GRAPHICAL METHODS

Citation:

K. Bagi (2018): Mechanics of Masonry Structures. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

In case of any question or problem, do not hesitate to contact Prof. K. Bagi, kbagi.bme@gmail.com .

This Lecture: Graphical Methods

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 … 2018

Wolfe's method for membrane forces in domes

O'Dwyer's funicular analysis $\Rightarrow$

Thrust Network Analysis (TNA)

Questions

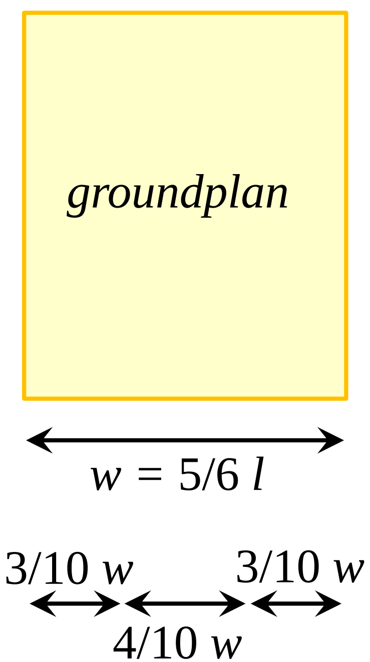
Historical times: Practical geometrical rules

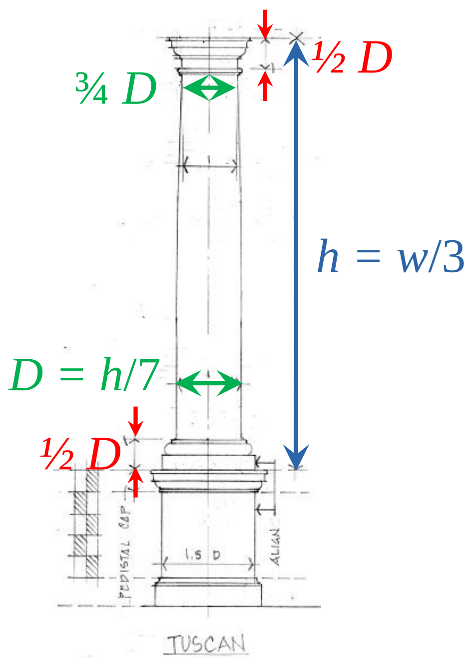
Roman era, Vitruvius (Roman Empire, BC 1st ct., army engineer & architect):

„Ten Books on Architecture"

→ inspired many architects, already from VIIIth century;
particularly important for Renaissance

e.g. in the „Tuscan" order, the design of the ***column*** of a temple:





St Paul's Church, London, XVIIth century,
flickr.com/photos/ddtmmm/1367084017

Historical times: Practical geometrical rules

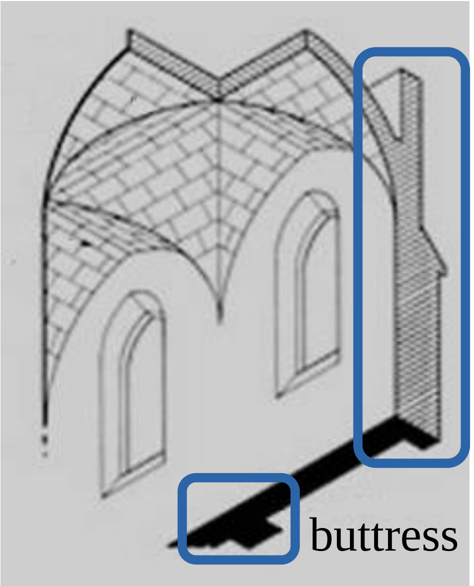
Gothic architecture terminology:

Felicity Lynch: Gothic Art History 1150 - 1500 A.D.

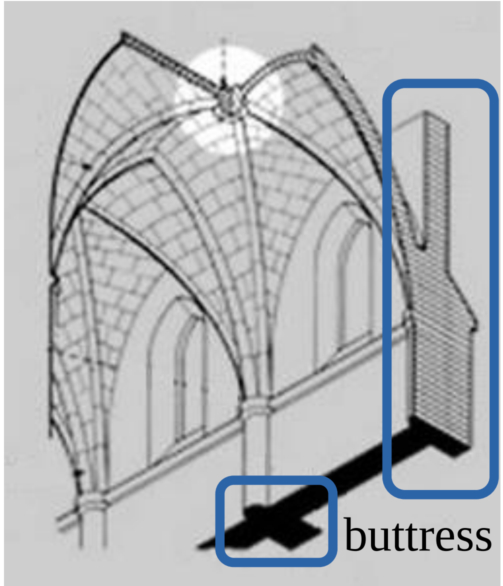
http://slideplayer.com/slide/5932736/

Cameron Daniels: Architecture of The Middle Ages

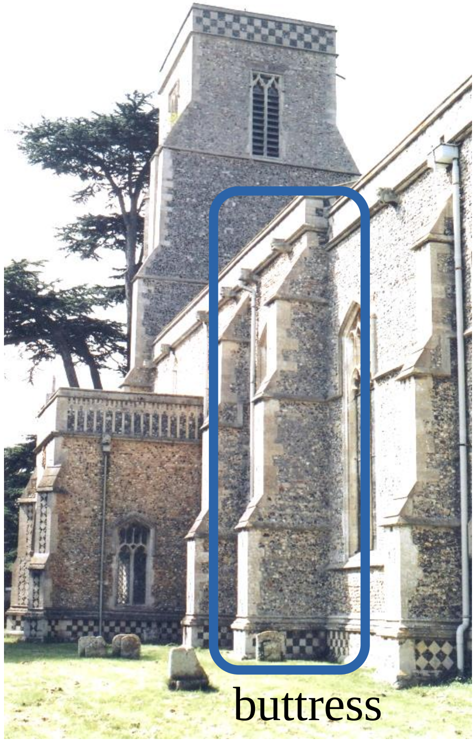
http://slideplayer.com/slide/8837376/



groin vault



ribbed cross vault

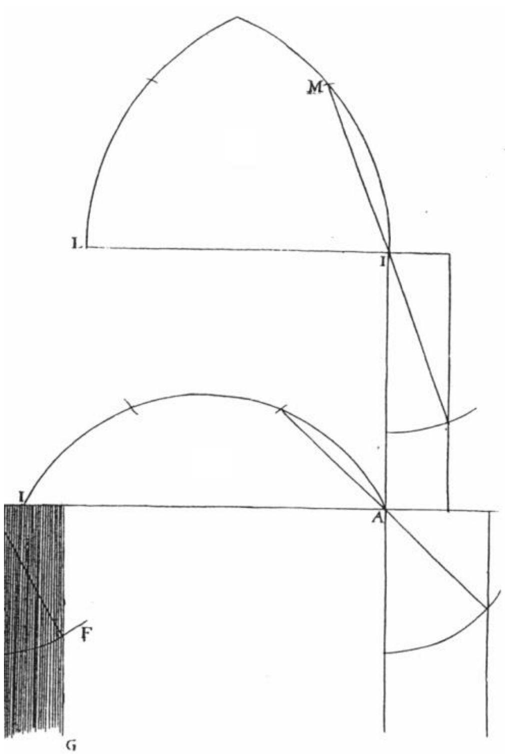


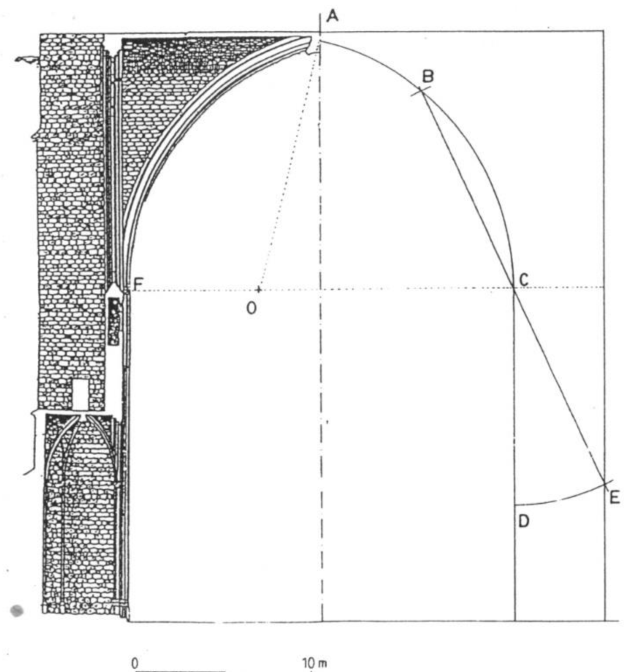
Historical times: Practical geometrical rules

Gothic rules:

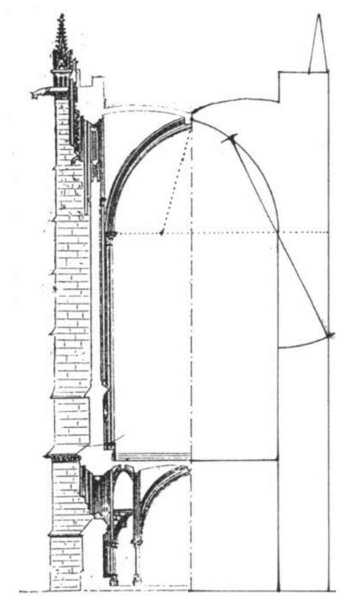
e.g. Derand's rule for ***buttress thickness***:

($\Rightarrow$. similarity of Gothic cathedrals of the same geographic area)





Gerona cathedral



Saint-Chapelle, Paris

Historical times: Practical geometrical rules

Gothic rules:

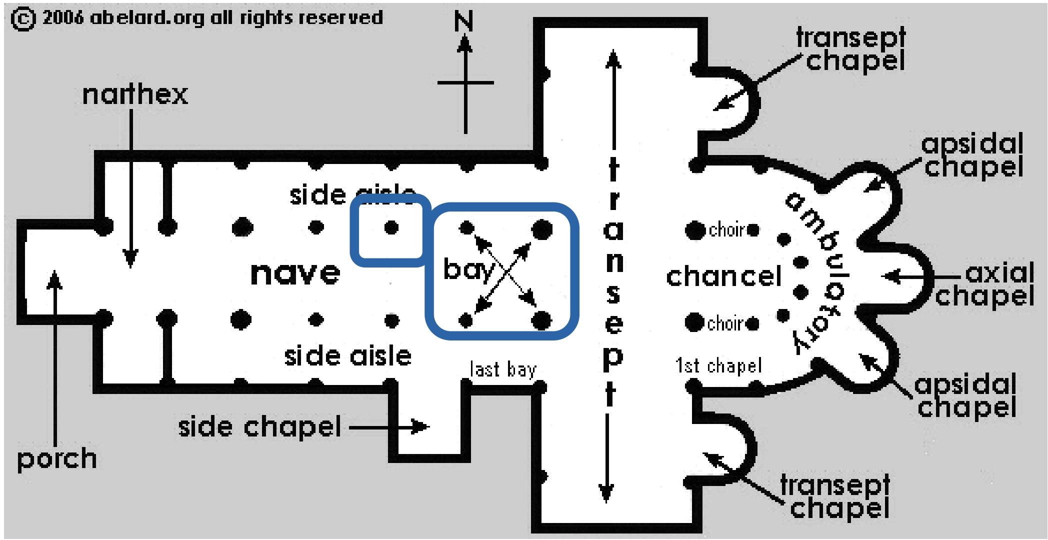
e.g. Rodrigo’s interior ***pier diameter*** design rule:

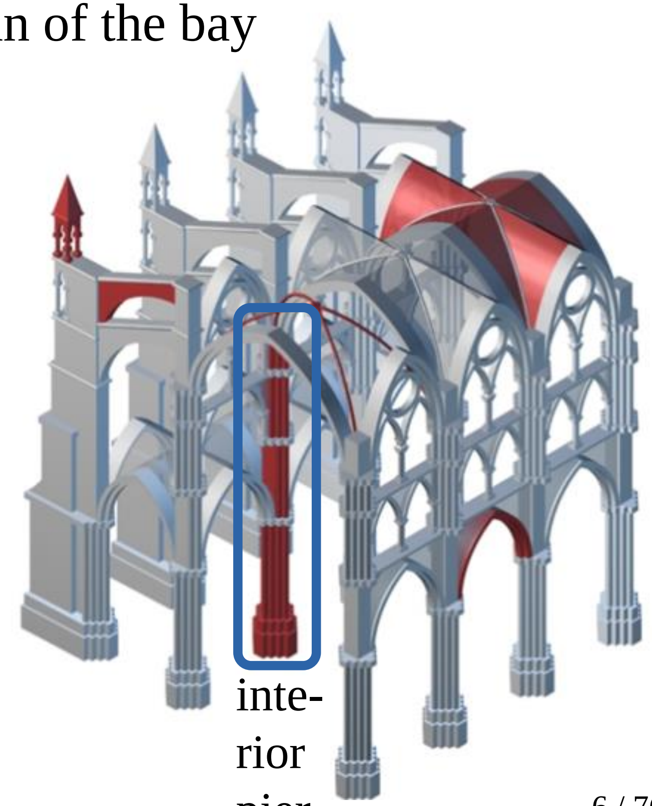
$$d := \frac{1}{2}\sqrt{h + w + s}$$

h: pillar height
w: length of the bay
s: span of the bay

dimension !!!!

$\Rightarrow$ works only in Castilian feet (0,28m)





Further reading: Huerta (2006); Aita et al (2018a)

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e.g. Gothic rules

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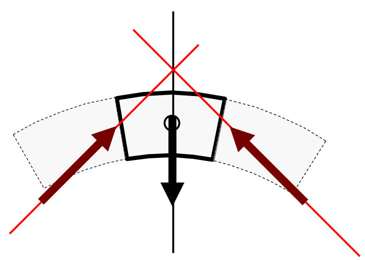
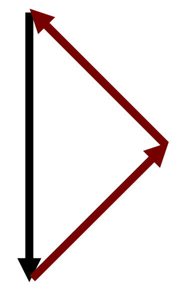
Questions

GRAPHICAL STATICS

Reminder to fundaments:

Equilibrium of three forces in 2D:





funicular diagram („form diagram"):
the three lines of action intersect

force diagram:
closed vector triangle

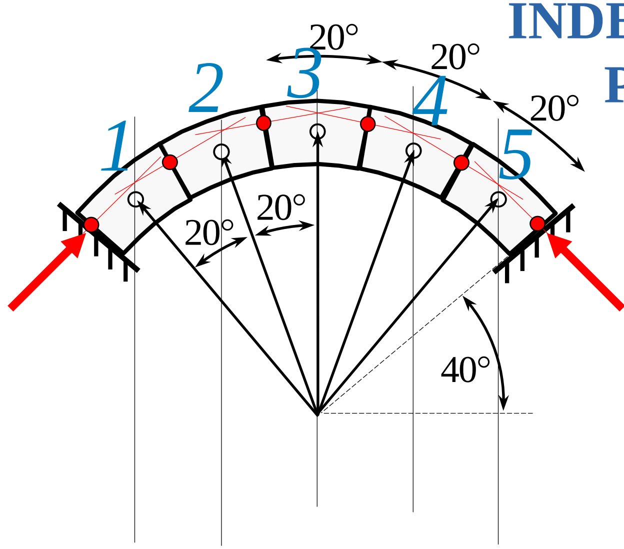
Equilibrium of four forces in 3D:
different projections, e.g. hoop view and top view
all views have to intersect / be closed

More than three (2D) or more than four (3D) forces: closed force diagram,
but: lines of action not necessarily intersect

The basic problem: Stability of an arch

Question: arch submitted to its selfweight; ?reactions? ?contact forces?

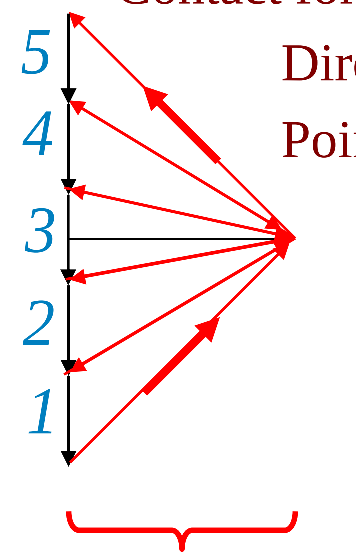
3× STATICALLY INDETERMINATE PROBLEM



Contact forces: H = ??

Direction?

Point of action?



e.g. $H := 2{,}5G$

„*loads are transmitted to the supports*"

Given: geometry: R_{inner} = 2,4 m ; R_{middle} = 2,7 m

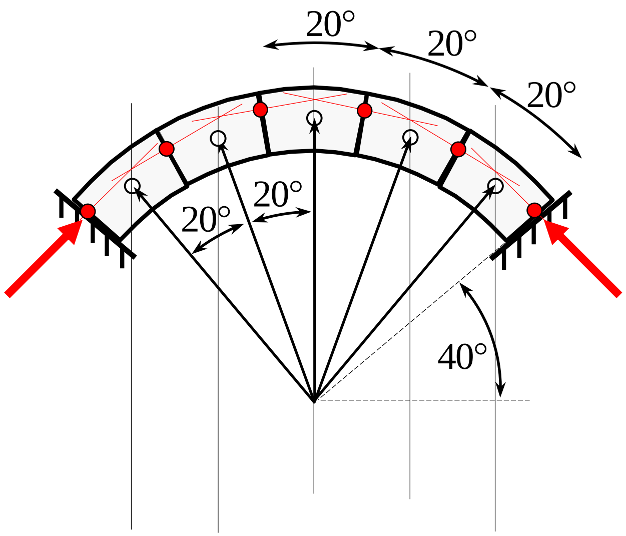
identical selfweight for each block: $G_1 = G_2 = G_3 = G_4 = G_5$

Try to find an equilibrated force system!

→ contact forces: compression & friction; inside the contact area

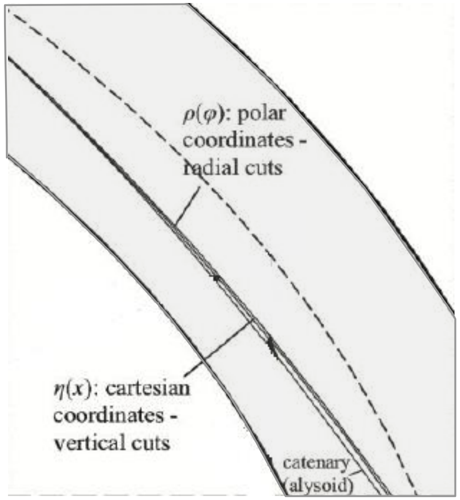
The basic problem: Stability of an arch

Thrust line: [intuitive concept; theoretical definition: Gáspár et al (2018)]



≈ „ the line determined by
the points of action of the
contact forces "

BUT: depends on the
orientation of contacts
(*Alexakis & Makris, 2015)*



stability criterion: [later: more details]

thrust line can be found so that
it runs everywhere inside the contacts

arch shape is „better", if it can be done with smaller thickness

→ e.g. pointed arch versus circular arch

The basic problem: Stability of an arch

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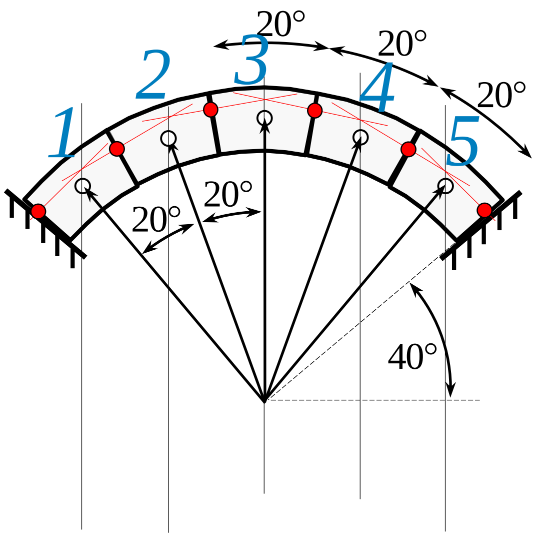

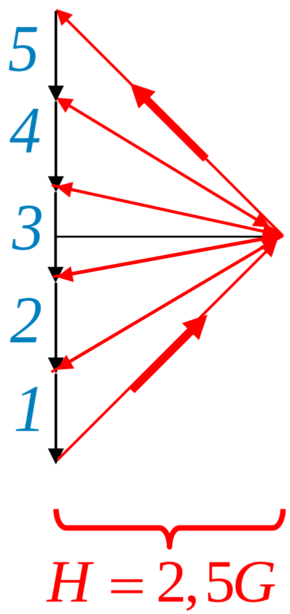

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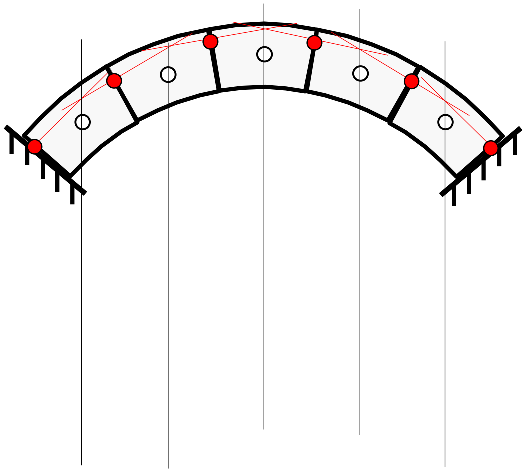

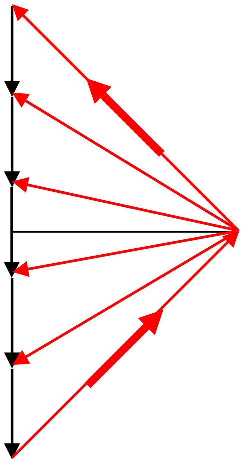

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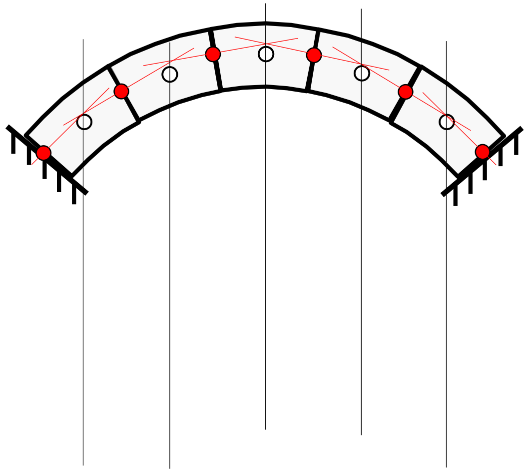

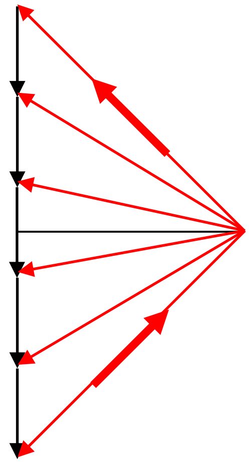

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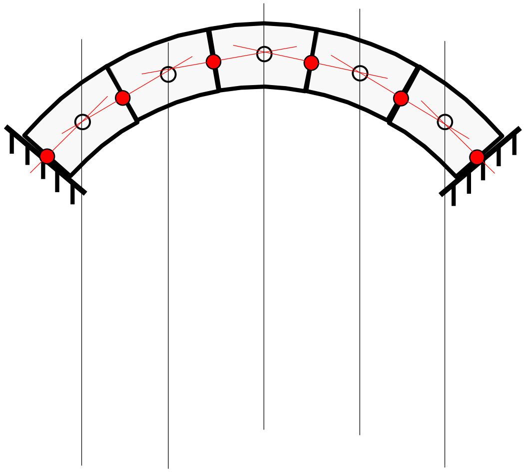

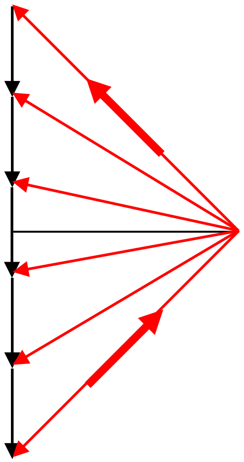

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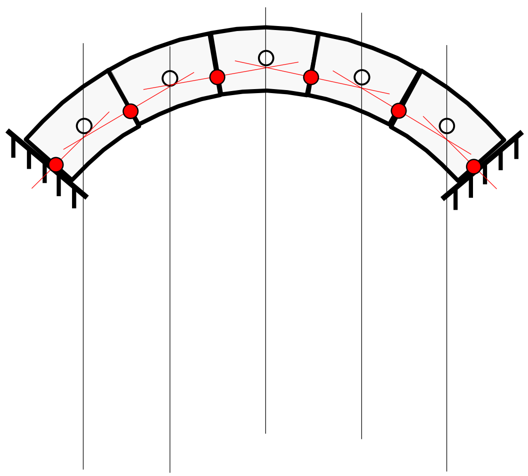

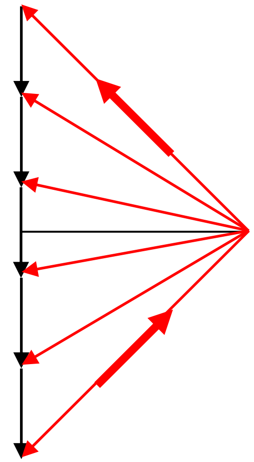

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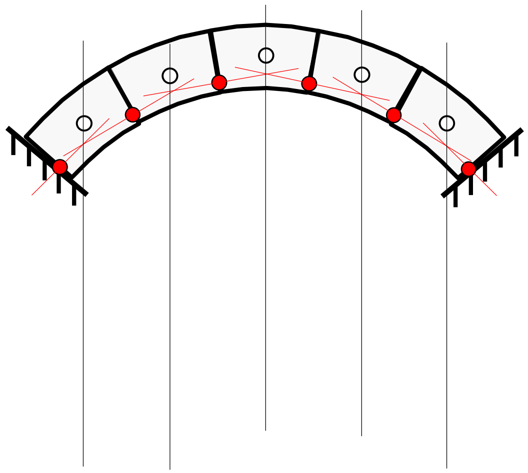

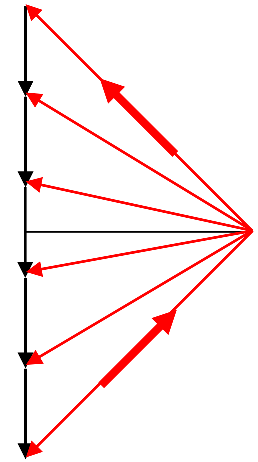

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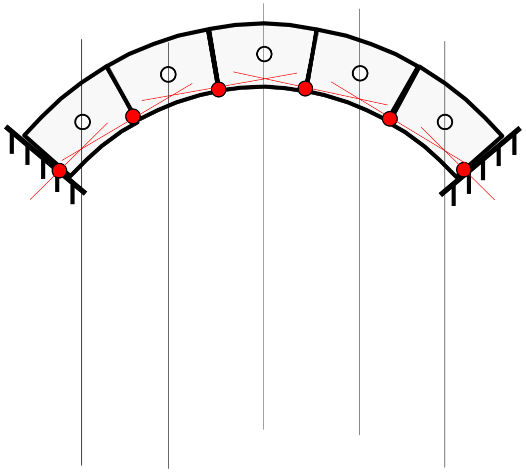

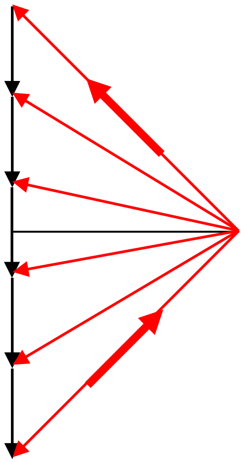

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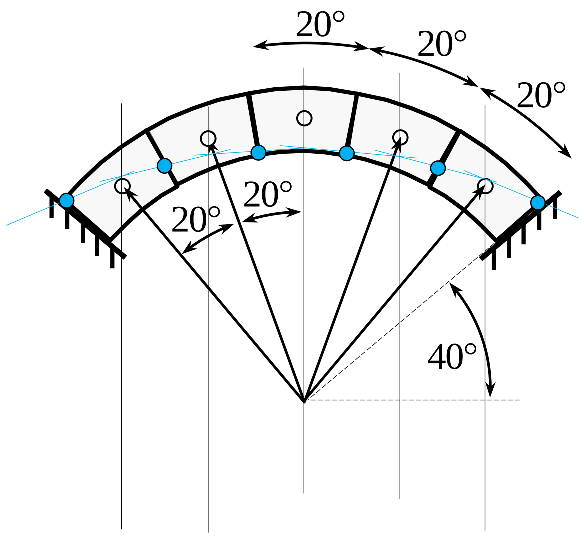

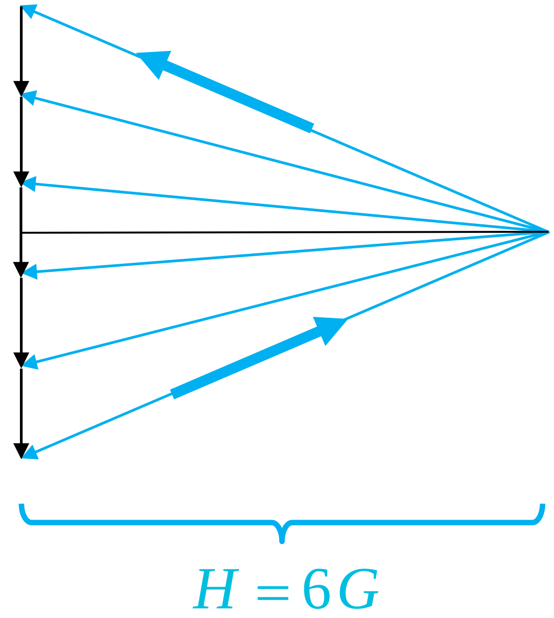

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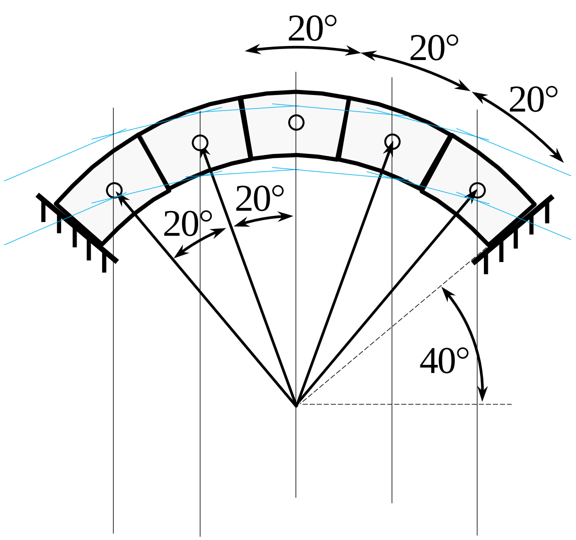

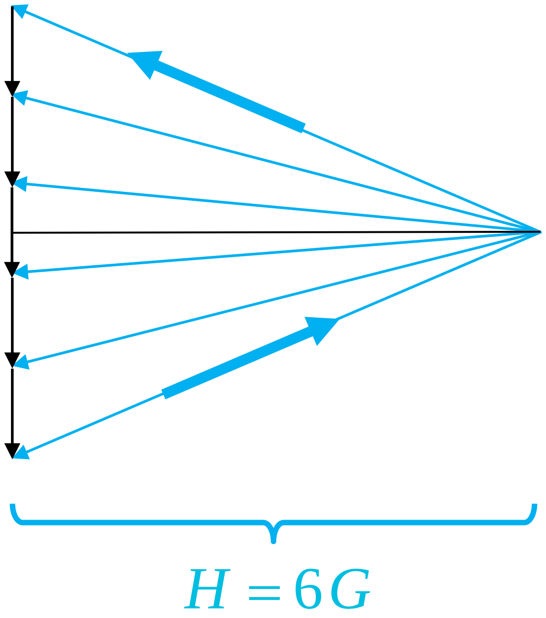

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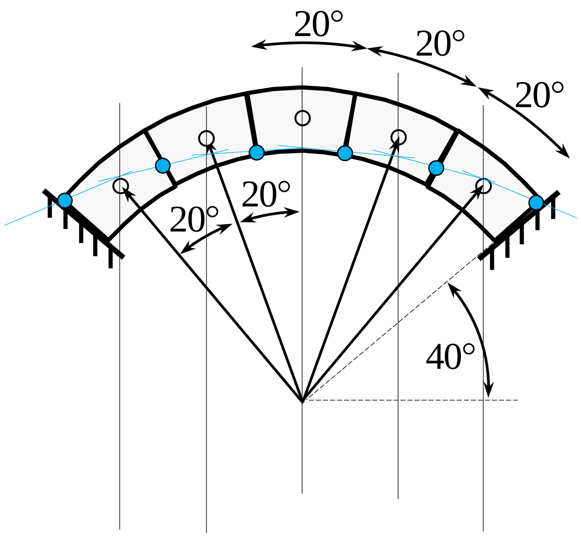

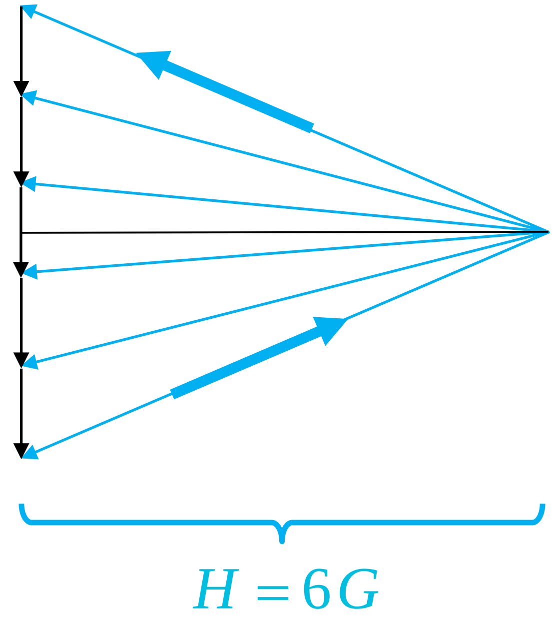

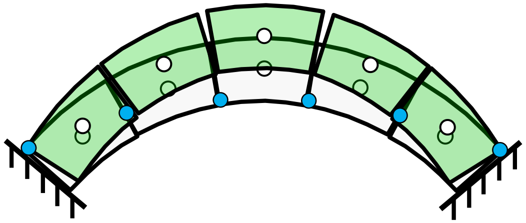

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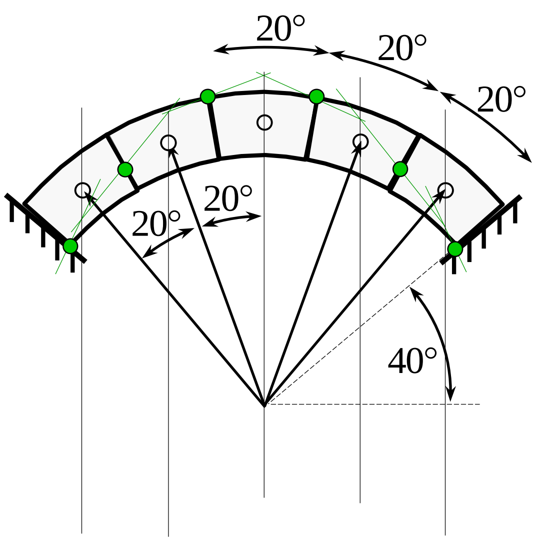
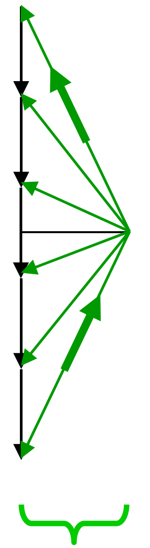
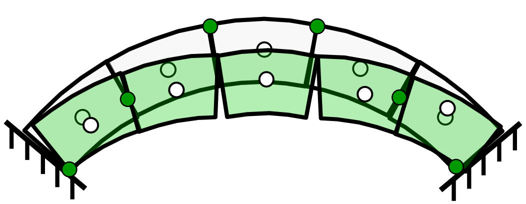




$H = 1{,}1G$



a wide range of equilibrium solutions: $\Leftarrow$ because the arch is thick enough !

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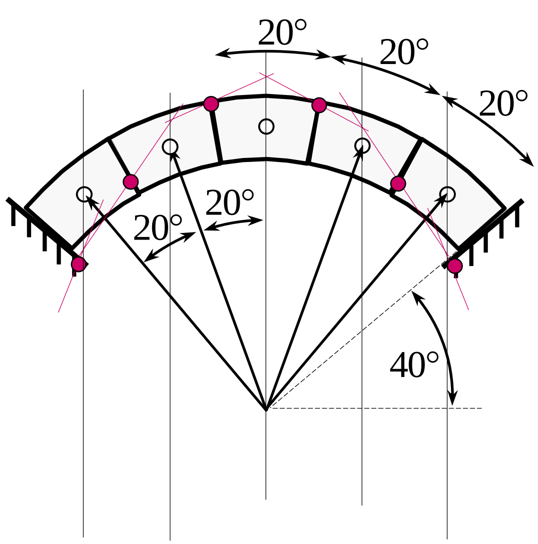

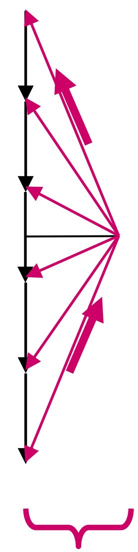

$H = 0{,}8G$

EQUILIBRIUM IS IMPOSSIBLE WITH THIS *H*

⇒ possible direction of the reactions is limited

The basic problem: Stability of an arch

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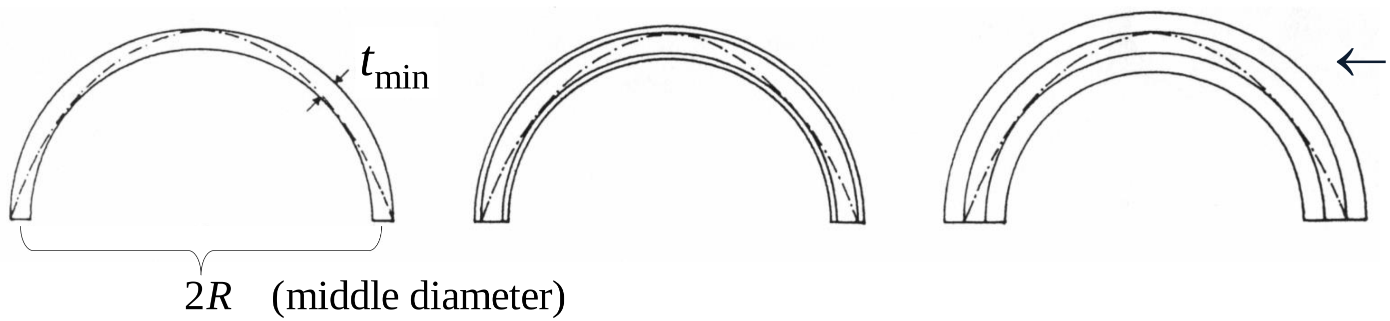
Solution for an arch having ***infinitely dense***, ***radially oriented*** contacts,
with ***zero tension resistance*** ? $\Rightarrow$ thrust line must run inside the arch

Ideal shape of the arch to produce thrust line through the contacts centroids:

ch-curve („chain curve")

Existing arches: typically circular middle curve (or composed of circular arcs)



← Geometrical factor of safety: t_{actual} / t_{min}

t_{min}: smallest uniform thickness for which the arch can carry its selfweight

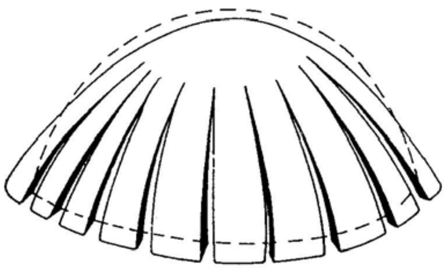
semicircle: Heyman (1966): $t_{\min} = 0{,}1059 \cdot R$
Milankovitch (1907): $t_{\min} = 0{,}1075 \cdot R$

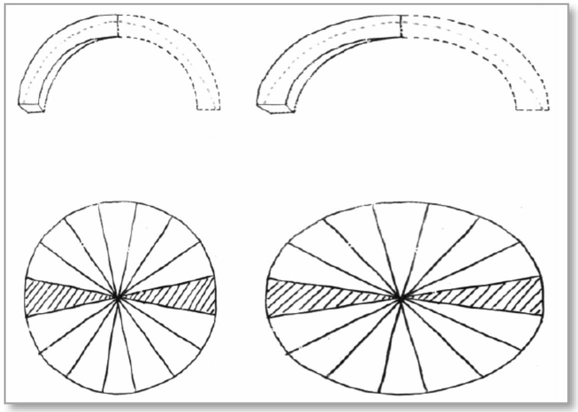
Stability of vaults under selfweight

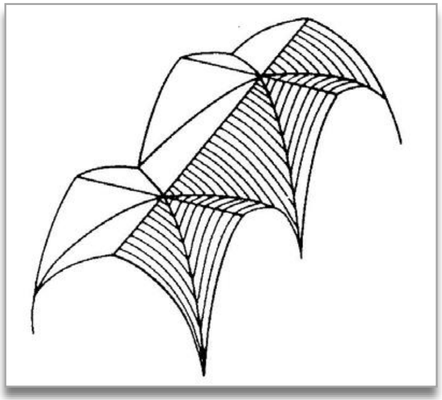
Slicing technique: cut into individual arches, and check them separately!

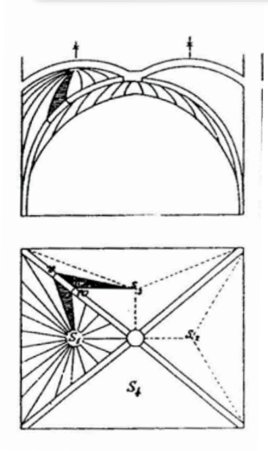
XIXth century: different assumptions on the internal force system

based on the inspection of typical crack patterns: e.g.









pictures from: Huerta, 2001 and the references therein

Stability of vaults under selfweight

N. Valencia, archdailiy.com

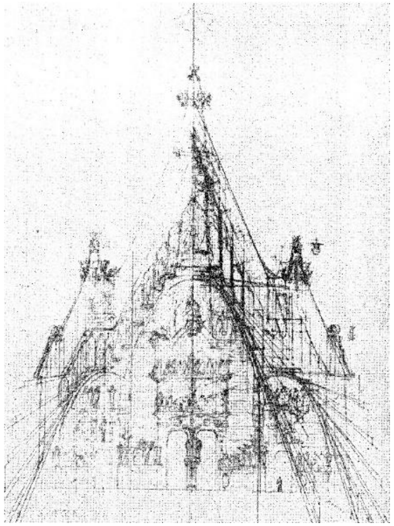
<u>Slicing technique:</u>

Gaudi, Sagrada Familia, Barcelona:

designed by:

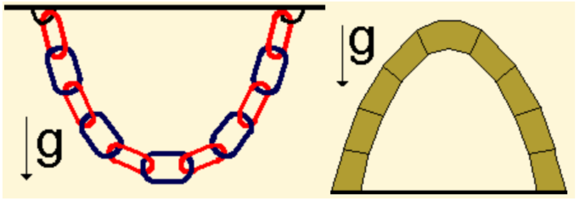
→ graphical statics:

slice of the structure:



Rafals, 1929

→ physical models:



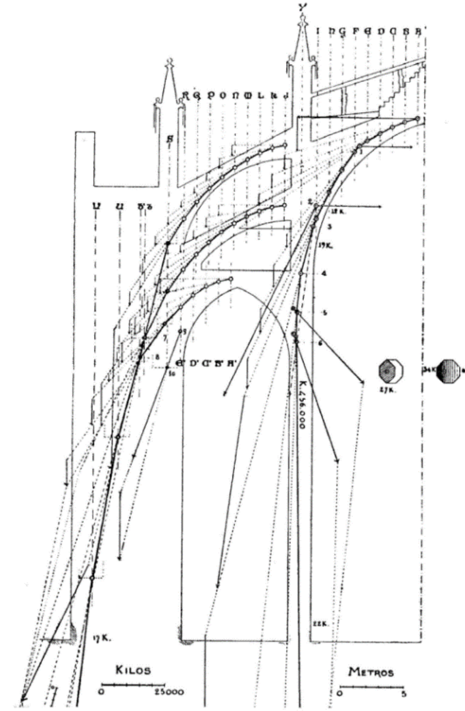
http://www.art-nouveau-around-the-world.org/en/villes/barcelona/models.htm

http://dataphys.org/list/gaudis-hanging-chain-models/

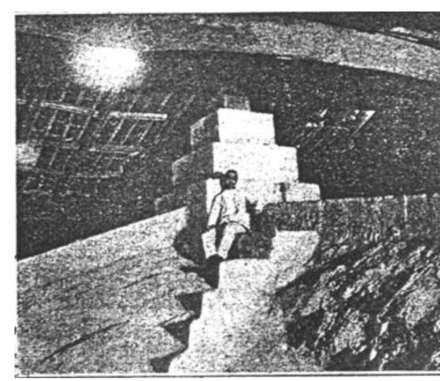
Stability of vaults under selfweight

Slicing technique:

https://spainattractions.es/palma-cathedral-mallorca/



Rubio Bellver, 1912



Problem:

extremely tall slender pillars of the main nave

$\rightarrow$ is it safe?

Rubio Bellver, 1912: graphical statics analysis

$\Rightarrow$ weights needed over the crown!

This Lecture: Graphical Methods

Historical times: Practical geometrical rules

- e.g. Vitruvius
- e.g. Gothic rules

Graphical statics

- The basic problem: Stability of an arch
- Durand-Claye's stability area method for arches
  - computerized & extended for domes: Aita et al 2003 … 2018
- Wolfe's method for membrane forces in domes
- O'Dwyer's funicular analysis $\Rightarrow$
  - Thrust Network Analysis (TNA)

Questions

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads

Admissible (P, e) pairs?

Consider a contact j ,

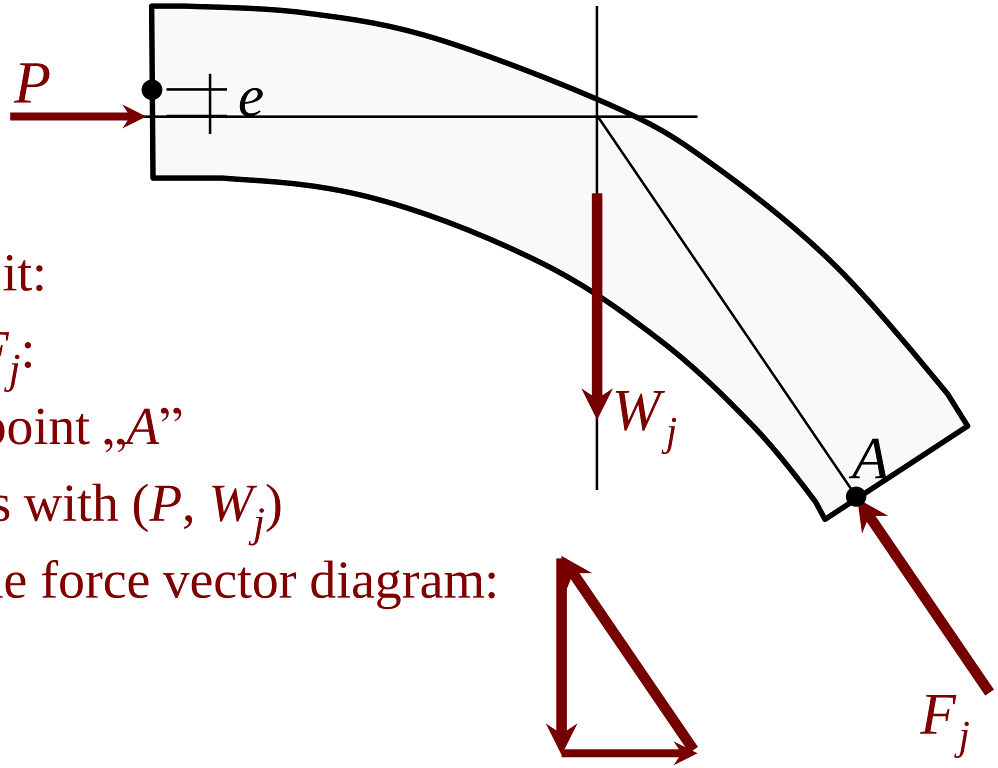
and a point „A" on it:

Contact force resultant, F_j:

acts at the chosen point „A"

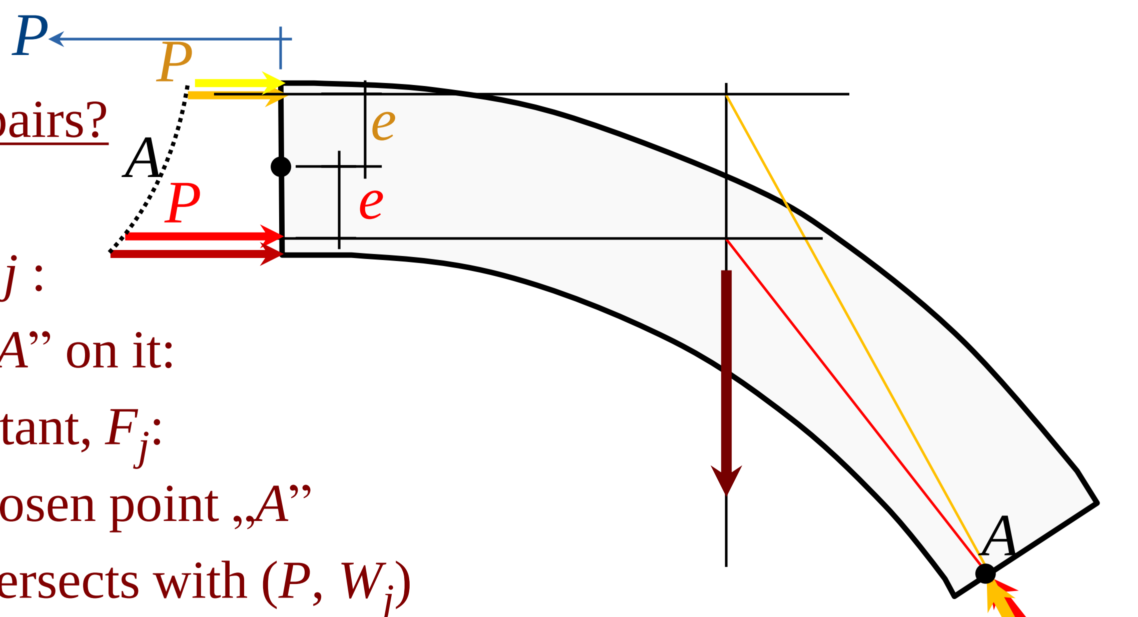
direction: intersects with (P, W_j)

magnitude: from the force vector diagram:



Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



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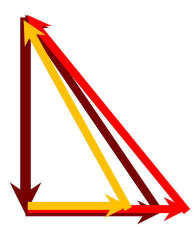
direction: intersects with (P, W_j)

magnitude: from the force vector diagram:

magnitude of P depends on e:

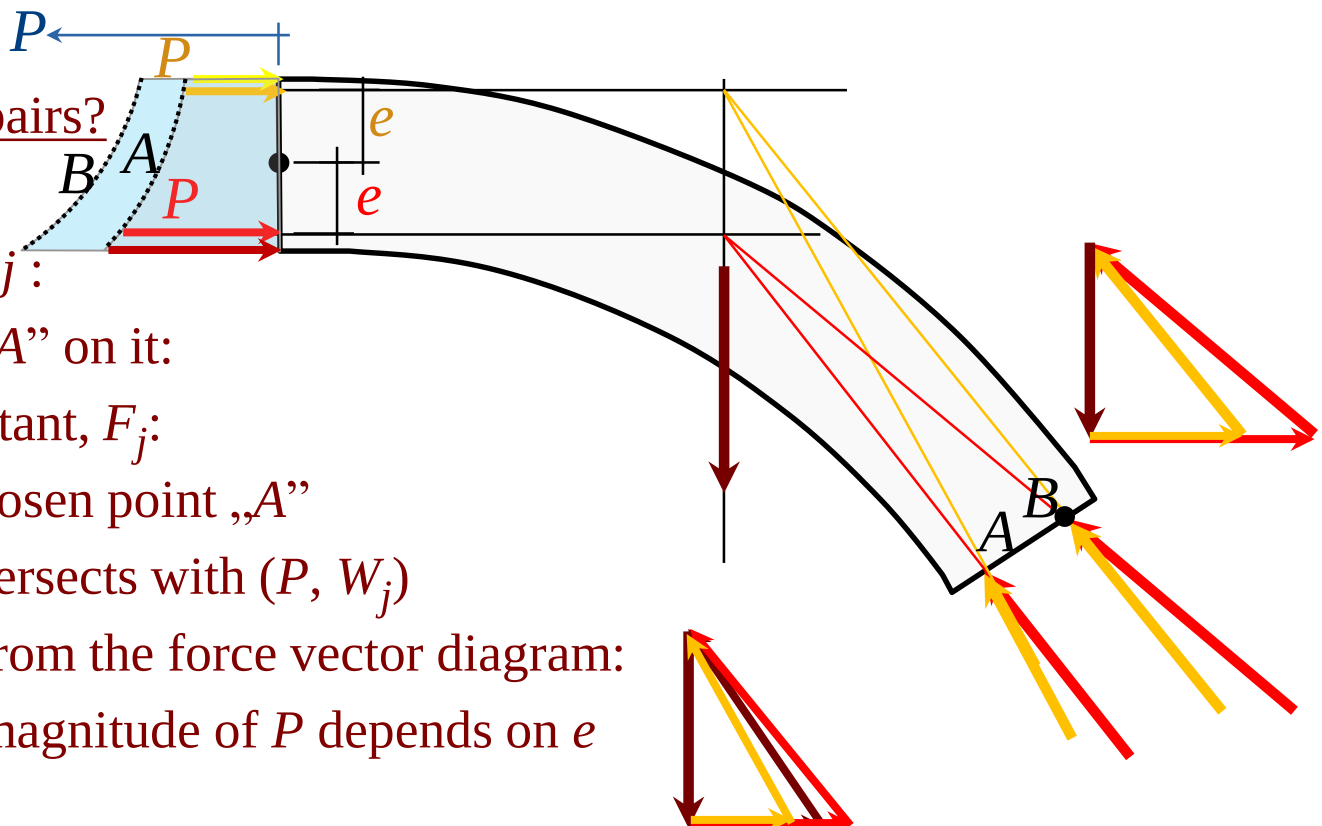


Possible magnitudes of P belonging to „A":

[see dotted line above]

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



<u>Admissible (P, e) pairs?</u>

Consider a contact j :

and a point „*A*" on it:

Contact force resultant, F_j:

acts at the chosen point „*A*"

direction: intersects with (P, W_j)

magnitude: from the force vector diagram:

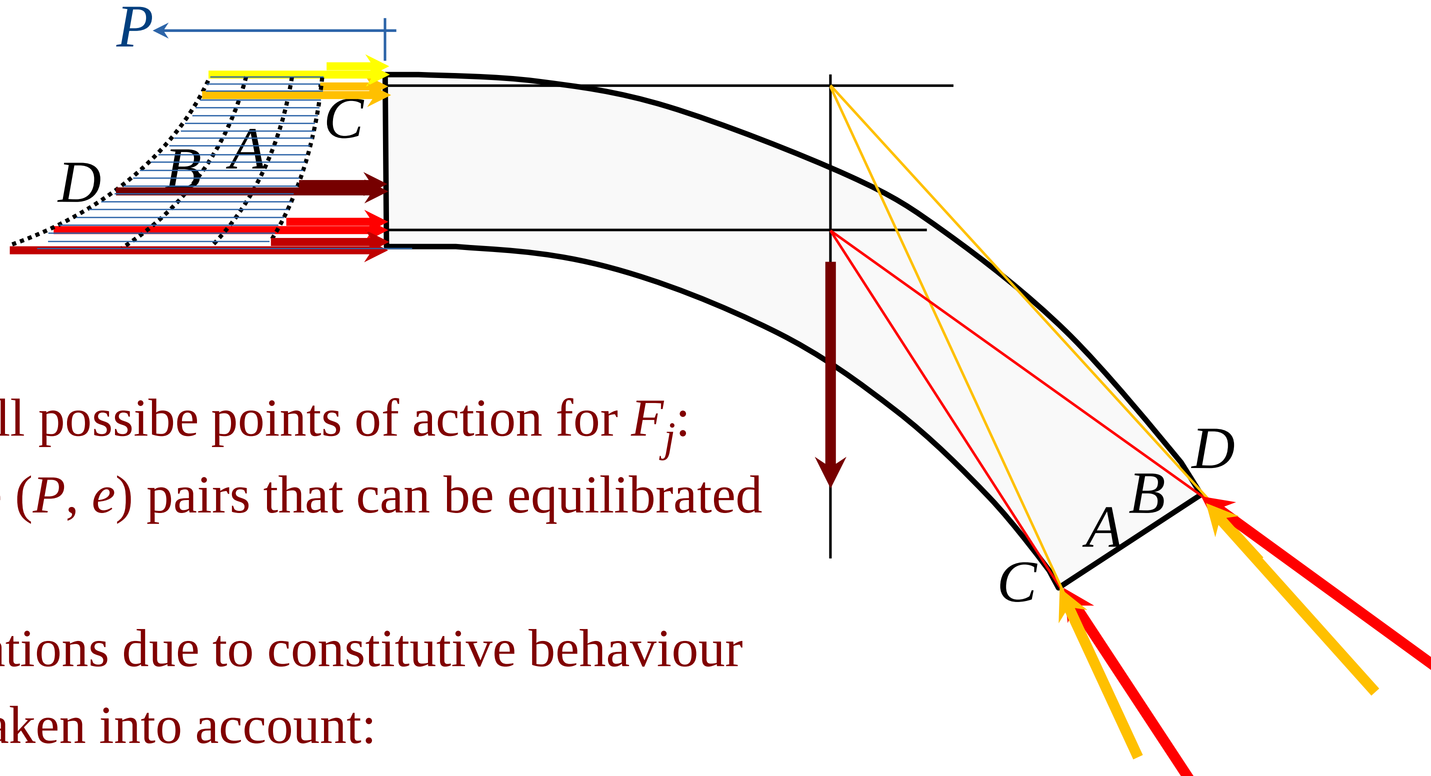
magnitude of P depends on e

Possible magnitudes of P belonging to *„A"*: [see grey domain horizontal sizes]

similarly to any „*B*": [see cyan domain horizontal sizes]

Durand-Claye’s stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Considering all possibe points of action for F_j:
found those (P, e) pairs that can be equilibrated

now the limitations due to constitutive behaviour
have to be taken into account:

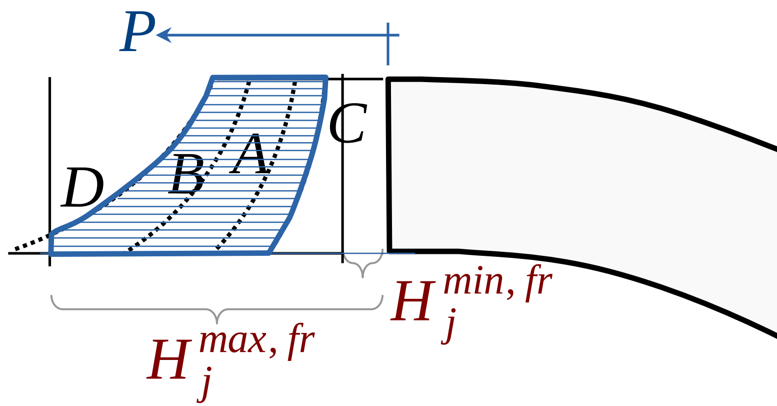
→ friction limit

→ compression strength

→ [in new versions: tension strength – will not be shown here]

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



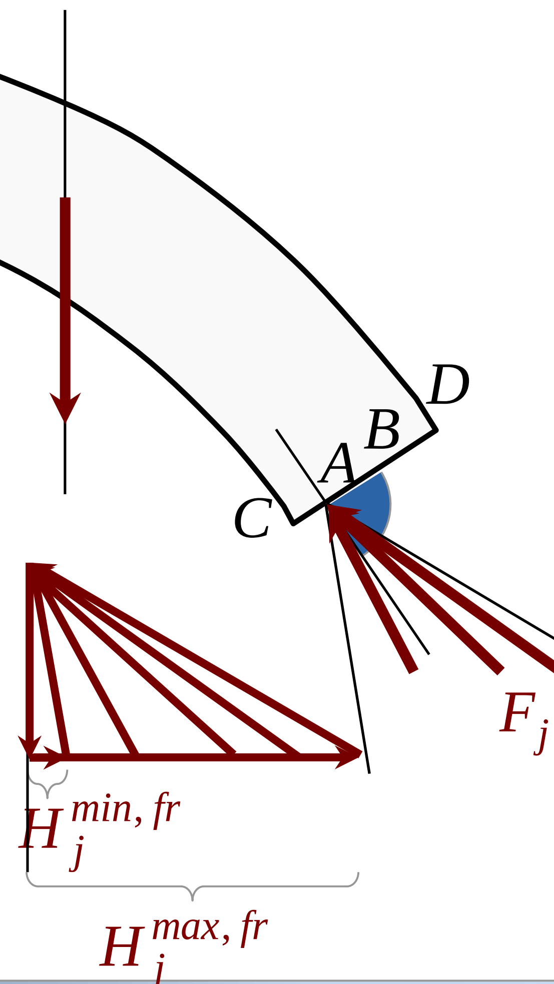
Friction limit:

if F_j acts at any chosen point „*A*":

friction cone

[remember: *P* is equal to the horizontal component of F_j]

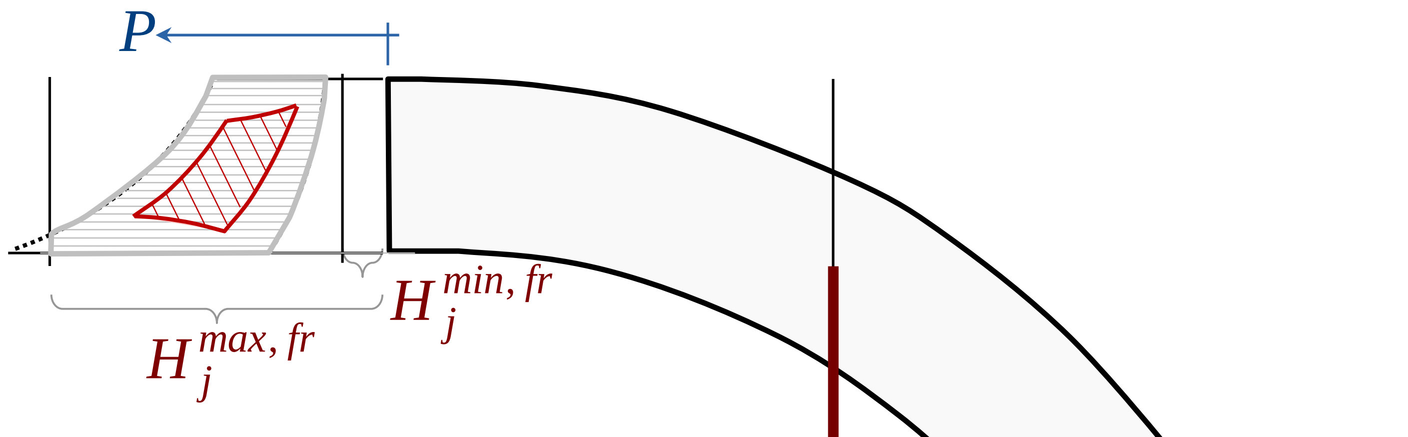
$\Rightarrow H_j^{min,fr}$ and $H_j^{max,fr}$ found



Equals for all other points from *C* to *D* !

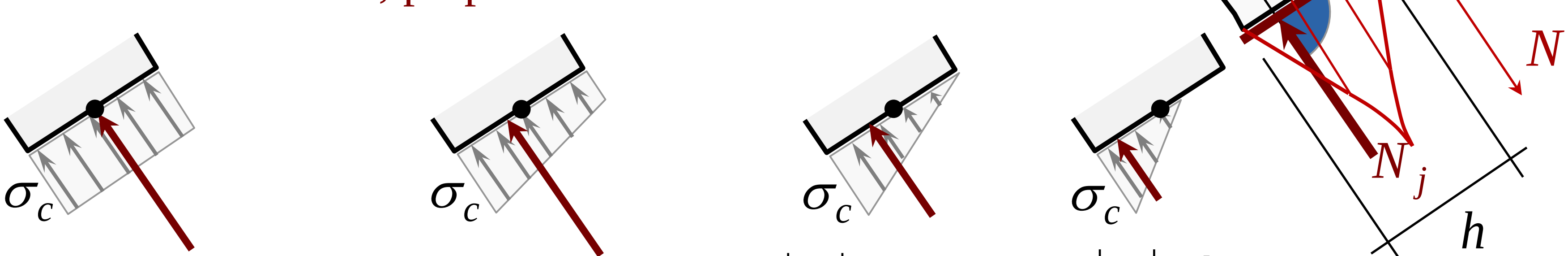
Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Compression strength: σ_c

thickness: h ; perpendicular width: b



$e_N = 0$:
$$N \le b \cdot h \cdot \sigma_c$$

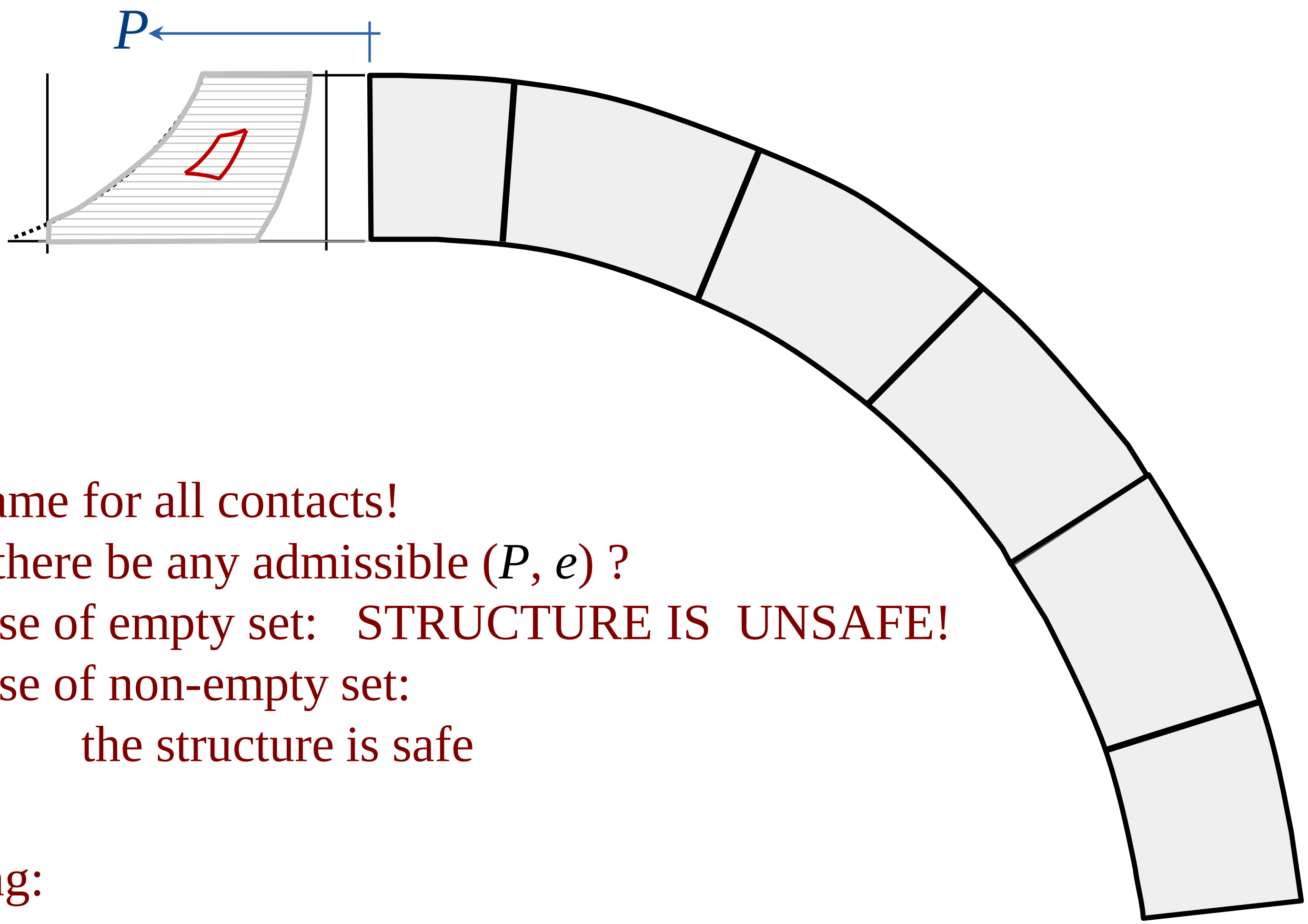
$|e_N| \le h/6$:
$$N \le \frac{h}{h + 6 \cdot |e_N|} \cdot b \cdot h \cdot \sigma_c$$

$|e_N| = h/6$:
$$N \le \frac{1}{2} b \cdot h \cdot \sigma_c$$

$|e_N| \ge h/6$:
$$N \le \frac{3}{2}\left(\frac{h}{2} - |e_N|\right) \cdot b \cdot \sigma_c$$

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Now do the same for all contacts!

→ will there be any admissible (P, e) ?

In case of empty set: STRUCTURE IS UNSAFE!

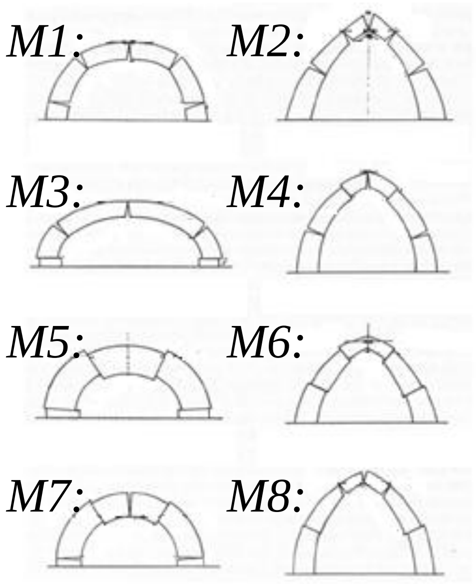
In case of non-empty set:

the structure is safe

Further reading:

Foce & Aita (2003); Aita et al (2017)

Durand-Claye's stability area method



Michon, 1857

Applications:

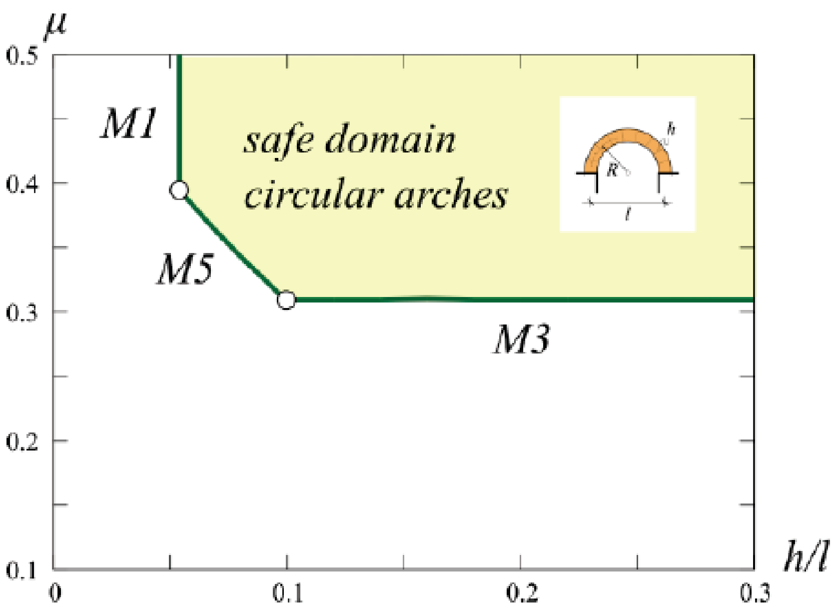
e.g. Barsotti et al (2017):

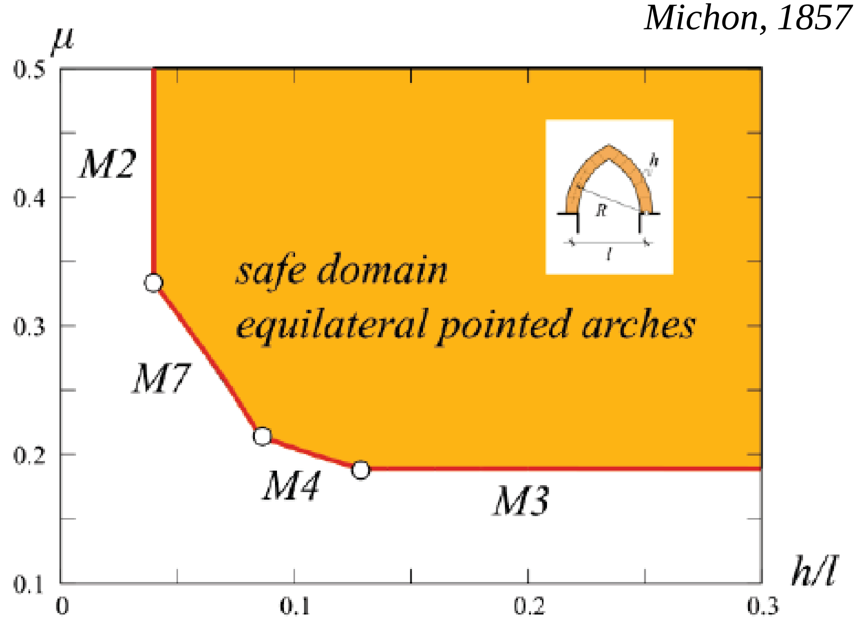
comparison of different arch types

and their possible collapse modes

μ : friction coefficient

h : arch thickness; l : span





Durand-Claye's stability area method

Applications:

e.g. Aita et al (2018a):

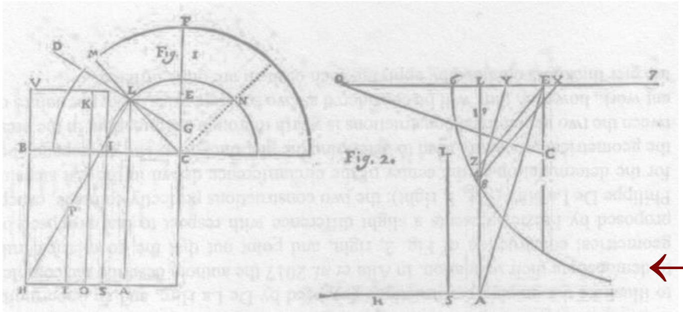
geometrical factor of safety for historic design rules:

→ find necessary minimum value of a certain size with Durand-Claye's;

→ find that size according to historic rule;

→ compare!

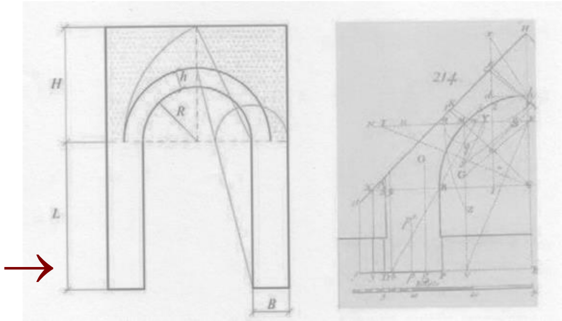
Comparison of different historical rules for pier thickness for the same arch-wall-pier system:



limit width according to the Durand-Claye's method: 2,54 m

← 2,81 m

3,61 m →



La Hire, 1731

Frézier, 1737

Durand-Claye's stability area method

<u>Applications:</u>

e.g. Aita et al (2018b): Safety assement of the dome of Pisa Cathedral

tripadvisor.co.za

constructed: XIth century

dome: oval groundplan,

≈ circular meridians

restoration of the dome going on recently

„*On the north side... at about eye level, is an original piece of Roman marble, on which are a series of small black marks. Legend says that these marks were left by the Devil when he climbed up to the dome attempting to stop its construction, and so they are referred to as the scratches of the devil. The legend also says that out of spite the number of scratches always changes when counted.*" (Wikipedia)

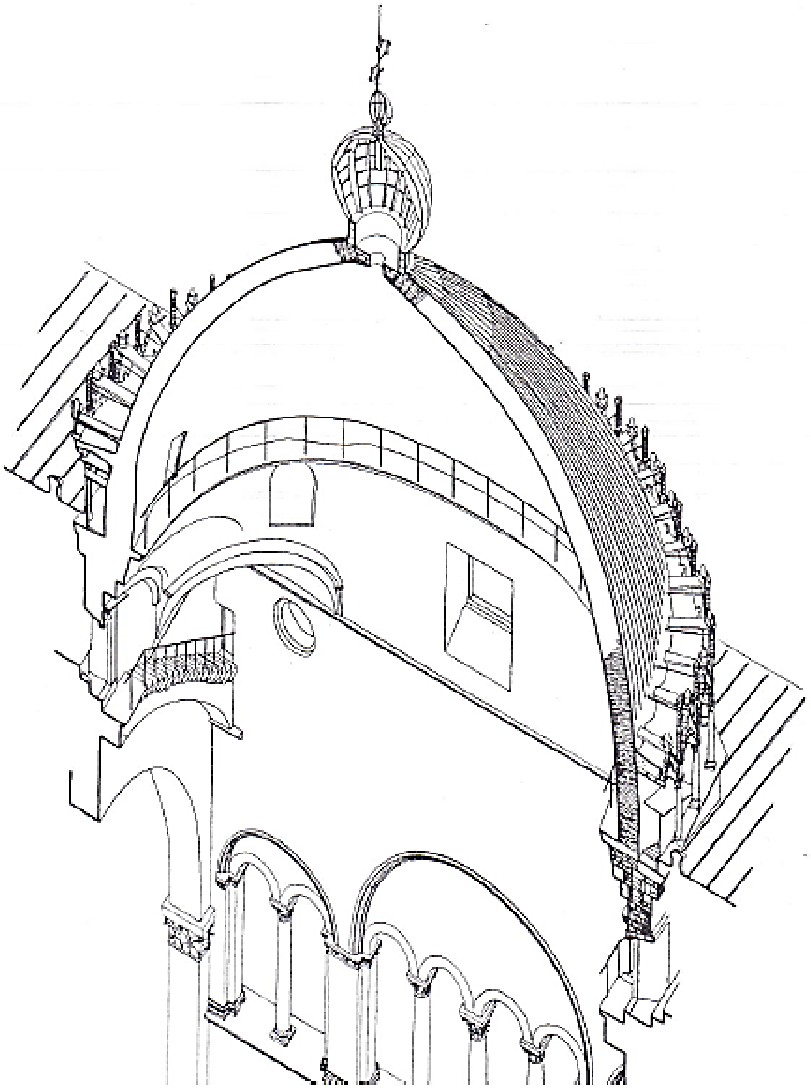
Durand-Claye's stability area method

Applications:

e.g. Aita et al (2018b):

→ D-C method extended for domes
with membrane forces
(Durand-Claye, 1880)

→ analysis of the dome



Result:

geometrical factor of safety ≈ 2

Further reading:

Aita (2018b)

This Lecture: Graphical Methods

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 … 2018

Wolfe's method for membrane forces in domes

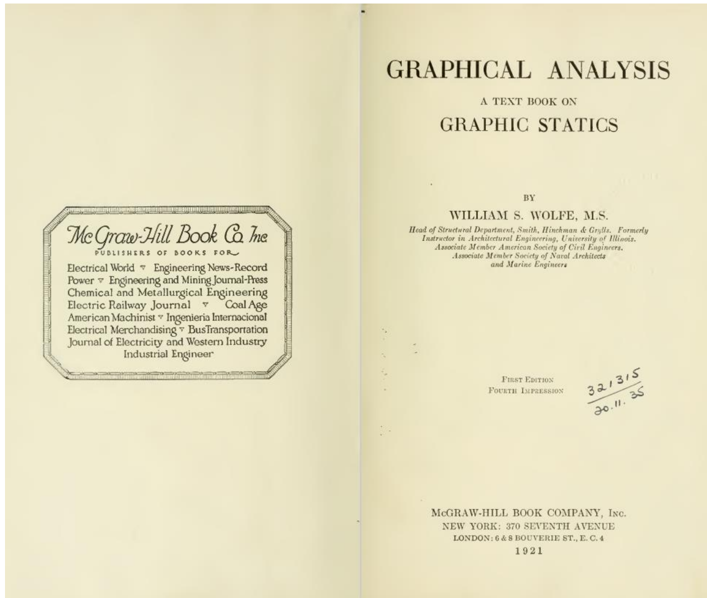
O'Dwyer's funicular analysis $\Rightarrow$

Thrust Network Analysis (TNA)

Questions

Wolfe's method

Wolfe (1921);



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GRAPHICAL ANALYSIS

A TEXT BOOK ON

GRAPHIC STATICS

BY

WILLIAM S. WOLFE, M.S.

Head of Structural Department, Smith, Hinchman & Grylls. Formerly Instructor in Architectural Engineering, University of Illinois. Associate Member American Society of Civil Engineers. Associate Member Society of Naval Architects and Marine Engineers

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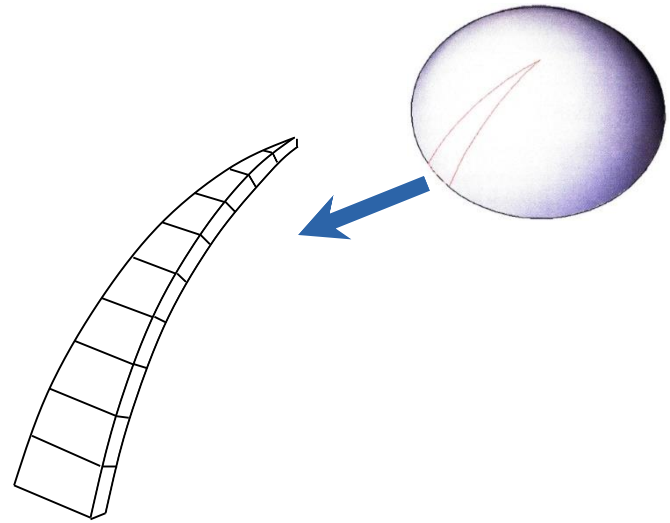


→ Version 1.: domes with tension resistance

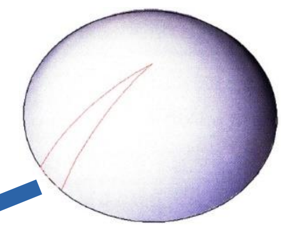
→ Version 2.: domes without tension resistance

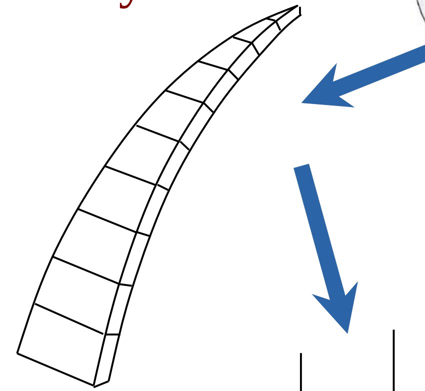
Wolfe’s method

→ Restricted to: domes with ***vertical axis*** of symmetry;

under ***vertical loads*** with vertical axis of symmetry

→ basic assumption: ***membrane state***





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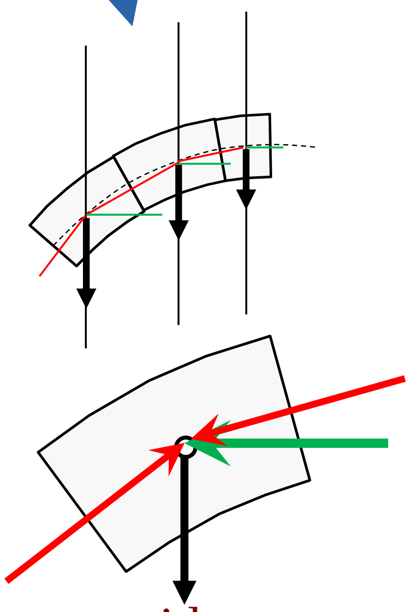
→ Version 2.: domes without tension resistance



Starting step:

weights of lune voussoirs; at centroids

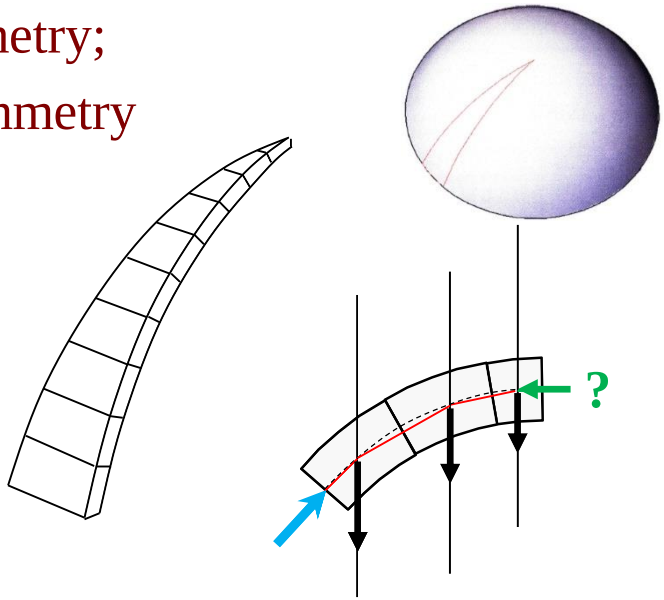
Assumption:

contact force: line of action joins the two neighbouring centroids

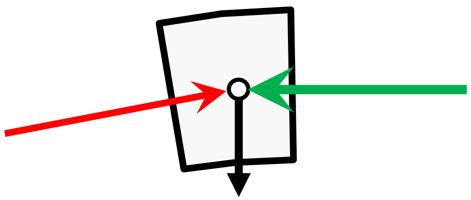
Wolfe's method

→ Restricted to: domes with ***vertical axis*** of symmetry;
under ***vertical loads*** with vertical axis of symmetry

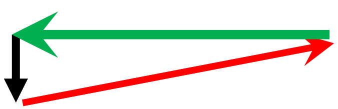
→ basic assumption: ***membrane state***



1. Analysis of the top segment:

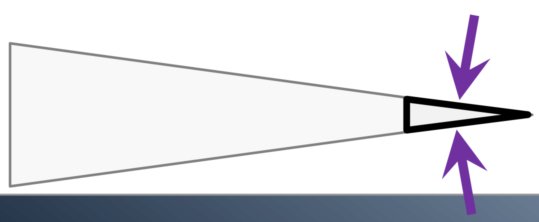


funicular diagram



force diagram, front view

top view:

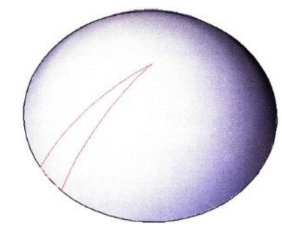


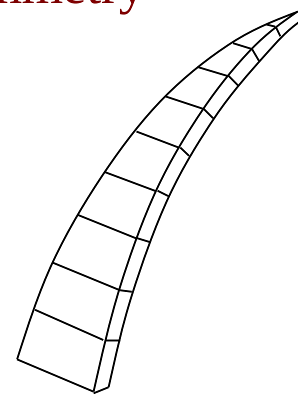
[later]

Wolfe’s method

→ Restricted to: domes with ***vertical axis*** of symmetry;

under ***vertical loads*** with vertical axis of symmetry

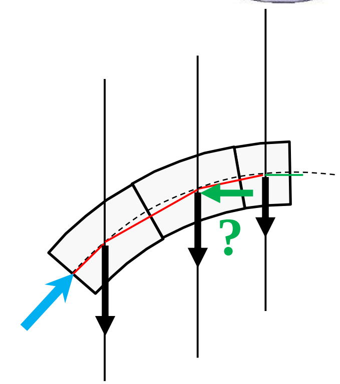




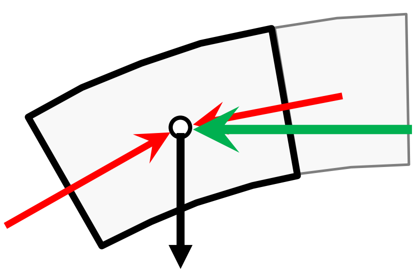
→ basic assumption: ***membrane state***

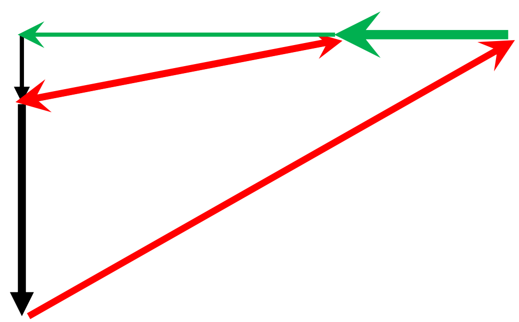
⇒ contact force intersect with weight

along the ***middle line***



2. Analysis of the 2nd segment:





Wolfe's method

→ Restricted to: domes with ***vertical axis*** of symmetry;

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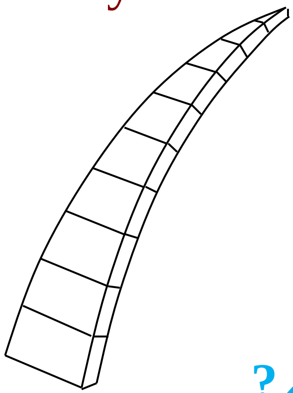
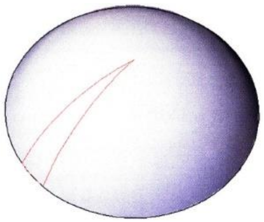
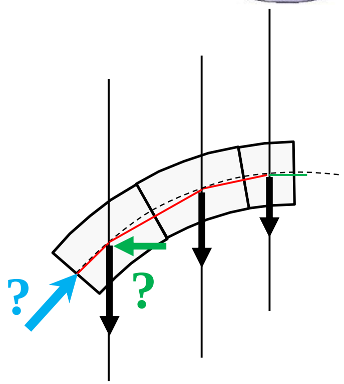
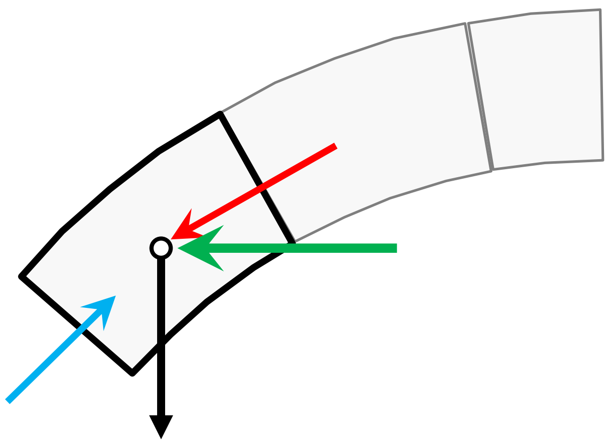
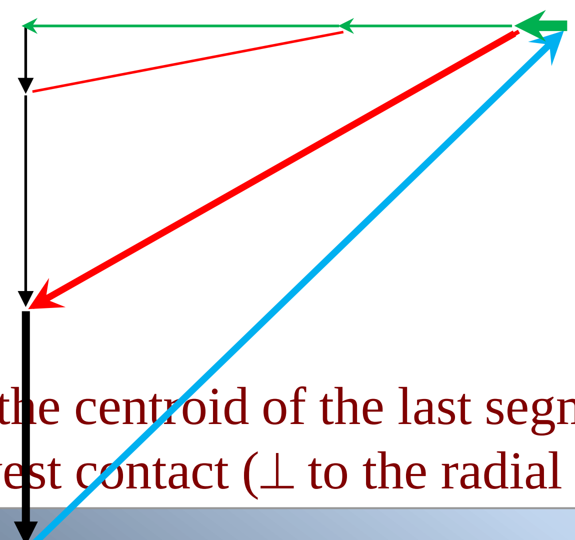






3. Analysis of the bottom segment:





Assumption: Reaction goes through the centroid of the last segment,
perpendicular to lowest contact (⊥ to the radial direction)

Wolfe's method

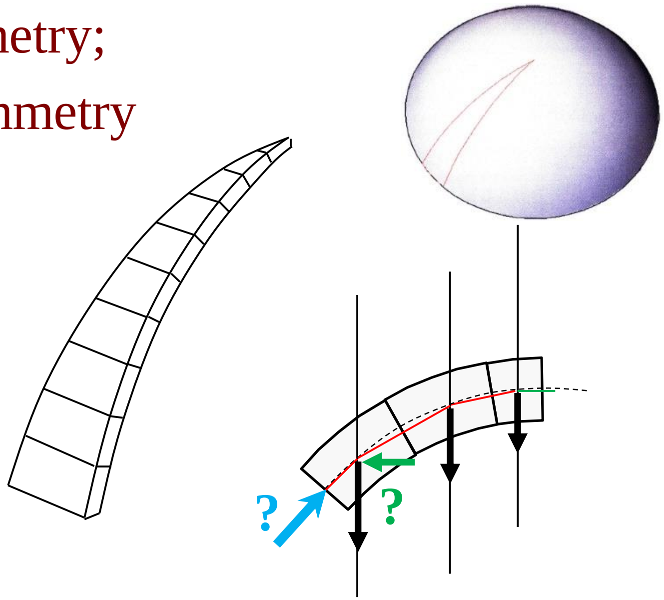
→ Restricted to: domes with ***vertical axis*** of symmetry;

under ***vertical loads*** with vertical axis of symmetry

→ basic assumption: ***membrane state***

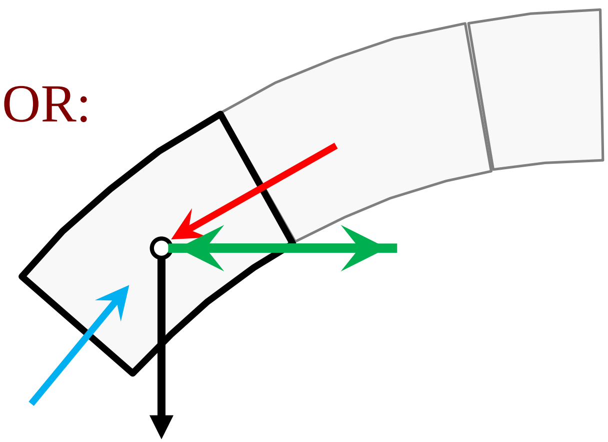
⇒ contact force intersect with weight

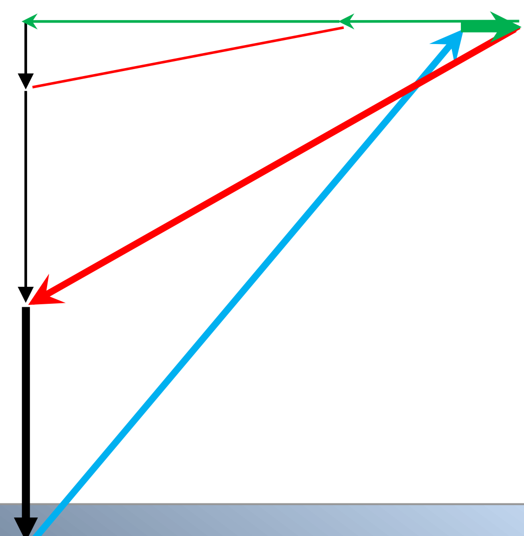
along the ***middle line***



<u>3. Analysis of the bottom segment:</u>

OR:





Wolfe’s method

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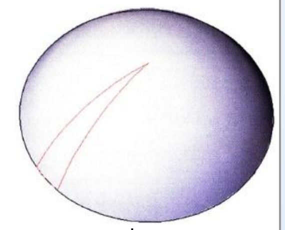
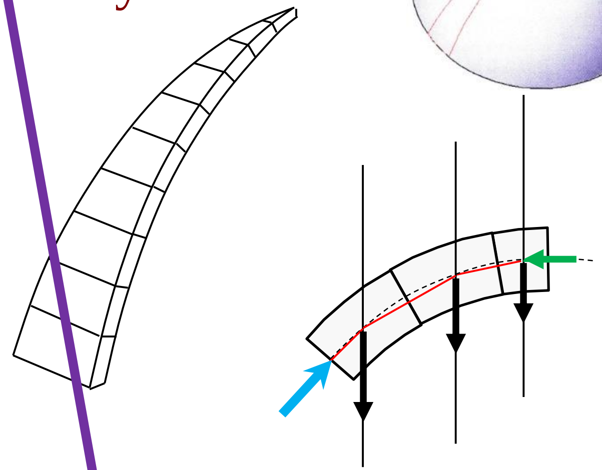
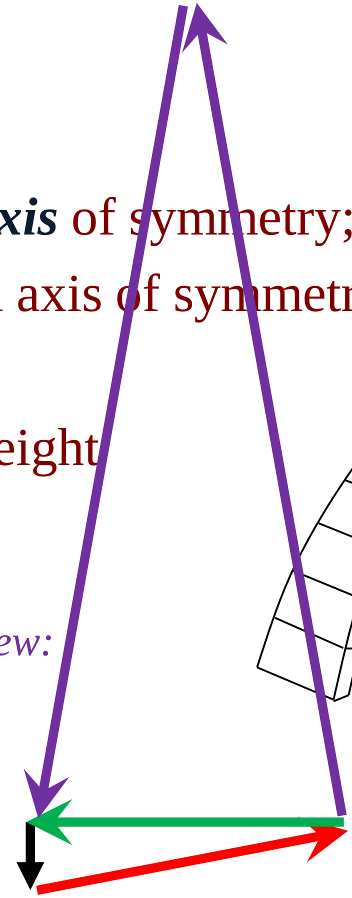
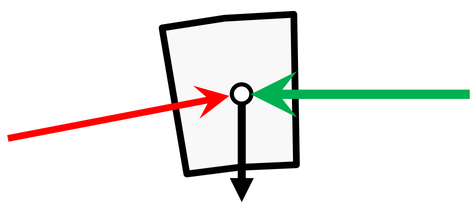




1. Analysis of the top segment:

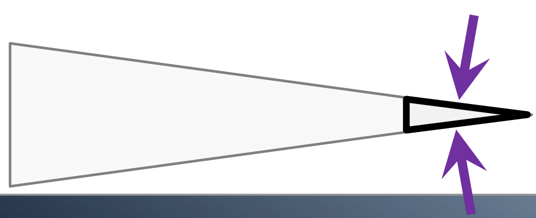
top view:





Hoop forces:

top view:



Wolfe's method

→ Restricted to: domes with ***vertical axis*** of symmetry;

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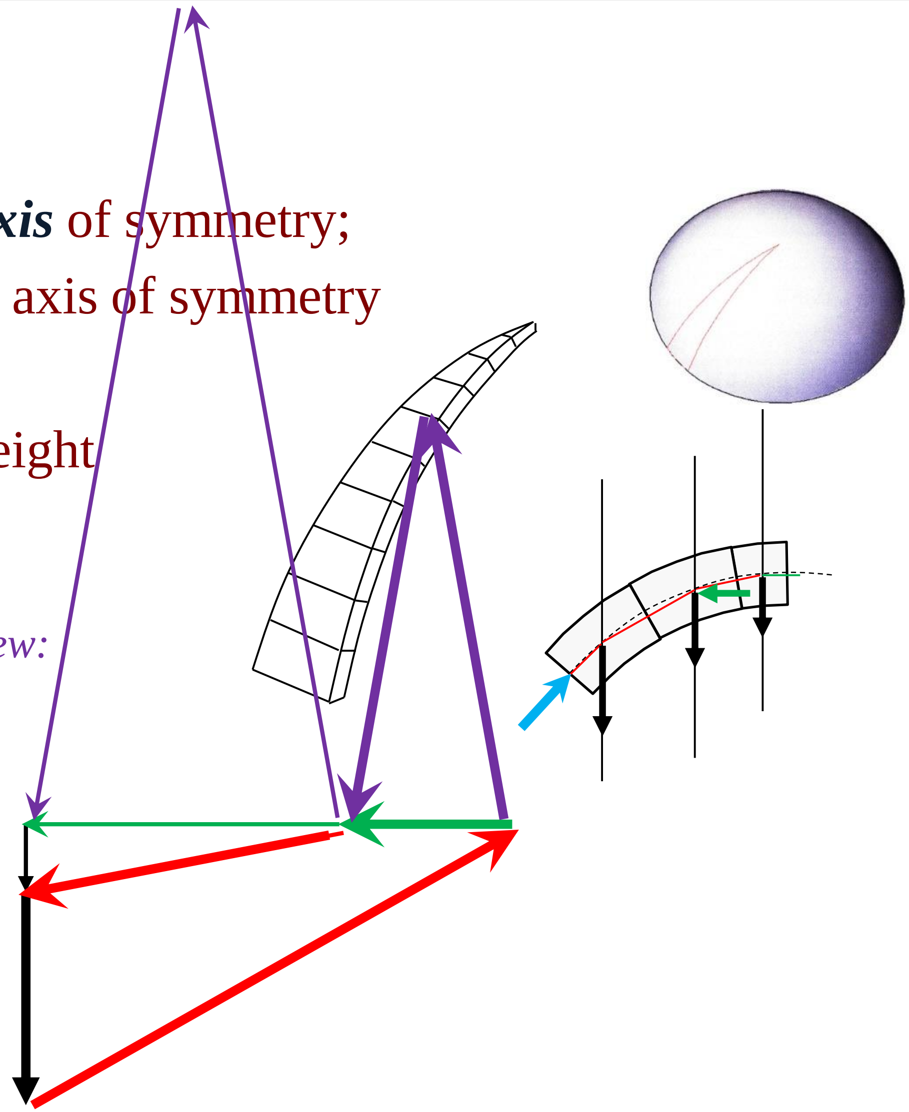
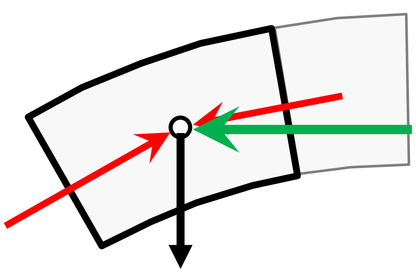
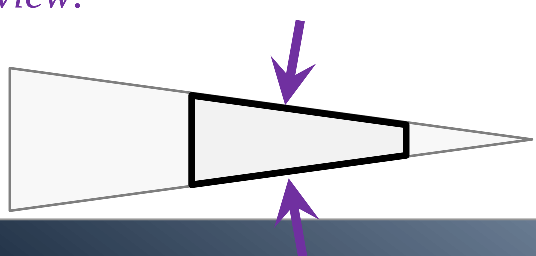


2. Analysis of the 2nd segment:



top view:



Wolfe's method

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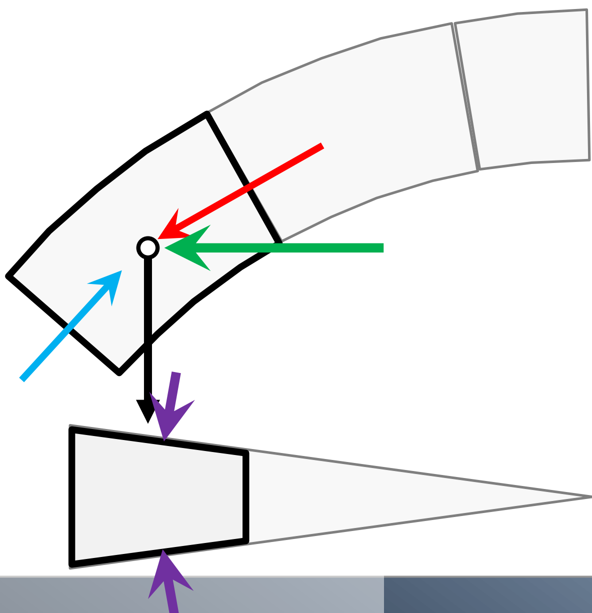
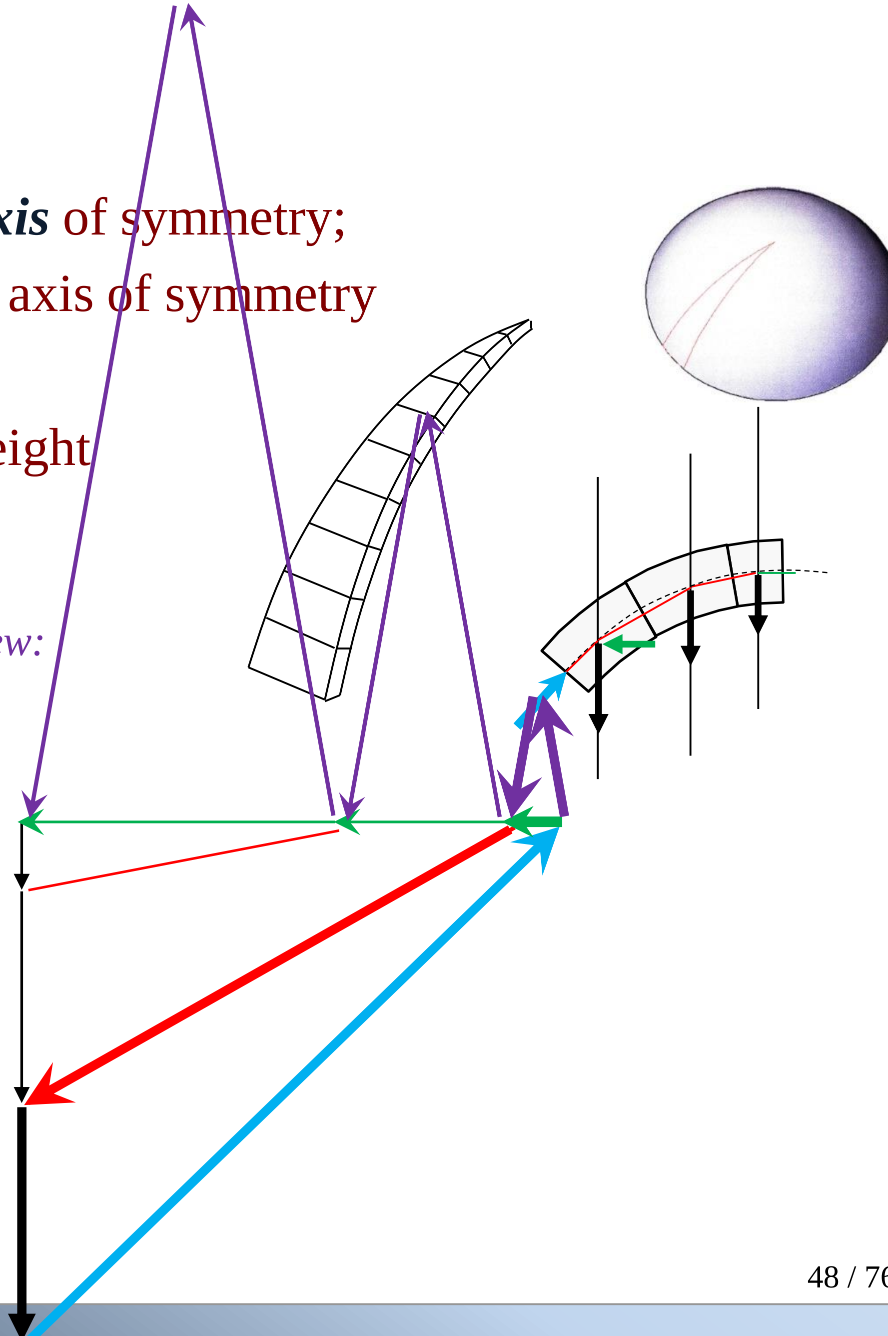
3. Analysis of the bottom segment:





Wolfe's method

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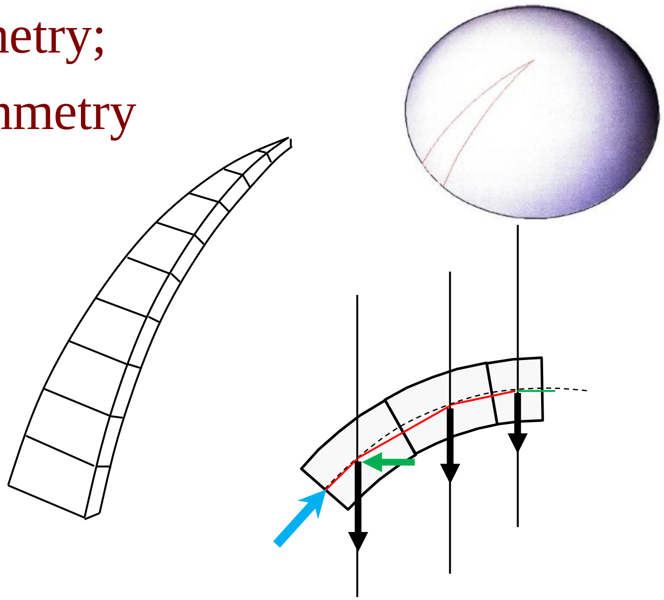
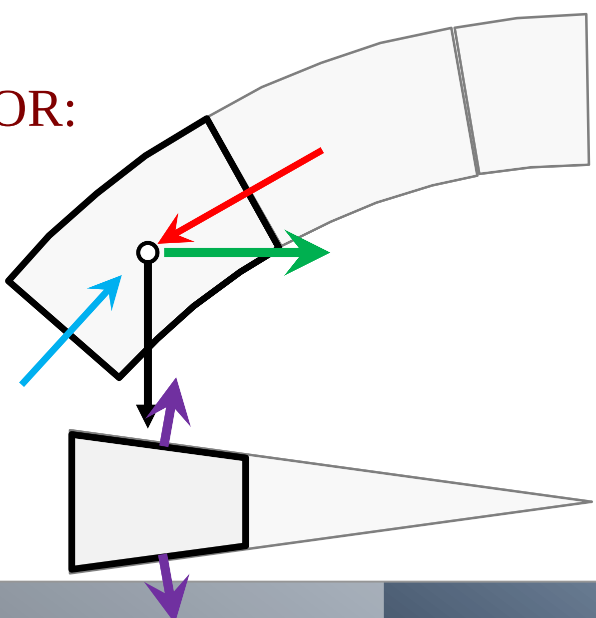
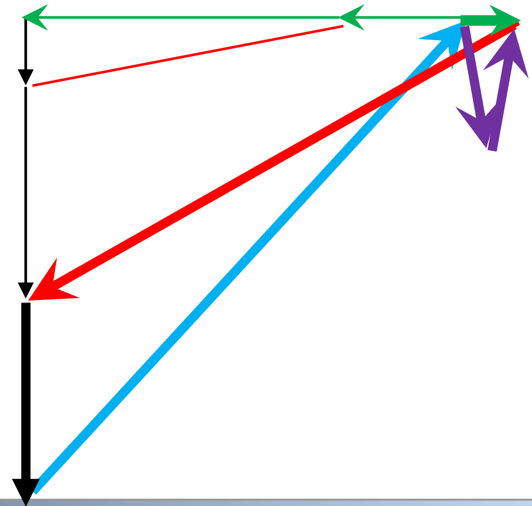


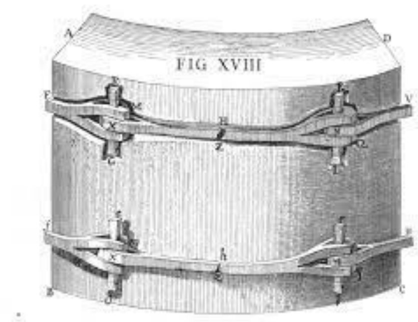
3. Analysis of the bottom segment:

OR:

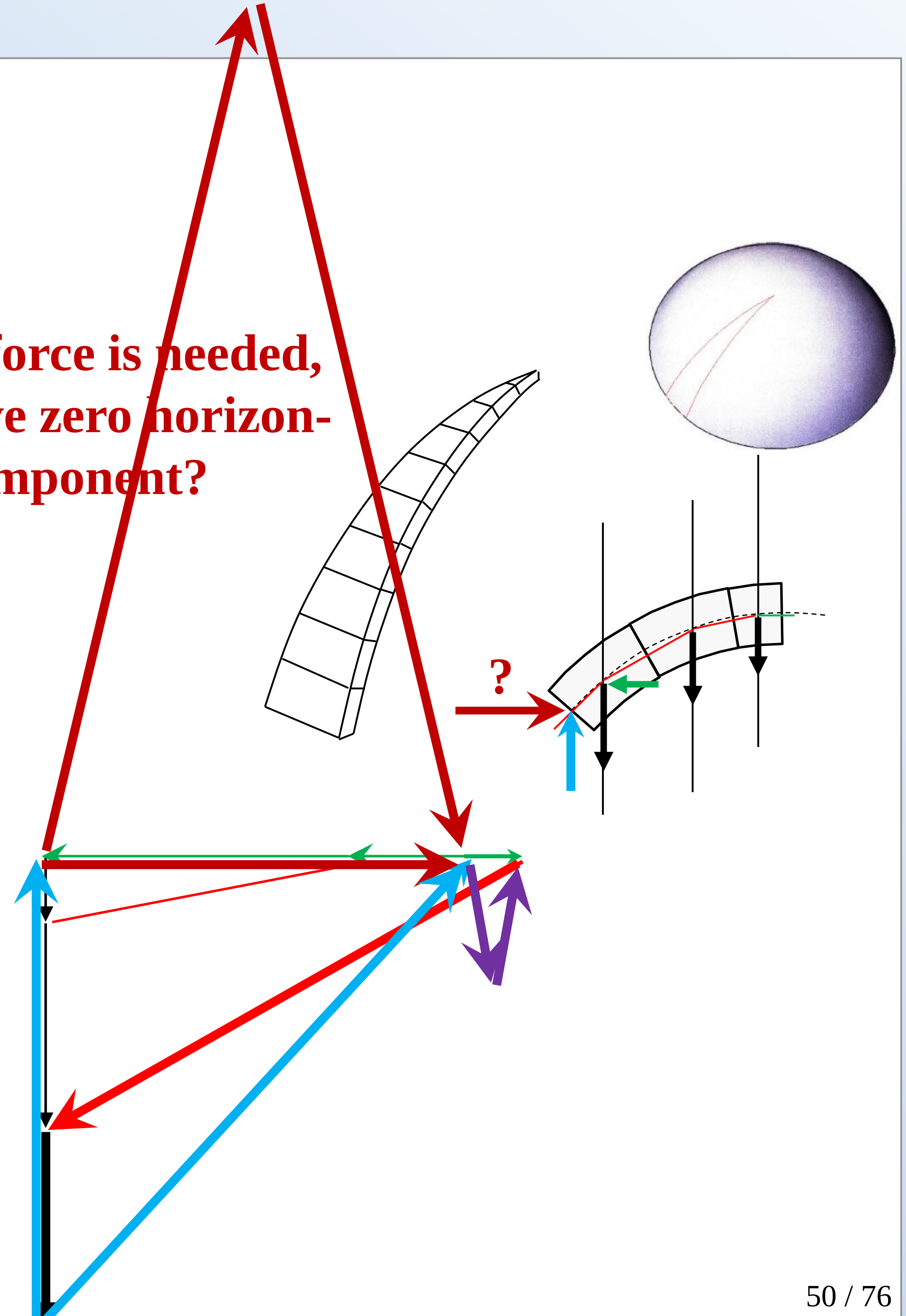




Wolfe’s method

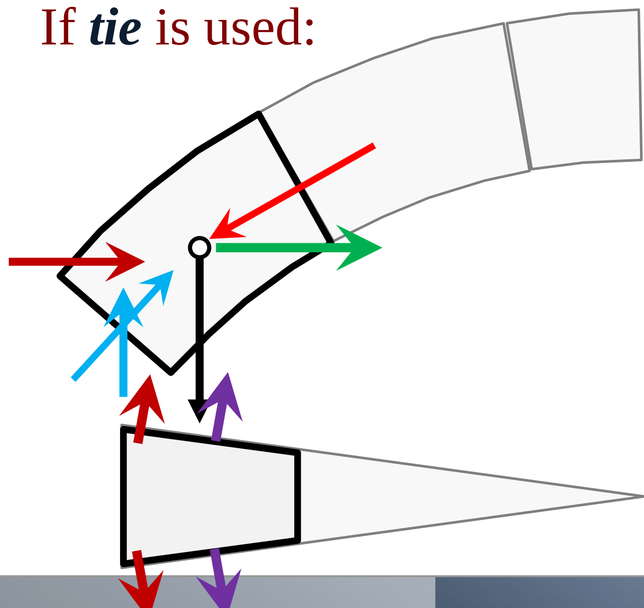


How large tie force is needed, in order to have zero horizontal reaction component?

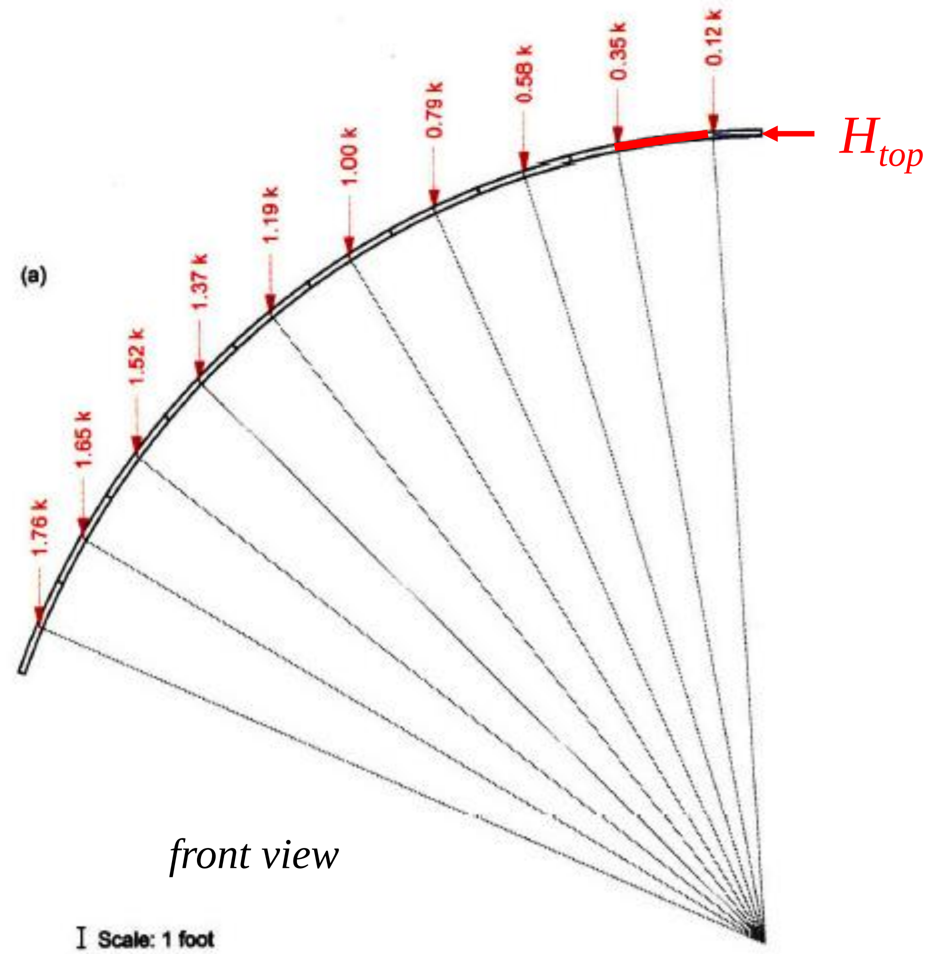


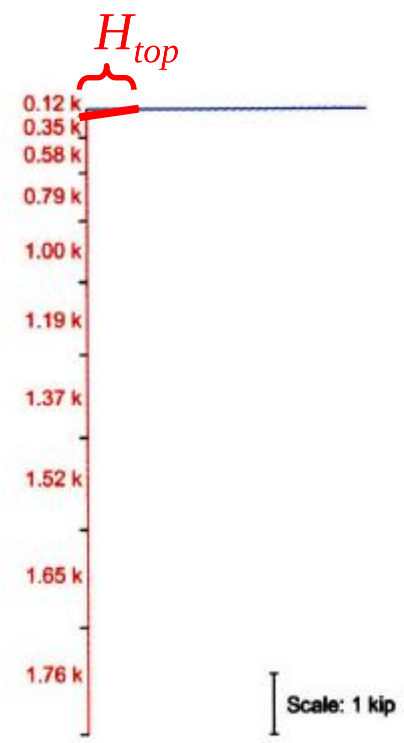
<u>3. Analysis of the 3rd segment:</u>

If ***tie*** is used:

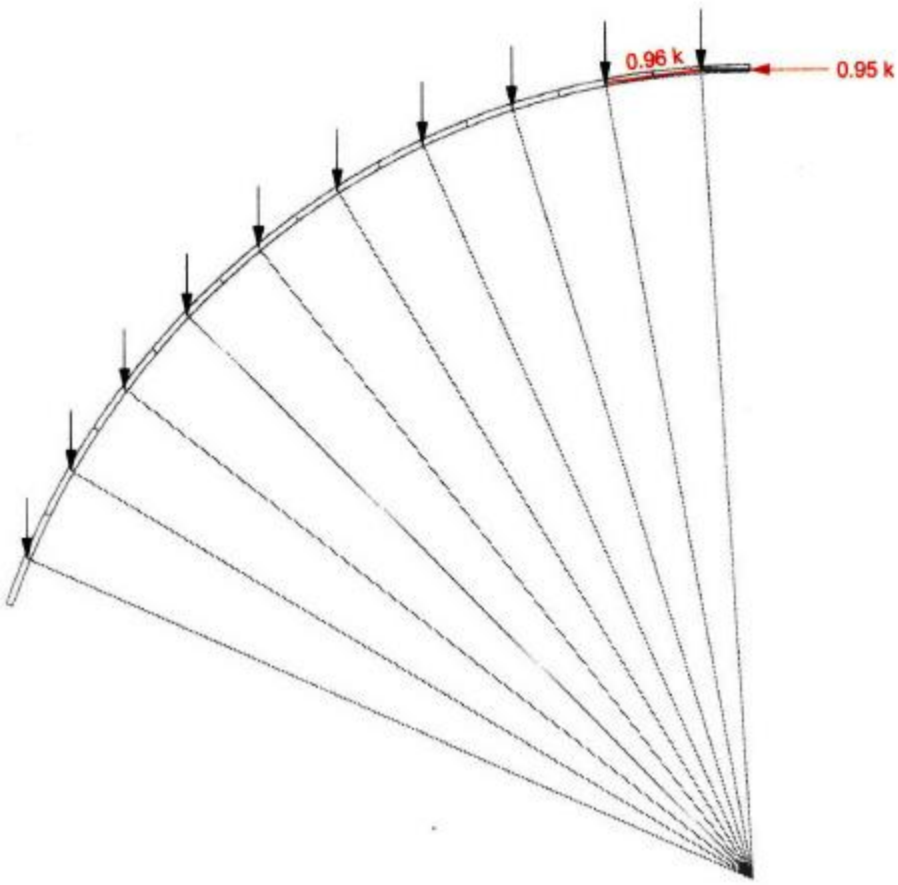


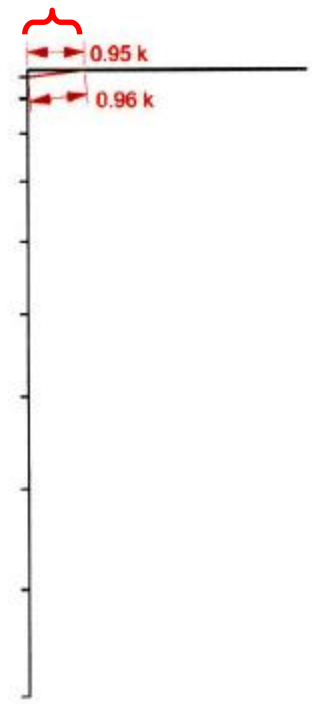
Wolfe's method, for tension-resisting domes



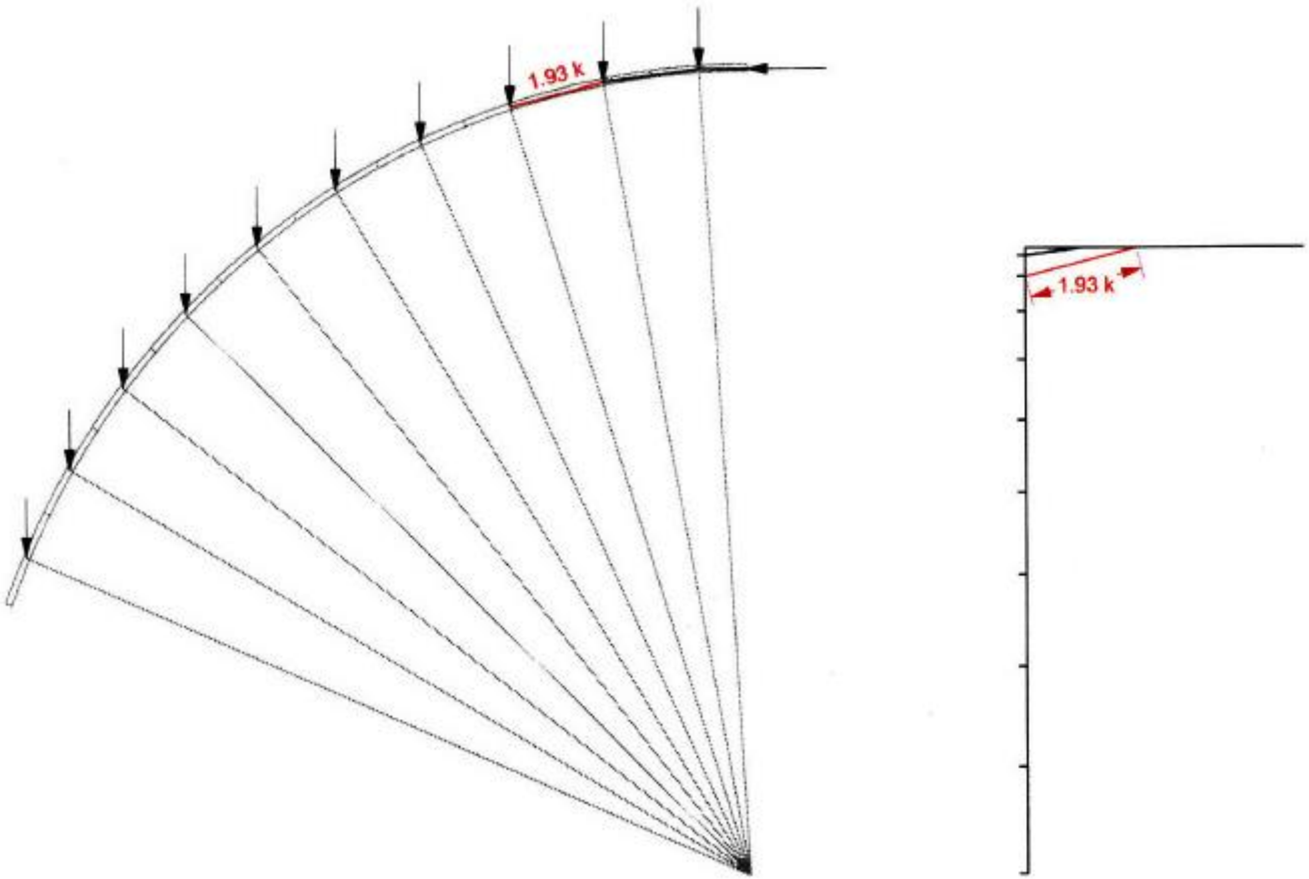


Wolfe's method, for tension-resisting domes





Wolfe’s method, for tension-resisting domes



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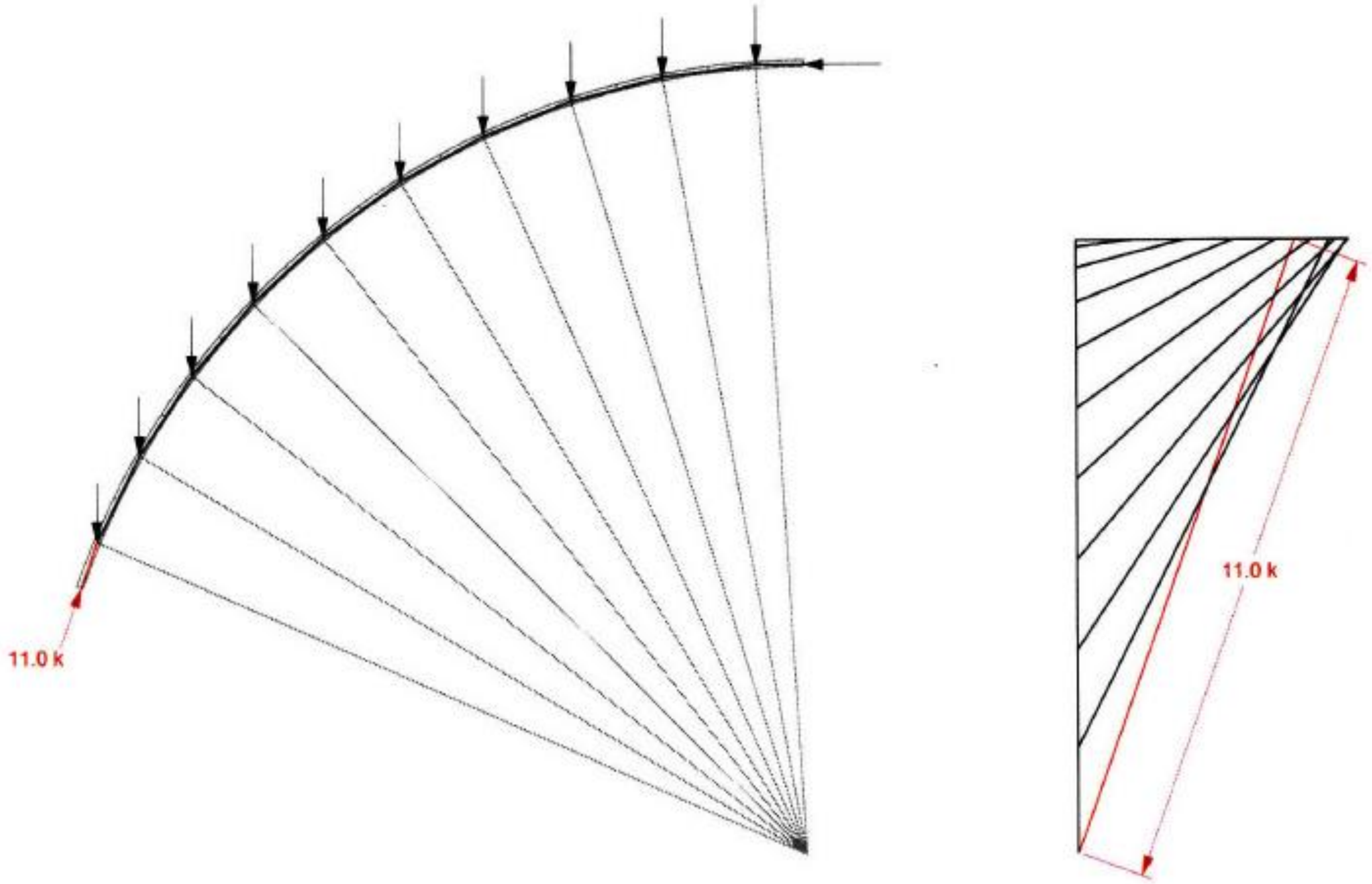
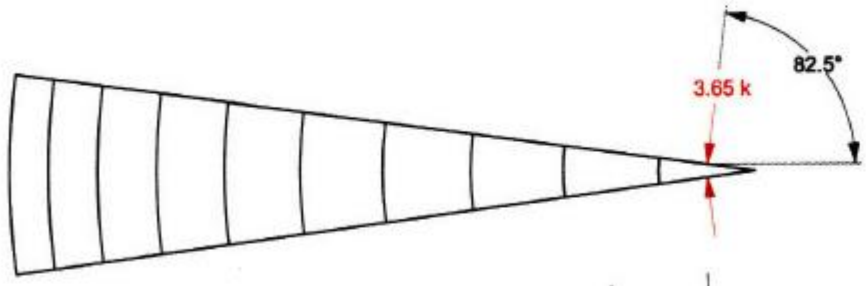
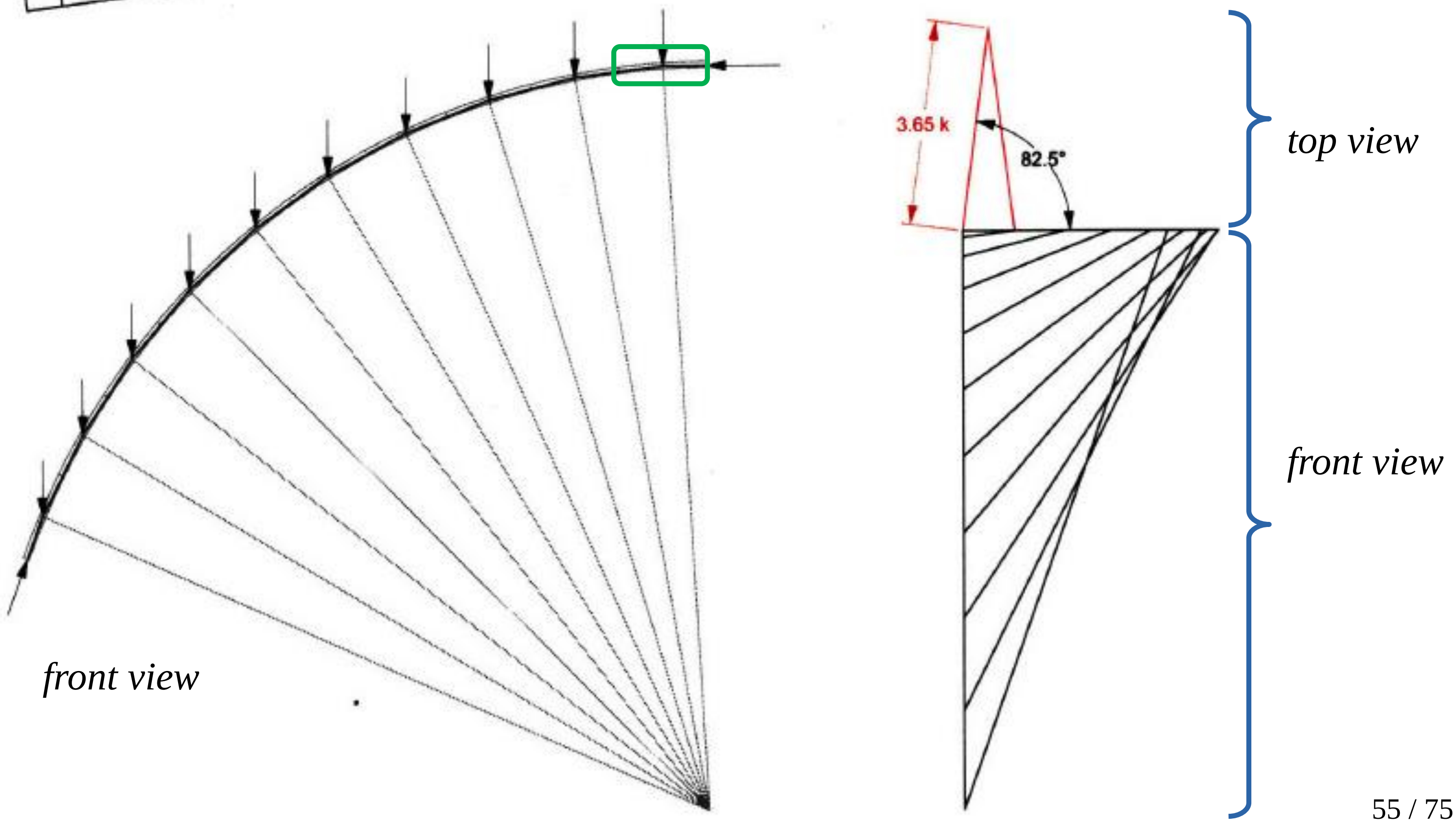


Wolfe's method, for tension-resisting domes

top view

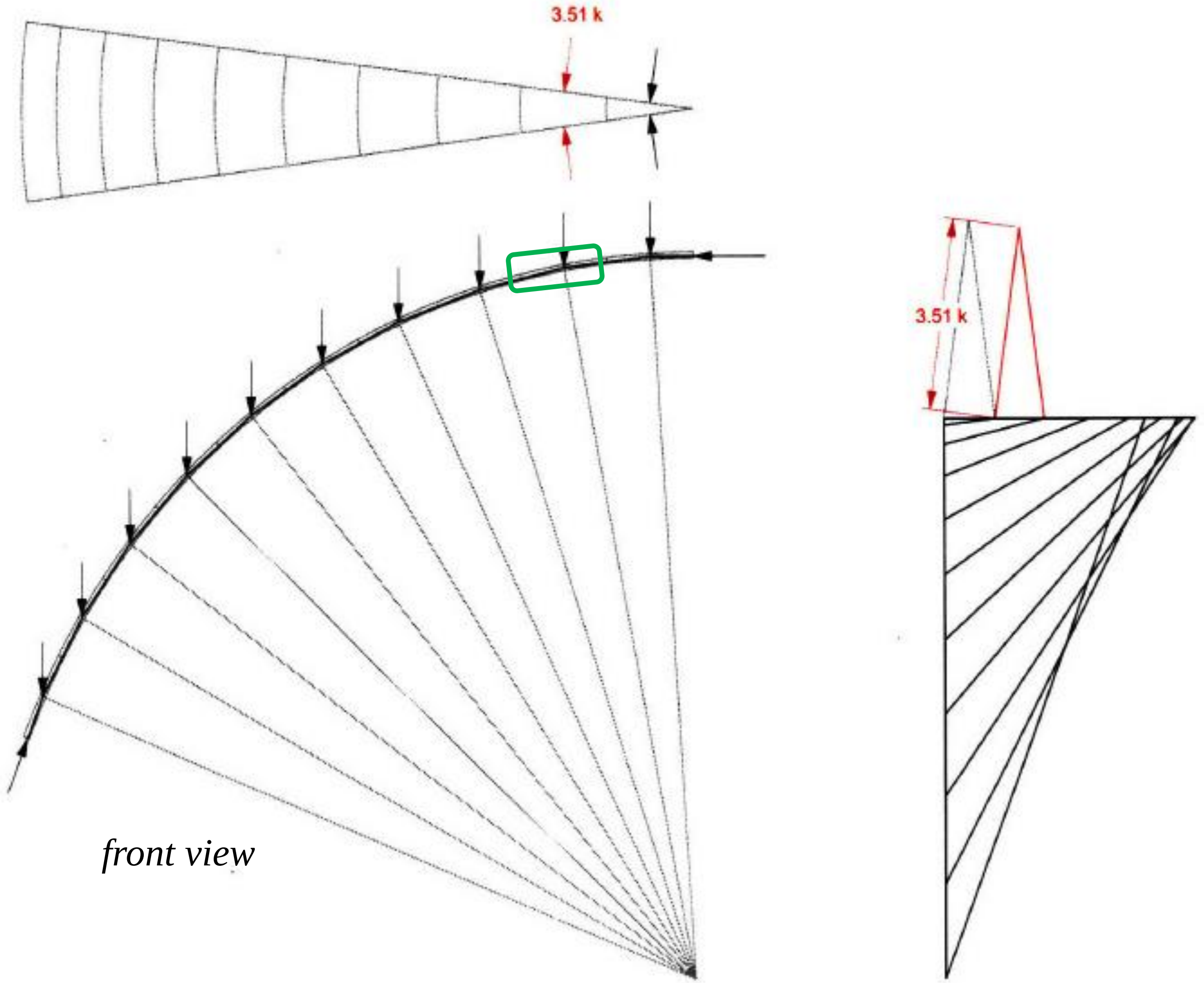


front view



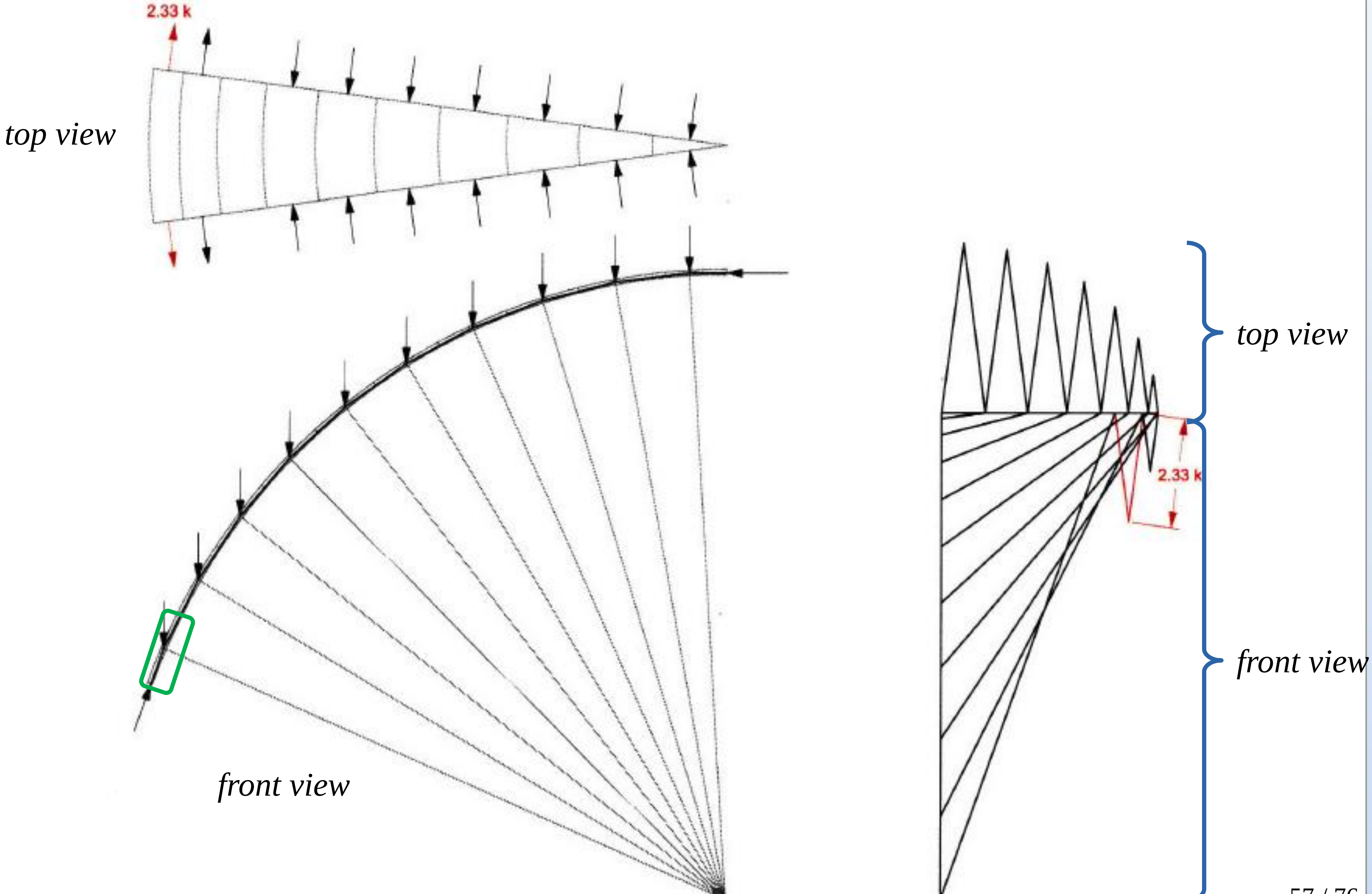
Wolfe's method, for tension-resisting domes

top view

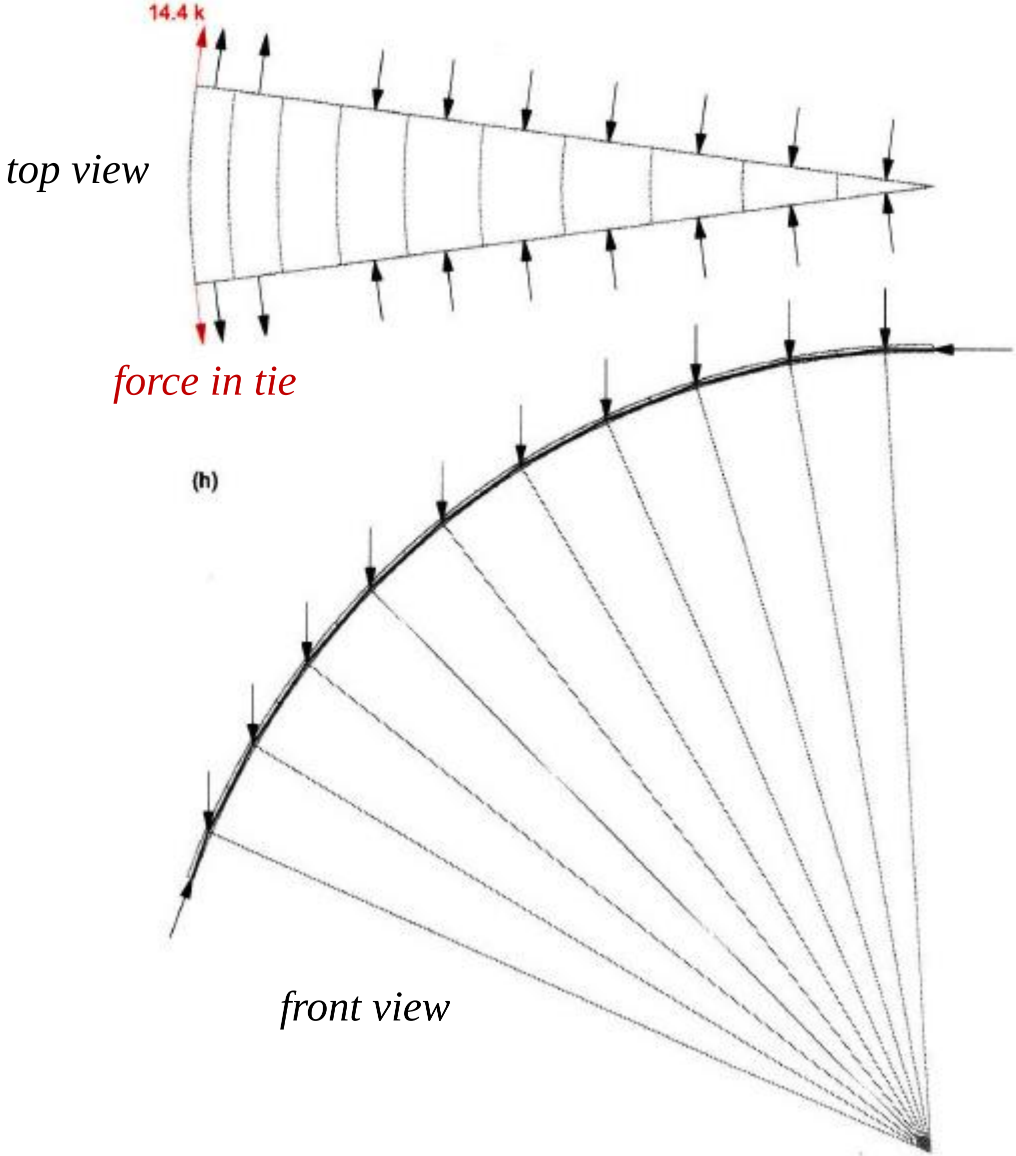


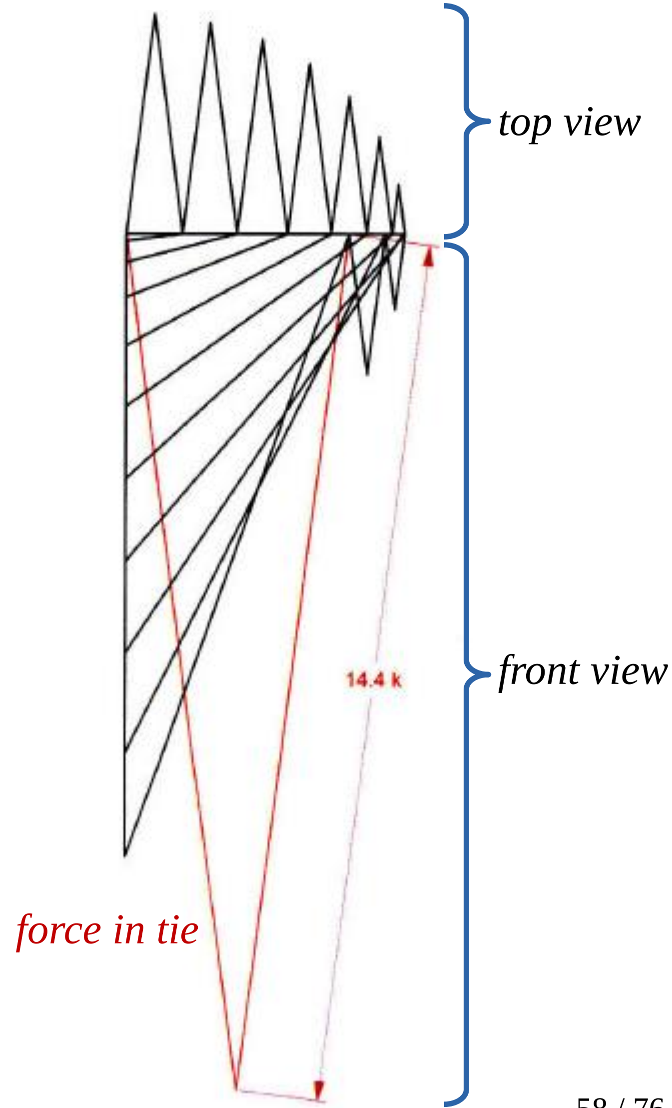
front view

Wolfe’s method, for tension-resisting domes

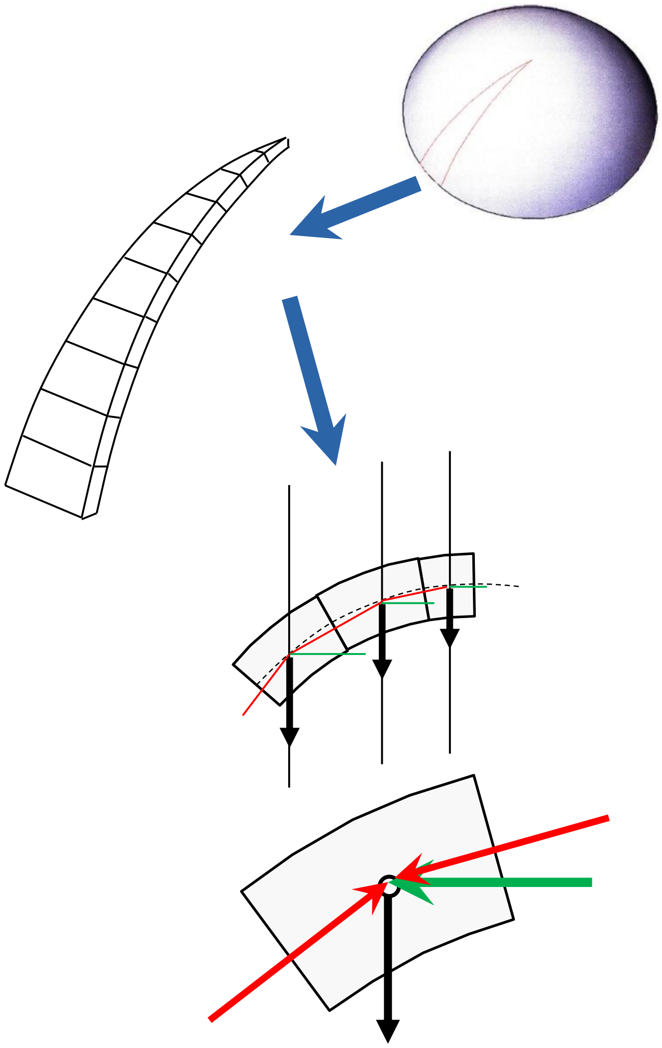


Wolfe's method, for tension-resisting domes





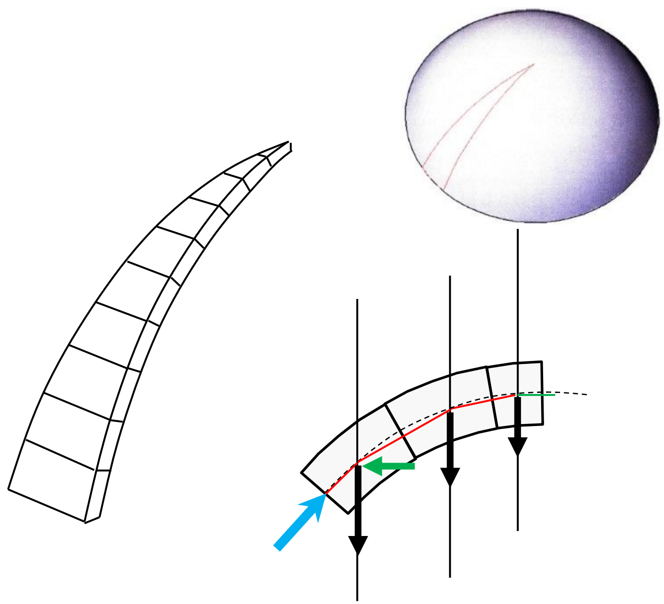
Wolfe's method



→ Version 1.: domes with tension resistance

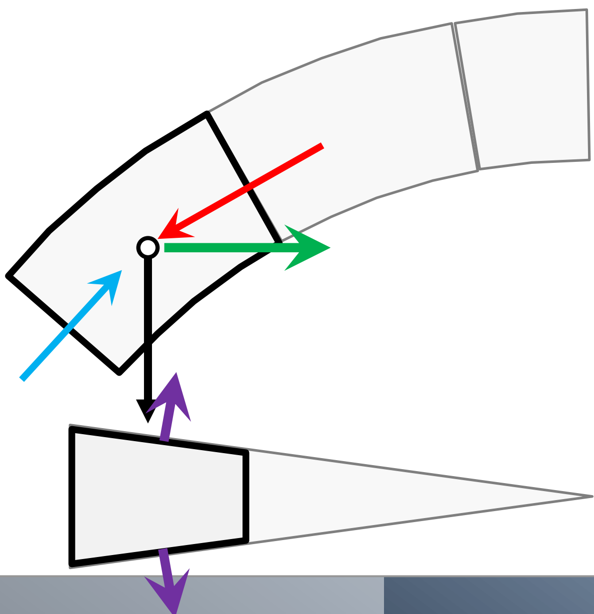
→ Version 2.: domes without tension resistance

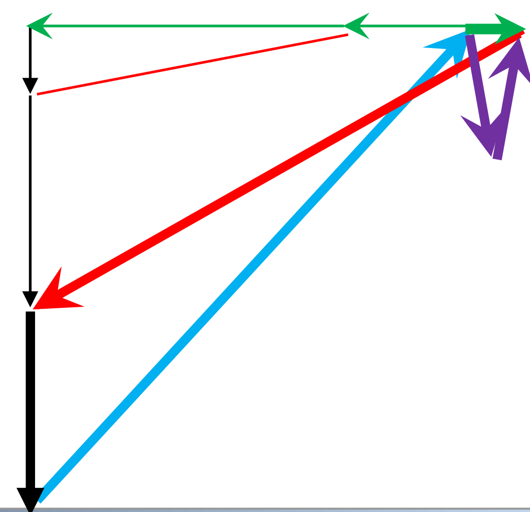
Wolfe's method



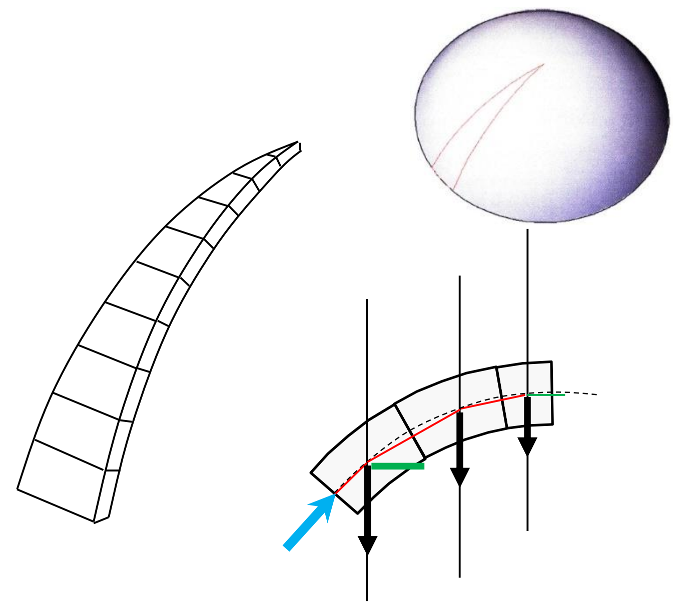
REMEMBER:

3. Analysis of the bottom segment:





Wolfe's method, for no-tension domes



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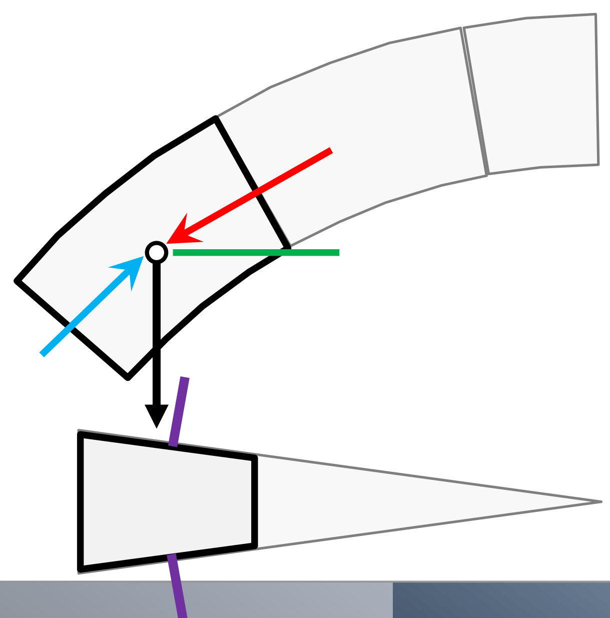
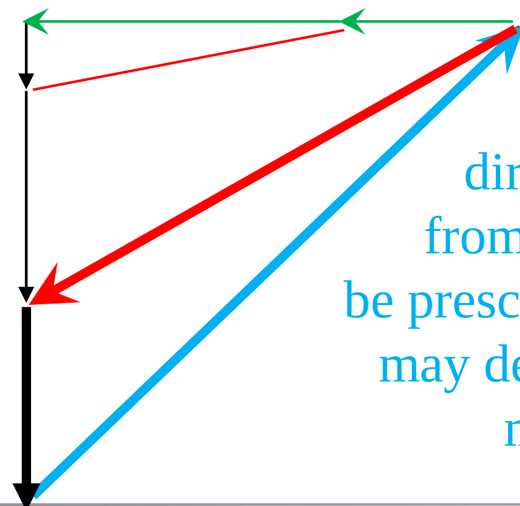
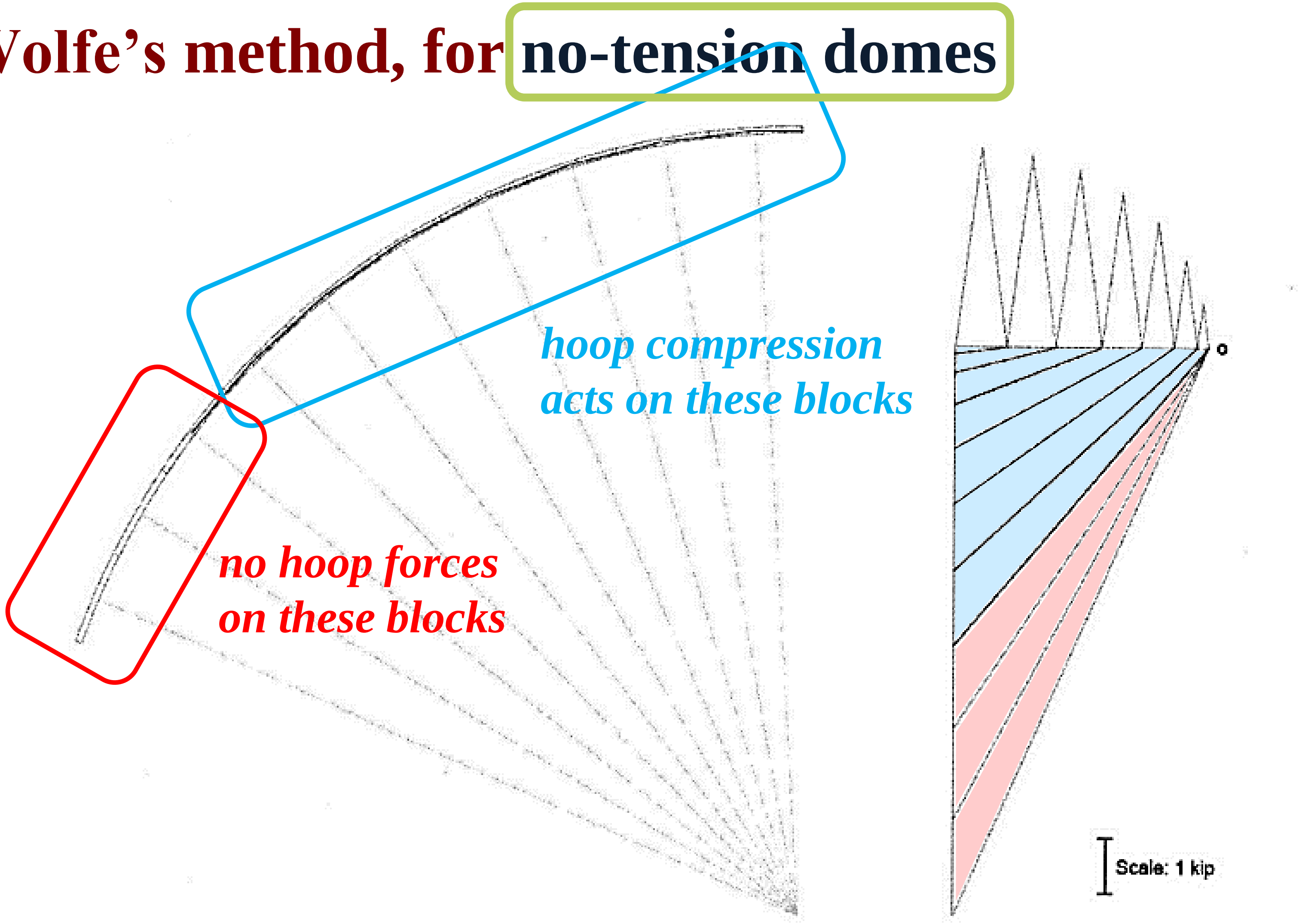




direction of force
from below: cannot
be prescribed/assumed:
may deviate from the
membrane state

Wolfe's method, for no-tension domes



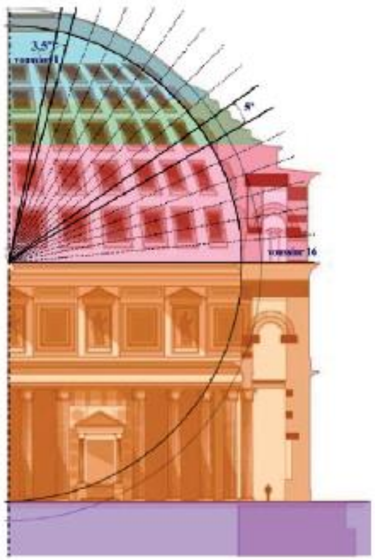
Wolfe's method

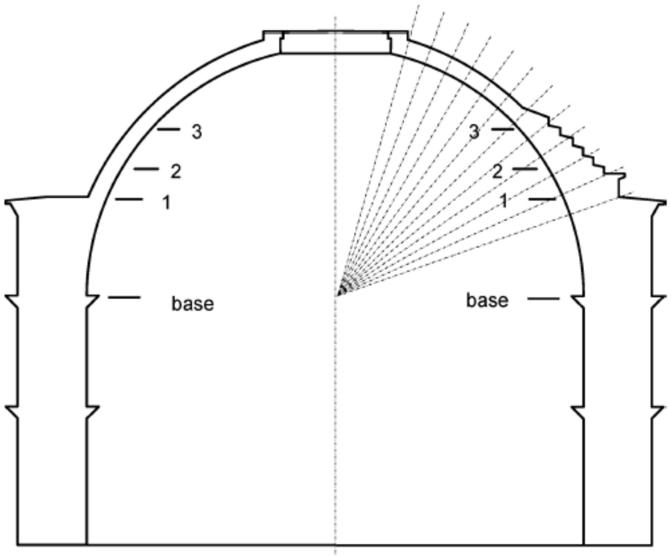
<u>Application:</u>

Morer & Goni (2008):

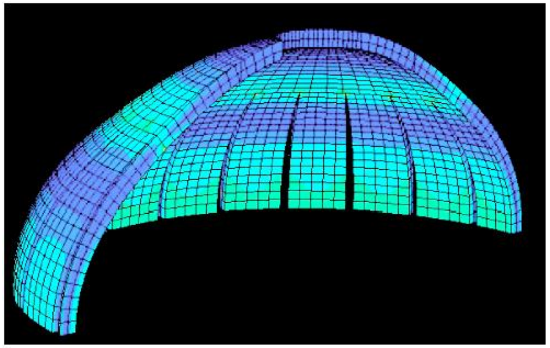
Pantheon in Rome, Italy

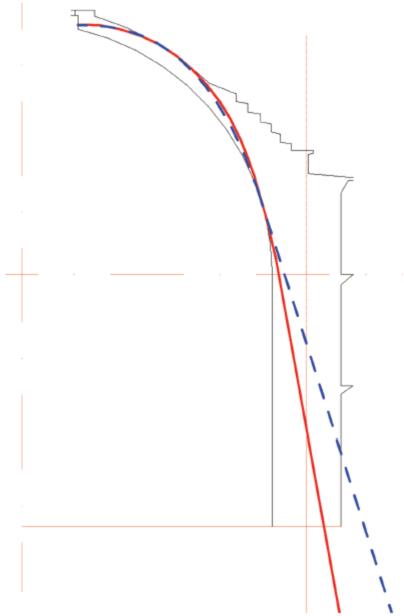
[not masonry!]





agreement with ABAQUS





method extended to find line of thrust: Lau (2006)

Wolfe’s method

Application:

Cavalagli & Gusella (2015)

Cavalagli et al (2016)

Dome of the „Santa Maria Degli Angeli”

Basilica, Assisi, Italy

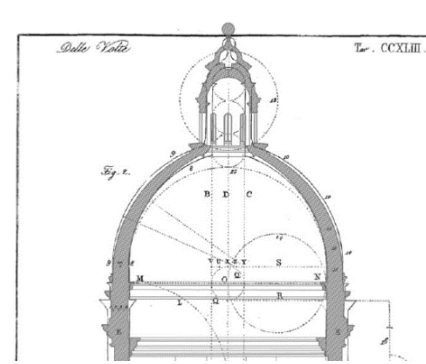
→ construction: 1569-1679; dome completed in 1677

→ dome diameter: ≈20 m; thickness: ≈180…90 cm

perimeter: inside circular, outside octagonal

→ several earthquakes; e.g. 1832

after that: iron rings were added

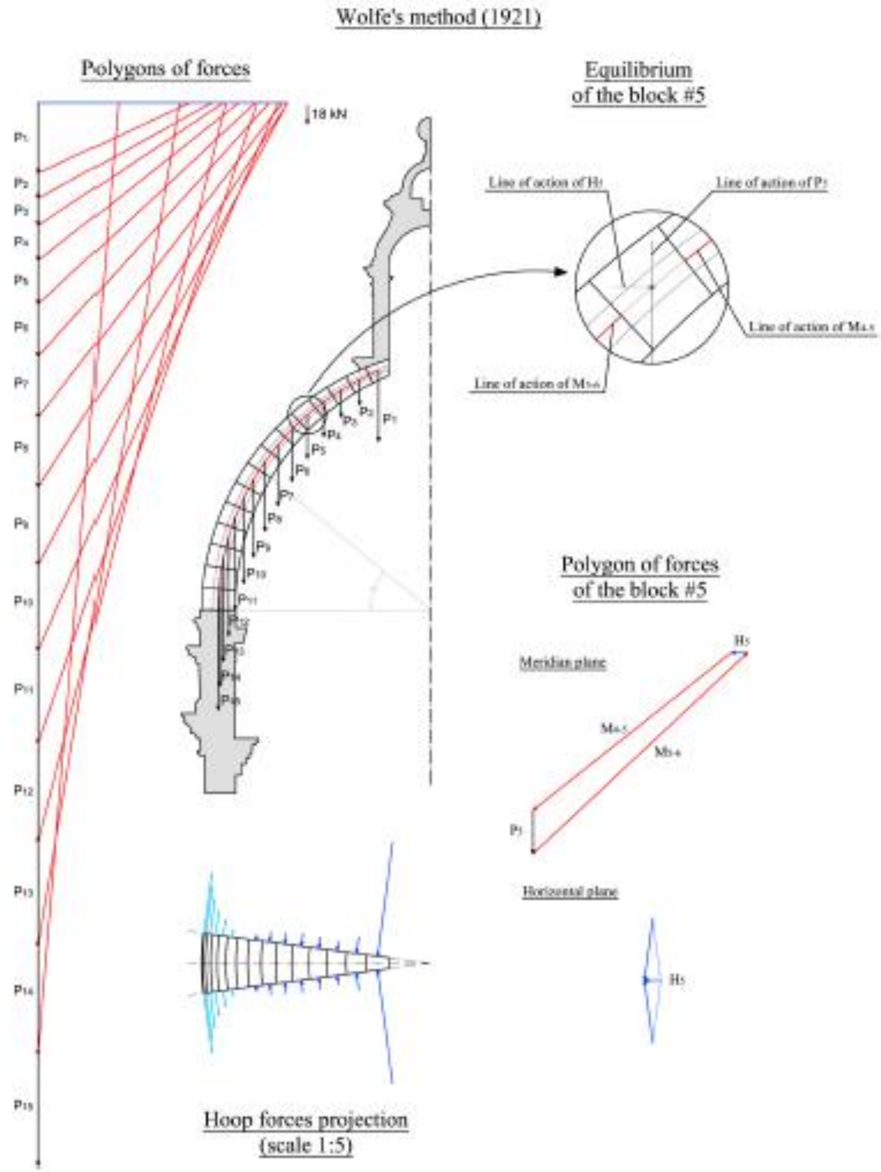


Cavalagli et al (2016)

Wolfe's method

<u>Application:</u>

Cavalagli & Gusella (2015)



Cavalagli & Gusella (2015):

Wolfe's method compared to:

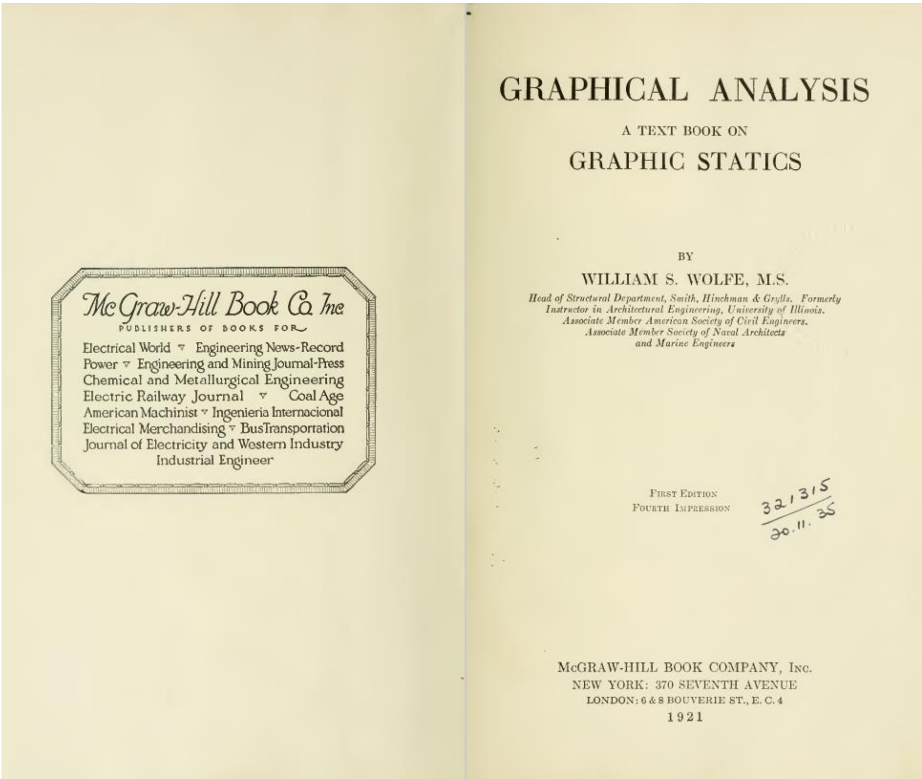
→ the Italian architect manual

→ another old graphical method;

Conclusion:

graphical methods predict slight crackings near the base

Wolfe’s method



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Industrial Engineer

GRAPHICAL ANALYSIS

A TEXT BOOK ON

GRAPHIC STATICS

BY

WILLIAM S. WOLFE, M.S.

Head of Structural Department, Smith, Hinchman & Grylls. Formerly Instructor in Architectural Engineering, University of Illinois. Associate Member American Society of Civil Engineers. Associate Member Society of Naval Architects and Marine Engineers

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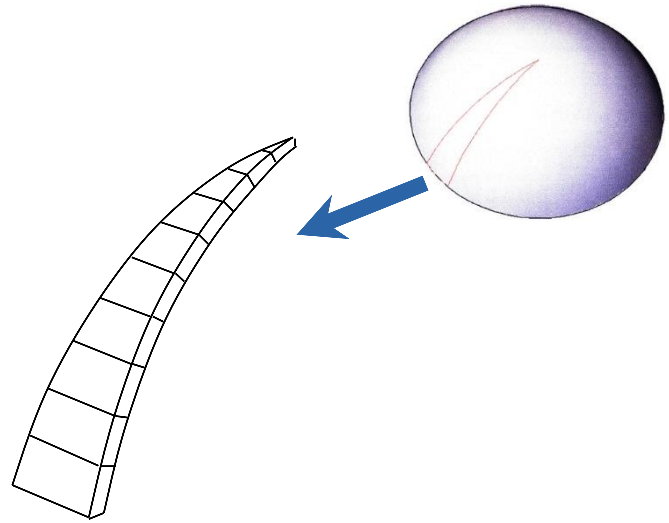


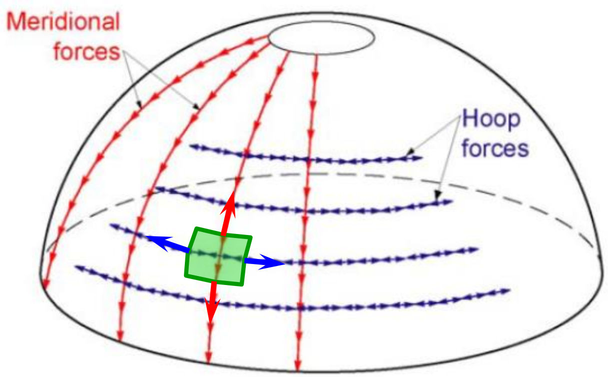
Further reading:

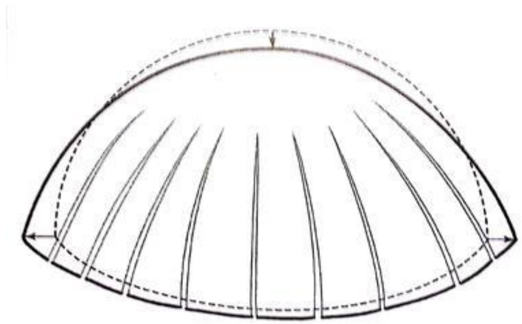
Wolfe (1921); Reese (2008); Lau (2006);

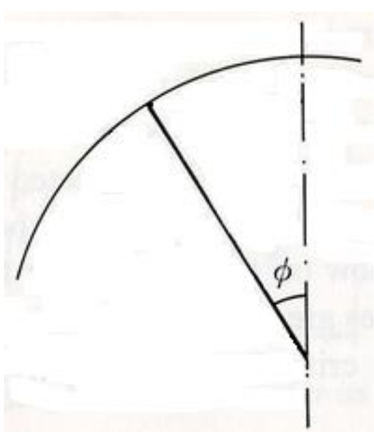
Cavalagli, N., Gusella, V. (2015); Morer & Goni (2008)

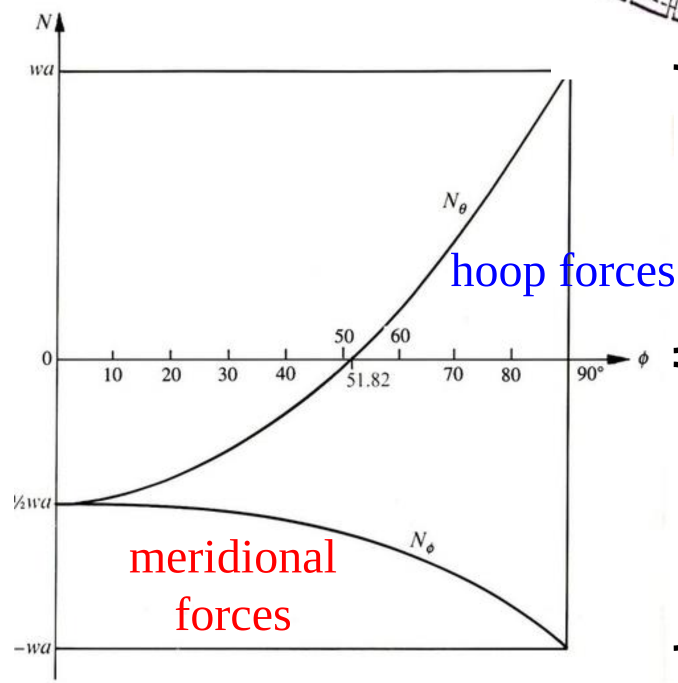
Remark: Membrane solution for spherical cap

Details: next lecture!









tension

compression

This Lecture:
Graphical Methods

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 … 2018

Wolfe's method for membrane forces in domes

O'Dwyer's funicular analysis $\Rightarrow$

Thrust Network Analysis (TNA)

Questions

Thrust Network Analysis

Preliminary: „Funicular Analysis”, O’Dwyer (1999)

~~masonry vault~~ → 3D truss: nodes ≈ stone block inner points

bars ≈ contacts between blocks

bar forces ≈ contact forces

Vertical loads only!

Approximative because:

→ all forces acting on a stone block intersect in the same point

→ the lines of action in top view must be assumed at the beginning

Given: geometry of the vault; loading forces (dead & live)

Unknowns: → vertical coordinates (z_i) of the nodes

→ some of the horizontal force magnitudes

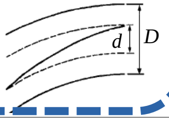
Equalities: equilibrium of the nodes

Inequalities: nodes fall inside the material: $z_i^{\text{intrados}} \leq z_i \leq z_i^{\text{extrados}}$

Objective function: either: live load multiplier → max!

or : deviation from middle surface → min!

linear optimization problem



Thrust Network Analysis

Preliminary: „Funicular Analysis”, O’Dwyer (1999)

~~masonry vault~~ → 3D truss:

nodes ≈ stone block inner points

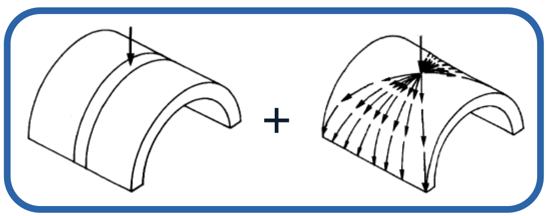
bars ≈ contacts between blocks

bar forces ≈ contact forces

Applications:

Problem Type 1:

Find maximum admissible live load on a given vault:



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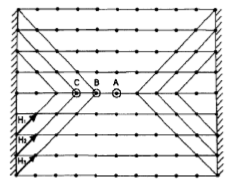
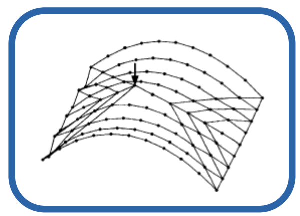
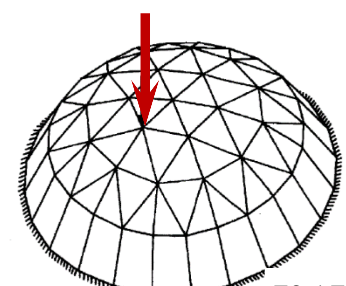




→ admissible max. load magnitudes were determined

Problem Type 2:

Find optimum network shape of a vault under a given load: minimize the vertical deviation of force lines of action from the vault middle surface



Thrust Network Analysis

Block & Ochsendorf (2007), (2008); Block & Lachauer (2014):

→ based on O'Dwyer's „Funicular Analysis"

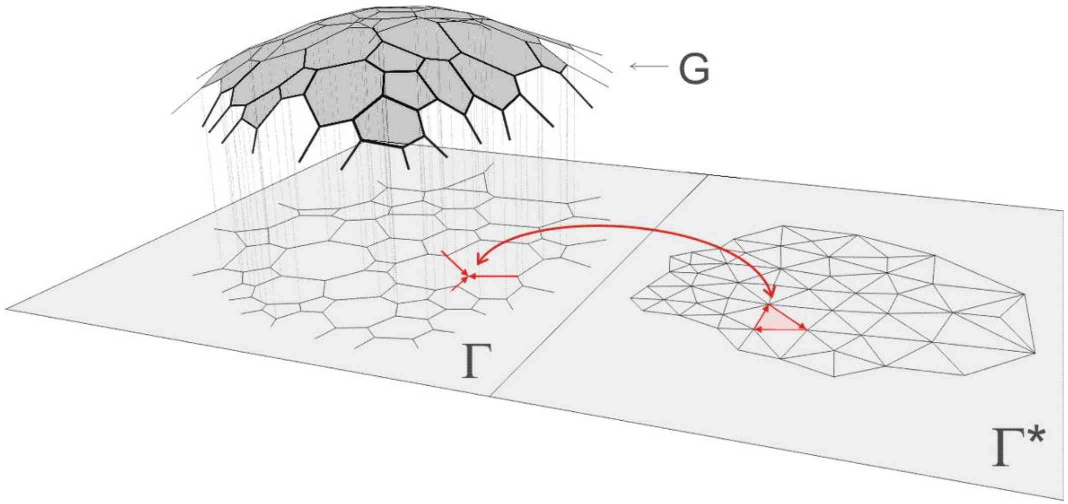
→ sophisticated computer coding; nice graphic representations

objective functions can be:

(1) minimize deviation from middle surface (max geometrical factor of safety)

(2) minimal / maximal horizontal thrust (deepest / shallowest force systems)

(3) maximize live load multiplier which can be added to the given selfweight



Thrust Network Analysis

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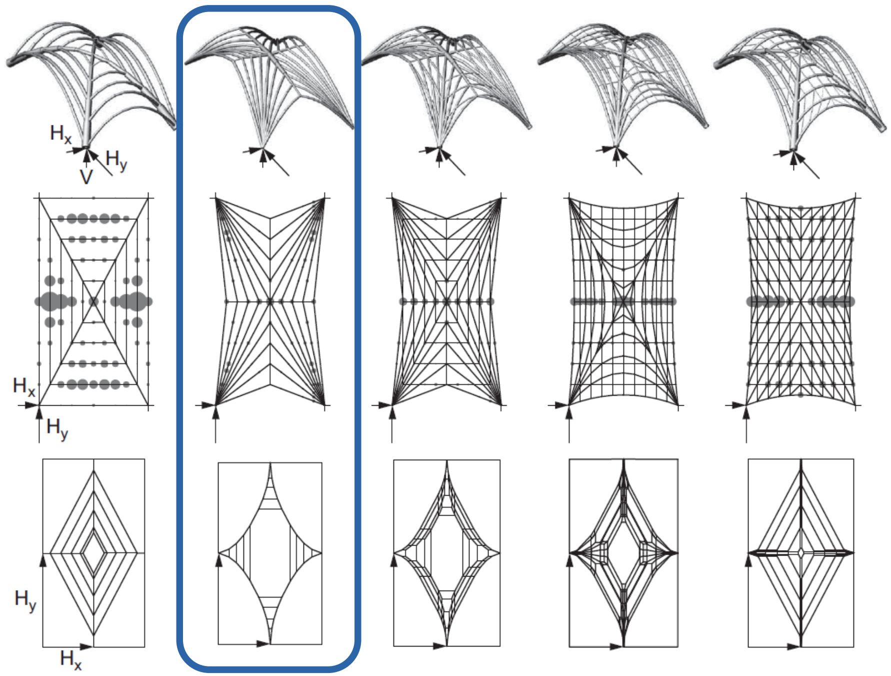
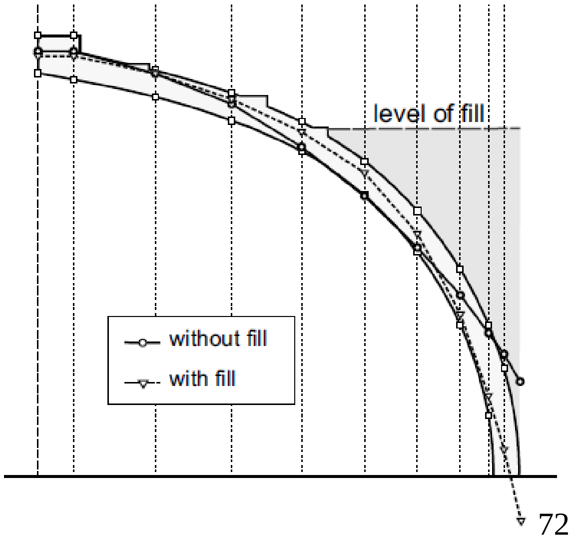
→ sophisticated computer coding; nice graphic representations

→ analysis of several Gothic structures

cross vaults:



fan vaults:



Thrust Network Analysis

Block & Ochsendorf (2007), (2008); Block & Lachauer (2014):

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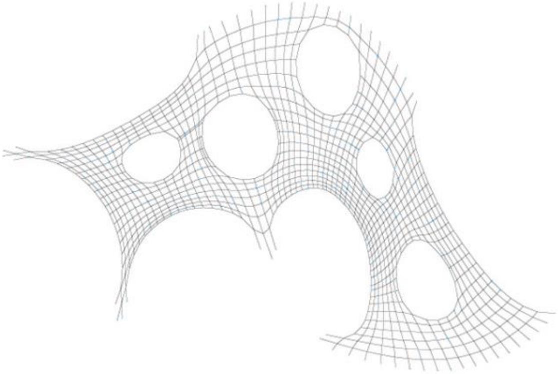
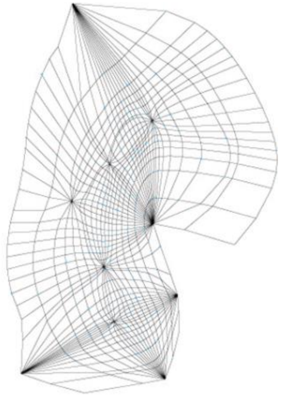
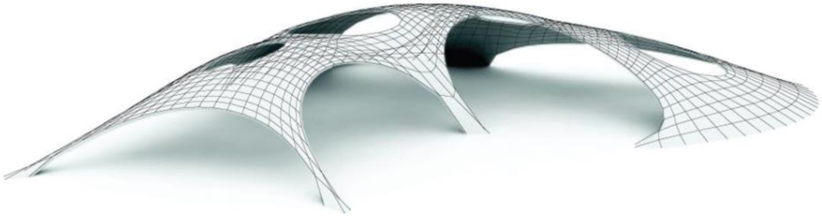
→ sophisticated computer coding; nice graphic representations

→ analysis of several Gothic structures

→ design optimal shapes for vaults







Thrust Network Analysis

Block Research Group:

e.g. The Red Line project, Rwanda:

drone port:

tile-vaulted (very thin) structures,

easy and cheap to construct

„Durabric” (earth + 8% cement, not burnt)

https://www.youtube.com/watch?v=mZwIIndTUow

block.arch.ethz.ch/brg/project/venice-biennale-2016_droneport

This Lecture: Graphical Methods

Historical times: Practical geometrical rules

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O'Dwyer's funicular analysis $\Rightarrow$

Thrust Network Analysis (TNA)

Questions

QUESTIONS

1. Introduce a chosen historic geometrical design rule. What is the background for this design rule?

2. How to determine the possible minimal and maximal horizontal thrust for an arch under selfeight, using graphical statics?

3. What is the geometrical factor of safety of an arch or vault?

4. Introduce Durand-Claye’s stability area method.

5. Introduce Wolfe’s method for domes. How is it used for no-tension material, and for determining the tie force?

6. Introduce the Thrust Network Analysis method. What objective functions can be used?