MŰEGYETEM 1782

Tartószerkezetek Mechanikája Tanszék

Erőmódszer III.

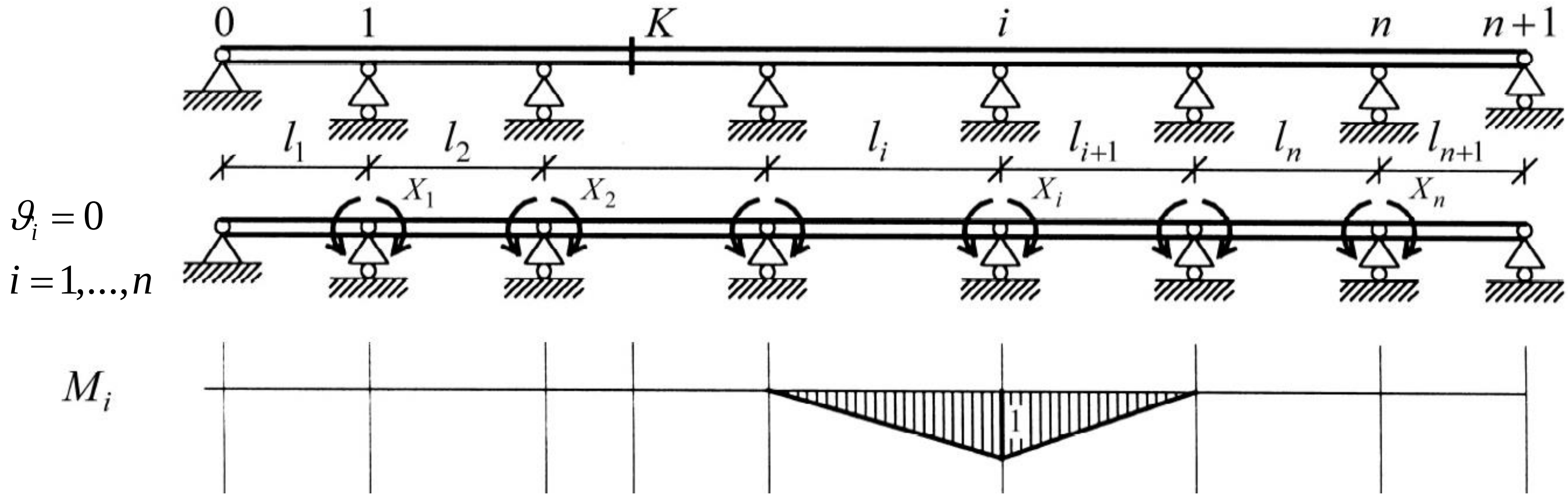
Dr. Hincz Krisztián

FOLYTATÓLAGOS TÖBBTÁMASZÚ TARTÓK VIZSGÁLATA ERŐMÓDSZERREL





Fix támasztású folytatólagos többtámaszú tartók



Egységtényezők:

$$\boxed{a_{ij} = \int_S \frac{M_i M_j}{EI(s)} ds}$$

és $a_{ij} = 0$ ha $|i-j| > 1$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$$

A terhelési tényezők meghatározása állóteher esetén

Erő jellegű teher:

$$\boxed{a_{i0} = \int_S \frac{M_i M_0}{EI(s)} ds}$$

Támaszelmozdulás-teher:

$$\boxed{a_{i0} = -\sum_{k=1}^{nd} R_{ki} e_{k0}}$$

Hőmérsékleti-teher:

$$\boxed{a_{i0} = \int_S M_i \kappa_t ds}$$

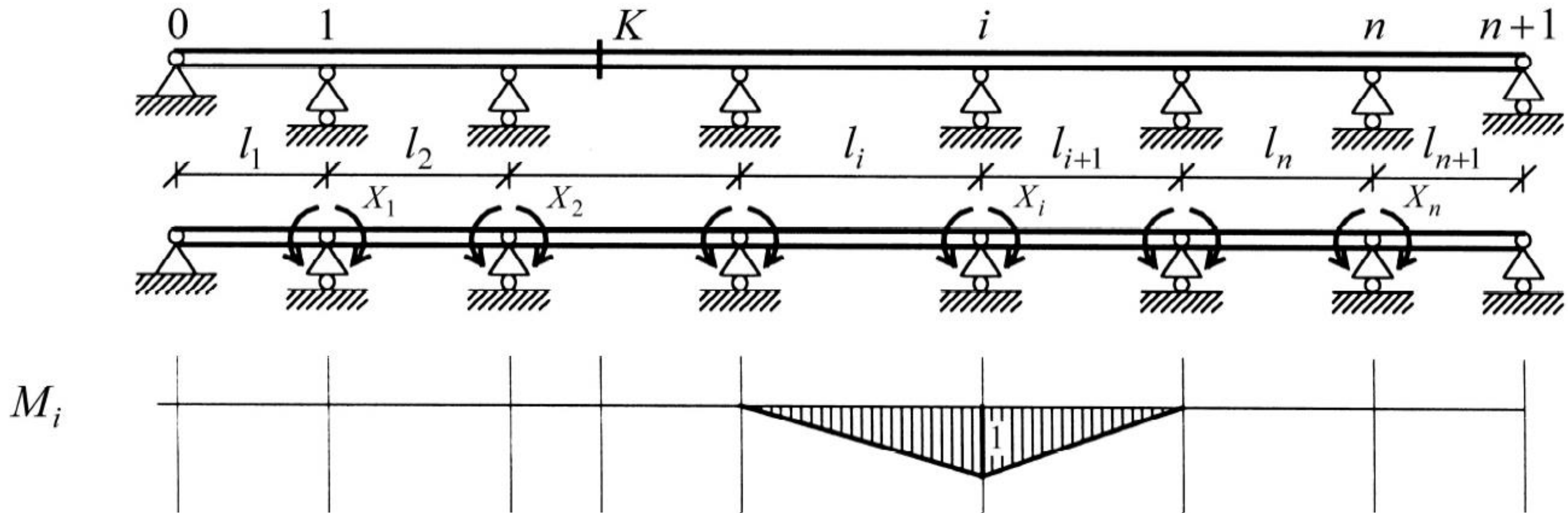
Az átvágási kapcsolati erők meghatározása:

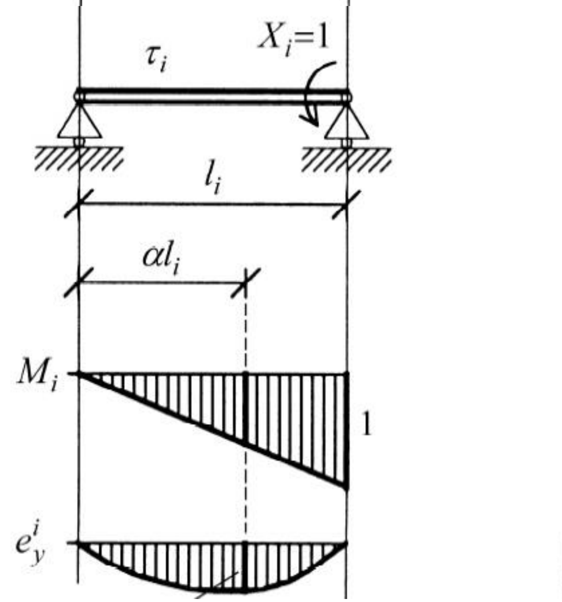
$$\mathbf{x} = -\mathbf{A}^{-1} \cdot \mathbf{a_0} = -\mathbf{Z} \cdot \mathbf{a_0}$$

A tartón fellépő hatások számítása szuperpozícióval:

pl. $M_K = M_{K0} + \sum_{i=1}^{n} M_{K,i} \cdot X_i$

Hatásábrák



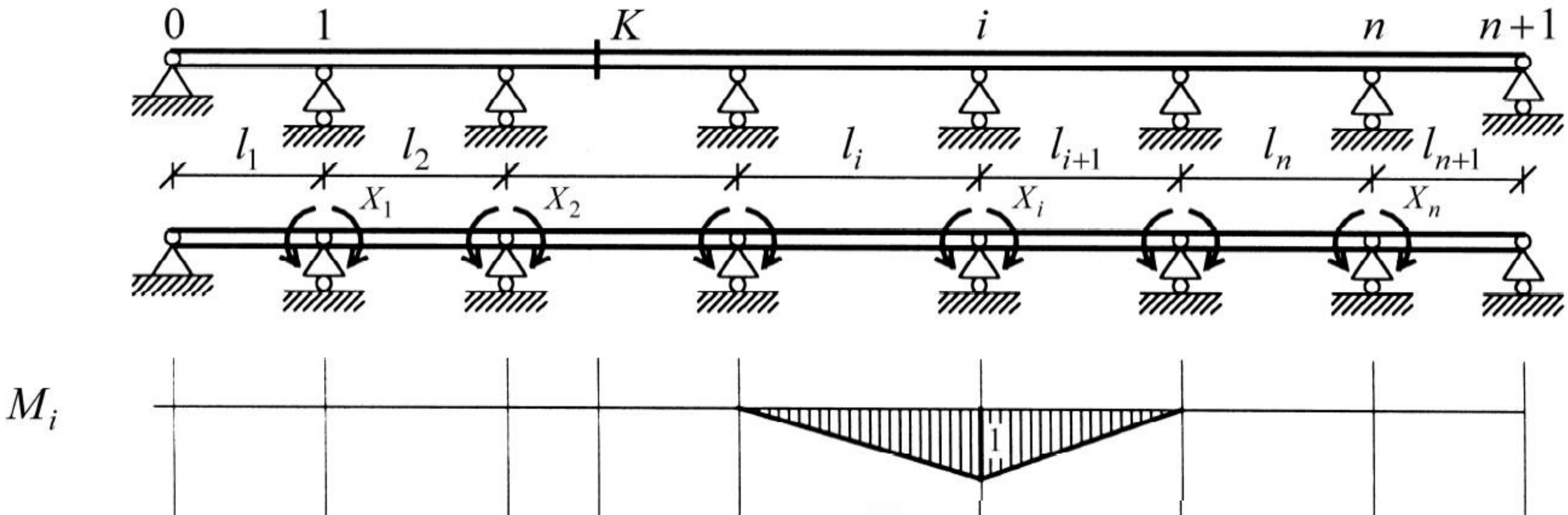


$$\eta(a_{i0}) = \eta(\vartheta_i)$$

$$\eta(\vartheta_i) = e_y \text{ - ábra} \left(\overline{X}_i = 1\right)$$

$$e_y^i(\alpha) = \frac{X_i l_i^2}{6EI_i}\left(\alpha - \alpha^3\right)$$

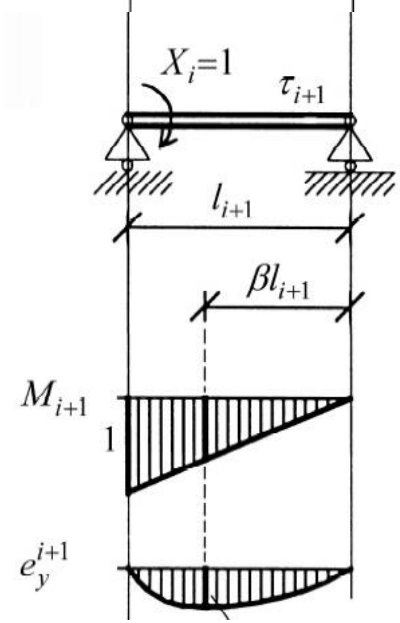
Hatásábrák



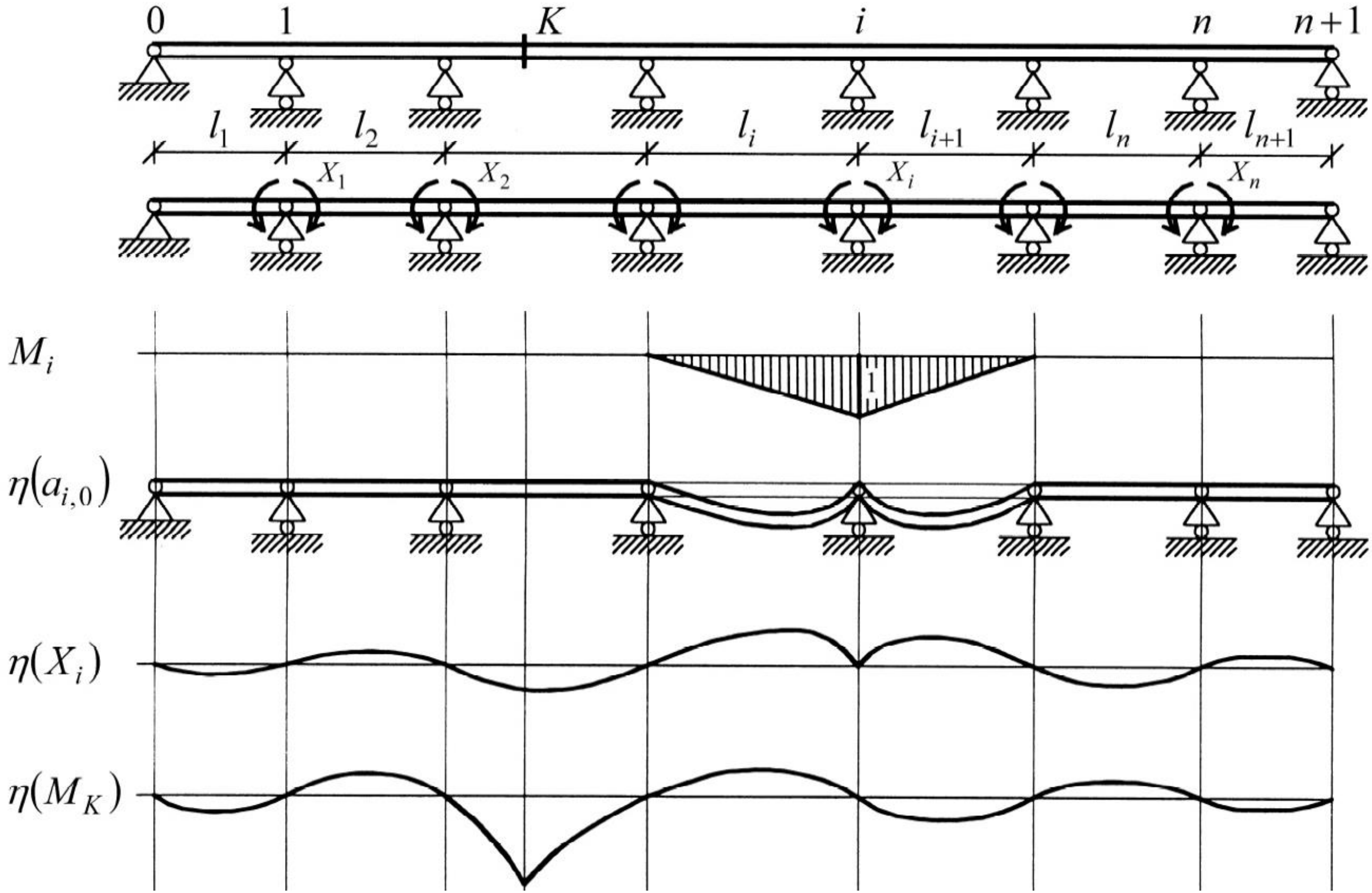
$$\eta(a_{i0}) = \eta(\vartheta_i)$$

$$\eta(\vartheta_i) = e_y \text{ - ábra} \left(\overline{X}_i = 1\right)$$

$$e_y^{i+1}(\beta) = \frac{X_i l_{i+1}^2}{6EI_{i+1}}\left(\beta - \beta^3\right)$$

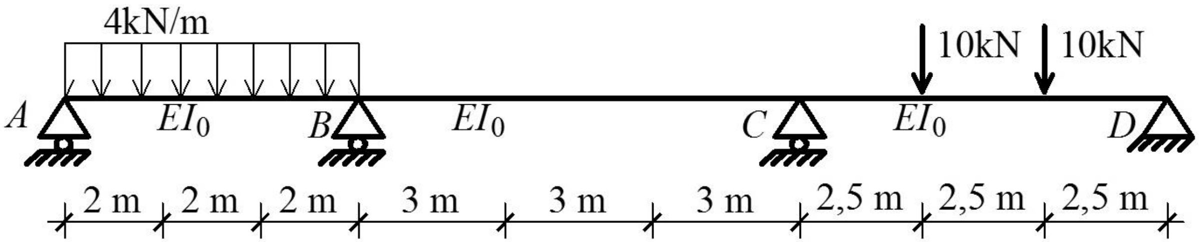


Hatásábrák



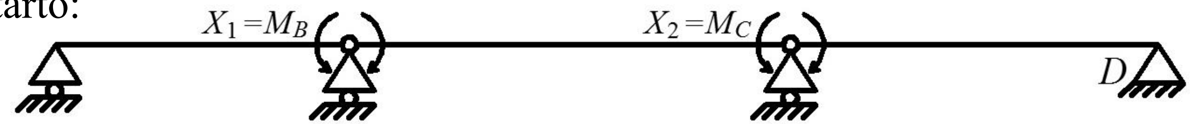
$$\eta(\mathbf{x}) = -\mathbf{A}^{-1} \cdot \eta(\mathbf{a}_0) = -\mathbf{Z} \cdot \eta(\mathbf{a}_0) \qquad \eta(X_i) = -\sum_{j=1}^{n} z_{ij}\eta(a_{j0}) \qquad \eta(M_K) = \eta(M_K)_0 + \sum_{i=1}^{n} M_{K,i} \cdot \eta(X_i)$$

1. mintapélda



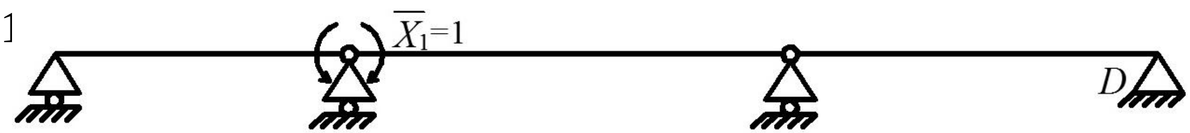
Határozzuk meg a nyomatéki ábrát!

Törzstartó:



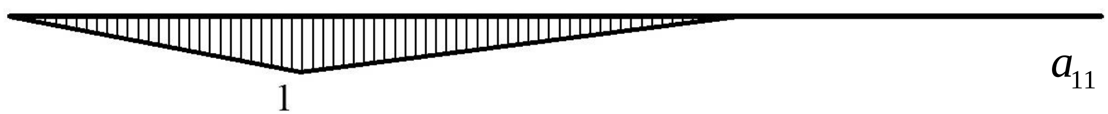
$\vartheta_B = 0 \qquad \vartheta_C = 0$

$\overline{X}_1 = 1$

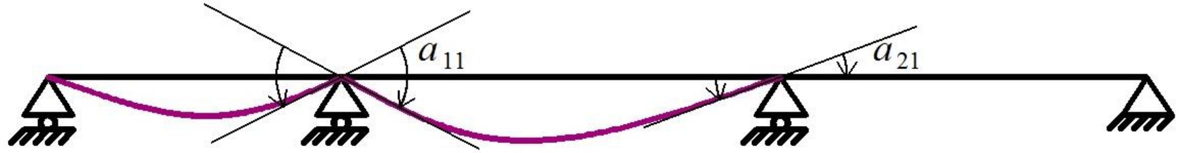


M_1

[-]

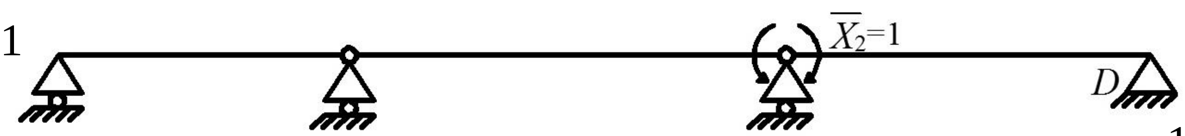


$$a_{11} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 6\cdot \frac{2}{3} + \frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3}\right) = \frac{5}{EI_0}$$



$$a_{12} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{1}{3}\right) = \frac{1,5}{EI_0}$$

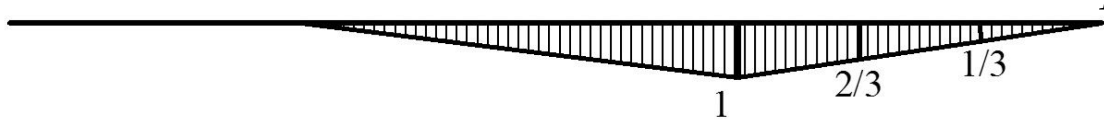
$\overline{X}_2 = 1$

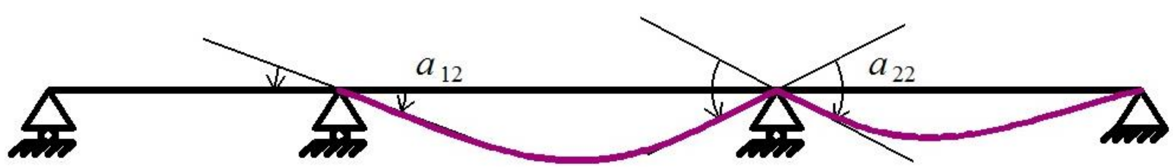


$$a_{22} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3} + \frac{1}{2}\cdot 1\cdot 7,5\cdot \frac{2}{3}\right) = \frac{5,5}{EI_0}$$

M_2

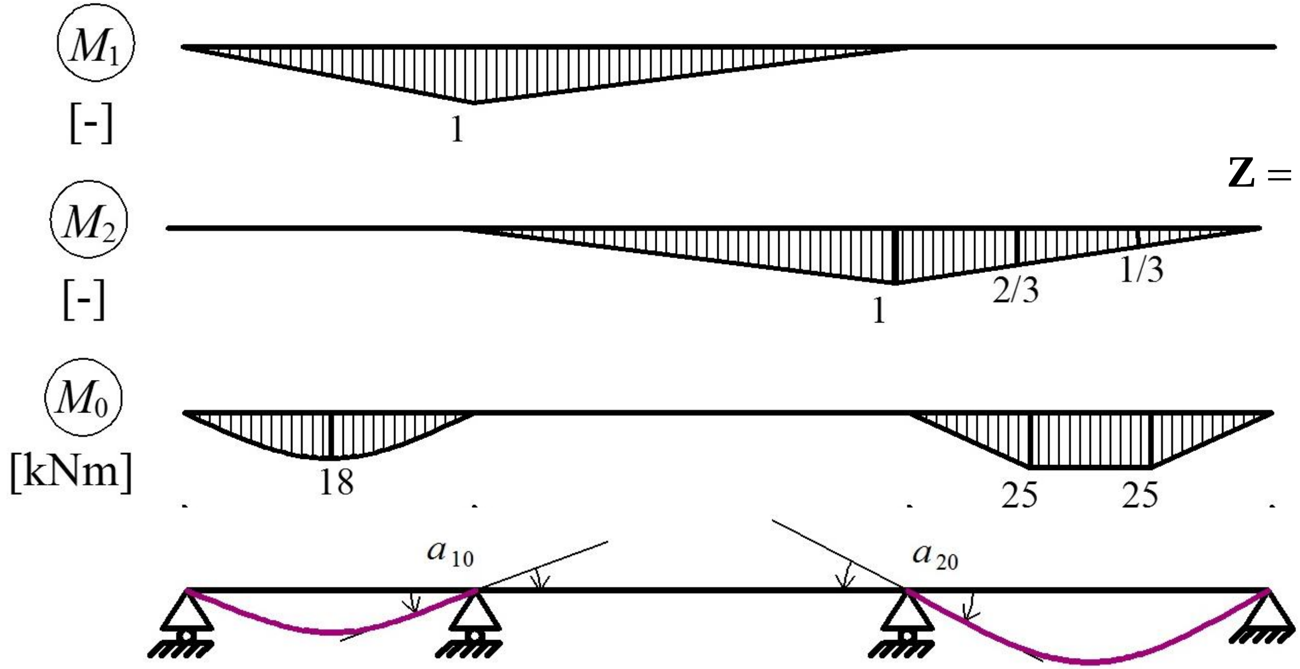
[-]





$$\mathbf{A} = \begin{bmatrix} 5 & 1,5 \\ 1,5 & 5,5 \end{bmatrix} \frac{1}{EI_0}$$

$$\mathbf{Z} = \mathbf{A}^{-1} = \begin{bmatrix} 0,2178 & -0,05941 \\ -0,05941 & 0,1980 \end{bmatrix} EI_0$$

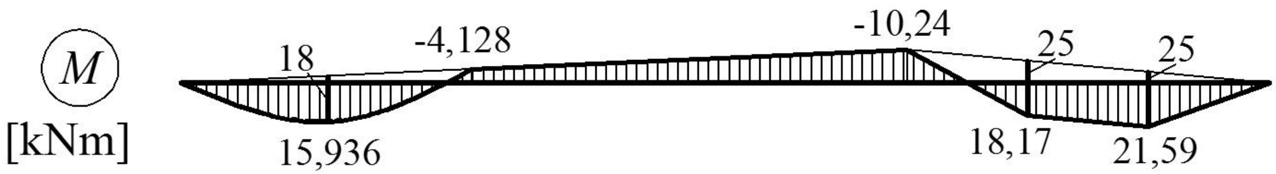


$$a_{i0} = \int_S \frac{M_i M_0}{EI(s)} ds$$

$$a_{10} = \frac{1}{EI_0}\left(\frac{2}{3}\cdot 18 \cdot 6 \cdot \frac{1}{2}\right) = \frac{36}{EI_0}$$

$$a_{20} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 25 \cdot 2,5 \cdot \left(\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{3}\right) + 25 \cdot 2,5 \cdot \frac{1}{2} + \frac{1}{2}\cdot 25 \cdot 2,5 \cdot \frac{2}{3}\cdot\frac{1}{3}\right) = \frac{62,50}{EI_0}$$

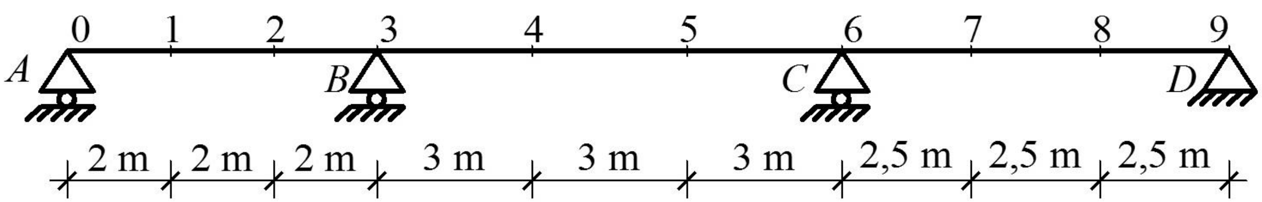
$$\mathbf{x} = -\mathbf{Z}\mathbf{a}_0 = \begin{bmatrix} -4,128 \\ -10,24 \end{bmatrix}$$



$$M = M_0 + X_1 M_1 + X_2 M_2$$

2. mintapélda

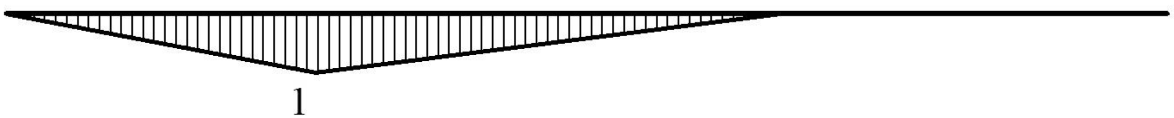
Határozzuk meg az $\eta(M_4)$ hatásábrát!



$$\mathbf{A} = \begin{bmatrix} 5 & 1,5 \\ 1,5 & 5,5 \end{bmatrix} \frac{1}{EI_0}$$

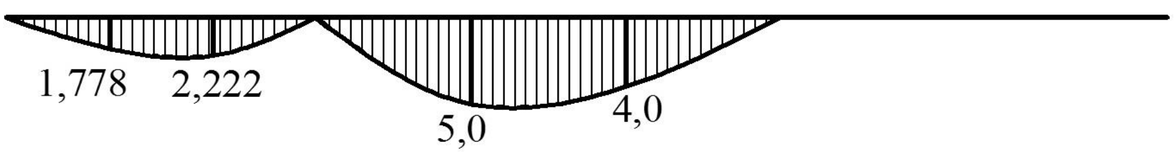
M_1

[-]

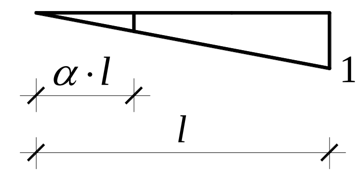


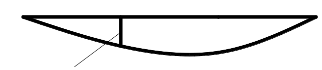
$\eta(a_{10})$

$\dfrac{1}{EI_0}$



M

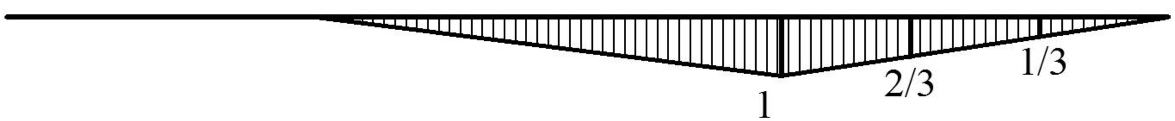




$$e_y = \frac{l^2}{6EI_0}(\alpha - \alpha^3)$$

M_2

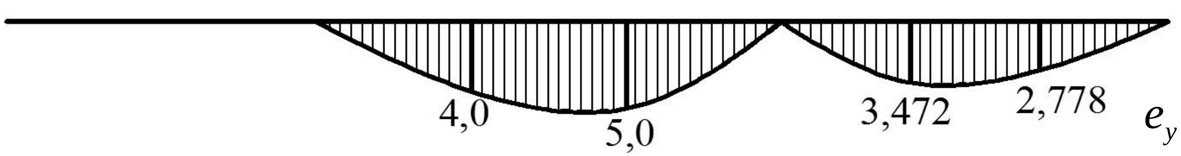
[-]



Például a K_2 –ben:

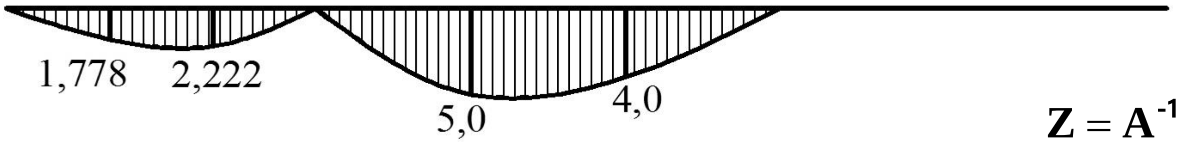
$\eta(a_{20})$

$\dfrac{1}{EI_0}$



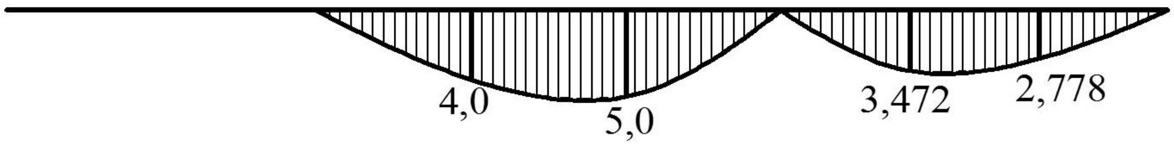
$$e_y = \frac{6^2}{6 \cdot EI_0}\left[\frac{2}{3} - \left(\frac{2}{3}\right)^3\right] = \frac{2,222}{EI_0}$$

$\eta(a_{10})$



$$\mathbf{Z} = \mathbf{A}^{-1} = \begin{bmatrix} 0,2178 & -0,05941 \\ -0,05941 & 0,1980 \end{bmatrix} EI_0$$

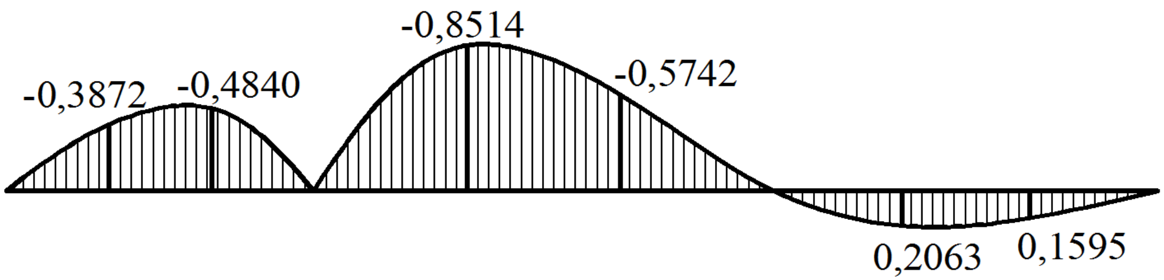
$\eta(a_{20})$



$$\eta(X_1) = -z_{11} \cdot \eta(a_{10}) - z_{12} \cdot \eta(a_{20})$$

Például a K_2 –ben: $\eta(X_1)_2 = -0{,}2178 \cdot 2{,}222 + 0{,}05941 \cdot 0 = -0{,}4840$

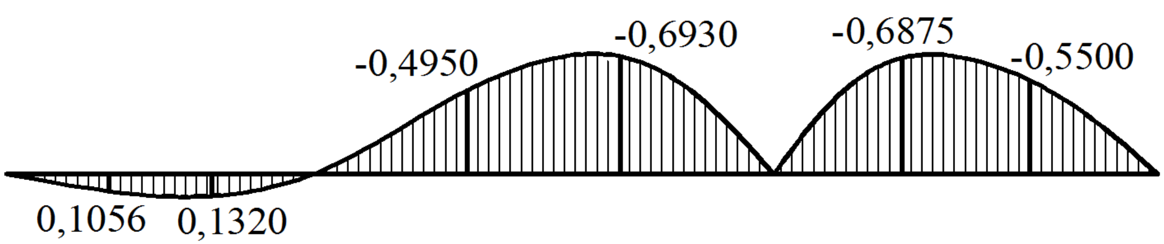
$\eta(X_1)$

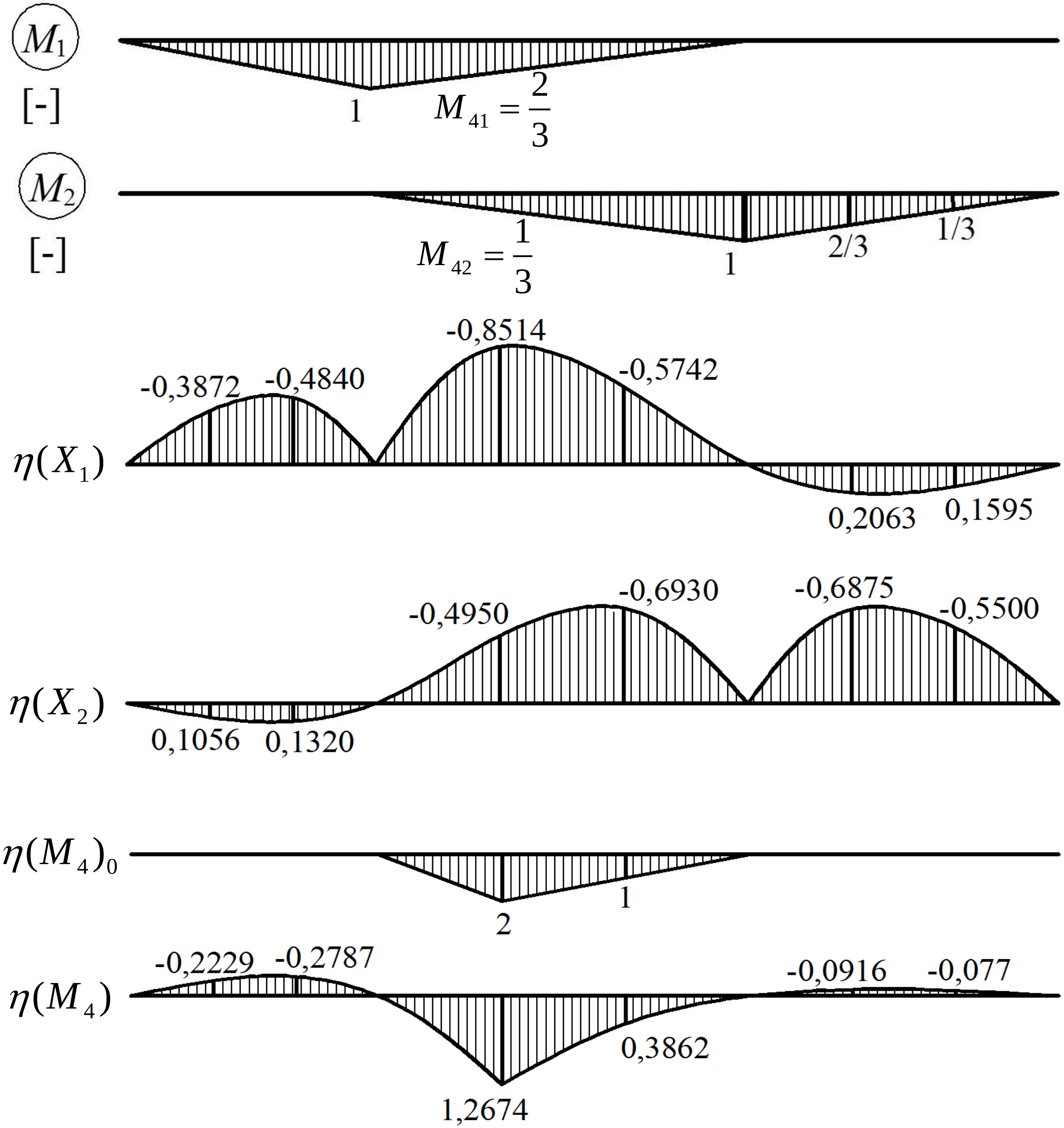


$$\eta(X_2) = -z_{21} \cdot \eta(a_{10}) - z_{22} \cdot \eta(a_{20})$$

Például a K_4 –ben: $\eta(X_2)_4 = 0{,}05941 \cdot 5{,}0 - 0{,}1980 \cdot 4{,}0 = -0{,}4950$

$\eta(X_2)$





$$\eta(M_4)=\eta(M_4)_0+$$

$$+M_{41}\cdot\eta(X_1)+M_{42}\cdot\eta(X_2)$$

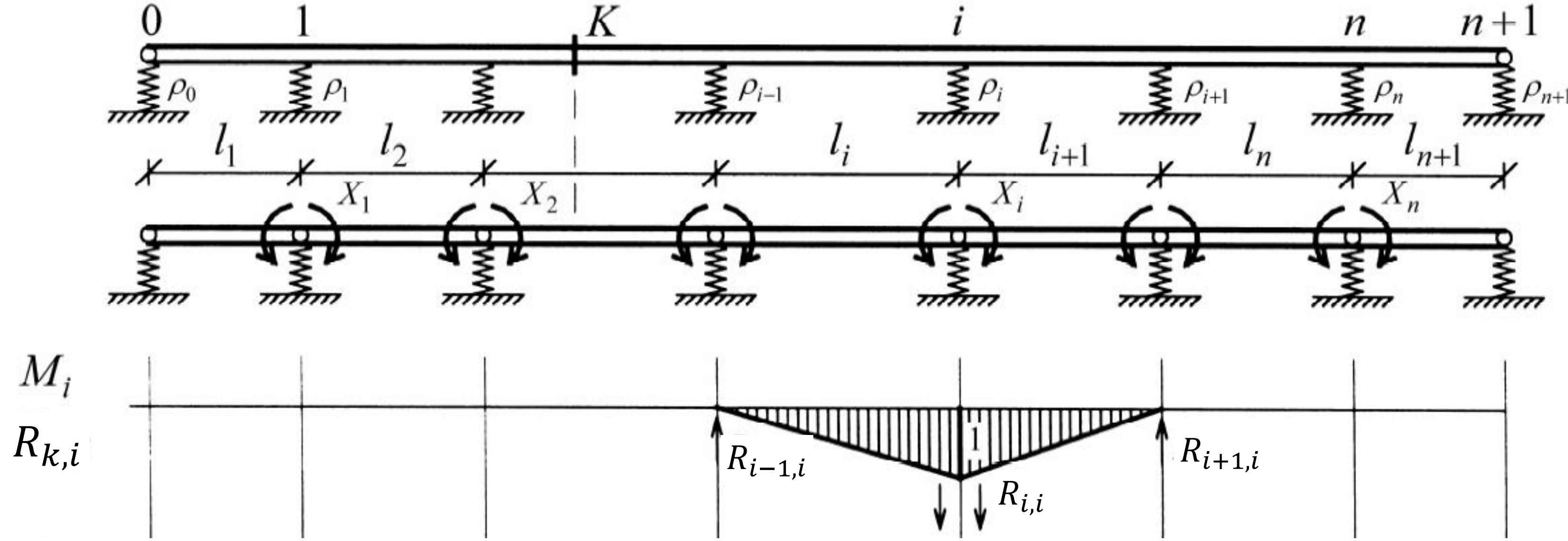
$$M_{41}=\frac{2}{3} \qquad M_{42}=\frac{1}{3}$$

Például a K_2 –ben:

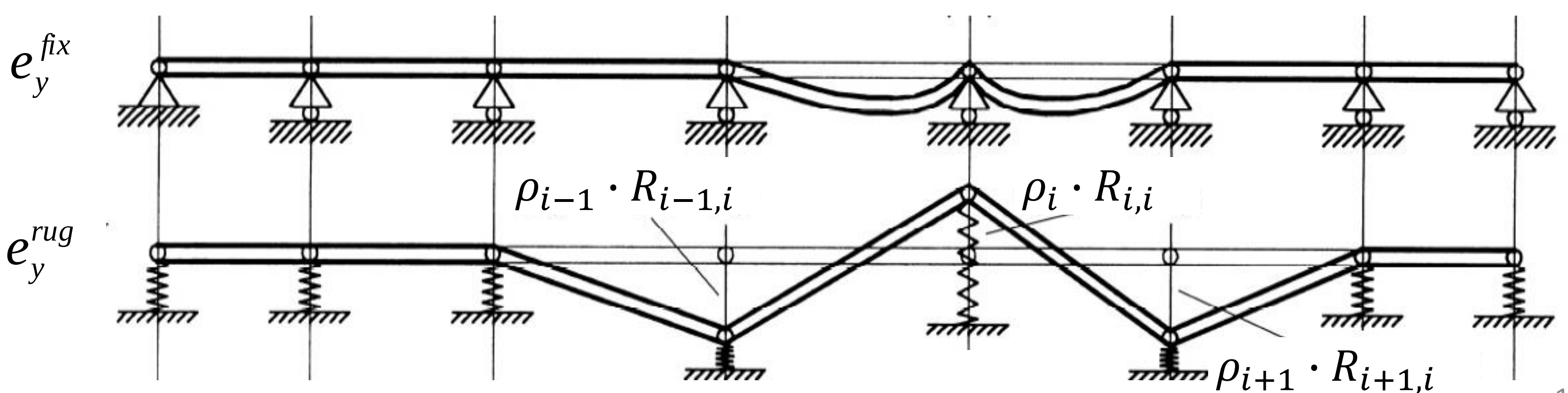
$$\eta(M_4)_2=0+\frac{2}{3}\cdot(-0{,}4840)+$$

$$+\frac{1}{3}(0{,}1320)=-0{,}2787$$

Rugalmas alátámasztású folytatólagos többtámaszú tartók



$$a_{ij} = a_{ij}^{fix} + a_{ij}^{rug} = \int \frac{M_i M_j}{EI} dx + \sum_{k=1}^{ns} R_{ki} \rho_k R_{kj}$$



Egységtényezők: $a_{ij} = a_{ij}^{fix} + a_{ij}^{rug} = \int \frac{M_i M_j}{EI} dx + \sum_{k=1}^{ns} R_{ki} \rho_k R_{kj}$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_{66} \end{bmatrix} \qquad a_{ij} = 0 \quad \text{ha} \quad |i - j| > 2$$

A terhelési tényezők meghatározása erő jellegű teher esetén:

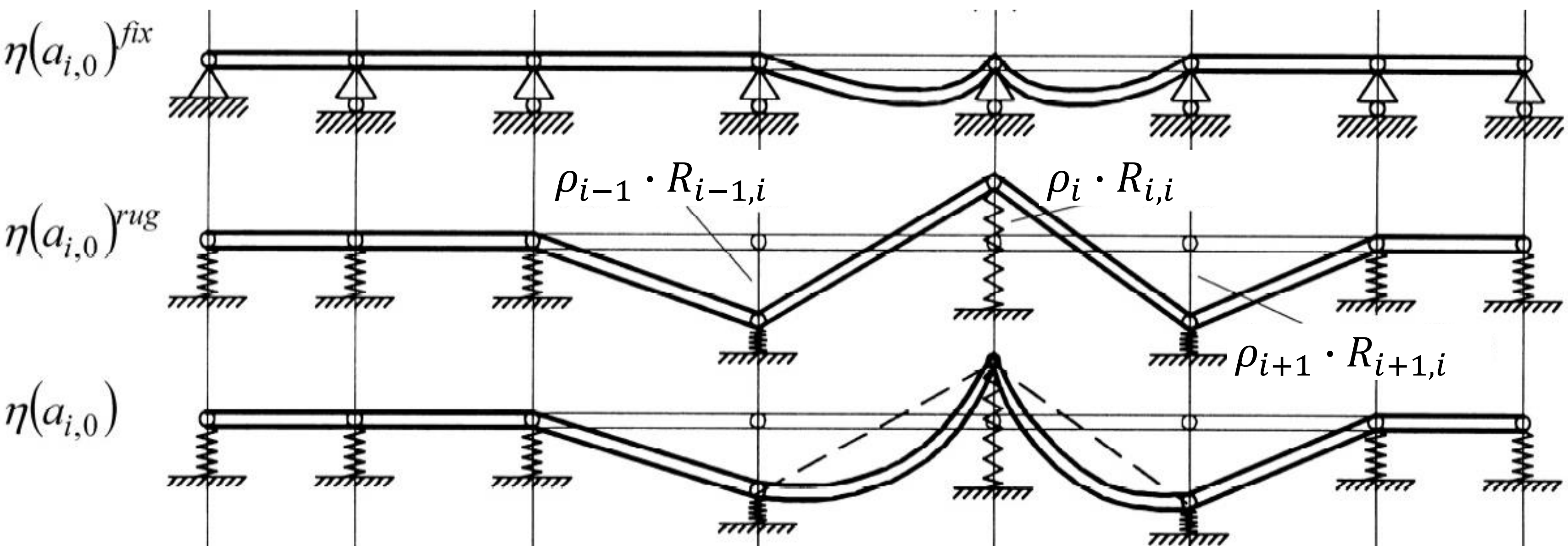
$$a_{i0} = \int \frac{M_i M_0}{EI} dx + \sum_{k=1}^{ns} R_{ki} \rho_k R_{k0}$$

Az átvágási kapcsolati erők meghatározása: $\mathbf{x} = -\mathbf{A}^{-1} \cdot \mathbf{a_0} = -\mathbf{Z} \cdot \mathbf{a_0}$

A tartón fellépő hatások számítása szuperpozícióval: pl. $M_K = M_{K0} + \sum_{i=1}^{n} M_{K,i} \cdot X_i$

Hatásábrák

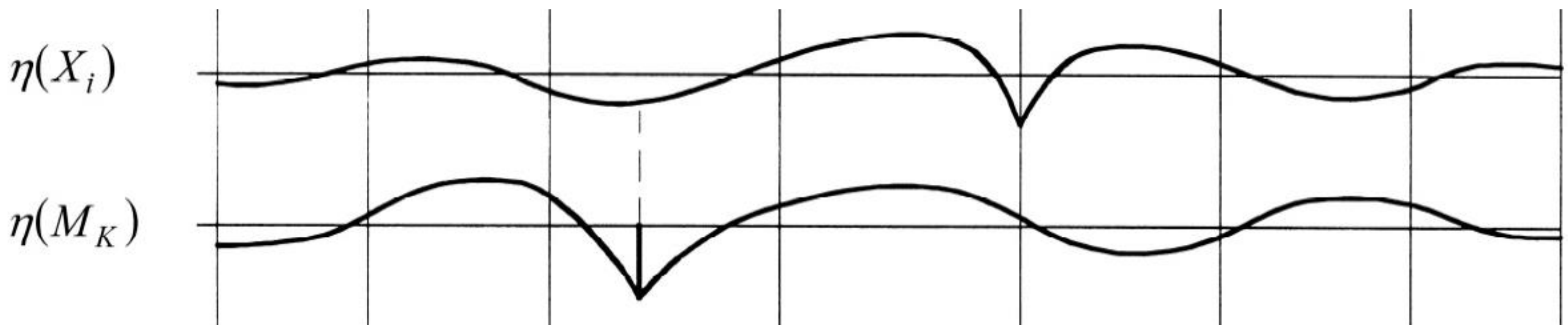
$\eta(a_{i0}) = \eta(\vartheta_i) = e_y$ - ábra $\left(\overline{X}_i = 1\right)$ $\eta(a_{i0}) = \eta(a_{i0})^{fix} + \eta(a_{i0})^{rug}$



Az átvágási kapcsolati erők hatásábráinak meghatározása:

$$\eta(\mathbf{x}) = -\mathbf{A}^{-1} \cdot \eta(\mathbf{a}_0) = -\mathbf{Z} \cdot \eta(\mathbf{a}_0)$$

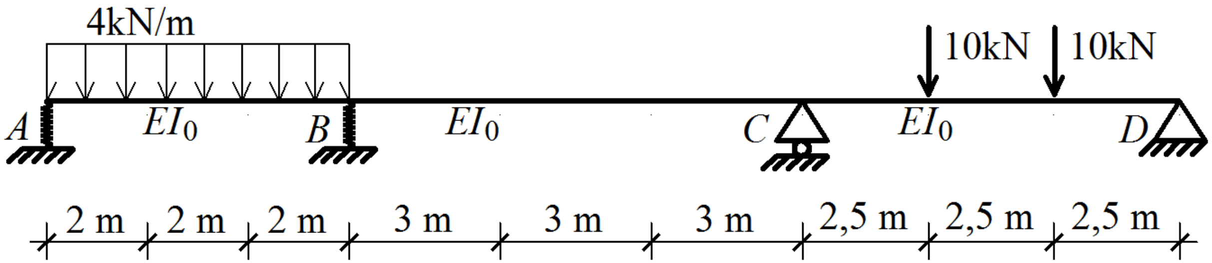
$$\eta(X_i) = -\sum_{j=1}^{n} z_{ij} \eta(a_{j0})$$



A tartón fellépő hatások számítása szuperpozícióval:

$$\eta(M_K) = \eta(M_K)_0 + \sum_{i=1}^{n} M_{K,i} \cdot \eta(X_i)$$

3. mintapélda



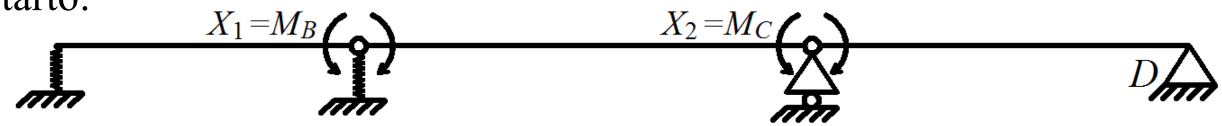
Határozzuk meg az nyomatéki ábrát!

$EI_0=10^5$ kN/m^2

$\rho_A=10^{-4}$ m/kN

$\rho_B=2\cdot10^{-4}$ m/kN

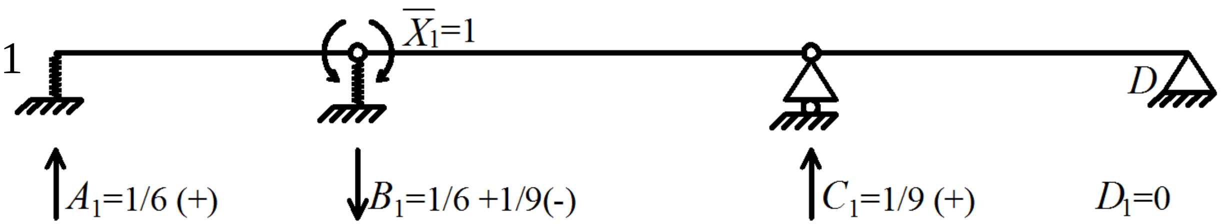
Törzstartó:



$\vartheta_B = 0$

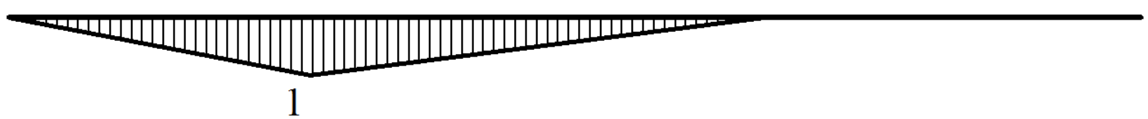
$\vartheta_C = 0$

$\overline{X}_1 = 1$

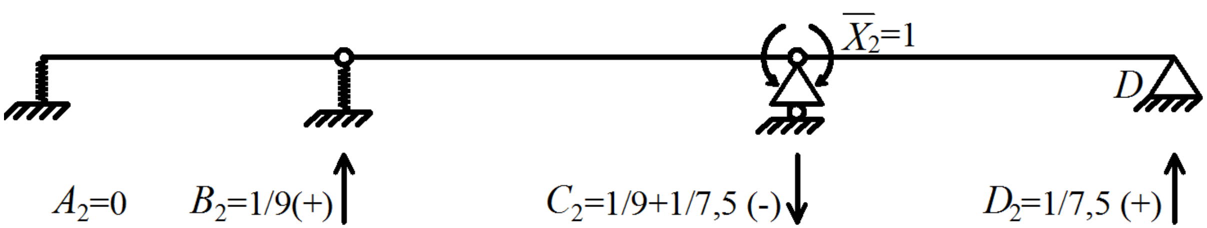


M_1

[-]

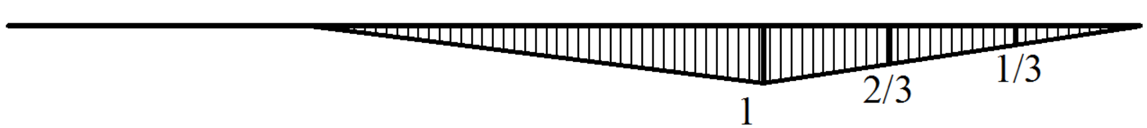


$\overline{X}_2 = 1$



M_2

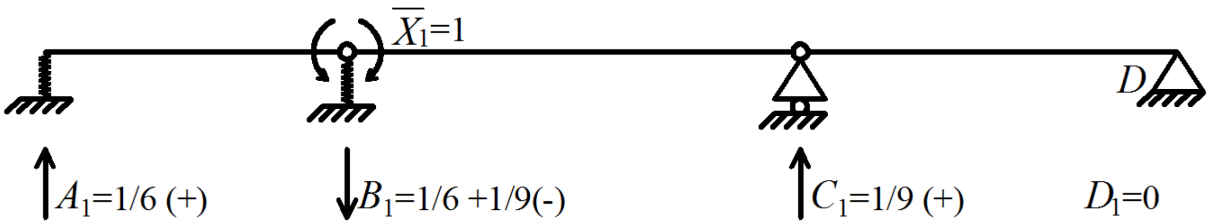
[-]

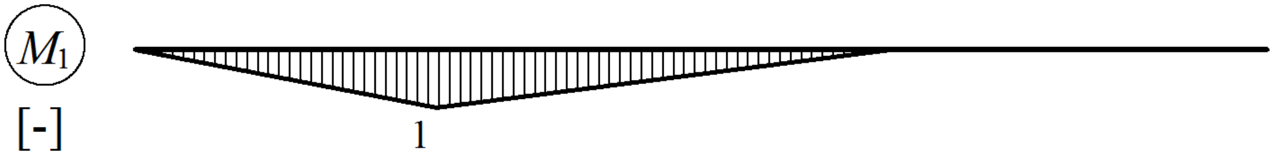


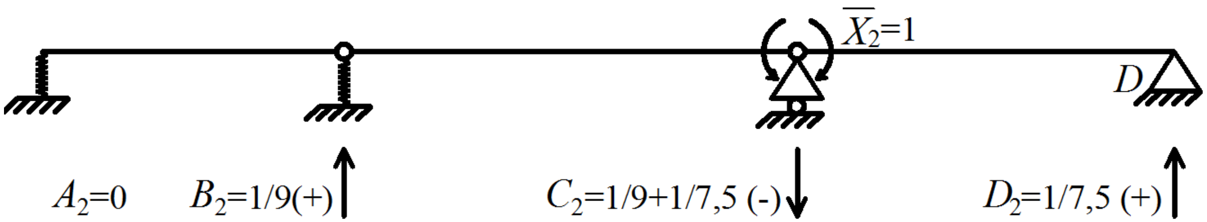
$EI_0 = 10^5$ kN/m^2

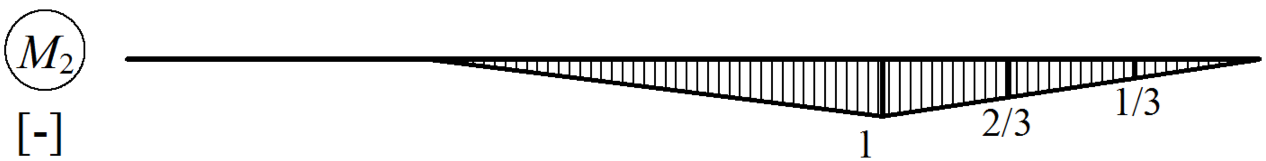
$\rho_A = 10^{-4}$ m/kN

$\rho_B = 2 \cdot 10^{-4}$ m/kN









$$a_{ij} = \int \frac{M_i M_j}{EI} dx + \sum_{k=1}^{ns} R_{ki} \rho_k R_{kj}$$

$$a_{11} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 6\cdot \frac{2}{3} + \frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3}\right) + \left(\frac{1}{6}\cdot\frac{1}{6}\cdot 10^{-4} + \left(\frac{1}{6}+\frac{1}{9}\right)^2 \cdot 2\cdot 10^{-4}\right) = \frac{5+1,821}{EI_0} = \frac{6,821}{EI_0}$$

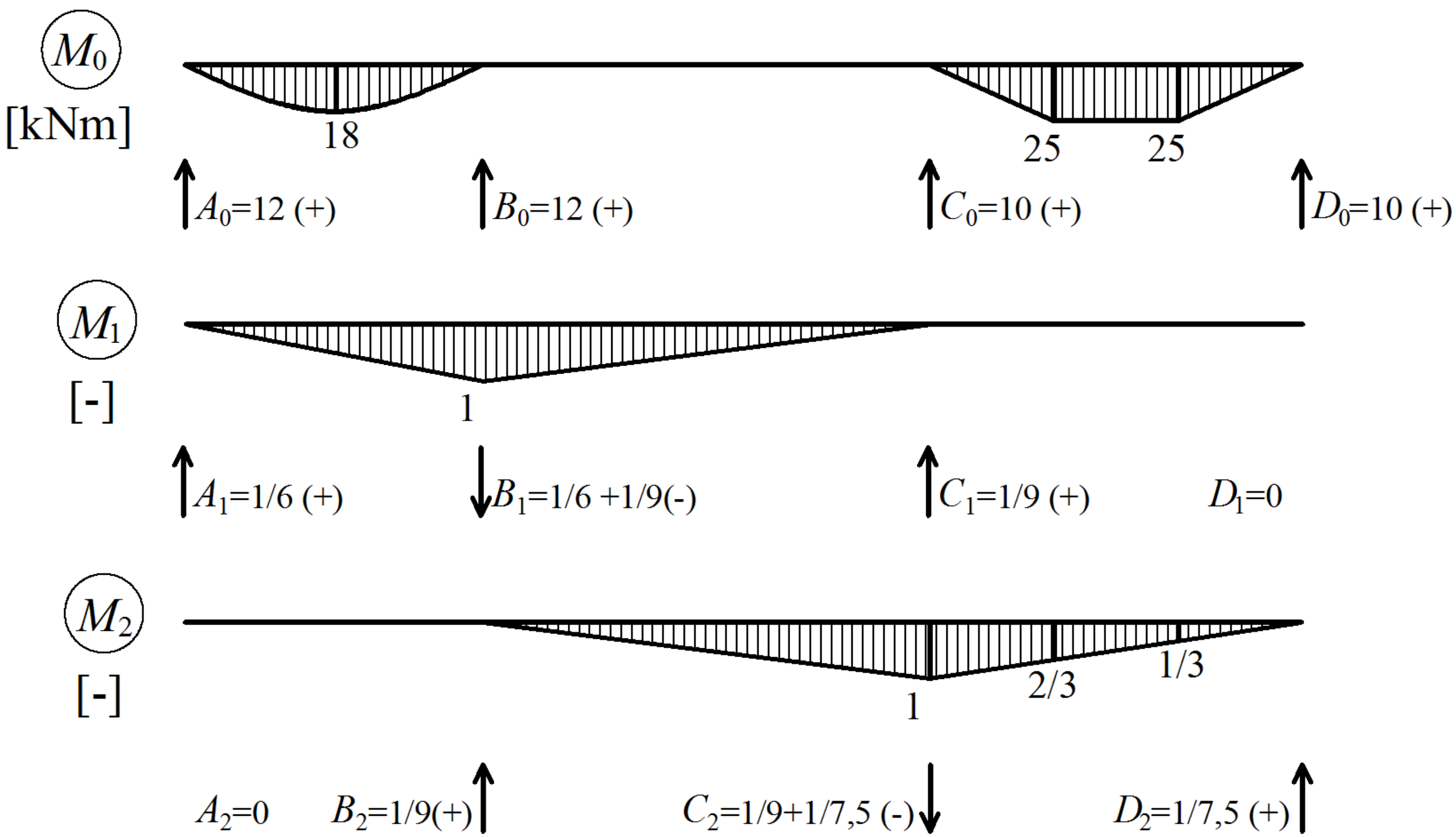
$$a_{12} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{1}{3}\right) + \left(-\left(\frac{1}{6}+\frac{1}{9}\right)\cdot\frac{1}{9}\cdot 2\cdot 10^{-4}\right) = \frac{1,5-0,6173}{EI_0} = \frac{0,8827}{EI_0}$$

$$a_{22} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3} + + \frac{1}{2}\cdot 1\cdot 7,5\cdot \frac{2}{3}\right) + \left(\frac{1}{9}\cdot\frac{1}{9}\cdot 2\cdot 10^{-4}\right) = \frac{5,5+0,247}{EI_0} = \frac{5,747}{EI_0}$$

$$\mathbf{A} = \begin{bmatrix} 6,821 & 0,8827 \\ 0,8827 & 5,747 \end{bmatrix} \frac{1}{EI_0}$$

$$\mathbf{A}=\begin{bmatrix} 6,821 & 0,8827 \\ 0,8827 & 5,747 \end{bmatrix}\frac{1}{EI_0} \quad \mathbf{Z}=\mathbf{A}^{-1}=\begin{bmatrix} 0,1496 & -0,0230 \\ -0,0230 & 0,1775 \end{bmatrix}EI_0 \qquad a_{i0}=\int\frac{M_i M_0}{EI}dx+\sum_{k=1}^{ns}R_{ki}\rho_k R_{k0}$$

$EI_0=10^5$ kN/m^2
$\rho_A=10^{-4}$ m/kN
$\rho_B=2\cdot 10^{-4}$ m/kN

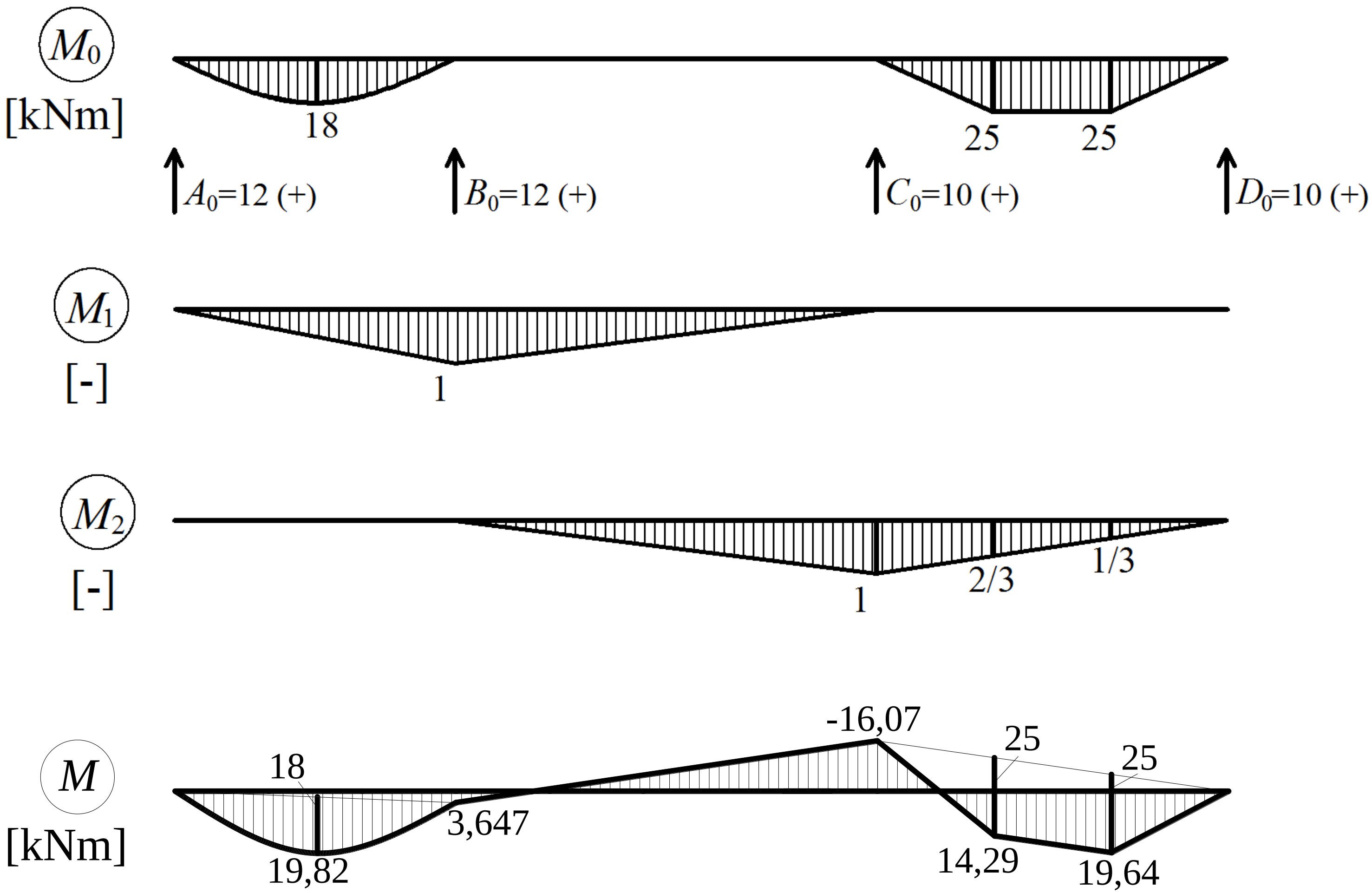


$$a_{10}=\frac{1}{EI_0}\left(\frac{2}{3}\cdot 18\cdot 6\cdot\frac{1}{2}\right)+\left(12\cdot\frac{1}{6}\cdot 10^{-4}-12\cdot\left(\frac{1}{6}+\frac{1}{9}\right)\cdot 2\cdot 10^{-4}\right)=\frac{36-46,67}{EI_0}=\frac{-10,67}{EI_0}$$

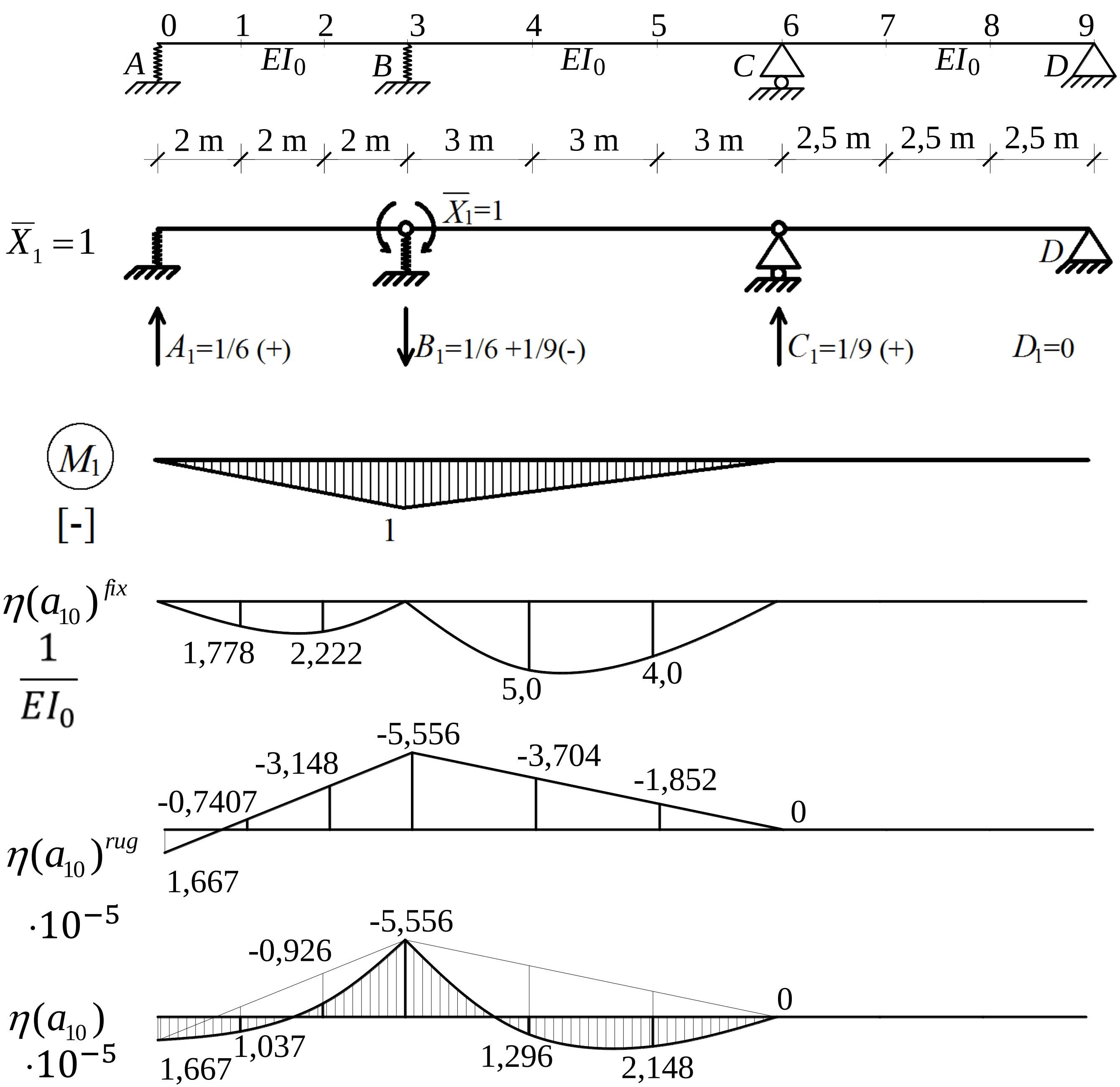
$$a_{20}=\frac{1}{EI_0}\left(\frac{1}{2}\cdot 25\cdot 2,5\cdot\left(\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{3}\right)+25\cdot 2,5\cdot\frac{1}{2}+\frac{1}{2}\cdot 25\cdot 2,5\cdot\frac{2}{3}\cdot\frac{1}{3}\right)+12\cdot\frac{1}{9}\cdot 2\cdot 10^{-4}=\frac{62,50+26,67}{EI_0}=\frac{89,17}{EI_0}$$

$$\mathbf{x} = -\mathbf{Z}\mathbf{a}_0 = -\begin{bmatrix} 0{,}1496 & -0{,}0230 \\ -0{,}0230 & 0{,}1775 \end{bmatrix}\begin{bmatrix} -10{,}67 \\ 89{,}17 \end{bmatrix} = \begin{bmatrix} 3{,}647 \\ -16{,}07 \end{bmatrix}$$

$$M = M_0 + X_1 M_1 + X_2 M_2$$



4. mintapélda



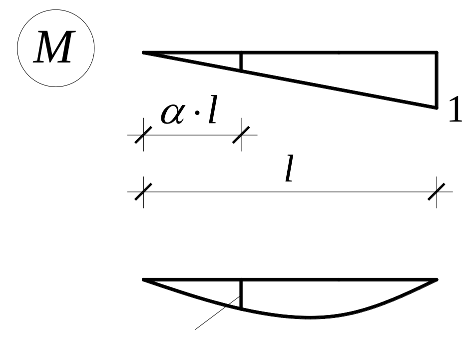
Határozzuk meg az $\eta(M_4)$ hatásábrát!

$EI_0=10^5$ kN/m²

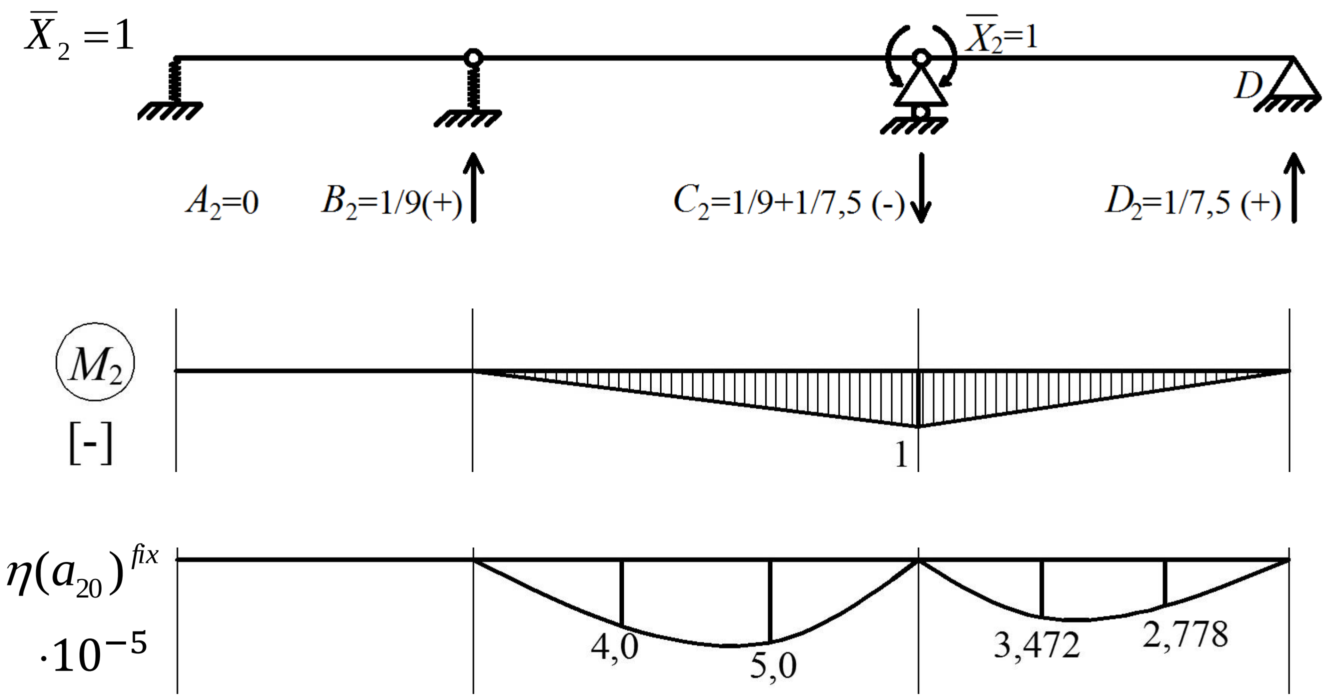
$\rho_A=10^{-4}$ m/kN

$\rho_B=2\cdot10^{-4}$ m/kN

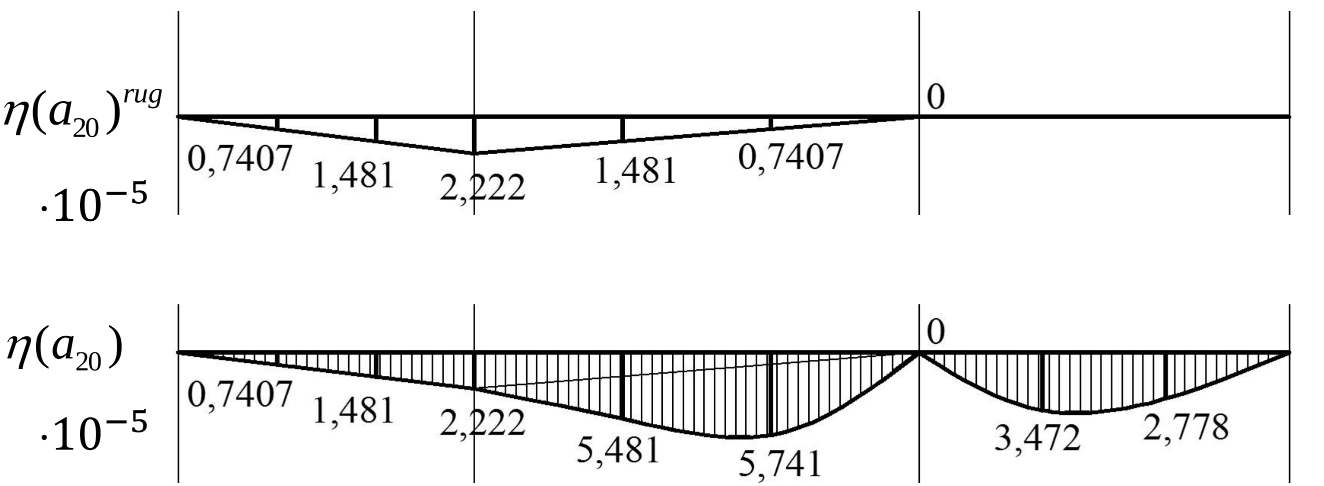
$$\mathbf{A}=\begin{bmatrix}6,821 & 0,8827\\ 0,8827 & 5,747\end{bmatrix}\frac{1}{EI_0}$$



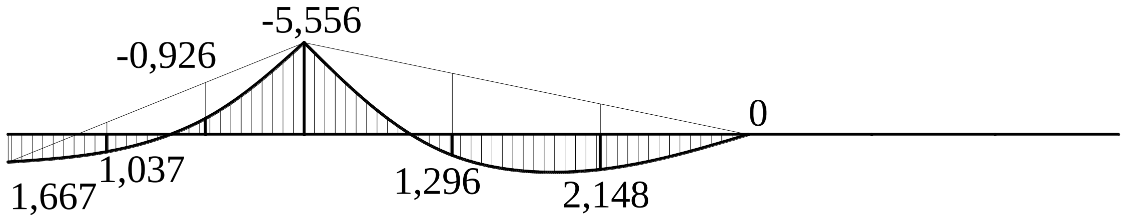
$$e_y^{fix}=\frac{l^2}{6EI}(\alpha-\alpha^3)$$



$$e_{3y}^{rug} = \frac{1}{9} \cdot 2 \cdot 10^{-4} = \frac{2,222}{EI_0}$$

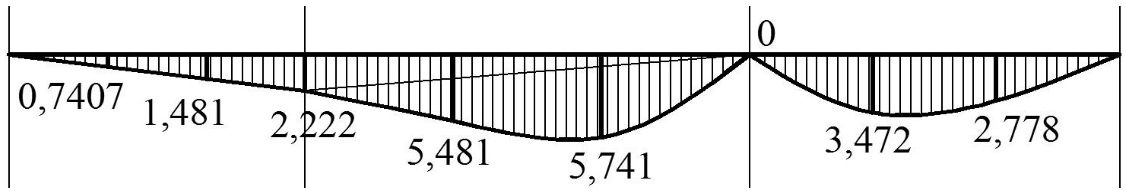


$\eta(a_{10})$
$\cdot 10^{-5}$



$$\mathbf{Z} = \begin{bmatrix} 0,1496 & -0,0230 \\ -0,0230 & 0,1775 \end{bmatrix} EI_0$$

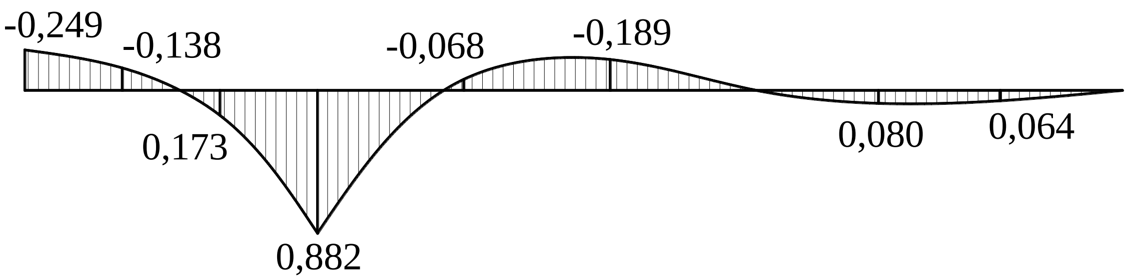
$\eta(a_{20})$
$\cdot 10^{-5}$



$$\eta(X_1) = -z_{11} \cdot \eta(a_{10}) - z_{12} \cdot \eta(a_{20})$$

Például a K_2 –ben: $\eta(X_1)_2 = -0,1496 \cdot (-0,926) + 0,0230 \cdot 1,481 = 0,173$

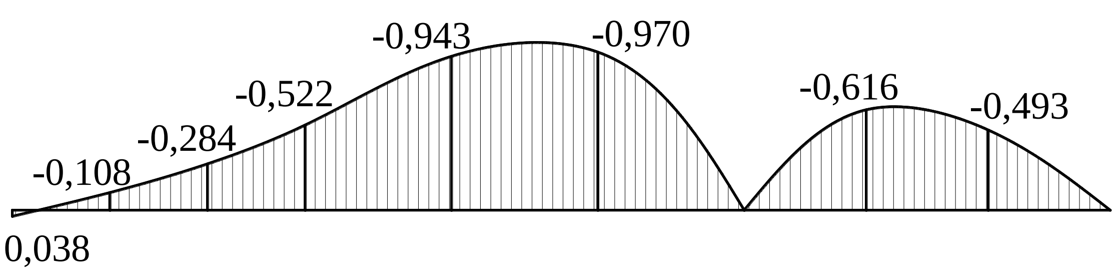
$\eta(X_1)$



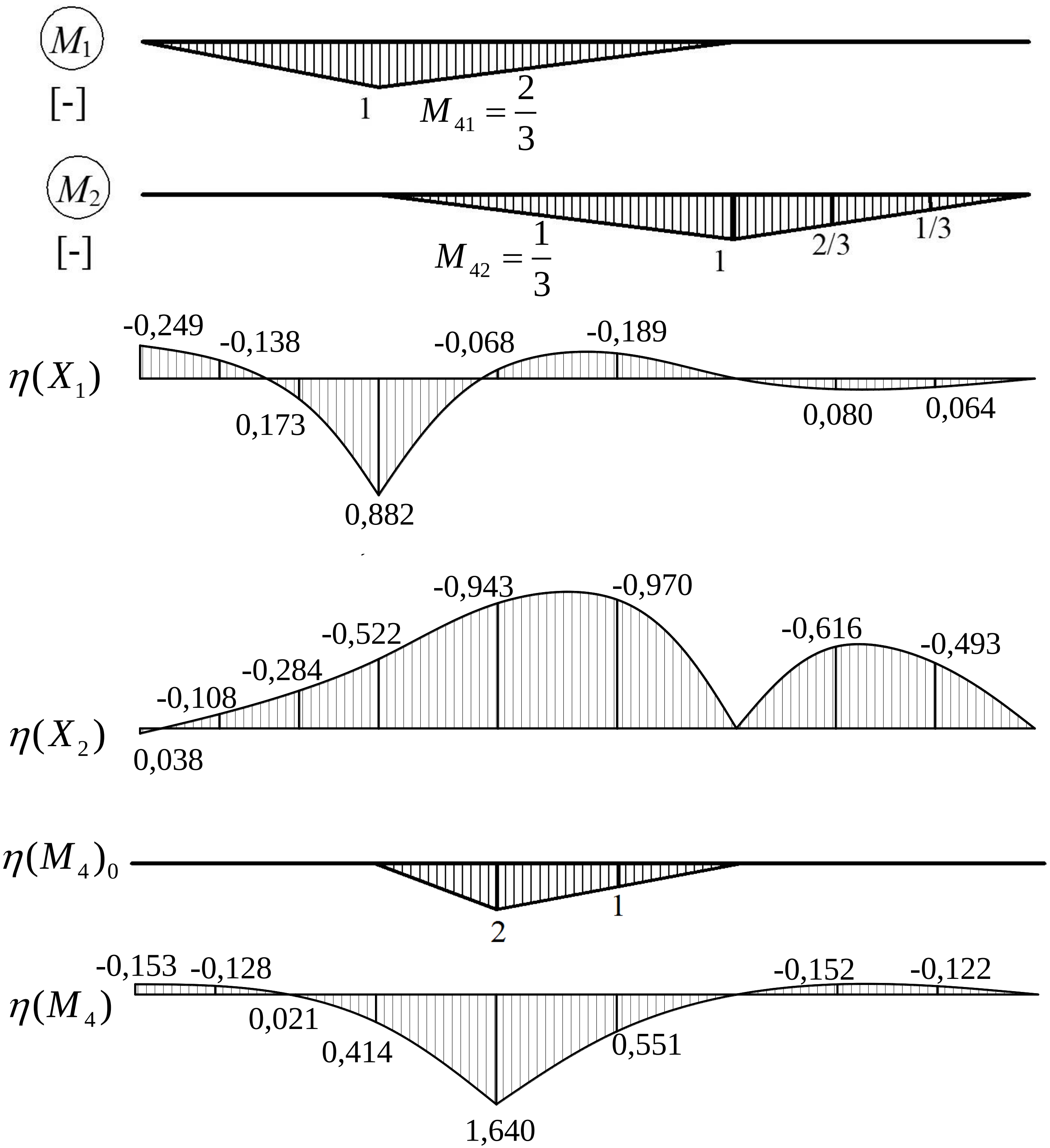
$$\eta(X_2) = -z_{21} \cdot \eta(a_{10}) - z_{22} \cdot \eta(a_{20})$$

Például a K_2 –ben: $\eta(X_2)_2 = 0,0230 \cdot (-0,926) - 0,1775 \cdot 1,481 = -0,284$

$\eta(X_2)$



Határozzuk meg az $\eta(M_4)$ hatásábrát!

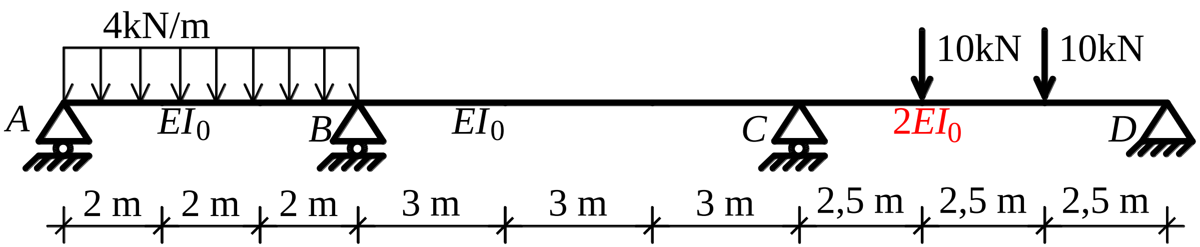


$$\eta(M_4) = \eta(M_4)_0 + $$
$$+ M_{41} \cdot \eta(X_1) + M_{42} \cdot \eta(X_2)$$
$$M_{41} = \frac{2}{3} \qquad M_{42} = \frac{1}{3}$$

Például a K_2 –ben:

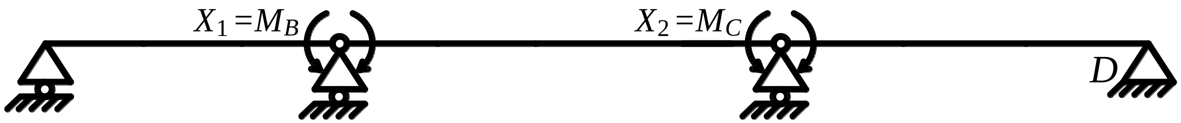
$$\eta(M_4)_2 = 0 + \frac{2}{3} 0{,}173 +$$
$$+ \frac{1}{3}(-0{,}284) = 0{,}021$$

5. Mintapélda (az 1. példában szereplő szerkezet a *CD* szakaszon nagyobb merevséggel



Határozzuk meg a nyomatéki ábrát!

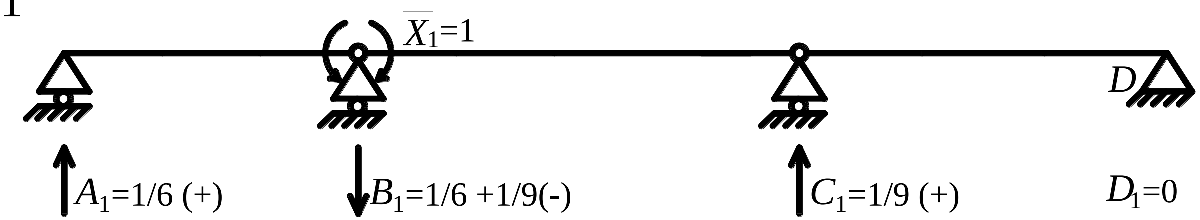
Törzstartó:

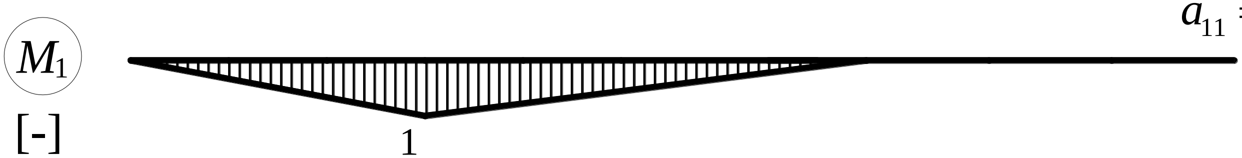


$$\vartheta_B = 0$$

$$\vartheta_C = 0$$

$$\overline{X}_1 = 1$$

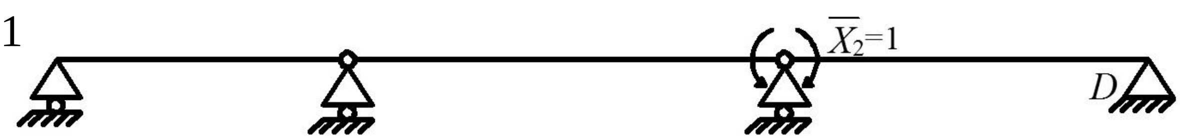




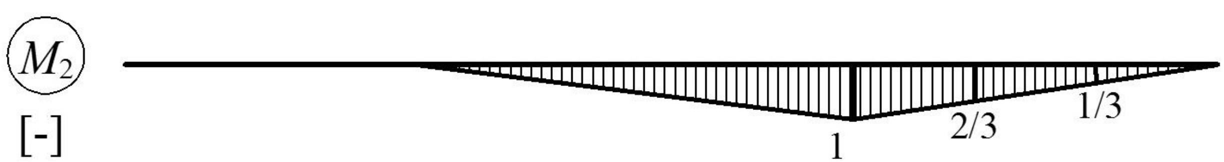
$$a_{11} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 6\cdot \frac{2}{3} + \frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3}\right) = \frac{5}{EI_0}$$

$$a_{12} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{1}{3}\right) = \frac{1,5}{EI_0}$$

$$\overline{X}_2 = 1$$



$$a_{22} = \frac{1}{EI_0}\left(\frac{1}{2}\cdot 1\cdot 9\cdot \frac{2}{3}\right) + \\ + \frac{1}{2EI_0}\cdot\left(\frac{1}{2}\cdot 1\cdot 7,5\cdot \frac{2}{3}\right) = \frac{4,25}{EI_0}$$

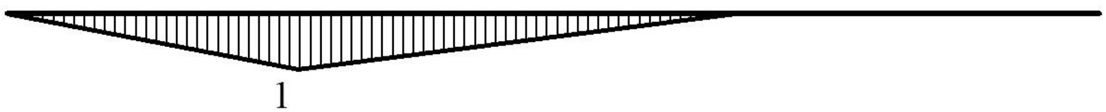


$$\mathbf{A} = \begin{bmatrix} 5 & 1,5 \\ 1,5 & 4,25 \end{bmatrix} \frac{1}{EI_0} \qquad \mathbf{Z} = \mathbf{A}^{-1} = \begin{bmatrix} 0,2237 & -0,0789 \\ -0,0789 & 0,2632 \end{bmatrix} EI_0$$

$$a_{i0} = \int_S \frac{M_i M_0}{EI} ds$$

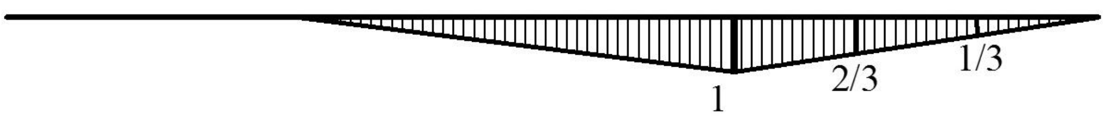
M_0
[kNm]

M_1
[-]



$$a_{10} = \frac{1}{EI_0}\left(\frac{2}{3}\cdot 18\cdot 6\cdot \frac{1}{2}\right) = \frac{36}{EI_0}$$

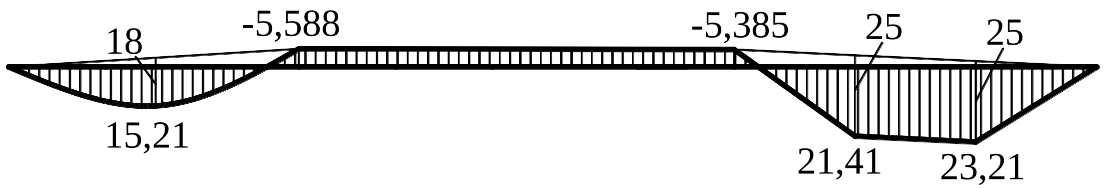
M_2
[-]



$$a_{20} = \frac{1}{2EI_0}\cdot\left[\frac{1}{2}\cdot 25\cdot 2,5\cdot\left(\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{3}\right)+25\cdot 2,5\cdot\frac{1}{2}+\frac{1}{2}\cdot 25\cdot 2,5\cdot\frac{2}{3}\cdot\frac{1}{3}\right] = \frac{31,25}{EI_0}$$

$$\mathbf{x} = -\mathbf{Z}\mathbf{a}_0 = \begin{bmatrix} -5,588 \\ -5,385 \end{bmatrix}$$

M
[kNm]



$$M = M_0 + X_1 M_1 + X_2 M_2$$

A statikailag határozatlan tartók erőjátéka a tartó merevségének is függvénye.