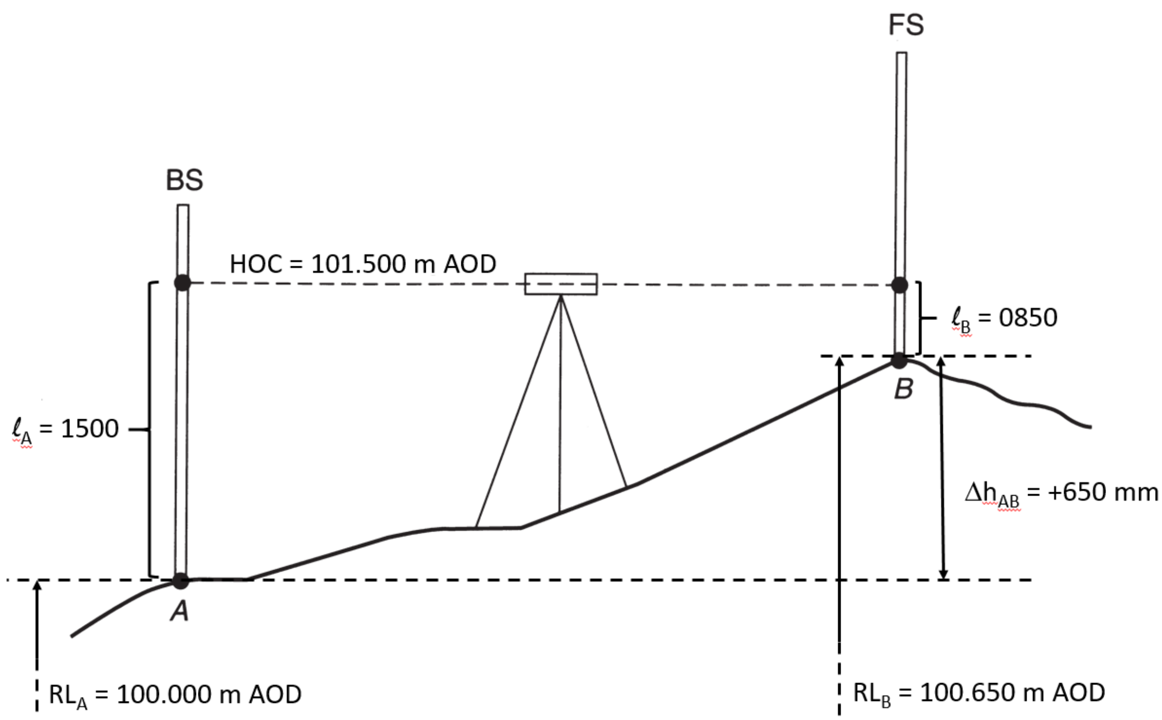
Height determination and mapping with the surveyor's level

1. Levelling in practice, determining the height difference between two points

The levelling instrument (called the level) is set up between two vertical levelling staves (A and B) at an equal distance from each staff and correctly levelled. Levelling the instrument means, that we use the circular and the levelling bubbles (or only the circular bubble in case of an automatic level) to make the line of sight of the instrument horizontal and therefore create a horizontal plane at the height of the instrument. If the direction of the levelling is from A to B, then point A is called the backsight point (BS) and B is called the foresight point (FS).



Let the reduced level (RL, the height of the point above a reference surface) of point A be 100.000 m AOD (above ordnance datum – a reference surface).

We start with taking a reading on the levelling staff on the BS point (λ_A) using the level. The readings are made in mm units and we always note down four digits. Let this reading be 1500 (meaning 1500 mm or 1.500 m). Then, we take a reading on the levelling staff on the FS point (λ_B). Let this reading be 0850 (meaning 850 mm or 0.850 m).

Adding the BS reading to the RL of point A gives us the height of collimation (HOC) or instrument height, which is the height of the reference plane set out by the instrument AOD. In this particular case, the HOC is 100.000 m + 1.500 m = 101.500 m AOD.

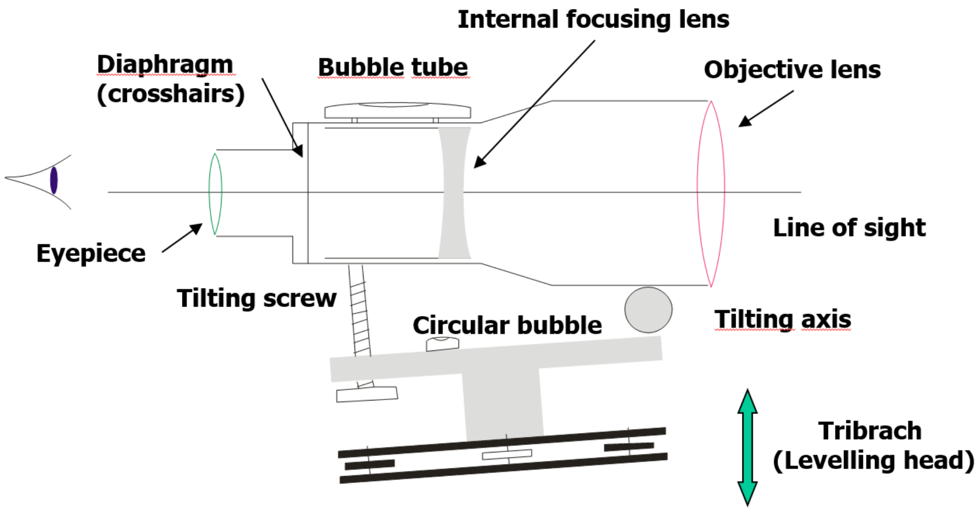
The height difference between the backsight point and the foresight point (points A and B respectively in this case) is always given by taking the BS reading and subtracting from it the FS reading: Δh_{AB} = 1500 – 0850 = +650 mm. If the difference is positive, it is called a rise, otherwise it's a fall. (We can visualize this calculation by imagining that we are first at point A, then we use the BS reading and add it to the RL of the point to rise up to the HOC. When we are the level of the HOC, we can use the FS reading to go down to the level of point B.)

Adding the rise/fall value to the RL of point A gives us the RL of point B: 100.000 m AOD + 0.650 m = 100.650 m AOD. We may also compute this as taking the HOC and subtracting the FS reading from it: 101.500 m AOD – 0.850 m = 100.650 m AOD.

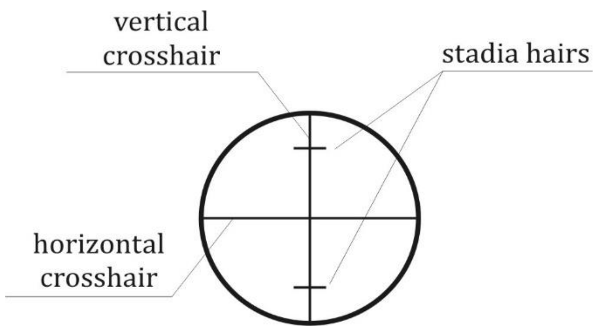
2. Levelling instruments (levels)

2.1. Tilting levels (or dumpy levels)

The tilting level consists of two main sections, the tribrach (or the levelling head) and the alidade, which is located above the tribrach. The tribrach connects the instrument to the tripod and it contains the foot screws that are used to approximately level the alidade. Mounted on the alidade, we have a surveyor's scope, which is fixed to the tribrach on one side with a tilting axis and on the other side with a levelling screw.



Source: Sz. Rózsa – Surveying I. lecture notes



The view inside the scope of the level

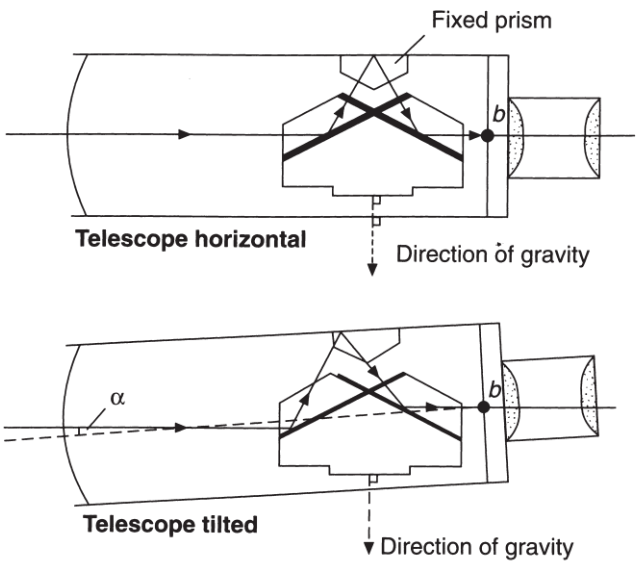
Fixed to the scope, we have the bubble tube, which is much more sensitive than the circular bubble on the alidade. The line of sight of the scope can be precisely levelled by first approximately levelling the alidade with the foot screws, then the using the levelling screw to adjust the bubble in the bubble tube. The bubble tube has to be adjusted before taking each reading, because when we turn the instrument from the BS point to the FS point, the line of sight diverges from the horizontal due to the tilting of the alidade (remember, we only set the alidade approximately horizontal with the circular bubble!).

The eyepiece can be adjusted to sharpen the image of the crosshairs inside the scope, while the parallax screw on the scope can be used to sharpen the image of the staff. The scope always creates a magnified image in order to make it possible to take accurate readings on the levelling staves. The horizontal crosshair inside the scope is used to take the readings, while the stadia lines can be used to calculate the distance between the instrument and the staff.

2.2. Automatic levels

Automatic levels are the most common instruments used in everyday surveying tasks concerning vertical control. An automatic level does not contain a bubble tube and a levelling screw. A compensator (a set of prisms) is built into the instrument which can compensate for moderate amounts of tilting. This means, that if we approximately level the instrument using the foot screws and the circular bubble on the alidade, the compensator can take care of the residual tilt due to the low sensitivity of the circular bubble.

Sokkia C320 automatic level



Source: W. Schofield, M. Breach: Engineering Surveying

The suspended prism is always forced into a vertical position due to the force of gravity. Combined with a fixed prism, this creates a compensation effect, so even when the telescope is tilted the line of sight remains horizontal. Naturally, the amount of tilt the compensator can handle is fixed, so the telescope has to be at least roughly horizontal.

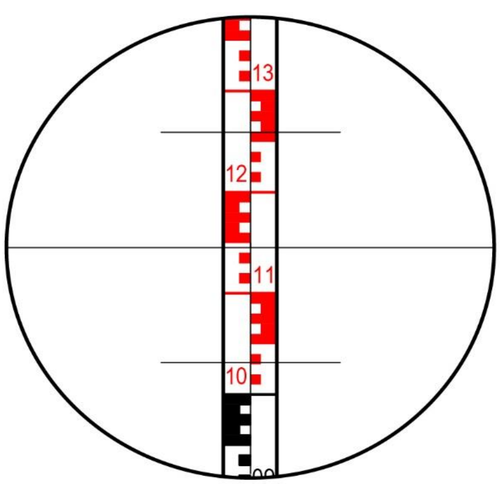
The advantage of the automatic level is that the measurements can be carried out much faster as there is no need to adjust the bubble tube before each reading. This also means that there is less chance of committing errors by forgetting to set the bubble tube.

The disadvantage of the automatic level is that under some circumstances (vibration due to traffic/construction or wind) the compensator prisms start to oscillate which prevents us from taking readings. The compensator can also get stuck so it is advisable to check whether it is working correctly before taking a reading. This can be done by carefully tapping the side of scope while looking through it and confirming that the image in the scope starts to shake slightly before become still again.

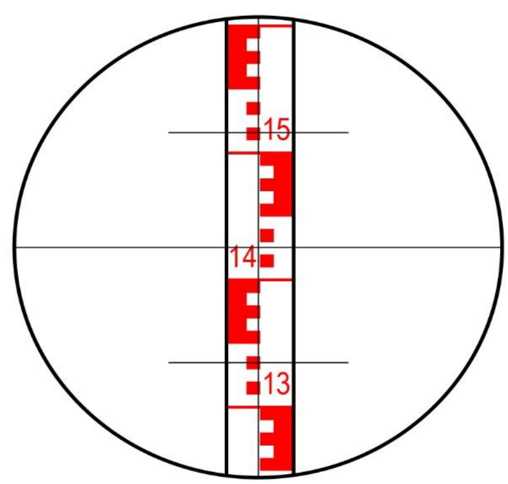
3. Calculation examples:

Example 1: calculate the height difference between points A and B! Calculate the distance between the level and the staff on point B!

Reading on point A:



Reading on point B:



Solution:

Reading on point A: $l_A = 1145$ Reading on point B: $l_B = 1427$

Height difference: $\Delta h_{AB} = l_A - l_B = 1145 - 1427 = -282$ mm $= -0.282$ m

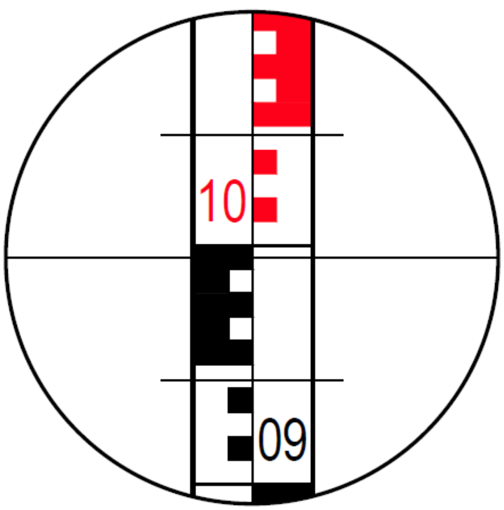
Distance from point B:

Reading on the upper stadia line: $U = 1516$ Reading on the lower stadia line: $L = 1335$

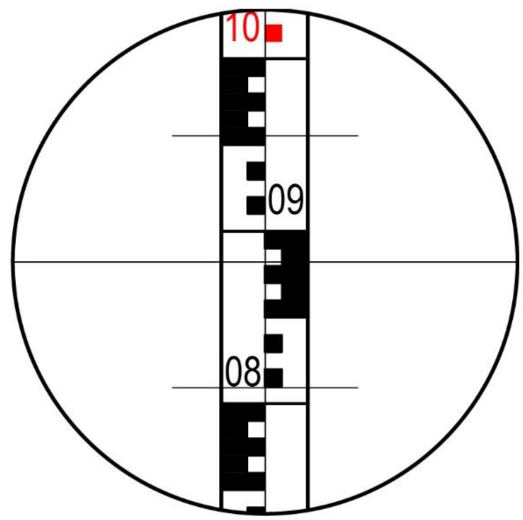
Distance: $d = (U - L) \cdot 100 = (1516 - 1335) \cdot 100 = 1810 \rightarrow 18.1$ m

Example 2: calculate the height difference between point C and point D! Calculate the distance between the instrument and the staff on point D!

Reading on point C



Reading on point D



Solution:

Reading on point C: $l_C = 0995$ Reading on point D: $l_D = 0882$

Height difference: $\Delta h_{CD} = l_C - l_D = 0995 - 0882 = 113$ mm $= +0.113$ m

Distance from point D:

Reading on the upper stadia line: $U = 0955$ Reading on the lower stadia line: $L = 0810$

Distance: $(U - L) \cdot 100 = (0955 - 0810) \cdot 100 = 1450 \rightarrow 14.5$ m

4. Mapping with the surveyor's level

We can use the surveyor's level to carry out basic mapping tasks that require low degrees of accuracy. Let's suppose that the level is set up on a control point (a marked point with known coordinates) and from this station both a benchmark (a point with known height above a reference surface) and the vertices of some object are visible.

In mapping, we usually differentiate between vertical (height determination) and horizontal (easting and northing coordinates) measurements. If we determine a points vertical and horizontal coordinates at the same time using one instrument (in this case the level), the method is called tacheometry (from the Greek word meaning 'quick measurement').

Determining the reduced level of the points is done as discussed previously, by taking a reading on the benchmark, computing the instrument height, taking a reading on the object's point and subtracting this reading from the instrument height.

To find the horizontal coordinates of the points, we first have to establish a reference direction from which we are measuring the angles. This reference direction is usually the Northing axis of the coordinate system we are working in. To measure angles, we can use the graduated circle on the bottom of level, by first aligning the zero with our reference direction, turning the telescope to our target point and reading the angle on the graduated circle. The accuracy of this angular measurement is of course somewhat limited, due to the fact that the graduated circle only has a resolution of 1°.

The coordinates of any new point can be found by using the first fundamental task of surveying with the measured whole circle bearing and distance from between the instruments known coordinates and the unknown point.

Example 3:

The level is set up on control point A with $E_A = 100.00$ m and $N_A = 100.00$ m and $H_A = 100.00$ m. The following field book contains the readings with the upper and lower stadia lines, the horizontal crosshair and the reading on the graduated circle. What are the Easting, the Northing and the height of point B?

Point ID	Upper	Middle	Lower	Angle	Distance
A	1836	1622	1532	0°	
B	1703	1567	1456	124°	

The height difference between A and B can be computed as the difference between the readings on the middle horizontal crosshair:

$$\Delta h_{AB} = l_A - l_B = 1622 - 1567 = +55 \text{ mm} = +0.055 \text{ m}$$

The height of point B from this height difference and the given height of point A:

$$H_B = H_A + \Delta h_{AB} = 100.00 + 0.055 = 100.55 \text{ m}$$

The distance between the instrument and B:

$$d_{AB} = (\text{Upper} - \text{Lower}) \cdot 100 = (1703 - 1456) \cdot 100 = 24700 \text{ mm} = 24.7 \text{ m}$$

The angle measured from the reference direction to point B can be considered a whole circle bearing in the reference system we are working in. The Easting and Northing coordinate differences between the instrument and B:

$$\Delta E_{AB} = d_{AB} \cdot \sin(\text{WCB}_{AB}) = 24.7 \cdot \sin(124°) = 20.48 \text{ m}$$

$$\Delta N_{AB} = d_{AB} \cdot \cos(\text{WCB}_{AB}) = 24.7 \cdot \cos(124°) = -13.81 \text{ m}$$

The coordinates of point B given the coordinates of point A:

$$E_B = E_A + \Delta E_{AB} = 100.00 + 20.48 = 120.48 \text{ m}$$

$$N_B = N_A + \Delta N_{AB} = 100.00 - 13.81 = \ 86.19 \text{ m}$$