MECHANICS OF MASONRY STRUCTURES

Citation:

K. Bagi (2018): Mechanics of Masonry Structures. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

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MASONRY STRUCTURES

Defintion:

Masonry is an ***artificial*** system that consists of individual ***solid units*** held together either by mortared or dry ***contacts.***

masonry units: stone, brick, adobe, …

contacts: frictional („dry”)

bonded (lime / cement mortar; gypsum…)

→ solid units occupy most volume, mortar: few

MASONRY STRUCTURES

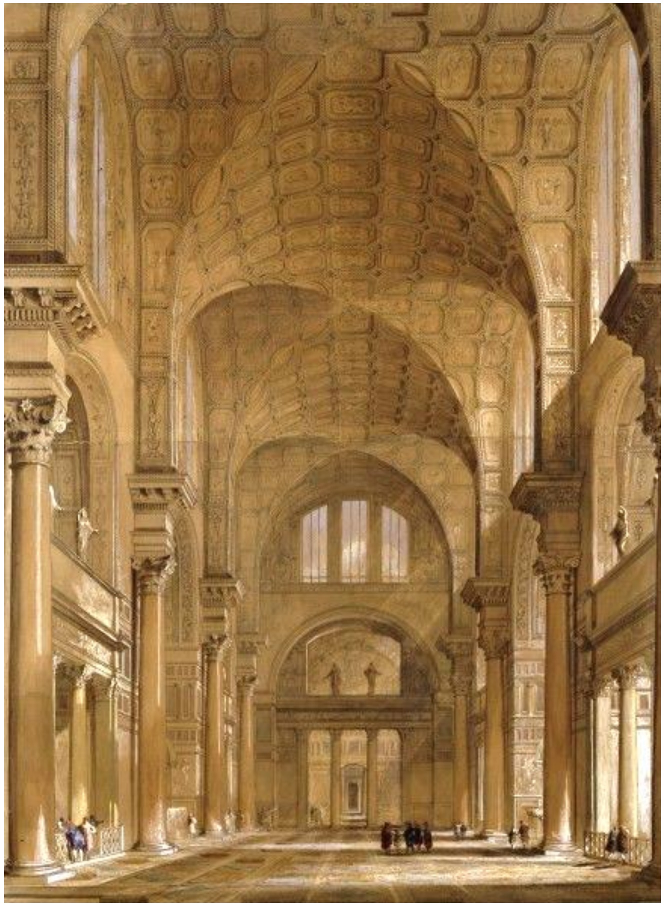
Defintion:

Masonry is an ***artificial*** system that consists of individual ***solid units*** held together either by mortared or dry ***contacts***

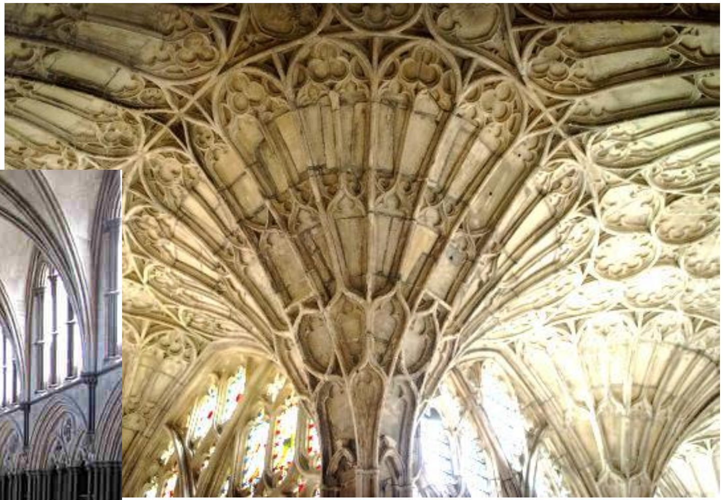
$\Rightarrow$ FLEXIBILITY OF FORM!

MASONRY STRUCTURES

MASONRY STRUCTURES



MASONRY STRUCTURES



MASONRY STRUCTURES

This Course

Part 1/2:

Lecture 01: Introduction & general overview

Lecture 02: Graphic methods

Lecture 03: Shell theories

Lecture 04: Limit state analysis

+ invited lecture: software introduction

Lecture 05: The Discrete Element Method

} Exam Part 1.

Part 2/2:

Lecture 07: Arches

Lecture 08: Domes

Lecture 09: Barrels and cross vaults

Lecture 10: Fan vaults

} Exam Part 2.

This Lecture: Introduction & Overview

Terminology and Material Basics

Basic types of masonry structures

special attention to: masonry arches

masonry vaults

masonry domes

Mechanical analysis of masonry structures

most important methods, short overview

Limit Analysis theorems

Questions

TERMINOLOGY BASICS

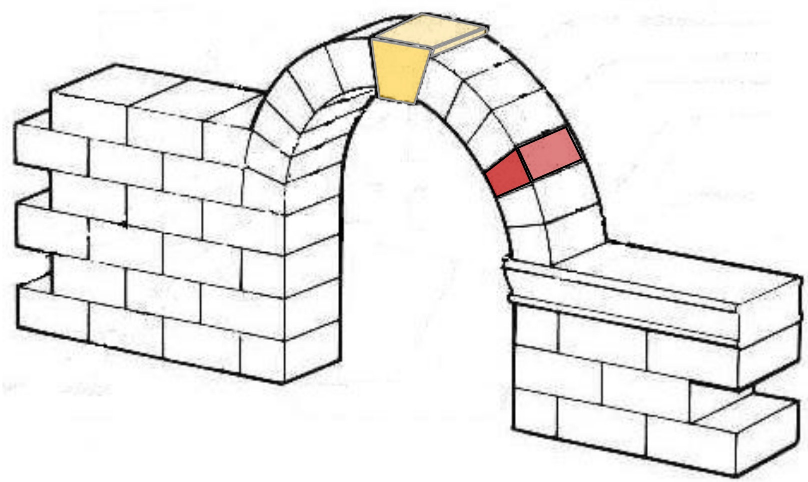
voussoir: a stone (ashlar) block being an element in a masonry wall or vault

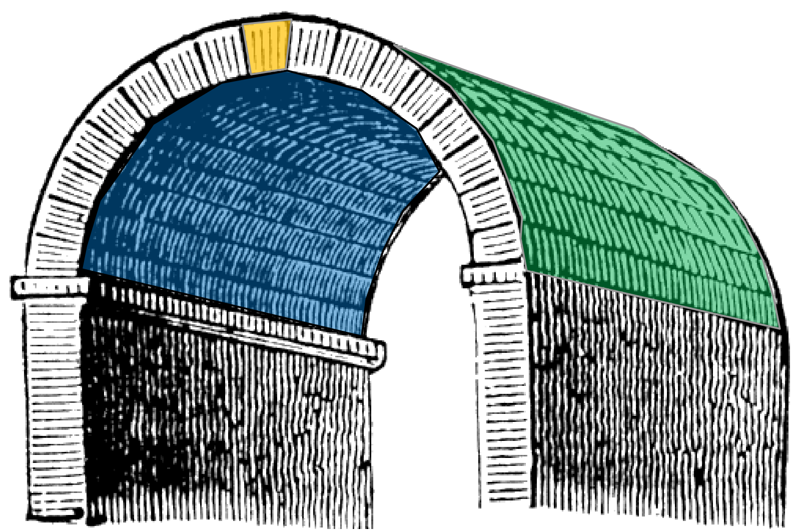
mortar/grout (cement/lime): material that glues masonry blocks to each other

plaster: mortar-like layer that covers the free surface of masonry

intrados/extrados: inner/outer surface of a vault or arch

crown: keystone of an arch / top line of a vault





VOUSSOIR MATERIAL BASICS

STONE MASONRY ↔ BRICK MASONRY

Marble

Limestone

Sandstone

Granite

mud brick

burnt clay brick

VOUSSOIR MATERIAL BASICS

STONE MASONRY ↔ BRICK MASONRY


Marble


Limestone


Sandstone

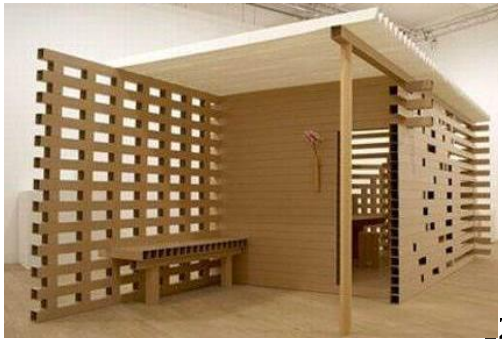

Granite


mud brick


burnt clay brick

Masonry units: ***stone*** or ***brick***

or any solid bodies!



VOUSSOIR MATERIAL BASICS

STONE MASONRY ↔ BRICK MASONRY


Marble


Limestone


Sandstone

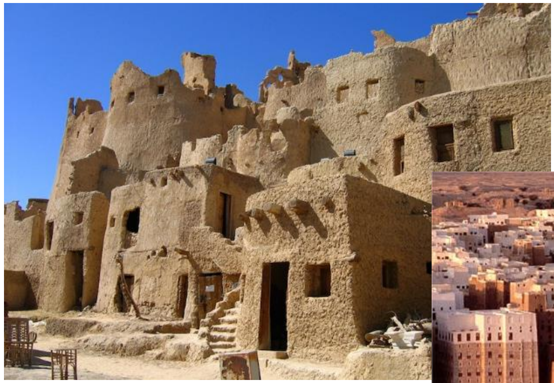

Granite


mud brick


burnt clay brick


Burial tower, Peru, ≈XIct, http://davidpratt .info/andes2.htm


Erechtheion, Athens, Greece, www.pinterest.com/iconofile/ greek-temple-models


Siwa Oasis, Egypt, touropia.com/amazing-mud-brick-buildings/

Yemen, thevintagenews .com/ 2016/ 04/11/city-full-500-year-old-skyscrapers-made-mud


VOUSSOIR MATERIAL BASICS

BRICK MASONRY

mud brick

burnt clay brick

Largest mud brick structure:
Djenné Mosque, Mali (1907)

Djenné Mosque, Mali, http://naturalhomes.org/img/great-mosque-djenne.jpg

VOUSSOIR MATERIAL BASICS

STONE MASONRY

Marble

Limestone

Sandstone

Granite

	origin	surface friction	Young modulus GPa	Poisson ratio	compressive strength MPa
Sandstone	sedimentary	0.6 - 0.9	5-20	0.2-0.4	20-150
Limestone	sedimentary		15-55	0.2-0.33	30-150
Granite	igneous		20-70	0.1-0.3	150-300
Marble	metamorphic		50-70	0.06-0.25	40-100
			(*steel: 200*)	(*steel: 0.3*)	(*steel: 250*)

https://flexiblelearning.auckland.ac.nz/rocks_minerals/rocks/

VOUSSOIR MATERIAL BASICS

BRICK MASONRY

Mud brick: „adobe”

earliest: Middle East, ≈ 7500 BC

Egypt, ≈ 4000 BC

sun-dried ⇒ small strength

mostly with straw etc. binding mat.

Burnt clay brick:

earliest: neolith China, ≈ 4400 BC

Indus Valley, ≈ 3000 BC

Rome ≈25 BC: (sundried) → fired

wet clay pressed into moulds;

dry then burn in kiln

Karaman et al (2006): →

Modern brick materials:

concrete / sand lime / fly ash…

molded / dry pressed / extruded …

mud brick

burnt clay brick

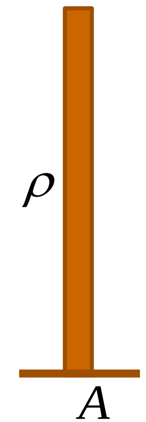
Firing temp °C		Firing time, min 120	240	360	480
		Compressive strength, MPa			
700	Mean	8.532	8.827	9.022	9.12
	Std. Deviation	0.211	0.288	0.266	0.220
800	Mean	11.7	11.4	11.7	11.9
	Std. Deviation	0.464	0.356	0.380	0.294
900	Mean	15.4	15.5	15.4	16.1
	Std. Deviation	0.535	0.200	0.267	0.200
1000	Mean	22.85	22.66	22.94	23.11
	Std. Deviation	0.700	0.306	0.366	0.238
1100	Mean	31.13	31.2	31.1	31.4
	Std. Deviation	0.048	0.194	0.105	0.29[illegible]

VOUSSOIR MATERIAL BASICS

STONE MASONRY ↔ BRICK MASONRY

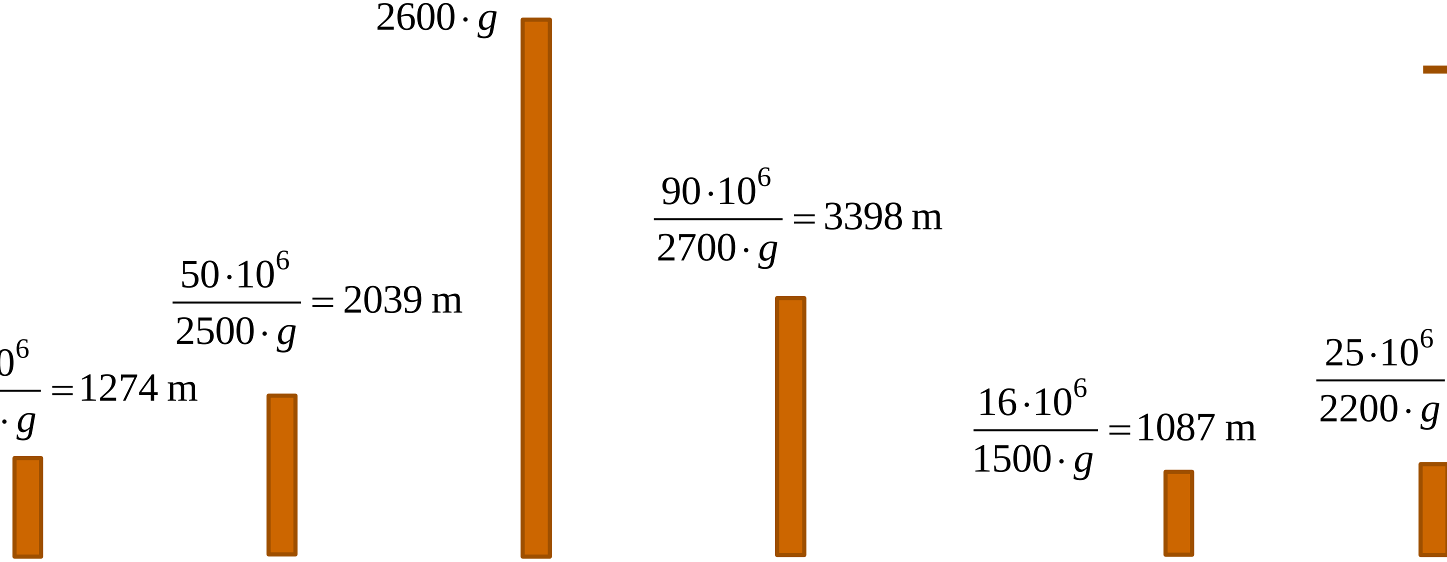
Admissible maximal height of a tower with uniform cross section:

$$\sigma_{bottom} = \frac{A \cdot h \cdot (\rho \cdot g)}{A} \leq \sigma_c^{failure} \quad \Rightarrow \quad h \leq \frac{\sigma_c^{failure}}{\rho \cdot g}$$



$$\frac{30 \cdot 10^6}{2400 \cdot g} = 1274 \text{ m} \qquad \frac{50 \cdot 10^6}{2500 \cdot g} = 2039 \text{ m} \qquad \frac{180 \cdot 10^6}{2600 \cdot g} = 7057 \text{ m} \qquad \frac{90 \cdot 10^6}{2700 \cdot g} = 3398 \text{ m}$$

$$\frac{16 \cdot 10^6}{1500 \cdot g} = 1087 \text{ m} \qquad \frac{25 \cdot 10^6}{2200 \cdot g} = 1158 \text{ m}$$

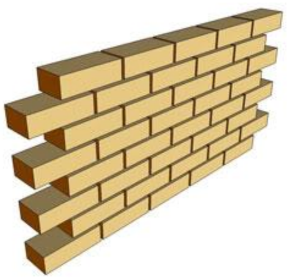


sandstone limestone granite marble mud brick fired clay brick

MASONRY BOND PATTERNS

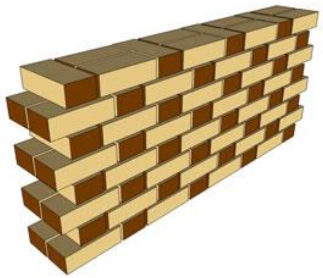
Basic types for bricks:

http://www.geograph.org.uk/ article/Brick-bonds



Running bond

English bond

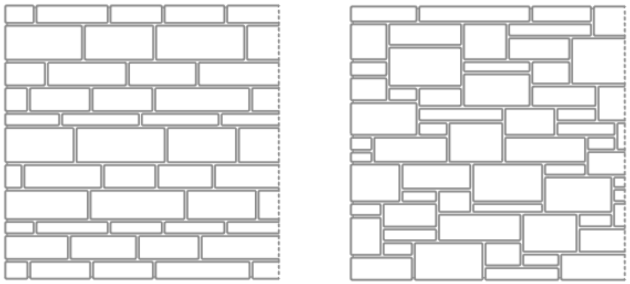


Flemish bond

Herringbone pattern

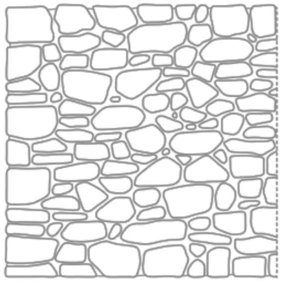
Basic types for stones:

https://www.dimensions.guide/ collection/stone-masonry



Ashlar masonry

↑↑ *cut into brick-shape*

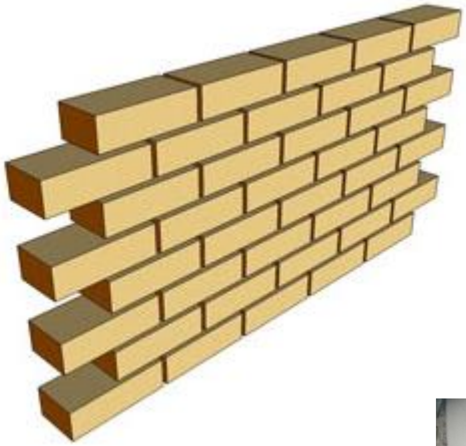


Rubble masonry

↑↑ *irregular shape*

MASONRY BOND PATTERNS

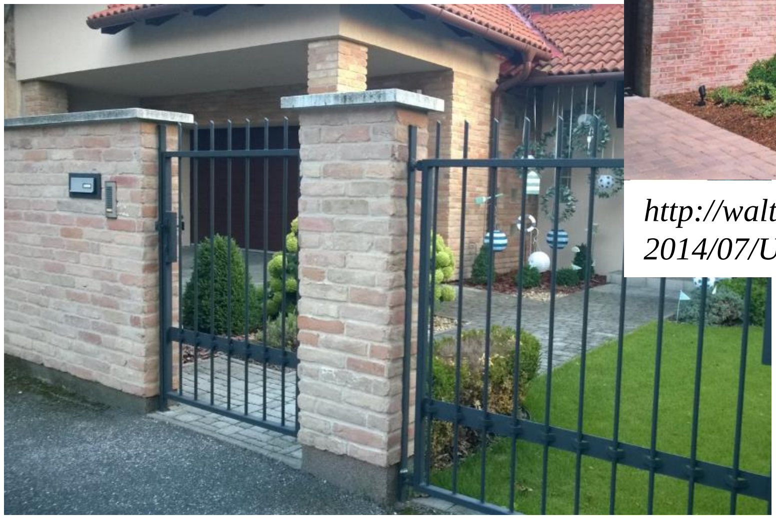
The running bond: „strecher bond”



→ simplest to construct; common

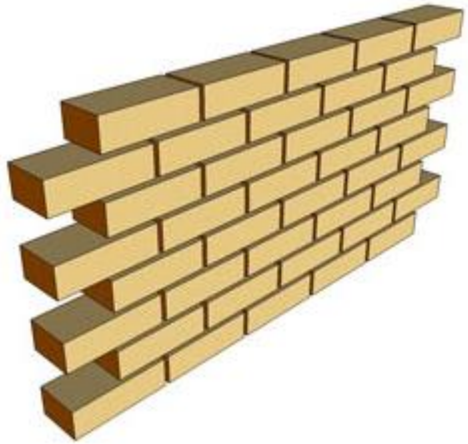
→ fields of application:
for visible surfaces:

http://waltonsons.com/wp-content/uploads/2014/07/UN-27-Serpentine-Wall.jpg



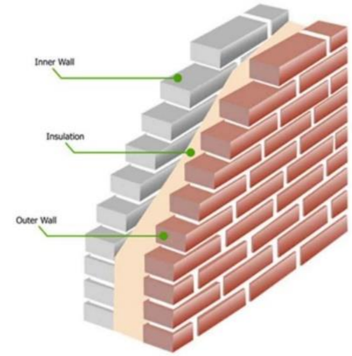
MASONRY BOND PATTERNS

The running bond: „strecher bond”



→ simplest to construct; common

→ fields of application:
for visible surfaces
for cavity walls:



https://www.dailycivil.com/ various-types-walls/

for partitioning walls

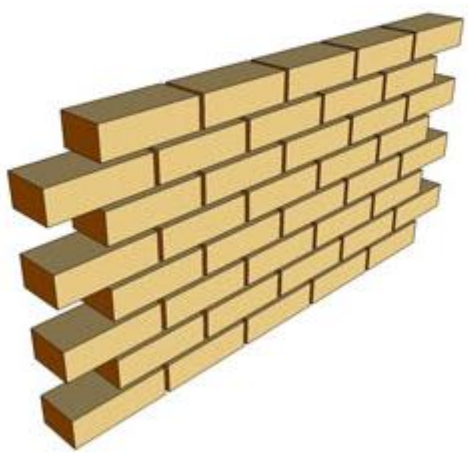
for shells:

Brick oven, http://www.stovemaster.com/ html_en/commercial_brick_ oven.html

http://www.natural buildingblog.com/ brick-barrel-vaults/

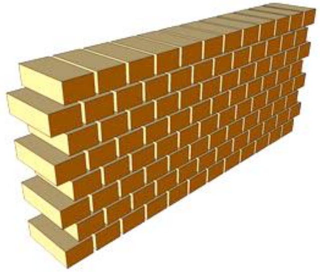
MASONRY BOND PATTERNS

The running bond: „strecher bond”



Its variations:

header bond



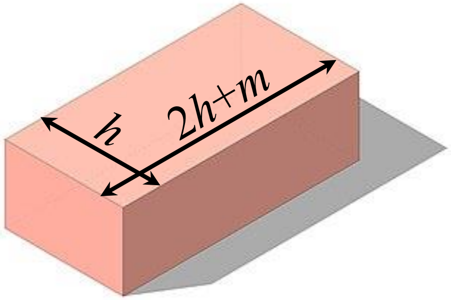
double thickness

shifted running bond

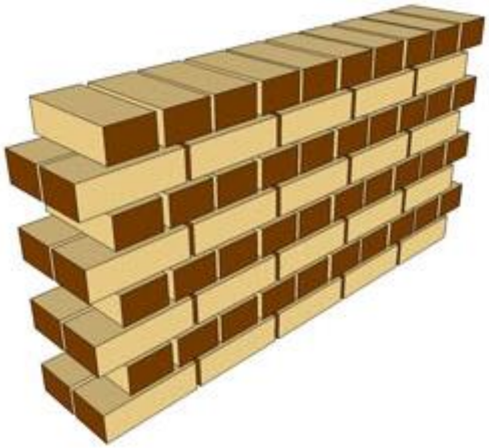
https://brickarchitecture.com/about-brick/why-brick/brickwork-bonds

nice visual appearance

MASONRY BOND PATTERNS



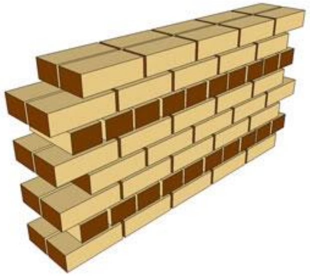
The English bond:



→ from late medieval buildings
(sometimes named „ancient bond”)
→ very strong ⇒ applied in main walls
→ „THE” typical bond pattern in English buildings
and then on the American colonies
→ easy to construct

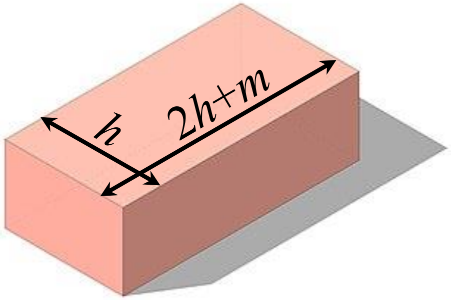
Its variation:

English garden bond

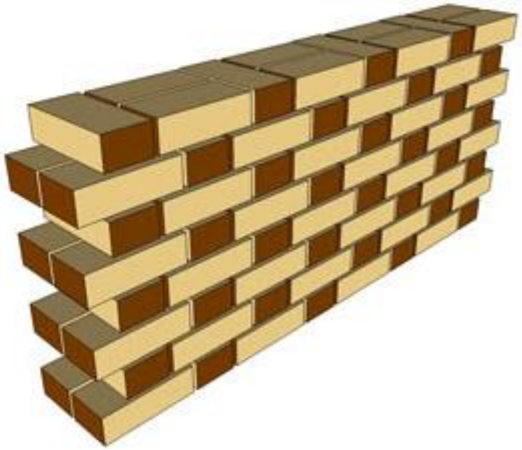


→ weaker; less durable
→ 5-course: „Scotch bond”
→ 7-course: „American bond”
→ decorative purposes

MASONRY BOND PATTERNS



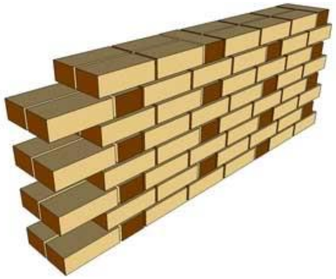
The Flemish bond:



→ from medieval Central and Northern Europe; from about XVIIth century
→ weak but beautiful ⇒ external & garden walls
→ more difficult to construct
→ very popular in US: many historic buildings

Its variation:

Flemish garden bond („Flemish strecher bond")



→ versions also with more strechers
→ mostly decorative purposes (but: hardly in garden walls !!!)
→ popular in the US

MASONRY BOND PATTERNS

The Flemish bond:

https://www.classicist.org/articles/flemish-bond-a-hallmark-of-traditional-architecture/

earliest American example: XVIIth cnt, St. Luke's church, Virginia

Recent example: XXth cnt., Reveille church, Virginia

Its variation:

Flemish garden bond

http://www.ehsmithclayproducts.co.uk/inspirations/details/11/bonding-pattern-flemish-garden-wall-bond

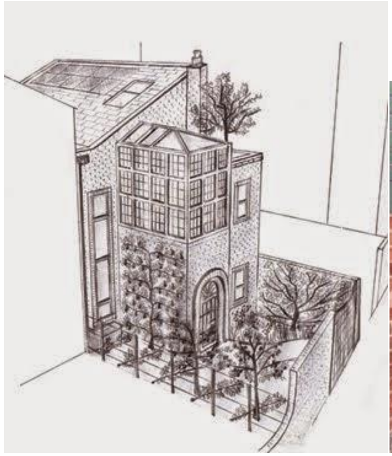
MASONRY BOND PATTERNS

The Flemish bond:

Its variation:

Flemish garden bond

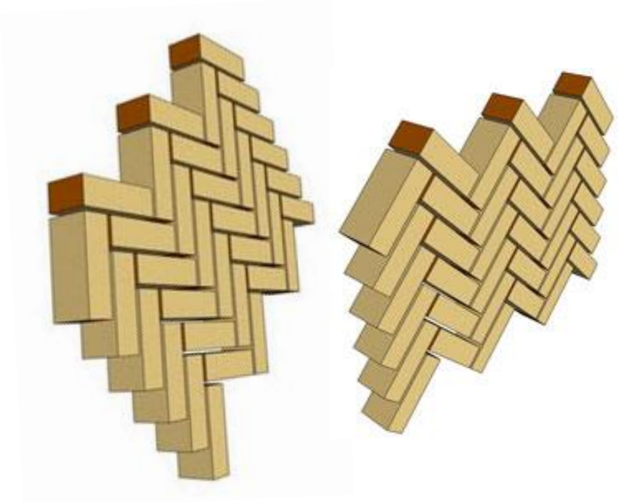
hardly for gardens ☺ but: an example from South London:



http://buildingtheorchard.blogspot.com/2015/01/59-in-flemish-garden.html

MASONRY BOND PATTERNS

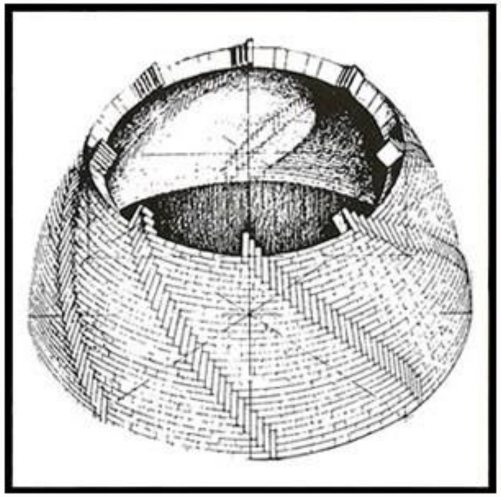
The herringbone pattern:



→ origin: Moslim architecture

→ most famous application:
Brunelleschi's dome in Florence
↑ constructional advantage !

https://traveltomatoes.com/duomo-florence-facts-and-history/



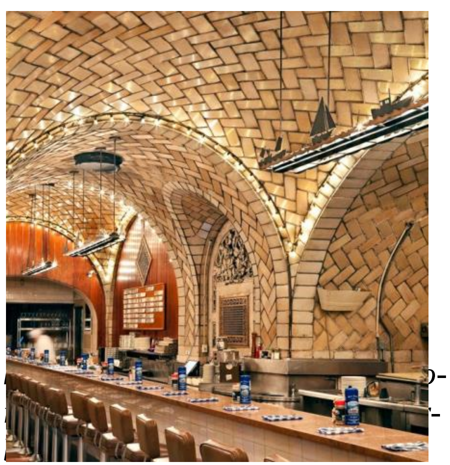
https://www.distilledhistory.com/brunelleschi/

MASONRY BOND PATTERNS

The herringbone pattern:

→ origin: Moslim architecture
→ most famous application:
Brunelleschi's dome in Florence
↑ constructional advantage

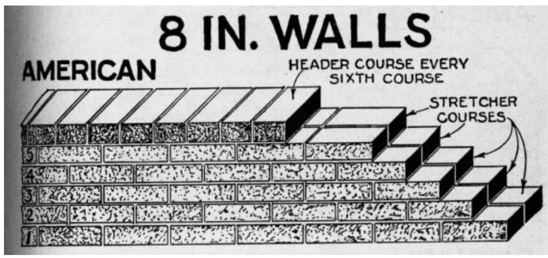
→ main advantage: picturesque!
applied in several Guastavino vaults:

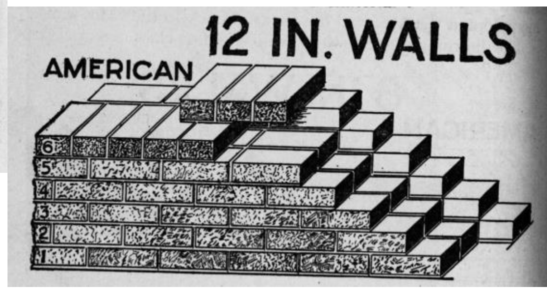


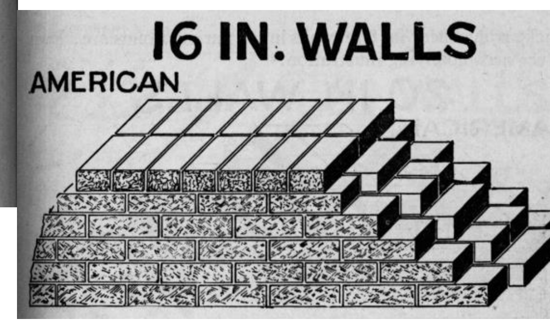
MASONRY BOND PATTERNS

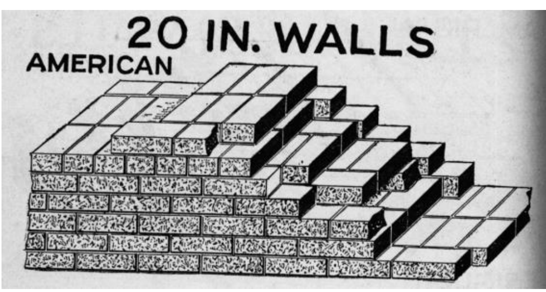
Several other patterns!

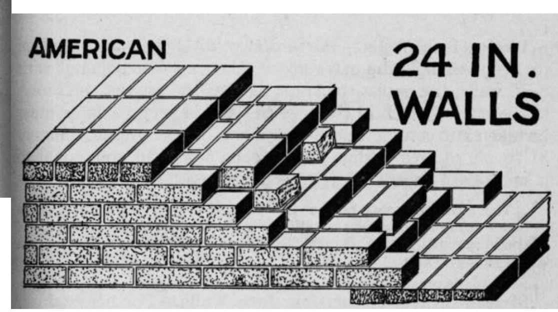
A few examples, for larger wall thicknesses:











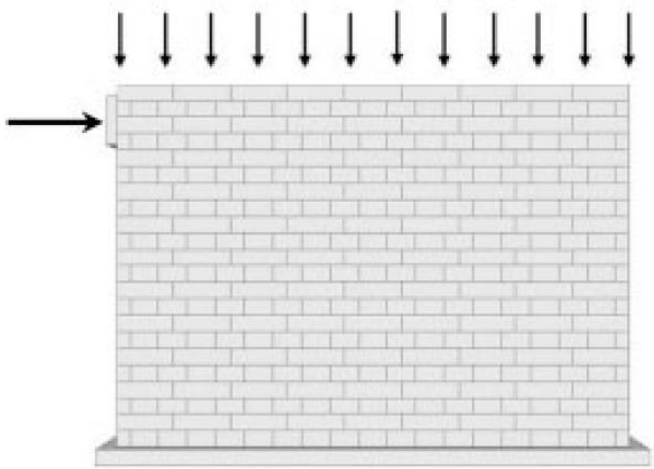
https://archinect.com/blog/article/150038321/a-history-of-the-new-york-city-brick-and-how-it-got-laid

MASONRY BOND PATTERNS

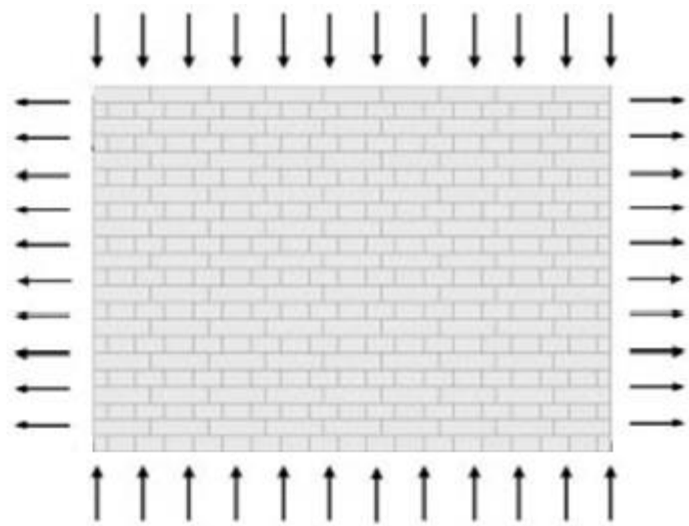
Several other patterns!

The core message:

there are innumerous different bond patterns, and
the choice of bond pattern has a mechanical effect



shear resistance

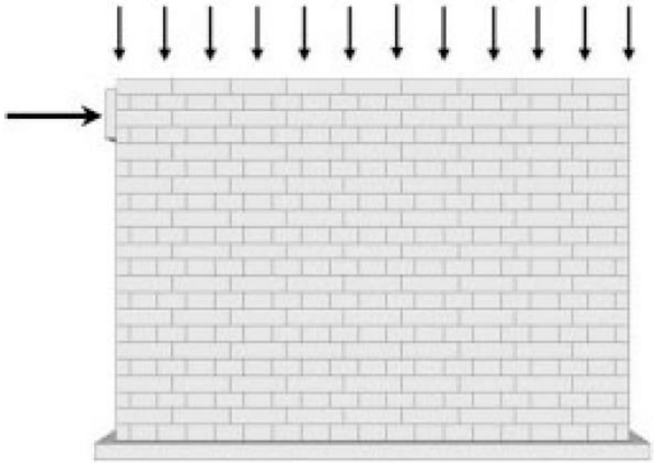


crosswise tensile resistance

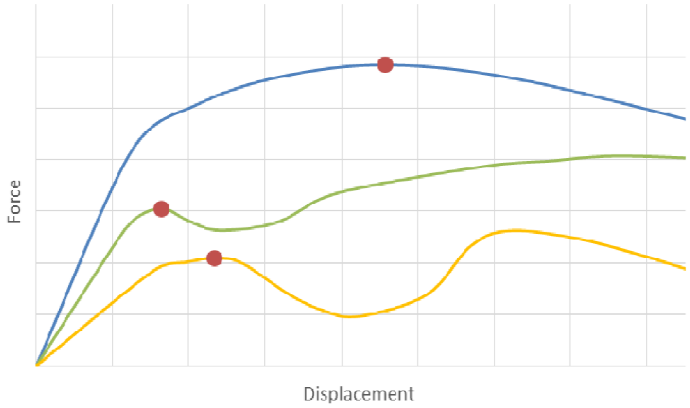
MASONRY BOND PATTERNS

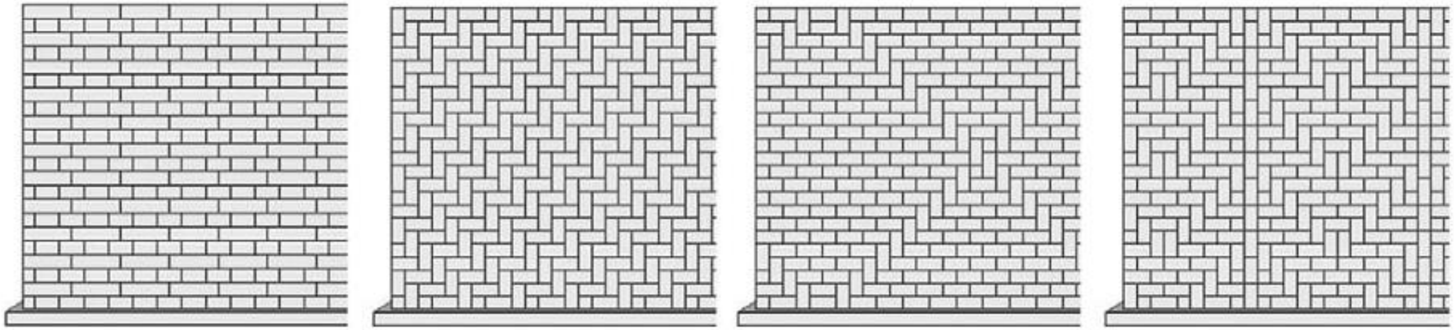
Vertical compression and horizontal shear:

Szakaly et al (2014):



Definition of load bearing capacity:

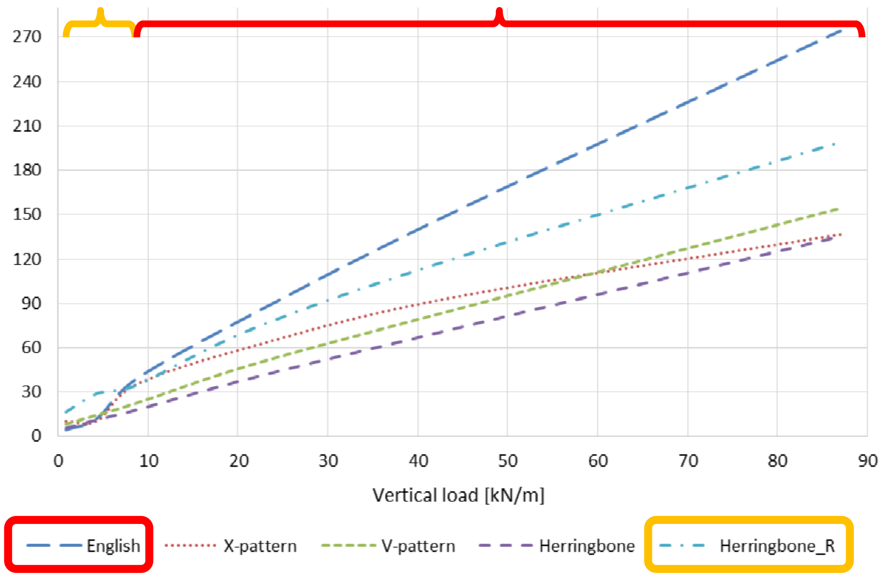




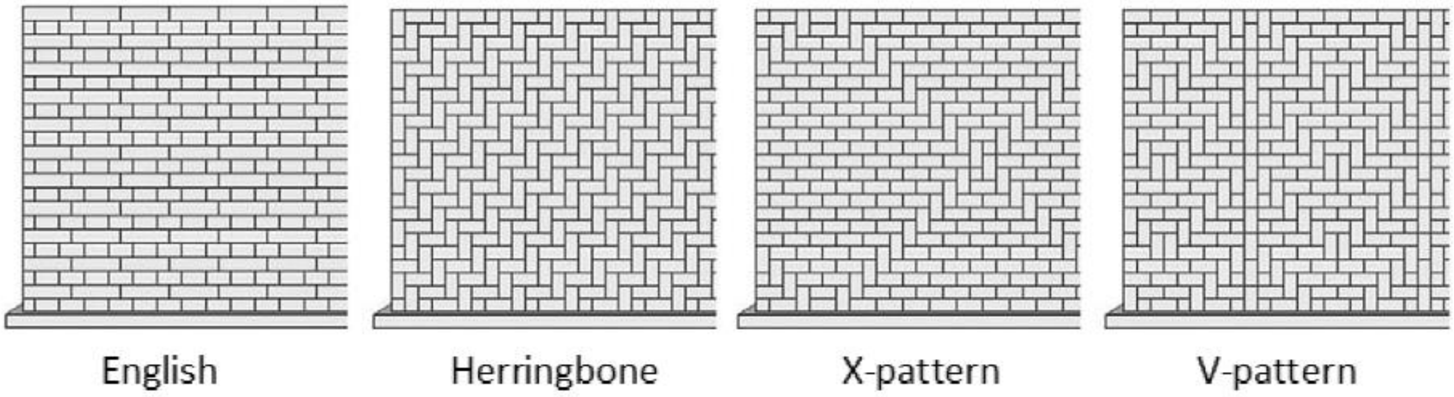
MASONRY BOND PATTERNS

Vertical compression and horizontal shear:

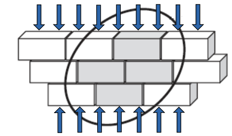
Load bearing capacity
in terms of the vertical
compression magnitude:

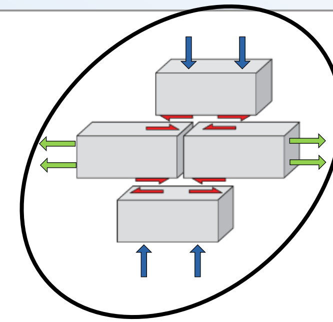


Note:
also depends on the
length-to-height ratio!



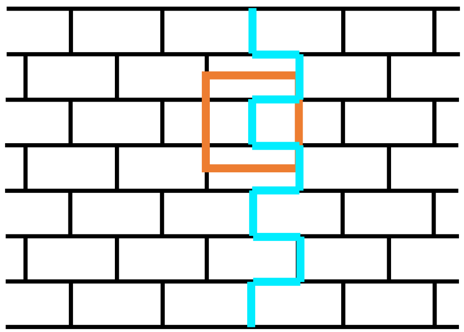
MASONRY BOND PATTERNS

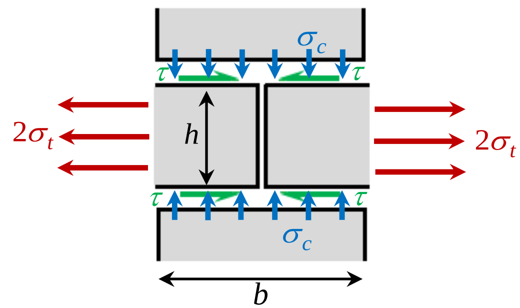


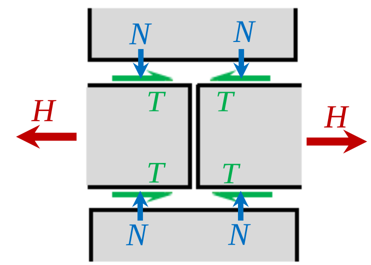


Vertical compression and crosswise tension: Chen & Bagi, 2020

Running bond:

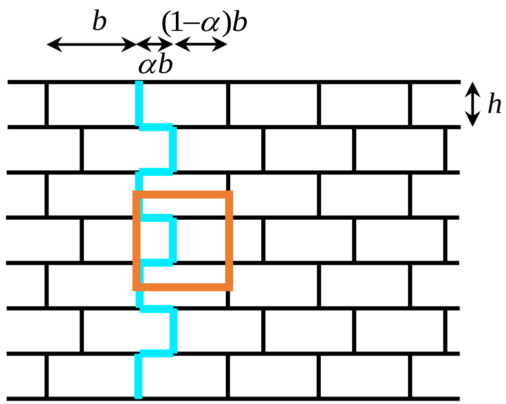


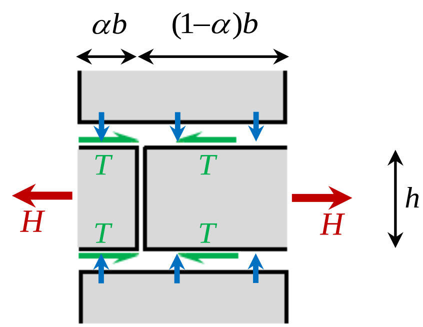




$$\sigma_t \leq \mu \cdot \frac{b}{2h} \cdot \sigma_c$$

Running bond, 1:3 shifted:



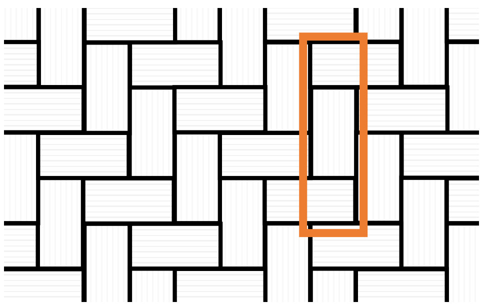


$$\sigma_t \leq \mu \cdot \frac{\alpha \cdot b}{h} \sigma_c$$

MASONRY BOND PATTERNS

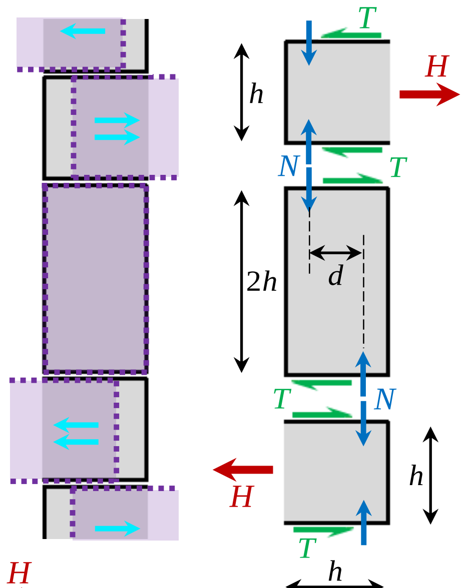
Vertical compression and crosswise tension:

Herringbone pattern:



small friction:

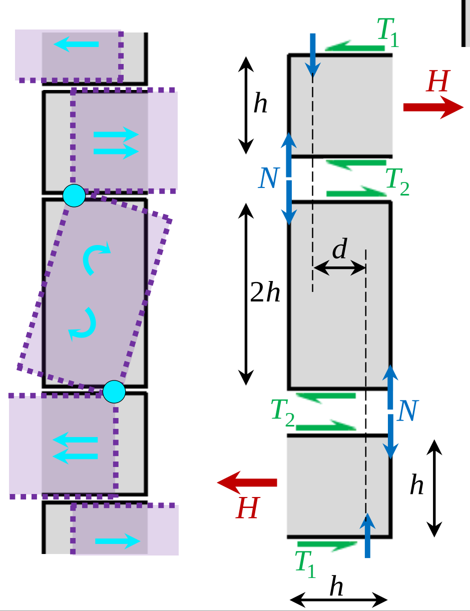
$(\mu \leq 0{,}5)$



$$\sigma_t \leq \frac{1}{2}\mu \cdot \sigma_c$$

large friction:

$(\mu \geq 0{,}5)$

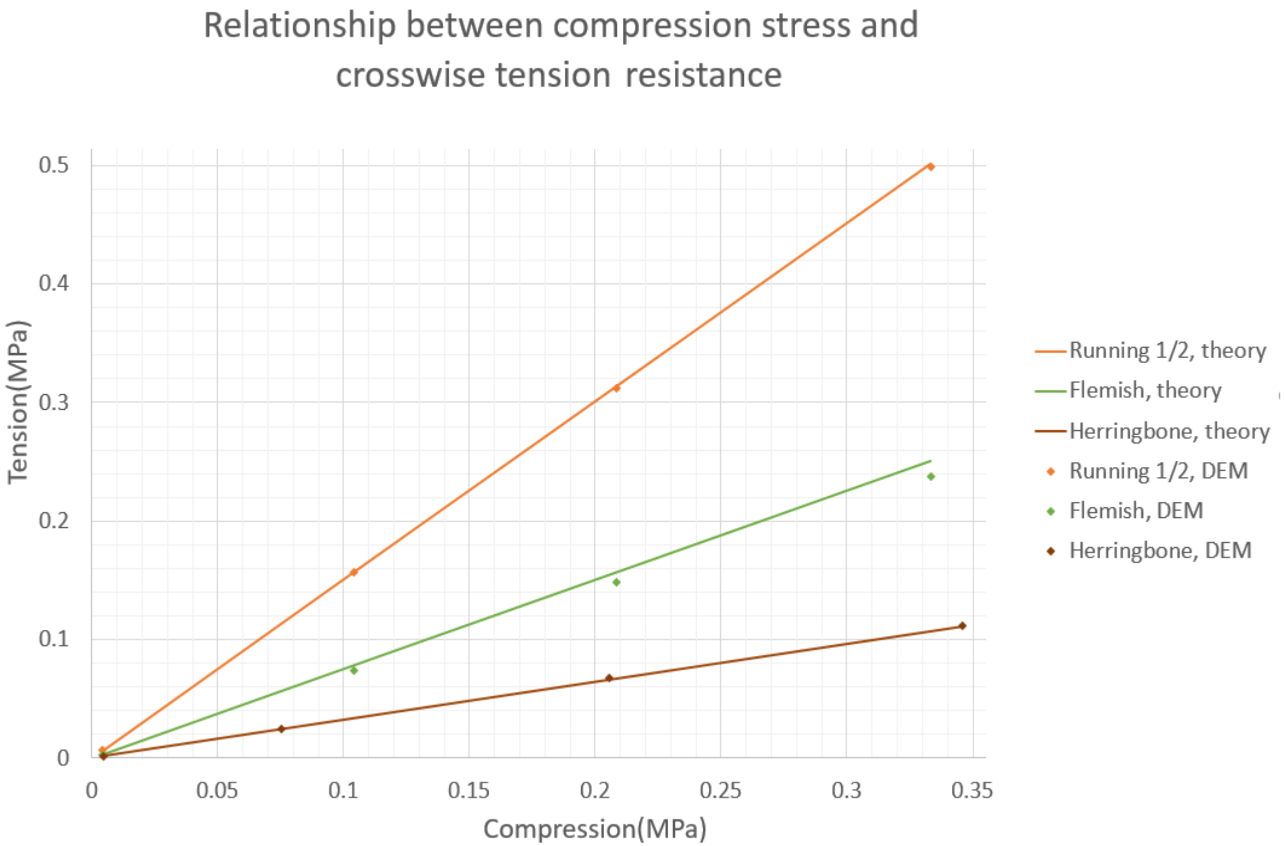


$$\sigma_t \leq \frac{0{,}5+\mu}{4} \cdot \sigma_c$$

MASONRY BOND PATTERNS

Vertical compression and crosswise tension:

Chen & Bagi, 2020:



MASONRY BOND PATTERNS

Mechanical behaviour, advantages / disadvantages:

MANY OPEN ISSUES !

VOUSSOIR MATERIAL BASICS

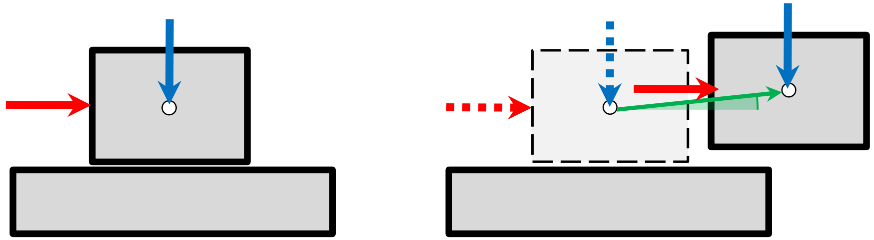
STONE MASONRY	↔	BRICK MASONRY
natural material		manufactured, e.g. mud or burnt clay
large blocks possible		densely arranged small pieces
$\approx$ higher strength		smaller strength
large pieces $\Rightarrow$ may resist some tension		several small pieces $\Rightarrow \approx$ no tension
higher resistance to weathering		less durable
too high thermal conductivity		better heat insulation
more skill and labour needed		less expertise needed to build it

CONTACTS MATERIAL BASICS

Contacts between the voussoirs: ***mortared*** or ***dry***

→ if ***mortared***: smoothened; easier to fit the masonry blocks together
low but nonzero tension resistance
(often neglected in calculations)
low but nonzero shear resistance

→ if ***dry***: careful stonecutting is needed; if not proper:
surface irregularities ⇒ local stress peaks may be present
Coulomb friction (≈ 35°–50°) + dilation (≈ 1°–10°)



MORTAR MATERIAL BASICS

cement mortar (stronger but shrinks more) ↔ ***lime*** mortar (more ductile)

THIS LECTURE:
INTRODUCTION & OVERVIEW

Terminology and Material Basics

Basic types of masonry structures

special attention to: masonry arches

masonry vaults

masonry domes

Mechanical analysis of masonry structures

most important methods, short overview

Limit Analysis theorems

Questions

BASIC TYPES OF MASONRY STRUCTURES

→ columns, towers (≈1D)

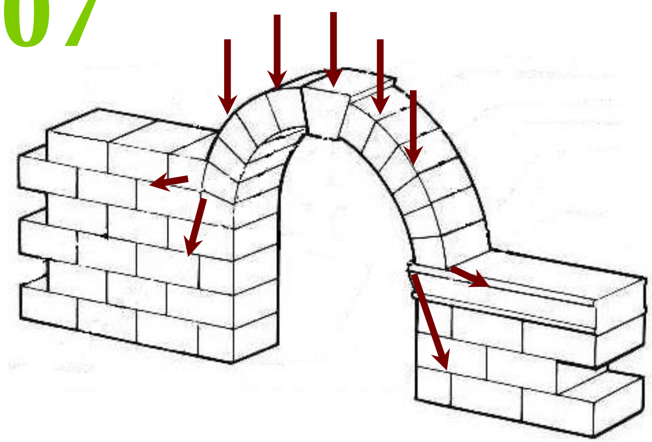
→ walls (≈2D)

→ arches, vaults, domes (≈1D, 2D)

→ others: pillars; stairs; dolmens, …

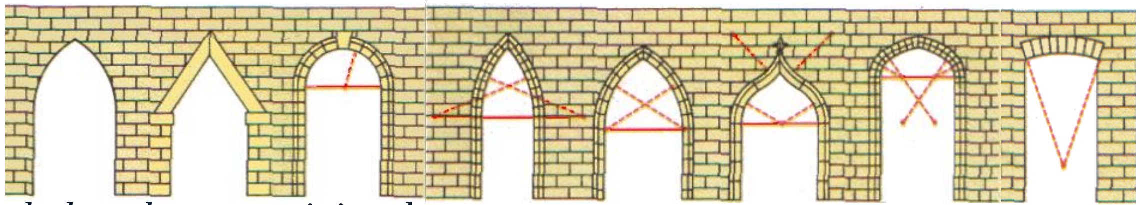
Masonry arch: ← **Lecture 07**

an upwards curved masonry line, for spanning an opening so that to transmit the downwards loads received from above, to the sides and to the supports



represented by: its middle line

main types: according to the middle line geometry



corbel arch *semicircular* *pointed* *ogee* *tudor* *segmental*

BASIC TYPES OF MASONRY STRUCTURES

- → columns, towers (≈1D)
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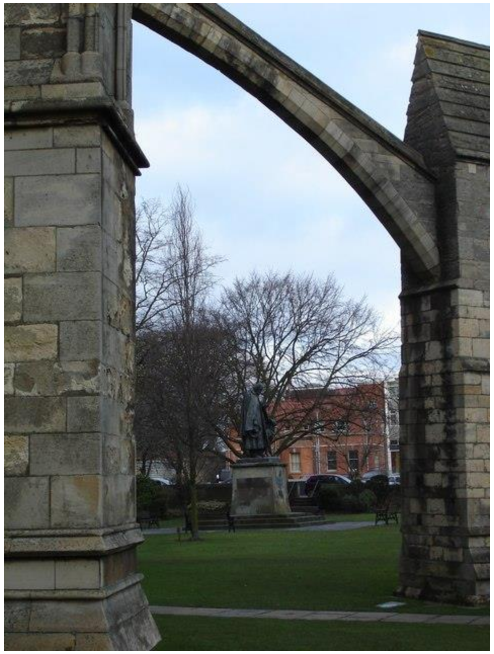
- → others: pillars; stairs; dolmens, …

Masonry arch: ← **Lecture 07**

an upwards curved masonry line, for spanning an opening so that to transmit the downwards loads received from above, to the sides and to the supports

San Angelo, Rome, 2nd century, https://bridgevalleyroad.wordpress.com/stone-bridges/

Aquitaine, France; in metmuseum.org



Lincolnshire, http://www.geograph.org.uk/photo/678859

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→ columns, towers (≈1D)

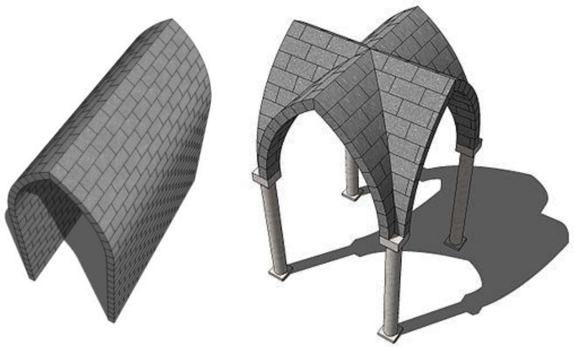
→ walls (≈2D)

→ arches, vaults, domes (≈1D, 2D)

→ others: pillars; stairs; buttresses; dolmens, …

Masonry vault:

a curved shell-like masonry construction, for covering a ground area so that to transmit the downwards loads to the sides and to the supports



Represented by:

its middle surface

Classification:

→ of the complete vault: according to the continuity of the support

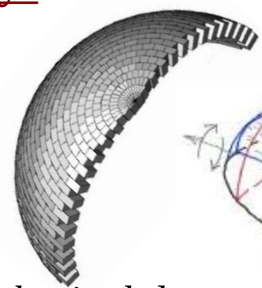
→ of its points: according to the middle surface principle curvatures

MOST IMPORTANT MASONRY VAULT TYPES

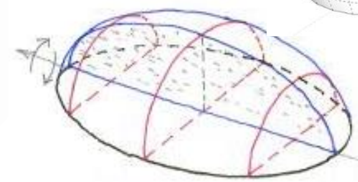
Classified according to support continuity:

→ closed vault:

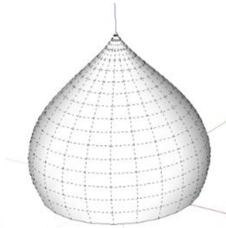
continuously supported along the boundary of the groundplan



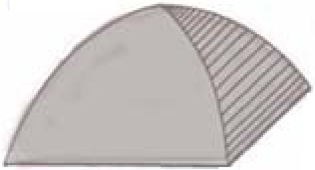
spherical dome



oval dome



bulbous dome

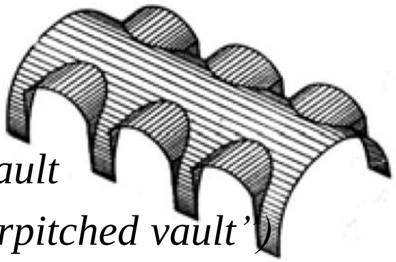


cloister vault

→ half-open vault:

continuously supported along a portion of groundplan boundary [usually along two opposite walls]

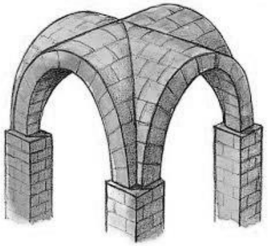
barrel vault



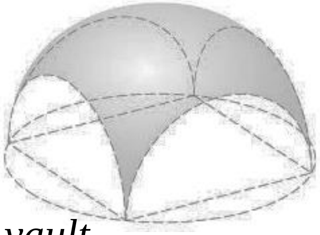
welsh vault („underpitched vault")

→ open vault:

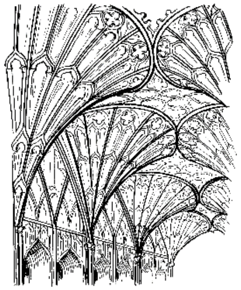
supported only in the groundplan corners



cross vault



sail vault („bohemian vault")



fan vault

MOST IMPORTANT MASONRY VAULT TYPES

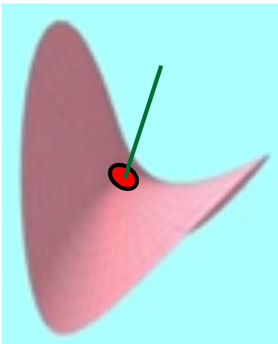
Points classified according to middle surface principle curvatures:

→ elliptical points

→ parabolic points

→ hyperbolic points:

Repetition from Maths: Principal curvatures

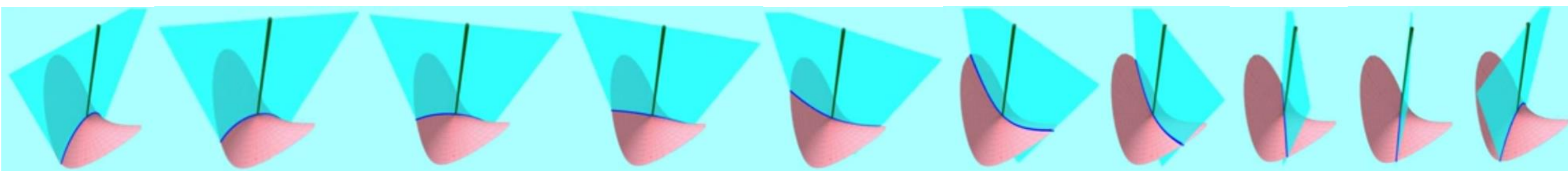


Principal curvatures at a point of a surface:

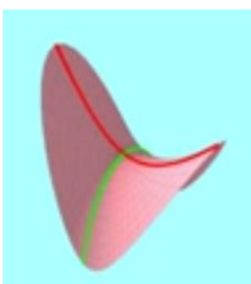
→ draw a straight axis;

→ lay a plane along this axis ⇒ intersection along a curve;

→ rotate the plane and produce these curves:



https://www.youtube.com/watch?v=HUzOPbZk8Pg



→ take that two curves having largest / smallest curvature:

radii of curvature at the point:

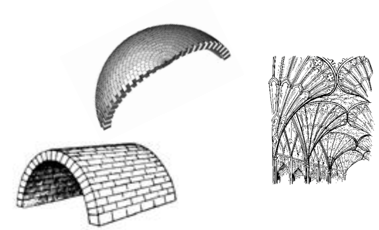
R_{min} and R_{max} are received

Product of the two principal curvatures: „Gaussian curvature”

→ if it is (+) : „elliptical point”

→ if it is (–) : „hyperbolic point”

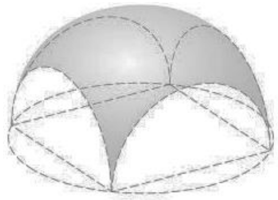
→ if it is 0 : „parabolic point”



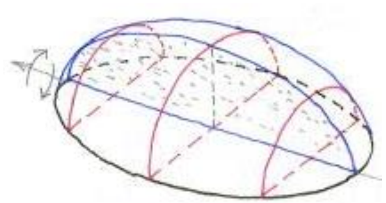
MOST IMPORTANT MASONRY VAULT TYPES

Points classified according to middle surface principle curvatures:

→ elliptical points



spherical vaults



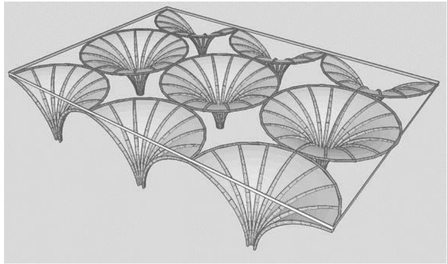
oval dome

→ parabolic points

barrel vault

→ hyperbolic points:

fan vaults



[mechanical consequences]

BASIC TYPES OF MASONRY STRUCTURES

→ columns, towers (≈1D)

→ walls (≈2D)

→ arches, vaults, domes (≈1D, 2D)

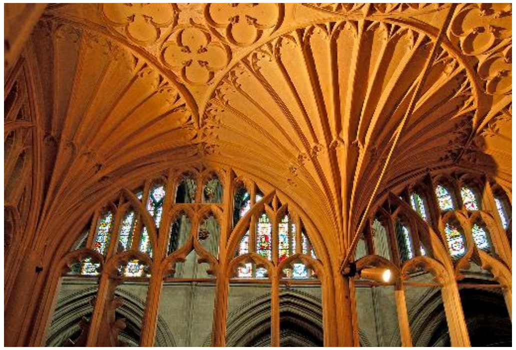
→ others: pillars; stairs; buttresses; dolmens, …

Masonry vault: ← **Lectures 09-10**

a curved shell-like masonry construction, for covering a ground area so that to transmit the downwards loads to the sides and to the supports

Great Herod's palace, 1st c, Judea, Israel; gettyimages.co

Monastery of Santa Maria de Alcobaca, Portugal, XIIth century, thoughtco.com



Holy Trinity Chantry Chapel, Tewkesbury Abbey, England, 14th c, flickr.com/photos/edk7/1407783359

BASIC TYPES OF MASONRY STRUCTURES

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Masonry dome:

generally accepted definition does not exist ⇒ let's see a few examples:

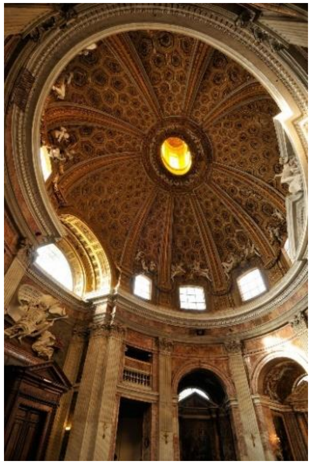
hemispherical:

St Paul's Cathedral, London (1710)

faceted:

Florence, dome of Santa Maria del Fiore (1436)

oval:



Rome, San Andrea al Quirinale (1670)

BASIC TYPES OF MASONRY STRUCTURES

→ columns, towers (≈1D)

→ walls (≈2D)

→ arches, vaults, domes (≈1D, 2D)

→ others: pillars; stairs; buttresses; dolmens, …

Masonry dome: ← **Lecture 08**

generally accepted definition does not exist

→ „closed vault whose middle surface is a surface of rotation of an arc concave-from-below, about a vertical axis which erects from below the arc”

BUT: e.g. oval domes; faceted domes

→ „closed vault whose points are elliptical, concave from below”

BUT: (onion domes) ← (not concave everywhere)

→ „a closed vault whose middle surface is an affine transformation of a spherical cap” BUT: irregular domes; faceted domes

→ …

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masonry vaults

masonry domes

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most important methods, short overview

Limit Analysis theorems

Questions

Mechanical analysis of masonry structures

→ architectural heritage;
→ traffic infrastructure

„Is the structure safe?"

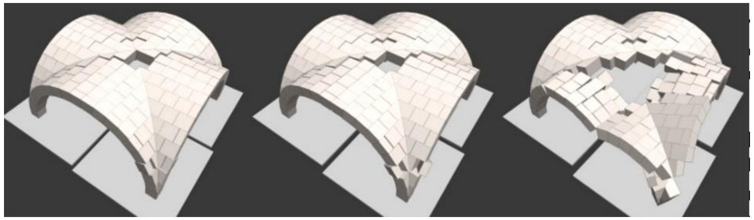
- selfweight
- weight of upfill etc
- support displacements [outwards; + sinkage]
- live loads

Load bearing limits of structures in general:

→ internal forces / stresses exceed strength limits of the material
→ deformations exceed the admissible deformations
→ stability: equilibrium cannot exist, or the equilibrium is unstable

Failure of masonry arches and vaults:

mostly a ***stability problem***



http://block.arch.ethz.ch/brg/research/collapse-of-masonry-structures

Mechanical analysis of masonry structures

Traditionally: ***geometric*** design rules based on ***trial-and-error*** experiences

XIXth century: ***graphostatics*** (force polygons & lines of action)

02

XXth century: continuum ***shell theories***; mostly ***membrane*** analysis **03**

Limit analysis of masonry structures: J. Heyman, from the 1960ies **04**

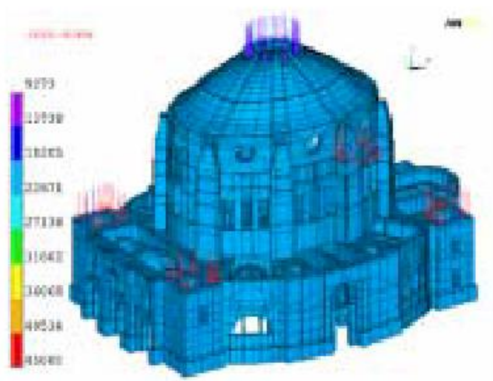
→ The ***static*** („Safe”) theorem
→ The kinematic theorem
(„Unsafe” or „Energy” theorem)

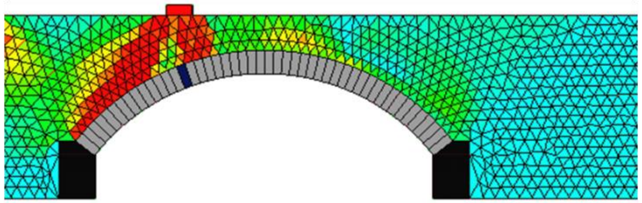
} With the given geometry, can the structure carry the loads or it will collapse?

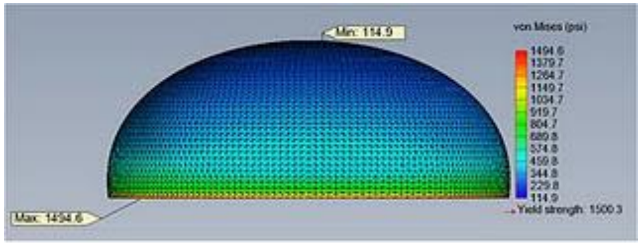
Nonlinear FEM

Discrete Element Method **05**

Problems about FEM:
Why do we need other methods?







Drawbacks of FEM:

→ initial geometry has to be known: often impossible

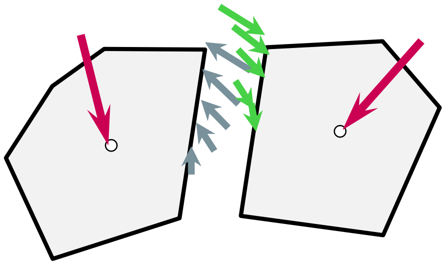
→ extreme sensitivity to small perturbations of the geometry

→ history dependence: the existing forces in the structure have to be known

→ contact separation; contact sliding; (can be done with contact elements, though difficult); BUT:

partial separation or reversible sliding or occurence of new contacts cannot be taken into account !!!

Limit analysis of masonry structures



The task considered by Heyman:

→ assembly of ***rigid blocks*** and ***dry contacts***;

→ given geometry, given dead load AND/OR one-parameter live load;

→ Can the structure (with the given geometry) ***balance the given load***?
OR: for ***what range*** of the load parameter will the structure stand?

→ simplifying assumptions:

(1): no tension resistance in the contacts

(2): blocks have infinite strength

(3): sliding does not occur

(+1): [implicitly also assumed:]
geometry remains the same for any load (infinite stiffness)

Limit analysis of masonry structures

The static theorem for masonry structures:

If a force system can be found for the given set of external loads which satisfies the material criteria and equilibrates the given external loads, then the structure with the given geometry is safe under these loads.

The kinematic theorem for masonry structures:

If a mechanism (a virtual displacement system**) can be found for the given set of external loads which satisfies the material criteria and produces non-negative work with the given external loads, then the structure will collapse under these loads.**

→ these theorems were given (Heyman, 1966) without proof
→ limited validity
→ details, limitations and applications will be given in Lecture 04
→ but still unconciously applied: Lectures 02 – 03

THE DISCRETE ELEMENT METHOD

Definition: a numerical method belongs to DEM if

← it consists of separate, finite-sized elements and their contacts

← its elements have independent degrees of frredom, with large displs

← contact separation and sliding considered; new contacts can be born

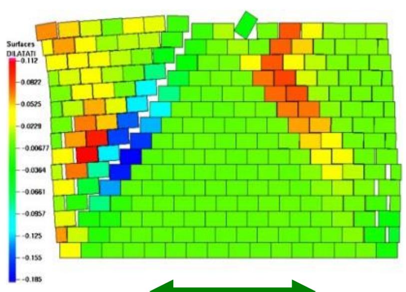
Main steps: define the elements (geometry);

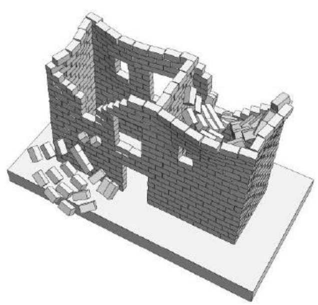
automatically recognize their contacts

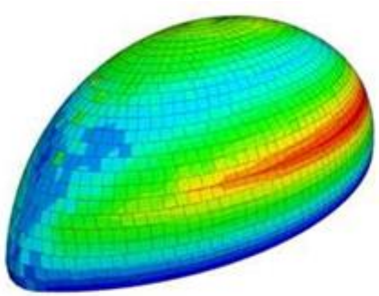
specify the material parameters (for elements; for contacts)

loading history: in small steps

Detailed introduction to DEM: will be given in Lecture 05







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QUESTIONS

1. Define „arch”, „vault” and „dome”. Define „closed vault”, „half-open vault” and „open vault”, and give examples for each type.
2. Recognize the following bond patterns: running / English / Flemish bond, herringbone pattern, ashlar masonry, rubble masonry
3. Recognize the following arch types: corbel, semicircular, pointed, segmental arch
4. Recognize the following vault types: barrel / cross / cloister vault, underpitched / sail / fan vault, spherical dome, oval dome
5. What were Heyman’s simplifying assumptions? What does the Static Theorem state?