike the mirror, as shown in the figure. The reflected rays are constructed using rules 1-4 above, and the rays are labelled accordingly. It can be seen that the reflected rays all come together at some point *T′*. Thus, *T′* is the image of T. A complete image of the object is produced between *S′* and *T′*. This image is termed a real image. According to the figure, the image is inverted with respect to the object, and is also magnified.

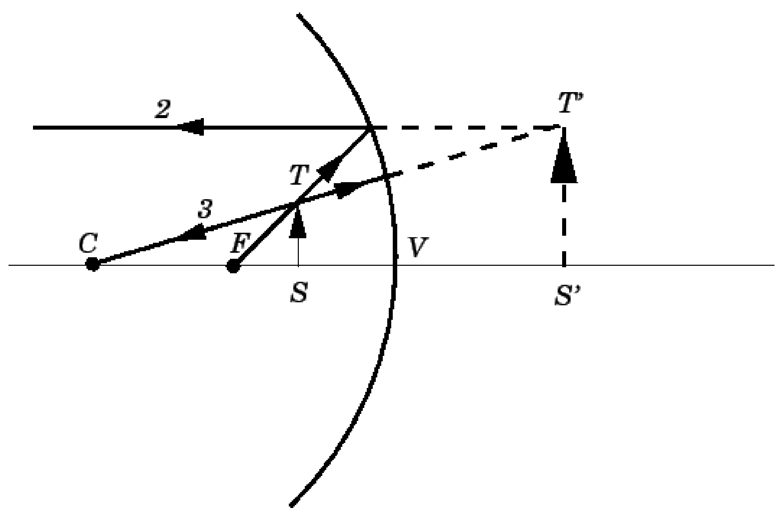

Figure 7. *Formation of a virtual image by a concave mirror*

Figure 7. shows what happens when the object distance *p* is less than the focal length *f*. In this case, the image appears to an observer looking straight at the mirror to be located *behind* the mirror. For instance, rays emanating from the tip *T* of the object appear, after reflection from the mirror, to come from a point *T'* which is behind the mirror. Note that only two rays are used to locate *T'*. *Two* is the minimum number of rays needed to locate a point image. Of course, the image behind the mirror cannot be viewed by projecting it onto a screen, because there are no real light-rays behind the mirror. This type of image is termed a *virtual image*. The characteristic difference between a real image and a virtual image is that, immediately after reflection from the mirror, light-rays emitted by the object *converge* on a real image, but *diverge* from a virtual image. According to Fig. 7., the image is *upright* with respect to the object, and is also *magnified*. (more details: https://farside.ph.utexas.edu/teaching/316/lectures/node137.html)

2.2. Analytical Image Formation by Concave Mirrors

The graphical method described above is fine for developing an intuitive understanding of image formation by concave mirrors, or for checking a calculation, but is a bit too cumbersome for everyday use. The analytic method described below is far more flexible.

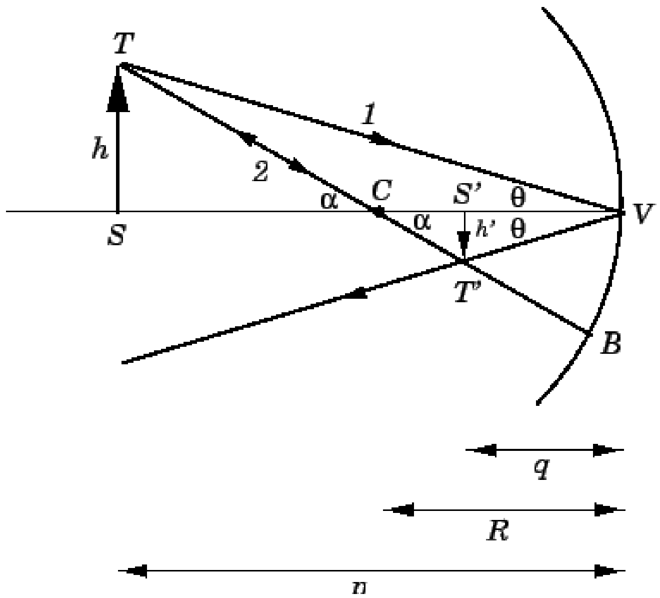

Figure 8. Image formation by a concave mirror

Consider an object *ST* placed at a distance *p* in front of a concave mirror of radius of curvature *R*. In order to find the image *S'T'* produced by the mirror, we draw two rays from *T* to the mirror (see Fig. 8.). The first, labelled *1*, travels from *T* to the vertex *V* and is reflected such that its angle of incidence θ equals its angle of reflection. The second ray, labelled *2*, passes through the center of curvature *C* of the mirror, strikes the mirror at point *B*, and is reflected back along its own path. The two rays meet at point *T'*. Thus, *S'T'* is the image of *ST*, since point *S'* must lie on the principal axis.

In the triangle *STV*, we have $\tan\theta = h/p$, and in the triangle *S'T'V* we have $\tan\theta = -h'/q$, where *p* is the object distance, and *q* is the image distance. Here, *h* is the height of the object, and *h'* is the height of the image. By convention, *h'* is a negative number, since the image is inverted (if the image were upright then *h'* would be a positive number). It follows that

$$\tan\theta = \frac{h}{p} = \frac{-h'}{q}. \quad (1)$$

Thus, the *magnification M* of the image with respect to the object is given by

$$M = \frac{-h'}{h} = \frac{q}{p} \quad (2)$$

By convention, *M* is negative if the image is inverted with respect to the object, and positive if the image is upright. It is clear that the magnification of the image is just determined by the ratio of the image and object distances from the vertex.

From triangles STC and $S'T'C$, we have $\tan\alpha = h/(p-R)$ and $\tan\alpha = -h'/(R-q)$, respectively. These expressions yield

$$\tan\alpha = \frac{h}{p-R} = -\frac{h'}{R-q}. \tag{3}$$

Equations (2) and (3) can be combined to give

$$\frac{-h'}{h} = \frac{R-q}{p-R} = \frac{q}{p}, \tag{4}$$

which easily reduces to

$$p\,R - p\,q = p\,q - q\,R$$

$$p\,R + q\,R = 2\,p\,q$$

$$p + q = \frac{2\,p\,q}{R}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \tag{5}$$

This expression relates the object distance, the image distance, and the radius of curvature of the mirror. For an object which is very far away from the mirror (*i.e.*, $p \to \infty$), so that light-rays from the object are parallel to the principal axis, we expect the image to form at the focal point F of the mirror. Thus, in this case, $q = f$, where f is the focal length of the mirror, and Eq. (5) reduces to

$$0 + \frac{1}{f} = \frac{2}{R} \tag{6}$$

The above expression yields

$$f = \frac{R}{2} \tag{7}$$

In other words, in the paraxial approximation, the focal length of a concave spherical mirror is *half* of its radius of curvature. Equations (5) and (7) can be combined to give

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{8}$$

The above expression was derived for the case of a real image. However, it also applies to virtual images provided that the following sign convention is adopted. For real images, which always form *in front* of the mirror, the image distance q is *positive*. For virtual images, which always form *behind* the mirror, the image distance q is *negative*. It immediately follows, from Eq. (2), that real images are always inverted, and virtual images are always upright. Table 1 shows how the location and character of the image formed in a concave spherical mirror depend on the location of the object, according to Eqs. (2) and (8). It is clear that the *modus operandi* of a shaving mirror, or a makeup mirror, is to place the object (*i.e.*, a face) between the mirror and the focus of the mirror. The image is upright, (apparently) located behind the mirror, and magnified.

Position of object	*Position of image*	*Character of image*
$p = \infty$	$q = f$	Real, zero size
$R < p < \infty$	$f < q < R$	Real, inverted, diminished
$p = R$	$q = R$	Real, inverted, same size
$f < p < R$	$R < q < \infty$	Real, inverted, magnified
$p = f$	$q = \infty$	
$p < f$	$-\infty < q < 0$	Virtual, upright, magnified
$p = 0$	$q = 0$	Virtual, upright, same size

Table 1: *Rules for image formation by concave mirrors.*

2.3. Image Formation by Convex Mirrors

The definitions of the principal axis, center of curvature *C*, radius of curvature *R*, and the vertex *V*, of a convex mirror are analogous to the corresponding definitions for a concave mirror. When parallel light-rays strike a convex mirror they are reflected such that they appear to emanate from a single point *F* located behind the mirror, as shown in Fig. 9. This point is called the virtual focus of the mirror. The focal length *f* of the mirror is simply the distance between *V* and *F*. As is easily demonstrated the focal length of a convex mirror is half of its radius of curvature.

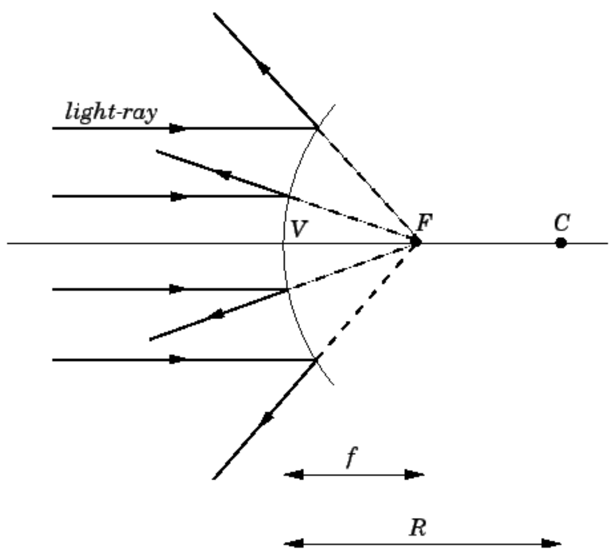

Figure 9 The virtual focus of a convex mirror.

There are, again, two alternative methods of locating the image formed by a convex mirror. The first is graphical, and the second analytical. According to the graphical method, the image produced by a convex mirror can always be located by drawing a ray diagram according to four simple rules:

1) An incident ray which is parallel to the principal axis is reflected as if it came from the virtual focus *F*.
2) An incident ray which is directed towards the virtual focus *F* is reflected parallel to the principal axis.
3) An incident ray which is directed towards the center of curvature *C* is reflected back along its own path (since it is normally incident on the mirror).
4) An incident ray which strikes the mirror at its vertex *V* is reflected such that its angle of incidence with respect to the principal axis is equal to its angle of reflection.

In the example shown in Fig. 10, two rays are used to locate the image *S'T'* of an object *ST* placed in front of the mirror. It can be seen that the image is virtual, upright, and diminished.

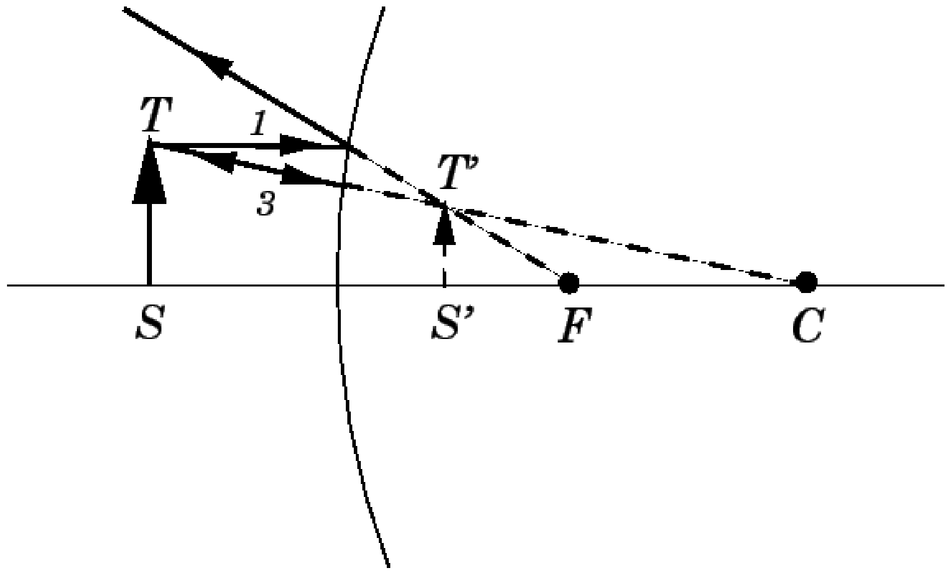

Figure 10. Image formation by a convex mirror

The focal length of a concave mirror, with a real focus, is always positive, and the focal length of a convex mirror, with a virtual focus, is always negative. Table 2 shows how the location and character of the image formed in a convex spherical mirror depend on the location of the object, according to Eqs. (2) and (8) (with $f<0$).

Position of object	*Position of image*	*Character of image*
$p = \infty$	$q = f$	Virtual, zero size
$0 < p < \infty$	$0 < q < f$	Virtual, upright, diminished
$p = 0$	$q = 0$	Virtual, upright, same size

Table 2: *Rules for image formation by convex mirrors.*

In summary, the formation of an image by a spherical mirror involves the crossing of light-rays emitted by the object and reflected off the mirror. If the light-rays actually cross in front of the mirror then the image is real. If the light-rays do not actually cross, but appear to cross when projected backwards behind the mirror, then the image is virtual. A real image can be projected onto a screen, a virtual image cannot. However, both types of images can be seen by an observer, and photographed by a camera. The magnification of the image is specified by Eq. (2), and the location of the image is determined by Eq. (8). These two formulae can be used to characterize both real and virtual images formed by either concave or convex mirrors, provided that the following sign conventions are observed:

1. The height h' of the image is positive if the image is upright, with respect to the object, and negative if the image is inverted.
2. The magnification M of the image is positive if the image is upright, with respect to the object, and negative if the image is inverted.
3. The image distance q is positive if the image is real, and, therefore, located in front of the mirror, and negative if the image is virtual, and, therefore, located behind the mirror.
4. The focal length f of the mirror is positive if the mirror is concave, so that the focal point F is located in front of the mirror, and negative if the mirror is convex, so that the focal point F is located behind the mirror.

Note that the front side of the mirror is defined to be the side from which the light is incident.

(more details: https://farside.ph.utexas.edu/teaching/316/lectures/node138.html)

Example 1:

An object of height h is in front of a spherical mirror with a focal length of $|f| = 40$ cm. The image produced by the mirror has the same orientation and its height is $h' = 0.20h$. (a) Is the image real or virtual and is it on the same side of the opposite side as the object? (b) Is the mirror concave or convex and what is the sign of the focal length?

(a) If a concave mirror produces an image which has the same orientation as the object, it is always virtual. All of these virtual images are located behind the mirror. The convex mirror can't produce real images as the light rays diverge from the mirror, therefore all of its images are virtual and behind the mirror. Using these facts, we can deduce that our image can only be virtual and behind the mirror.

(b) We can use the two equations for the lateral magnification and the equation of the spherical mirrors to find the sign of the focal point.

From

$$|M| = \frac{h'}{h} = 0.20$$

we get that the absolute value of the magnification is 0.20. As our image and object have the same orientation, the sign of m is positive. Substituting this into the other equation for calculating the magnification, we get:

$$m = -\frac{q}{p} \rightarrow +0.20 = -\frac{q}{p} \rightarrow q = -0.20p\ .$$

Now we can use the main equation of spherical mirrors to find the sign of f:

$$\frac{1}{f} = \frac{1}{q} + \frac{1}{p} = \frac{1}{-0.20p} + \frac{1}{p} = \frac{1}{p} \cdot (-5 + 1) \rightarrow f = -\frac{p}{4}.$$

As the value of p is positive, f must be negative with $f = -40$ cm.