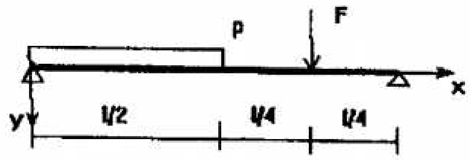
1.



$$EI = Const.$$

$$v = c \cdot \sin\left(\frac{\Pi \cdot x}{l}\right)$$

$$\int_0^l \left(\sin\frac{i\cdot\pi}{l}\right)^2 = \int_0^l \left(\cos\frac{i\cdot\pi}{l}\right)^2 = \frac{l}{2}$$

$$\Pi(c) = \frac{EI}{2}\int_0^l \left(\frac{d^2v}{dx^2}\right)^2 dx - \int_0^{l/2} pv dx - Fv(x = 3l/4)$$

$$\Pi(c) = \frac{EI}{2}c^2\frac{\Pi^4}{l^4}\int_0^l \sin^2\frac{\Pi x}{l}dx - cq\int_0^{l/2}\sin\frac{\Pi x}{l}dx - Fc\sin\frac{\Pi 3l/4}{l}$$

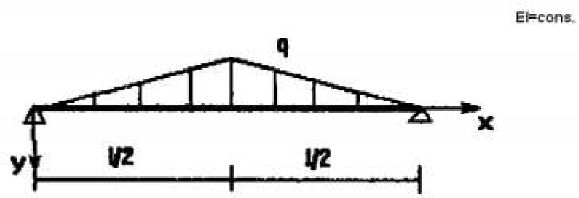
$$\Pi(c) = \frac{1}{4}\frac{EI\cdot\Pi^4c^2}{l^3} - \frac{lqc}{\Pi} - Fc\frac{\sqrt{2}}{2}$$

$$\frac{\delta\Pi(c)}{\delta c} = \frac{EI\Pi^4 c}{2l^3} - \frac{1}{2}F\sqrt{2} - \frac{lq}{\Pi} = 0 \quad \Rightarrow \quad c = \frac{1}{EI}\left(\frac{F\sqrt{2}\cdot l^3}{\Pi^4} + \frac{2l^4 q}{\Pi^5}\right)$$

$$v(x) = \frac{1}{EI}\left(2l^4\frac{q}{\Pi^5} + \frac{F\sqrt{2}l^3}{\Pi^4}\right)\sin\frac{\Pi x}{l}$$

$$M(x) = -EI\frac{d^2v}{dx^2} = \left(\frac{F\sqrt{2}\cdot l}{\Pi^2} + \frac{2\cdot l^2\cdot q}{\Pi^3}\right)\sin\frac{\Pi x}{l}$$

2.



$$v = c \cdot \sin\frac{\Pi x}{l}$$

$$\Pi(u) = \frac{1}{2} EI \int_0^l \left(-c\frac{\Pi^2}{l^2} \sin\frac{\Pi x}{l} \right)^2 dx - \int_0^{l/2} \frac{2xq}{l} \cdot c \cdot \sin\frac{\Pi x}{l} dx -$$

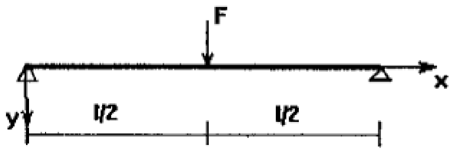
$$- c \int_{l/2}^{l} \left(2 - \frac{2x}{l} \right) q \sin\frac{\Pi x}{l} dx = \frac{1}{4} EI \frac{\Pi^4 c^2}{l^3} - 4\frac{lqc}{\Pi^2}$$

$$\frac{\delta \Pi(u)}{\delta c} = \frac{1}{2}\frac{EI\Pi^4 c}{l^3} - 4\frac{lq}{\Pi^2} = 0 \qquad \Rightarrow \qquad c = 8\frac{l^4 q}{EI\Pi^6}$$

$$v(x) = \frac{8l^4 q}{EI\Pi^6} \sin\frac{\Pi x}{l}$$

$$\int x \cdot \sin a \cdot x = \frac{\sin a \cdot x}{a^2} - \frac{x \cdot \cos a \cdot x}{a}$$

3.



$$v(x) = c_1 \sin\frac{\Pi x}{l} + c_2 \sin\frac{3\Pi x}{l} + c_3 \sin\frac{5\Pi}{l} x$$

$EJ = \text{const.}$

$$\frac{d^2 v}{dx^2} = -\frac{\Pi^2}{l^2}\left(c_1 \sin\frac{\Pi x}{l} + 9c_2 \sin\frac{3\Pi x}{l} + 25c_3 \sin\frac{5\Pi x}{l}\right)$$

$$F(u) = \frac{EI}{2}\int_0^l \left(\frac{d^2 v}{dx^2}\right)^2 dx - Fv(x = l/2)$$

$$F(u) = \frac{1}{4} EI \frac{\Pi^4}{l^3}\left(c_1^2 + 81c_2^2 + 625c_3^2\right) - F(c_1 - c_2 + c_3) = \min!$$

$$\frac{\delta F(u)}{\delta c_1} = \frac{1}{2}\frac{EI\Pi^4 c_1}{l^3} - F = 0 \qquad c_1 = 2\frac{l^3 F}{EI\Pi^4}$$

$$\frac{\delta F(u)}{\delta c_2} = \frac{81}{2}\frac{EI\Pi^4 c_2}{l^3} + F = 0 \qquad \Rightarrow \qquad c_2 = -\frac{2}{81}\frac{l^3 F}{EI\Pi^4}$$

$$\frac{\delta F(u)}{\delta c_3} = \frac{625}{2}\frac{EI\Pi^4 c_3}{l^3} - F = 0 \qquad c_3 = \frac{2}{625}\frac{l^3 F}{EI\Pi^4}$$

$$M(x) = -EI\sum c_i \frac{\delta^2 \sin\frac{i\Pi x}{l}}{\delta x^2}$$