

MECHANICS OF MASONRY STRUCTURES







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In case of any question or problem, do not hesitate to contact Prof. K. Bagi, kbagi.bme@gmail.com.

Defintion:

Masonry is an *artificial* system that consists of individual *solid units* held together either by mortared or dry *contacts*.

masonry units: stone, brick, adobe, ...

contacts: frictional (,,dry")

bonded (lime / cement mortar; gypsum...)

 \rightarrow solid units occupy most volume, mortar: few







Defintion:

Masonry is an *artificial* system that consists of individual *solid units* held together either by mortared or dry *contacts*

\Rightarrow FLEXIBILITY OF FORM!

















THIS COURSE

Part 1/2: METHODS OF THE ANALYSIS

Lecture 01: Introduction & general overview

Lecture 02: Graphic methods

Lecture 03: Shell theories

Lecture 04: Limit state analysis

Lecture 05: The Discrete Element Method

Part 2/2: MAIN TYPES OF ARCHES AND VAULTS

Lecture 07: Arches

Lecture 08: Domes

Lecture 09: Barrels and cross vaults

Lecture 10: Fan vaults

Exam Part 1.

Exam Part 2.

THIS LECTURE: 1. INTRODUCTION & OVERVIEW

Terminology and Material Basics

Basic types of masonry structures special attention to: masonry arches masonry vaults masonry domes

Mechanical analysis of masonry structures most important methods, short overview Limit Analysis theorems

Questions

TERMINOLOGY BASICS

voussoir: a stone (ashlar) block being an element in a masonry wall or vault mortar (cement/lime): material that glues masonry blocks to each other plaster: mortar-like layer that covers the free surface of masonry intrados/extrados: inner/outer surface of a vault or arch crown: keystone of an arch / top line of a vault





STONE MASONRY

 \leftrightarrow

BRICK MASONRY





Marble

Limestone



Sandstone



Granite



mud brick

burnt clay brick

STONE MASONRY

 \leftrightarrow

BRICK MASONRY





Marble

Limestone





Granite



mud brick



burnt clay brick

Masonry units: *stone* or *brick*

or any solid bodies!







STONE MASONRY

 \leftrightarrow

BRICK MASONRY





Marble

Limestone



Sandstone



Granite



mud brick



burnt clay brick





Burial tower, Peru,Erechtheion, Athens, Greece,≈XIct, http://davidprattwww.pinterest.com/iconofile/.info/andes2.htmgreek-temple-models



Siwa Oasis, Egypt, touropia.com/amazingmud-brick-buildings/

Yemen, thevintagenews .com/2016/04/11/cityfull-500-year-oldskyscrapers-made-mud



STONE MASONRY

 \leftrightarrow

BRICK MASONRY





Marble

Limestone



Sandstone



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mud brick



burnt clay brick

Yemen, thevintagenews .com/2016/04/11/cityfull-500-year-oldskyscrapers-made-mud

Shibam, in Yemen: some of them >30 m



BRICK MASONRY

Largest mud brick structure: Djenné Mosque, Mali completed: 1907 16 m tall, 75x75 m groundplan





mud brick

burnt clay brick



Djenné Mosque, Mali, http://naturalhomes.org/img/ great-mosque-djenne.jpg 16/60



https://flexiblelearning.auckland.ac.nz/rocks_minerals/rocks/

BRICK MASONRY

Mud brick: ,,adobe" earliest: Middle East, \approx 7500 BC Egypt, \approx 4000 BC sun-dried \Rightarrow small strength mostly with straw etc. binding mat.

Burnt clay brick: earliest: neolith China, ≈ 4400 BC Indus Valley, ≈ 3000 BC Rome ≈ 25 BC: (sundried) $\rightarrow \underline{\text{fired}}$ wet clay pressed into moulds; dry then burn in kiln *Karaman et al (2006):* \rightarrow

Modern brick materials: concrete / sand lime / fly ash... molded / dry pressed / extruded ...



mud brick

burnt clay brick

Firing temp		Firing time, min			
°C		120	240	360	480
	Compressive strength, MPa				
700	Mean	8.532	8.827	9.022	9.12
	Std. Deviation	0.211	0.288	0.266	0.220
800	Mean	11.7	11.4	11.7	11.9
	Std. Deviation	0.464	0.356	0.380	0.294
900	Mean	15.4	15.5	15.4	16.1
	Std. Deviation	0.535	0.200	0.267	0.200
1000	Mean	22.85	22.66	22.94	23.11
	Std. Deviation	0.700	0.306	0.366	0.238
1100	Mean	31.13	31.2	31.1	31.4
	Std. Deviation	0.048	0.194	0.105	0.294 18 /

VOUSSOIR MATERIAL BASICS STONE MASONRY ↔ BRICK MASONRY

Admissible maximal height of a tower with uniform cross section:



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Basic types for bricks:







Running bond

English bond

Flemish bond



Herringbone pattern

Basic types for stones:





Ashlar masonry ↑↑ *cut into brick-shape*





Rubble masonry ↑ *irregular shape*

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http://www.geograph.org.uk/ article/Brick-bonds

The running bond: "strecher bond"



 \rightarrow simplest to construct; common

→ fields of application: for visible surfaces:



http://waltonsons.com/wp-content/uploads/ 2014/07/UN-27-Serpentine-Wall.jpg

The running bond: "strecher bond"



 \rightarrow simplest to construct; common

→ fields of application: for visible surfaces for cavity walls:



https://www.dailycivil.com/ various-types-walls/

for partitioning walls

for shells:





http://www.natural buildingblog.com/ brick-barrel-vaults/

Brick oven, http://www.stovemaster.com/ html_en/commercial_brick_ oven.html

The running bond: "strecher bond"





shifted running bond



https://brickarchitecture.com/about-brick/why-brick/brickwork-bonds

nice visual appearance

The English bond:



→ from late medieval buildings
 (sometimes named ,,ancient bond")
 → very strong ⇒ applied in main walls
 → ,,THE" typical bond pattern in English buildings
 and then on the American colonies
 → easy to construct

Its variation: English garden bond



- \rightarrow weaker; less durable
- \rightarrow 5-course: "Scotch bond"
- \rightarrow 7-course: "American bond"
- \rightarrow decorative purposes

The Flemish bond:





- → from medieval Central and Northern Europe; from about XVIIth century
- \rightarrow weak but beautiful \Rightarrow external & garden walls
- \rightarrow more difficult to construct
- \rightarrow very popular in US: many historic buildings

Its variation:

Flemish garden bond ("Flemish strecher bond")



→ versions also with more strechers
 → mostly decorative purposes

 (but: hardly in garden walls !!!)
 → popular in the US

The Flemish bond:

https://www.classicist.org/articles/ flemish-bond-a-hallmark-oftraditional-architecture/



earliest American example: XVIIth cnt, St. Luke's church, Virginia



Recent example: XXth cnt., Reveille church, Virginia

Its variation: Flemish garden bond

http://www.ehsmithclayproducts.co. uk/inspirations/details/11/bondingpattern-flemish-garden-wall-bond



The Flemish bond:

Its variation: Flemish garden bond

hardly for gardens ⁽²⁾ but: an example from South London:



http://buildingtheorchard.blogspot.com /2015/01/59-in-flemish-garden.html



The herringbone pattern:



- \rightarrow origin: Moslim architecture
- → most famous application: Brunelleschi's dome in Florence ↑ constructional advantage !



https://traveltomatoes.com/duomoflorence-facts-and-history/





https://www.distilledhistory.com/brunelleschi/

The herringbone pattern:



- \rightarrow origin: Moslim architecture
- → most famous application:
 Brunelleschi's dome in Florence
 ↑ constructional advantage
- → main advantage: picturesque! applied in several Guastavino vaults:







Several other patterns!

A few examples, for larger wall thicknesses:



https://archinect.com/blog/article/150038321/ahistory-of-the-new-york-city-brick-and-how-it-got-laid

Several other patterns!

The core message:

there are innumerous different bond patterns, and the choice of bond pattern has a mechanical effect



shear resistance



Vertical compression and horizontal shear:

Szakaly et al (2014):



Definition of load bearing capacity:











English

Herringbone

X-pattern

V-pattern

Vertical compression and horizontal shear:

Load bearing capacity in terms of the vertical compression magnitude:

Note:

also depends on the length-to-height ratio!











English

Herringbone

X-pattern

V-pattern

Vertical compression and crosswise tension: Chen & Bagi, 2020 Running bond:



ΠИ

Running bond, 1:3 shifted:



Vertical compression and crosswise tension: Herringbone pattern:



large friction: $(\mu \ge 0,5)$



Vertical compression and crosswise tension:

Chen & Bagi, 2020:



Mechanical behaviour, advantages / disadvantages:

MANY OPEN ISSUES !

STONE MASONRY \leftrightarrow

natural material large blocks possible ≈ higher strength large pieces ⇒ may resist some tension higher resistance to weathering too high thermal conductivity more skill and labour needed

manufactured, e.g. mud or burnt clay densely arranged small pieces smaller strength several small pieces ⇒ ≈ no tension less durable better heat insulation less expertise needed to build it

BRICK MASONRY

CONTACTS MATERIAL BASICS

Contacts between the voussoirs: *mortared* or *dry*

→ if *mortared*: smoothened; easier to fit the masonry blocks together low but nonzero tension resistance (often neglected in calculations) low but nonzero shear resistance

→ if *dry*: careful stonecutting is needed; if not proper: surface irregularities \Rightarrow local stress peaks may be present Coulomb friction ($\approx 35^{\circ}-50^{\circ}$) + dilation ($\approx 1^{\circ}-10^{\circ}$)



 \leftrightarrow

MORTAR MATERIAL BASICS

cement mortar (stronger but shrinks more) *lime* mortar (more ductile)

THIS LECTURE: 1. INTRODUCTION & OVERVIEW

Terminology and Material Basics

Basic types of masonry structures

special attention to: masonry arches masonry vaults

masonry domes

Mechanical analysis of masonry structures most important methods, short overview Limit Analysis theorems

Questions

- \rightarrow columns, towers ($\approx 1D$)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes ($\approx 1D, 2D$)

 \rightarrow others: pillars; stairs; dolmens, ...

Masonry arch:

 $\leftarrow Lecture 07$

an upwards curved masonry line, for spanning an opening so that to transmit the downwards loads received from above, to the sides and to the supports

represented by: its middle line

main types: according to the middle line geometry



- \rightarrow columns, towers (\approx 1D)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes (\approx 1D, 2D)
- \rightarrow others: pillars; stairs; dolmens, ...

Masonry arch:

← Lecture 07

an upwards curved masonry line, for spanning an opening so that to transmit the downwards loads received from above, to the sides and to the supports



San Angelo, Rome, 2nd century, https://bridge valleyroad.wordpress.com/stone-bridges/



Aquitaine, France; in metmuseum.org



Lincolnshire, http://www.geog raph.org.uk/photo/678859 42/60

- \rightarrow columns, towers ($\approx 1D$)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes (\approx 1D, 2D)
- \rightarrow others: pillars; stairs; dolmens, ...

Masonry vault:

a curved shell-like masonry construction, for covering a ground area so that to transmit the downwards loads to the sides and to the supports

Represented by:

its middle surface

Classification:

- \rightarrow of the complete vault: according to the continuity of the support
- \rightarrow of its points: according to the middle surface principle curvatures



MOST IMPORTANT MASONRY VAULT TYPES

<u>Classified according to support continuity:</u>

 \rightarrow closed vault:

continuously supported along the boundary of the groundplan

\rightarrow half-open vault:

continuously supported along aportion of groundplan boundary[usually along two opposite walls]

barrel vault

spherical dome

welsh vault (,, underpitched vault)

cloister vault

bulbous

dome

 \rightarrow open vault:

supported only in the groundplan corners



cross vault



oval dome

(,,bohemian vault")



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MOST IMPORTANT MASONRY VAULT TYPES

Points classified according to middle surface principle curvatures:

 \rightarrow elliptical points

 \rightarrow parabolic points

 \rightarrow hyperbolic points:

Repetition from Maths: Principal curvatures

Principal curvatures at a point of a surface:

- \rightarrow draw a straight axis;
- \rightarrow lay a plane along this axis \Rightarrow intersection along a curve;
- \rightarrow rotate the plane and produce these curves:



→ take that two curves having largest / smallest curvature: radii of curvature at the point: R_{min} and R_{max} are received

https://www.youtube.com/watch?v=HUzOPbZk8Pg

Product of the two principal curvatures: "Gaussian curvature"

→ if it is (+) : ,,elliptical point"
→ if it is (-) : ,,hyperbolic point"
→ if it is 0 : ,,parabolic point"



MOST IMPORTANT MASONRY VAULT TYPES

Points classified according to middle surface principle curvatures:

 \rightarrow elliptical points



spherical vaults



oval dome

 \rightarrow parabolic points



barrel vault

 \rightarrow hyperbolic points:

fan vaults



[mechanical consequences]

- \rightarrow columns, towers ($\approx 1D$)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes ($\approx 1D, 2D$)
- \rightarrow others: pillars; stairs; buttresses; dolmens, ...

Masonry vault:

← Lectures 09-10

a curved shell-like masonry construction, for covering a ground area so that to transmit the downwards loads to the sides and to the supports



Great Herod's palace, 1st c, Judea, Israel; gettyimages.co



, Monastery of Santa Maria de Alcobaca, co Portugal, XIIth century, thoughtco.com



Holy Trinity Chantry Chapel, Tewkesbury Abbey, England, 14th c, flickr.com/photos/edk7/1407783359

- \rightarrow columns, towers (\approx 1D)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes ($\approx 1D, 2D$)
- \rightarrow others: pillars; stairs; buttresses; dolmens, ...

Masonry dome:

generally accepted definition does not exist \Rightarrow let's see a few examples:

Florence, dome of

Santa Maria del Fiore (1436)

hemispherical:



St Paul's Cathedral, London (1710)

faceted:





Rome, San Andrea al Quirinale (1670)

- \rightarrow columns, towers ($\approx 1D$)
- \rightarrow walls (\approx 2D)
- \rightarrow arches, vaults, domes ($\approx 1D, 2D$)
- \rightarrow others: pillars; stairs; buttresses; dolmens, ...

Masonry dome:

← Lecture 08

generally accepted definition does not exist

→ ,,closed vault whose middle surface is a surface of rotation of an arc concave-from-below, about a vertical axis which erects from below the arc"

BUT: e.g. oval domes; faceted domes

- \rightarrow ,,closed vault whose points are elliptical, concave from below"
 - BUT: (onion domes) \leftarrow (not concave everywhere)
- → ,,a closed vault whose middle surface is an affine transformation of a spherical cap"
 BUT: irregular domes; faceted domes

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Mechanical analysis of masonry structures

most important methods, short overview Limit Analysis theorems

Questions

Mechanical analysis of masonry structures





- \rightarrow architectural heritage; \rightarrow traffic infrastructure
- -,,Is the structure safe?"
- selfweight
- weight of upfill etc
- support displacements [outwards; + sinkage]
- live loads

Load bearing limits of structures in general:

- \rightarrow internal forces / stresses exceed strength limits of the material
- \rightarrow deformations exceed the admissible deformations
- \rightarrow stability: equilibrium cannot exist, or the equilibrium is unstable

Failure of masonry arches and vaults: mostly a *stability problem*



http://block.arch.ethz.ch/brg/research/collapse-ofmasonry-structures 52 / 60

Mechanical analysis of masonry structures

Traditionally: geometric design rules based on trial-and-error experiences

XIXth century: graphostatics (force polygons & lines of action)

XXth century: continuum shell theories; mostly membrane analysis

Limit analysis of masonry structures:

J. Heyman, from the 1960ies

→ The *static* (,,Safe") theorem
→ The kinematic theorem
(,,Unsafe" or ,,Energy" theorem)

With the given geometry,can the structure carry the loads or it will collapse?

Nonlinear FEM



Problems about FEM: Why do we need other methods?





Drawbacks of FEM:

- \rightarrow initial geometry and the loads have to be known: often impossible
- \rightarrow extreme sensitivity to small perturbations of the geometry
- \rightarrow history dependence: the existing forces in the structure have to be known
- → contact separation; contact sliding; (can be done with contact elements, though difficult); BUT: partial separation or reversible sliding or occurence of new contacts cannot be taken into account !!!

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Limit analysis of masonry structures

Jacques Heyman, from the 1960s:

The task considered:





 \rightarrow assembly of *rigid blocks* and *dry contacts*;

- \rightarrow given geometry, given dead load AND/OR one-parameter live load;
- → Can the structure (with the given geometry) *balance the given load*?
 OR: for *what range* of the load parameter will the structure stand?
- \rightarrow simplifying assumptions:
 - (1): no tension resistance in the contacts
 - (2): blocks have infinite strength
 - (3): sliding does not occur
 - (+1): [implicitly also assumed:]

geometry remains the same for any load (infinite stiffness)

Limit analysis of masonry structures

The static theorem for masonry structures:

If a force system can be found for the given set of external loads which satisfies the material criteria and equilibrates the given external loads, then the structure with the given geometry is safe under these loads.

The kinematic theorem for masonry structures:

- If a mechanism (a virtual displacement system) can be found for the given set of external loads which satisfies the material criteria and produces non-negative work with the given external loads, then the structure will collapse under these loads.
- \rightarrow these theorems were given (Heyman, 1966) without proof
- \rightarrow limited validity
- \rightarrow details, limitations and applications will be given in Lecture 04
- \rightarrow but still unconciously applied: Lectures 02 03

THE DISCRETE ELEMENT METHOD

Definition: a numerical method belongs to DEM if

- \leftarrow it consists of separate, finite-sized elements and their contacts
- \leftarrow its elements have independent degrees of frredom, with large displs
- \leftarrow contact separation and sliding considered; new contacts can be born

Main steps:define the elements (geometry);
automatically recognize their contactsspecify the material parameters (for elements; for contacts)
loading history: in small steps

Detailed introduction to DEM: will be given in Lecture 05







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QUESTIONS

- 1. Define ,,arch", ,,vault" and ,,dome". Define ,,closed vault", ,,half-open vault" and ,,open vault", and give examples for each type.
- 2. Recognize the following bond patterns: running / English / Flemish bond, herringbone pattern, ashlar masonry, rubble masonry
- 3. Recognize the following arch types: corbel, semicircular, pointed, segmental arch
- 4. Recognize the following vault types: barrel / cross / cloister vault, underpitched / sail / fan vault, spherical dome, oval dome
- 5. What were Heyman's simplifying assumptions? What does the Static Theorem state?







