



GRAPHICAL METHODS



Citation:

K. Bagi (2024): **Mechanics of Masonry Structures**. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

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THIS LECTURE:

2. GRAPHICAL METHODS

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches


computerized & extended for domes: Aita et al 2003 ... 2018


Wolfe's method for membrane forces in domes

O'Dwyer's funicular analysis \Rightarrow

Thrust Network Analysis (TNA)

Questions

 Közlemények

 Subject Datasheet


This content is public

 **MasonryMech References** Uploaded 11/07/22, 12:16

Virtual presentation


Teams join code: **odjgfvx**

Introduction

 **01. Fundamentals and General Overview** Uploaded 23/09/22, 00:18

This content is public

Methods of Mechanical Analysis

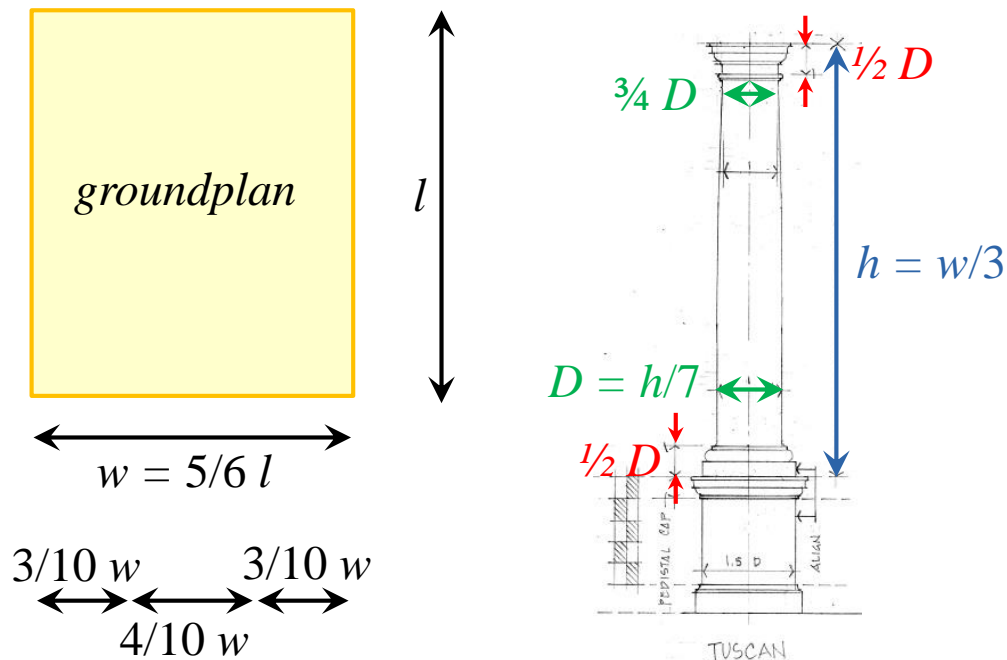
 **02. Graphical Methods** Uploaded 26/07/20, 12:46

Historical times: Practical geometrical rules

Roman era, Vitruvius (Roman Empire, BC 1stct., army engineer & architect):
„Ten Books on Architecture”

→ inspired many architects, already from VIIIth century;
particularly important for Renaissance

e.g. in the „Tuscan” order, the design of the *column* of a temple:



<https://www.northernarchitecture.us/architectural-theory/the-tuscan-order.html>

Historical times: Practical geometrical rules

Gothic architecture terminology:



vault of a Gothic cathedral



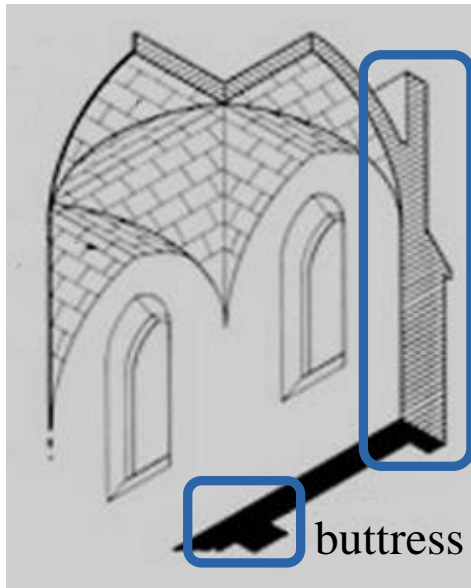
one bay of the vault

Stephen Ressler (2015): The Rise and Fall of the Gothic Cathedral. Fontana Regional Library, <https://fontanalib.org/books/rise-and-fall-gothic-cathedral>

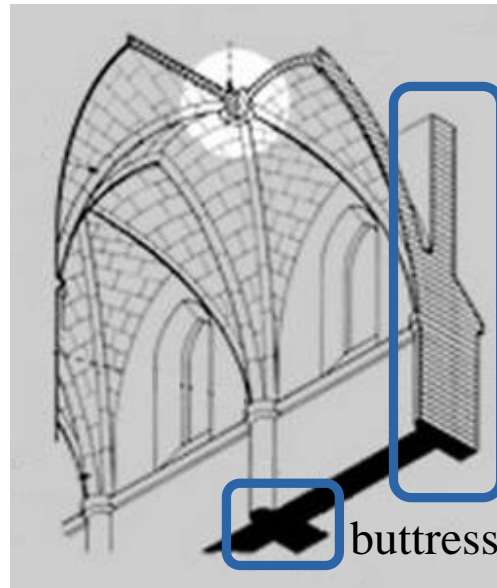
Historical times: Practical geometrical rules

Gothic architecture terminology:

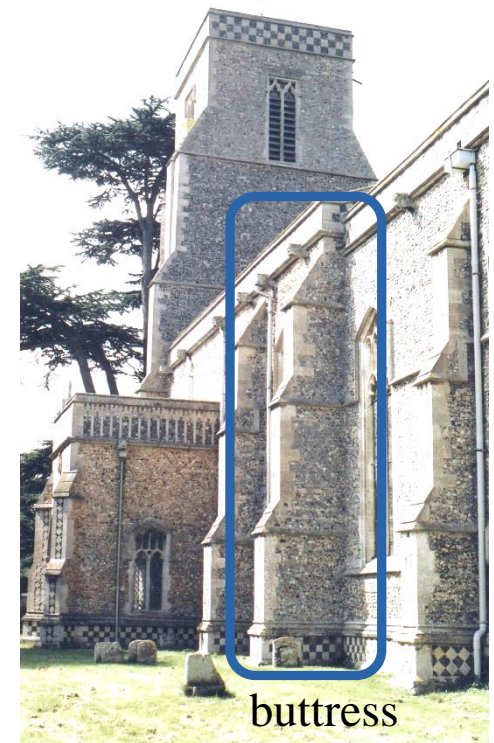
The buttress:



groin vault



ribbed cross vault



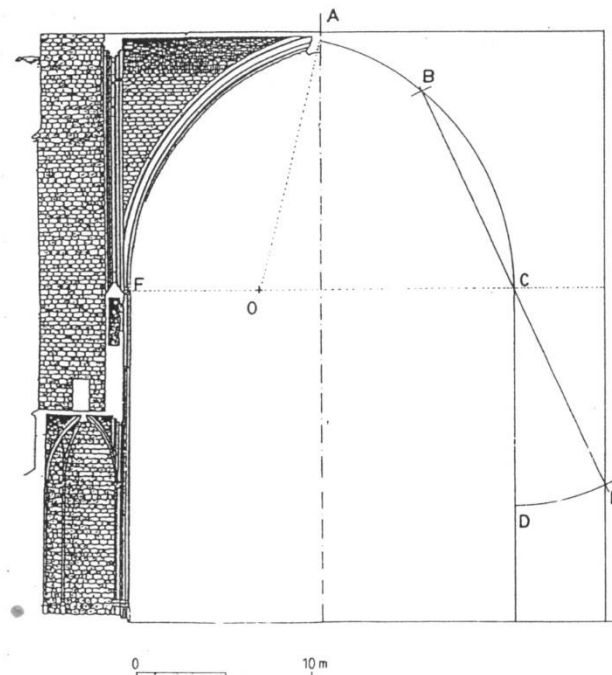
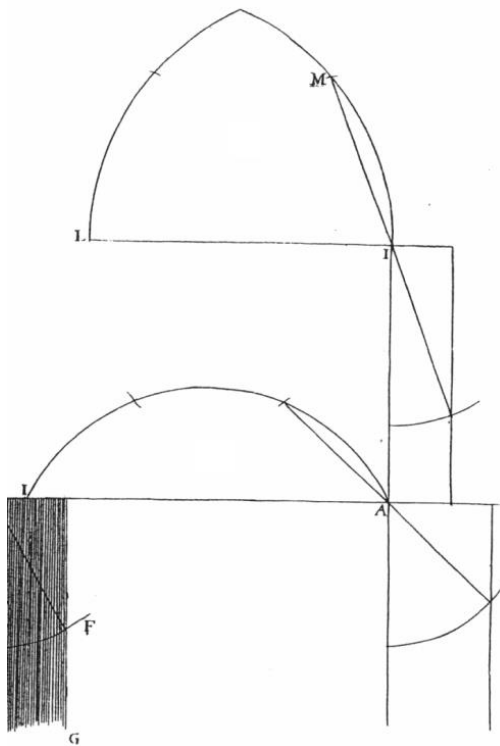
Historical times: Practical geometrical rules

Gothic rules:

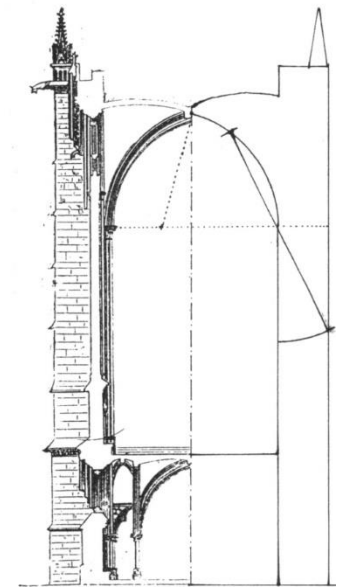
e.g. Derand's rule for *buttress thickness*:

(Derand, 1643)

(\Rightarrow . *similarity of Gothic cathedrals of the same geographic area*)



Gerona cathedral



Saint-Chapelle,

Paris

Historical times: Practical geometrical rules

Gothic rules:

e.g. Rodrigo's interior *pier diameter* design rule: (Rodrigo 1500s)

$$d := \frac{1}{2} \sqrt{h + w + s}$$

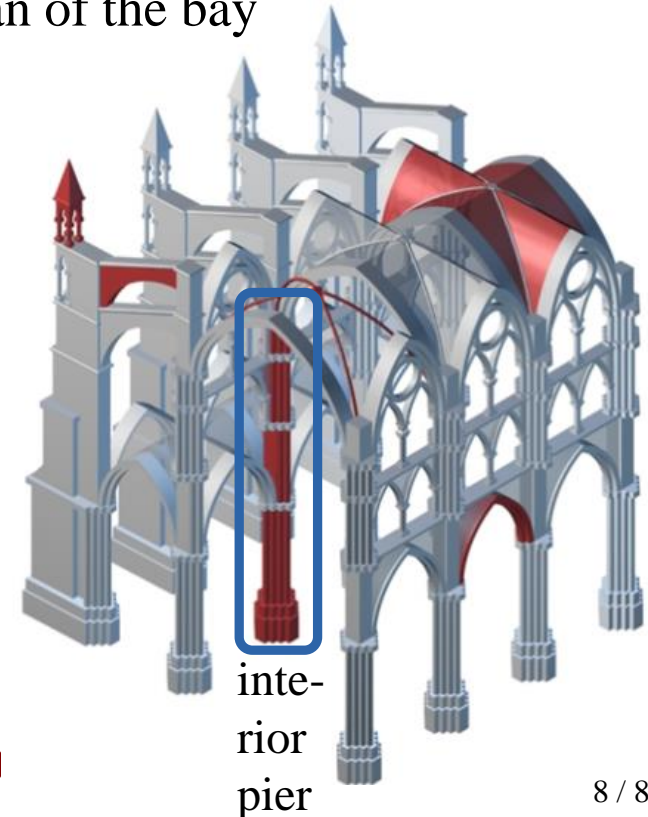
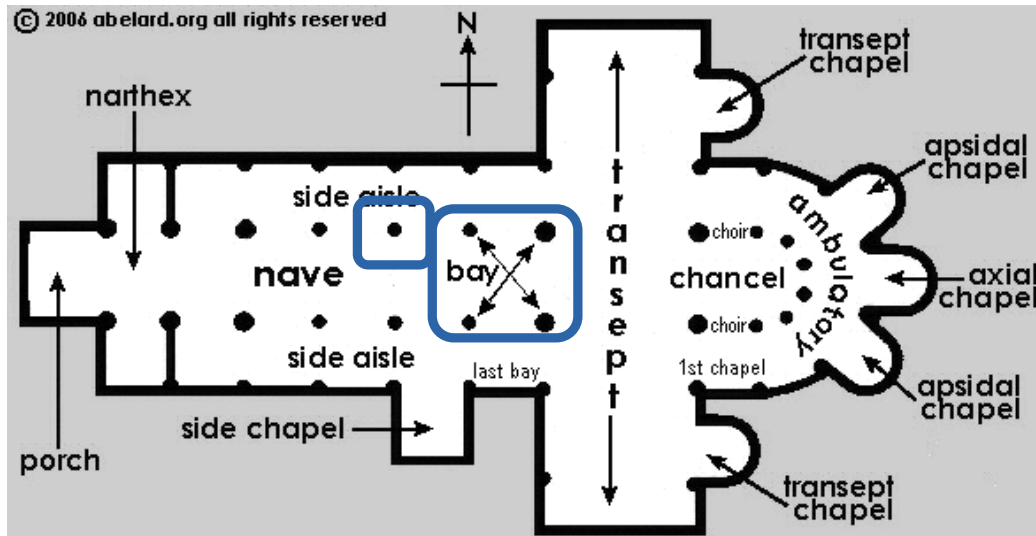
h : pillar height

w : length of the bay

s : span of the bay

dimension !!!!

⇒ works only in Castilian feet (0,28m)



Further reading: Huerta (2006); Aita et al (2018a)

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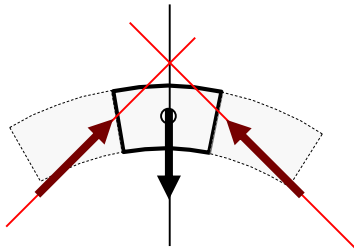
Thrust Network Analysis (TNA)

Questions

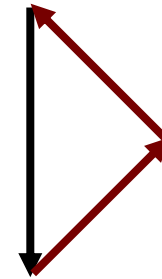
GRAPHICAL STATICS

Reminder to fundamentals:

Equilibrium of three forces in 2D:



funicular diagram („form diagram“):
the three lines of action intersect



force diagram:
closed vector triangle

Equilibrium of four forces in 3D:

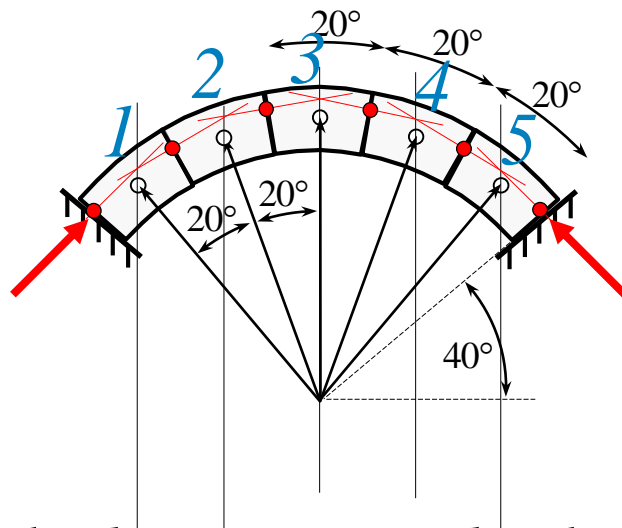
different projections, e.g. hoop view and top view
all views have to intersect / be closed

More than three (2D) or more than four (3D) forces: closed force diagram,
but: lines of action not necessarily intersect

The basic problem: Stability of an arch

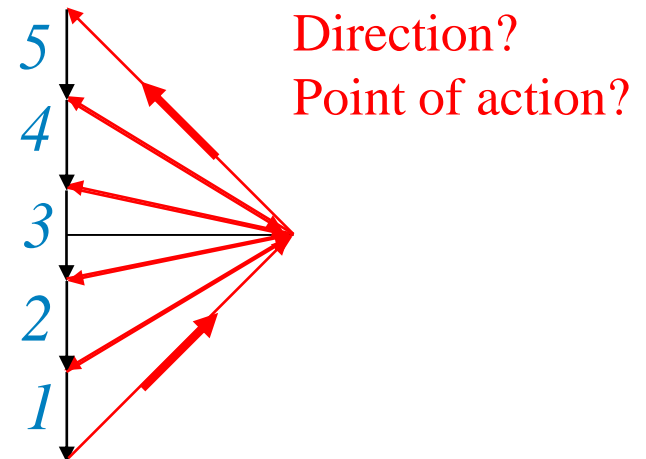
Question: arch submitted to its selfweight; ?reactions? ?contact forces?

3× STATICALLY INDETERMINATE



„loads are transmitted to the supports”

Contact forces: $H = ??$



Direction?

Point of action?

e.g. $H := 2,5G$

Given: geometry: $R_{inner} = 2,4 \text{ m}$; $R_{middle} = 2,7 \text{ m}$

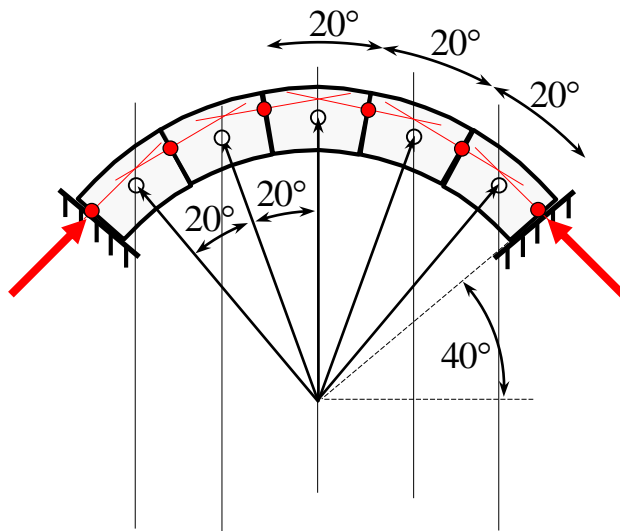
identical selfweight for each block: $G_1 = G_2 = G_3 = G_4 = G_5$

Try to find an equilibrated force system!

→ contact forces: compression & friction; inside the contact area

The basic problem: Stability of an arch

Thrust line: [intuitive concept; theoretical definition: Gáspár et al (2018)]



≈ „ the line determined by the points of action of the contact forces ”

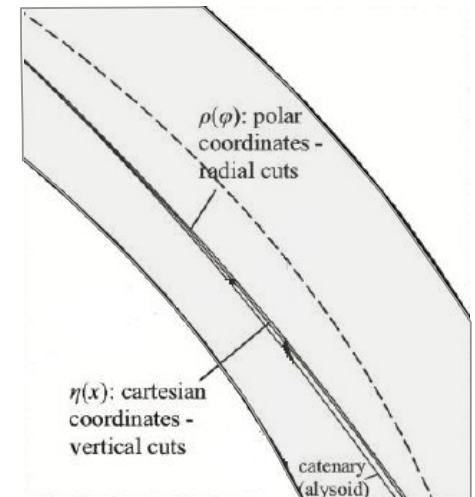
BUT: depends on the orientation of contacts
(Alexakis & Makris, 2015)

stability criterion: [later: more details]

thrust line can be found so that
it runs everywhere inside the contacts

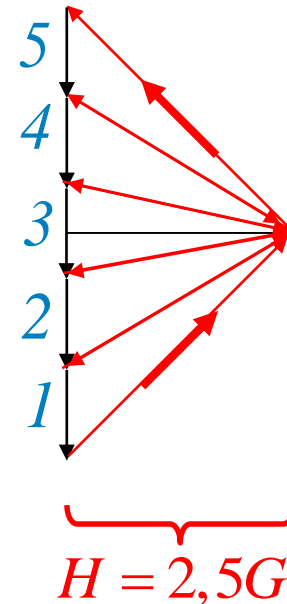
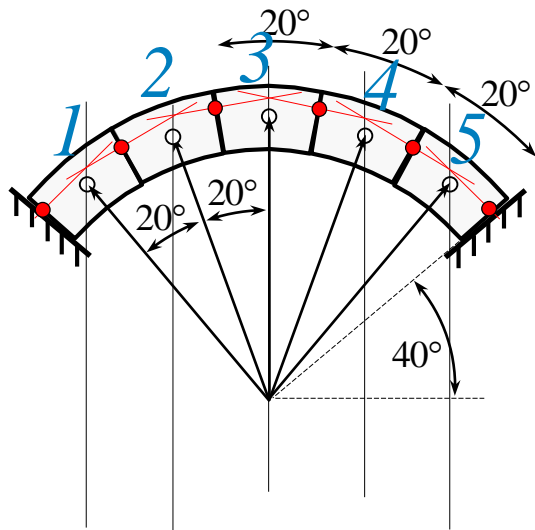
arch shape is „better”, if it can be done with smaller thickness

→ e.g. pointed arch versus circular arch



The basic problem: Stability of an arch

Question: arch submitted to its selfweight; ?reactions? ?contact forces?



Given: geometry: $R_{inner} = 2,4 \text{ m}$; $R_{middle} = 2,7 \text{ m}$

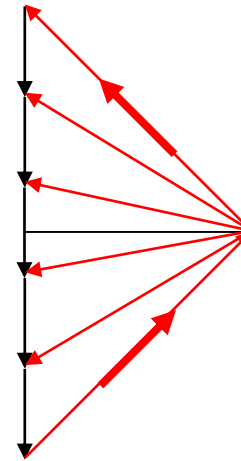
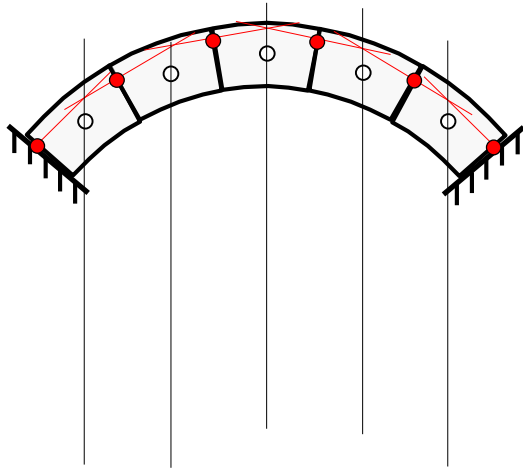
identical selfweight for each block: $G_1 = G_2 = G_3 = G_4 = G_5$

Try to find an equilibrated force system!

→ contact forces: compression & friction; inside the contact area [kernel?]

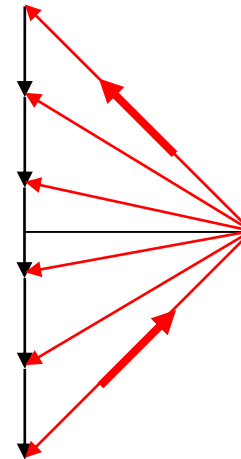
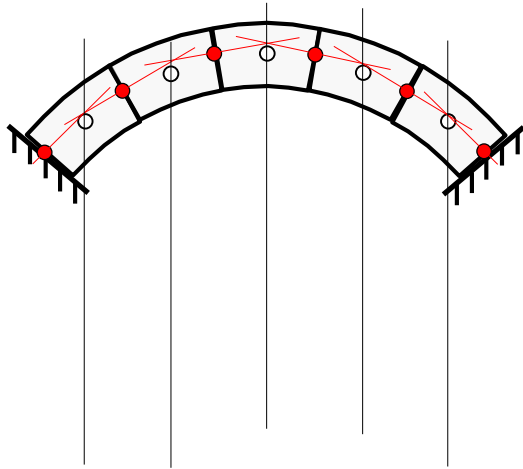
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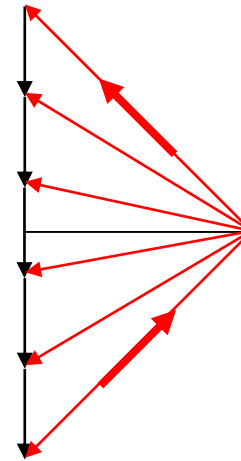
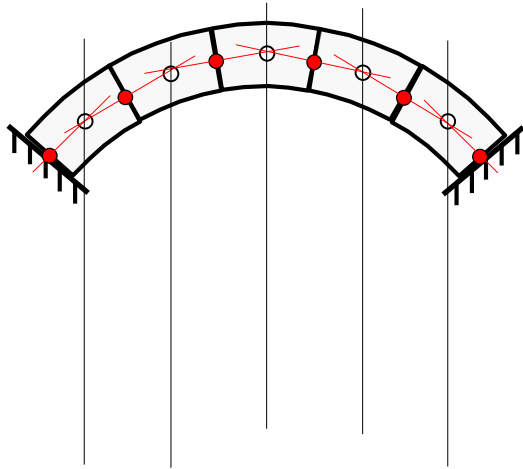
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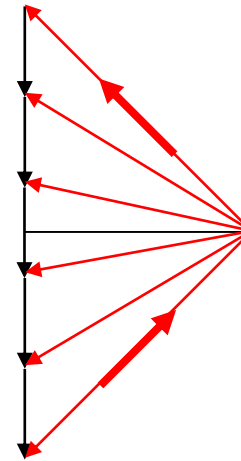
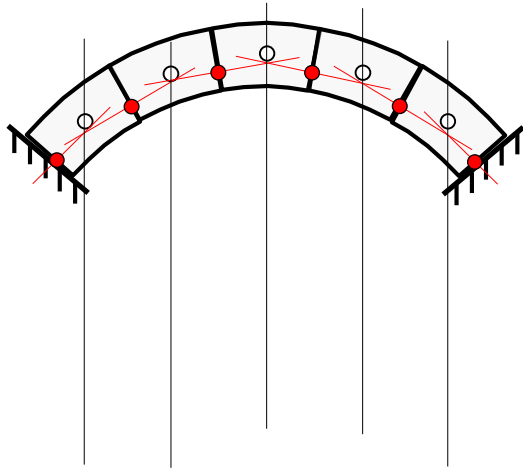
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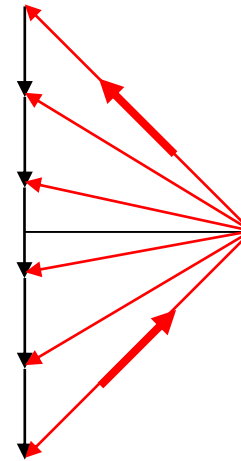
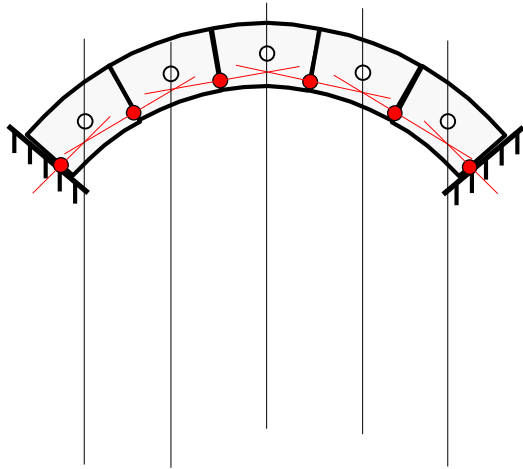
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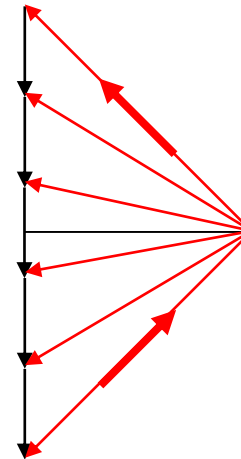
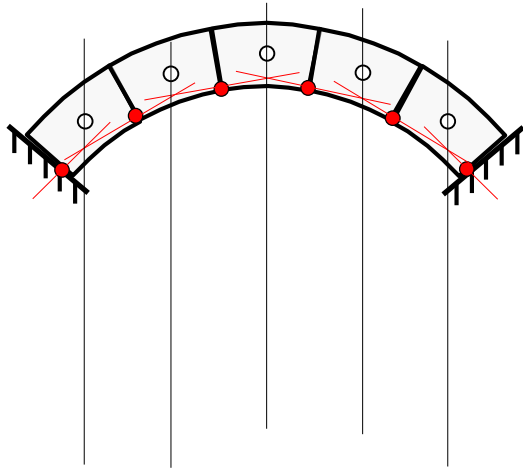
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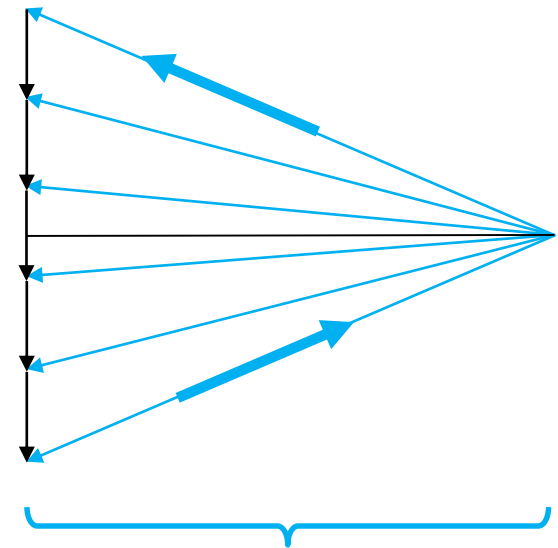
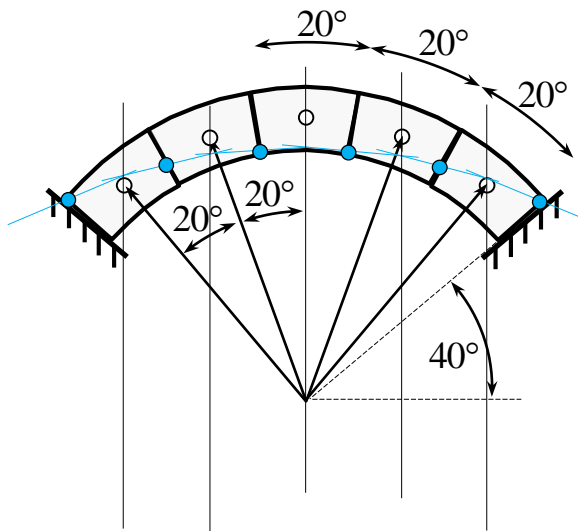
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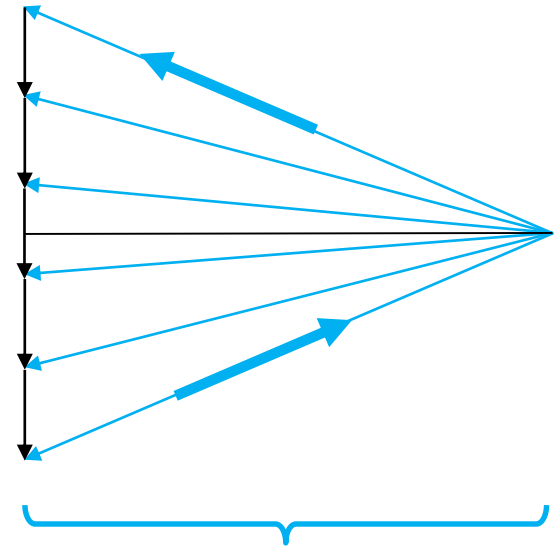
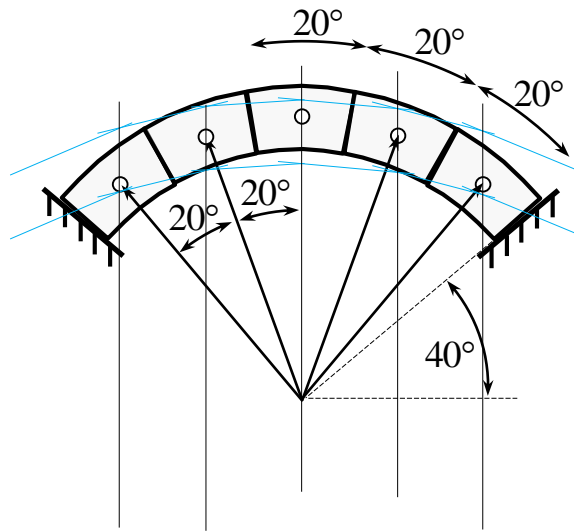
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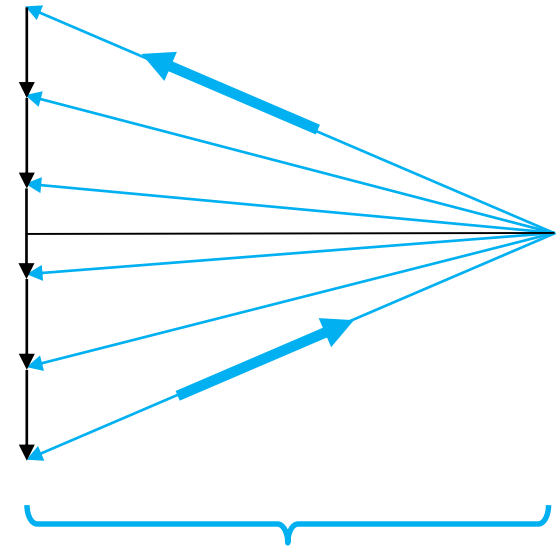
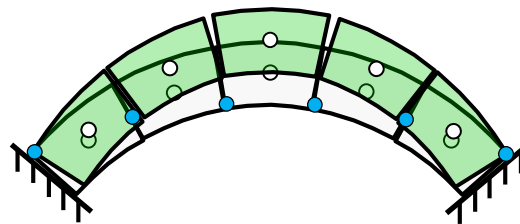
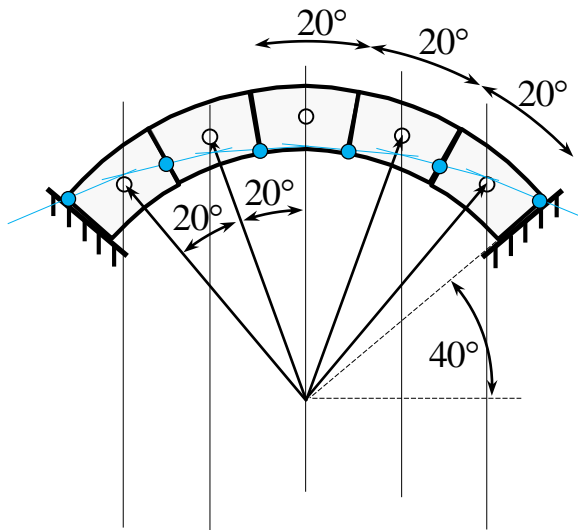
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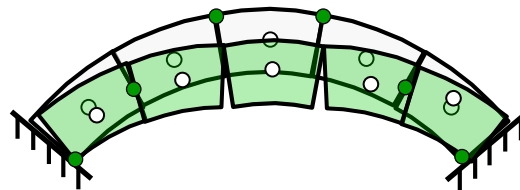
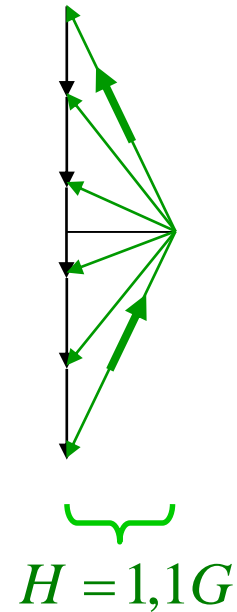
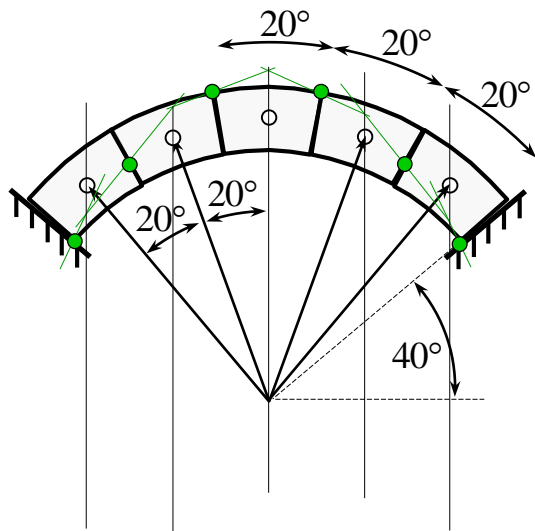
The basic problem: Stability of an arch

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The basic problem: Stability of an arch

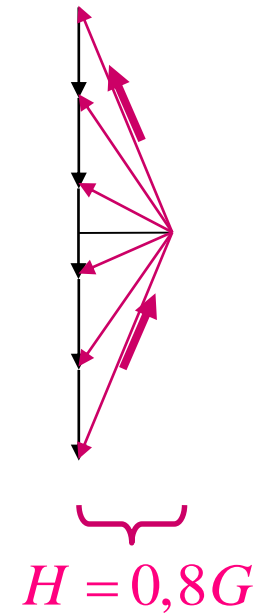
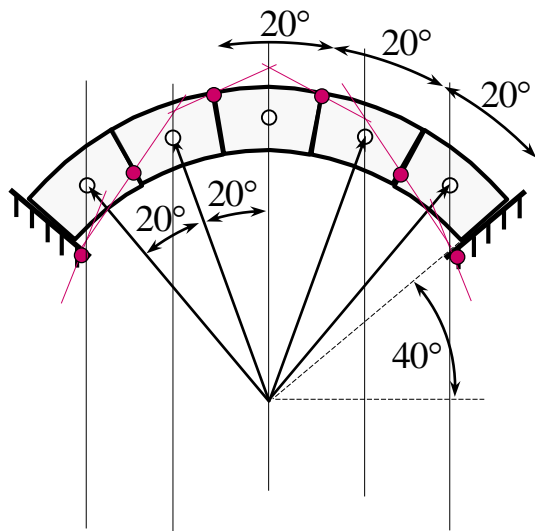
Question: arch submitted to its selfweight; ?reactions? ?contact forces?



a wide range of equilibrium solutions: \Leftarrow because the arch is thick enough !

The basic problem: Stability of an arch

Question: arch submitted to its selfweight; ?reactions? ?contact forces?



EQUILIBRIUM IS IMPOSSIBLE WITH THIS H

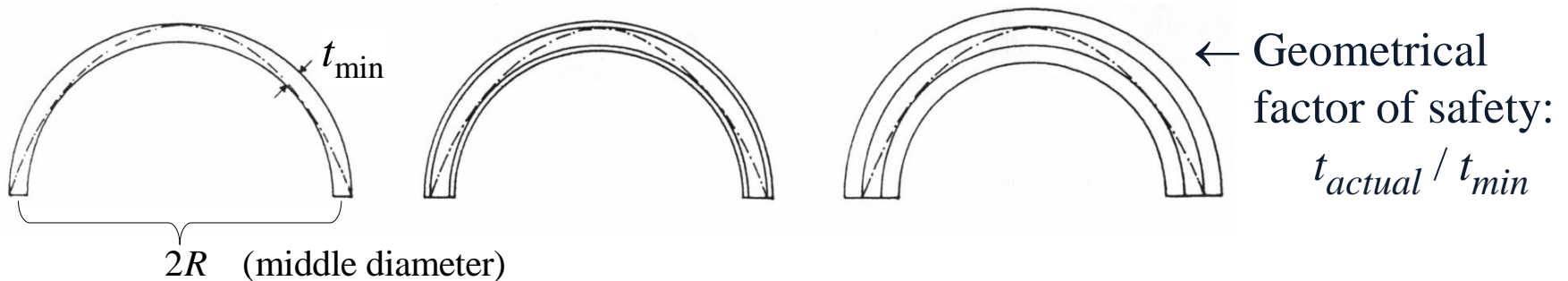
⇒ possible direction of the reactions is **limited**

The basic problem: Stability of an arch

Question: arch submitted to its selfweight; ?reactions? ?contact forces ?

Solution for an arch having *infinitely dense, radially oriented* contacts, with *zero tension resistance* ? \Rightarrow thrust line *must run inside the arch*

Usual arches: typically circular middle curve or composed of circular arcs



t_{min} : smallest uniform thickness for which the arch can carry its selfweight

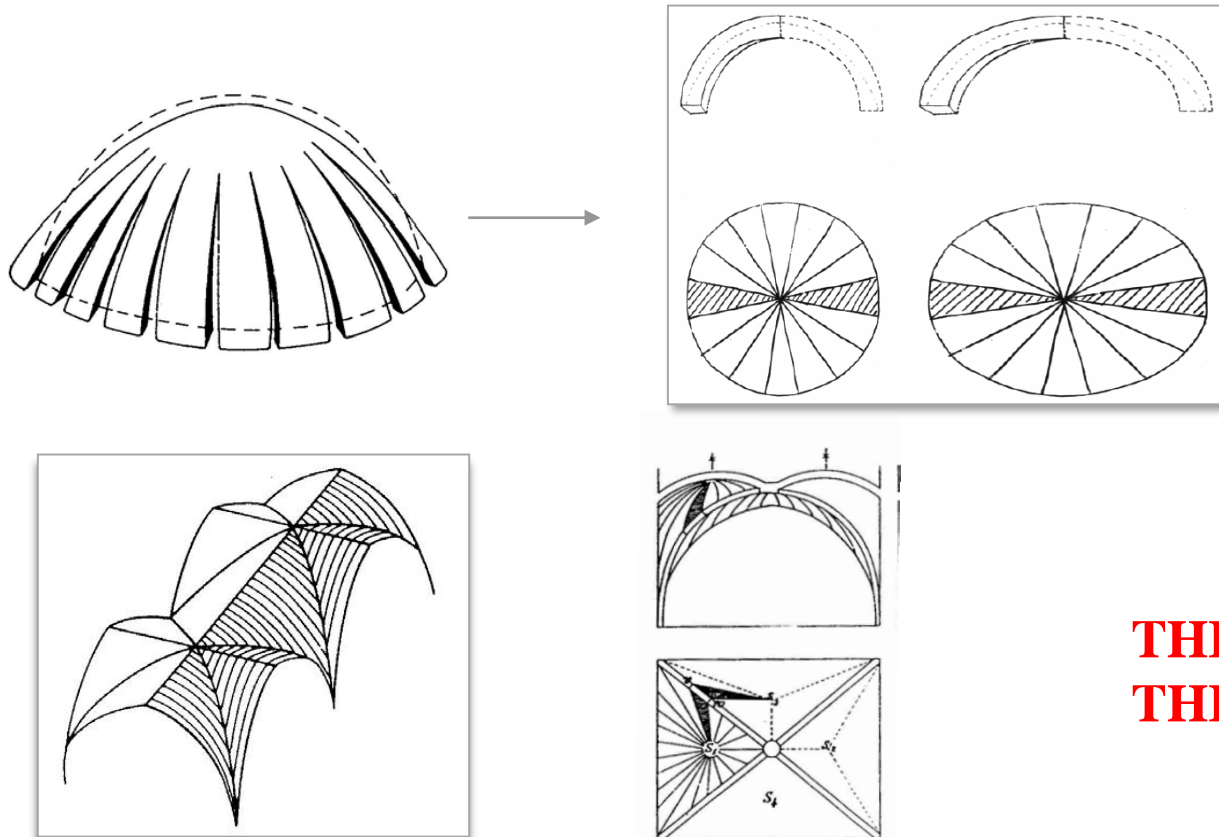
For semicircle: Heyman (1966): $t_{min} = 0,1059 \cdot R$

Milankovitch (1907): $t_{min} = 0,1075 \cdot R$

Stability of vaults under selfweight

Slicing technique: cut into individual arches, and check them separately!

XIXth century: different assumptions on the internal force system
based on the inspection of typical crack patterns: e.g.



**THE STATIC
THEOREM**

Limit analysis of masonry structures

The static theorem for masonry structures:

If a force system can be found for the given set of external loads which satisfies the material criteria and equilibrates the given external loads, then the structure with the given geometry is safe under these loads.

→ if the engineer found an equilibrium force system to the loads,
the structure is **safe** !

Stability of vaults under selfweight

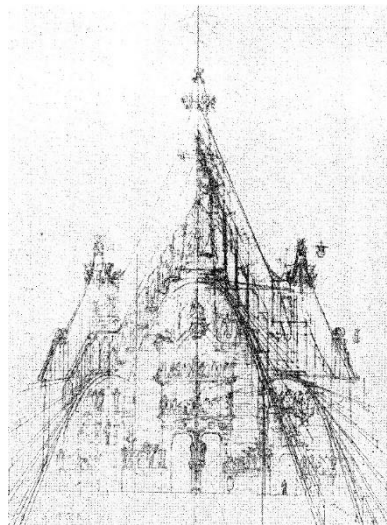
Slicing technique:

Gaudi, Sagrada Familia, Barcelona:

designed by:

→ graphical statics:

slice of the structure:

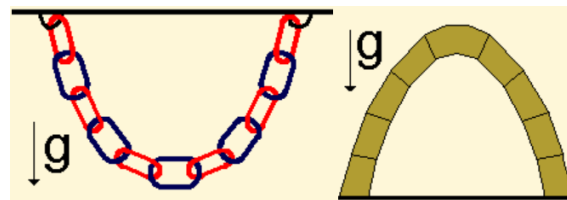


Rafals, 1929



N. Valencia, archdaily.com

→ physical models:



<http://www.art-nouveau-around-the-world.org/en/villes/barcelona/models.htm>



<http://dataphys.org/list/gaudis-hanging-chain-models/>

Stability of vaults under selfweight

Slicing technique:



<https://spainattractions.es/palma-cathedral-mallorca/>

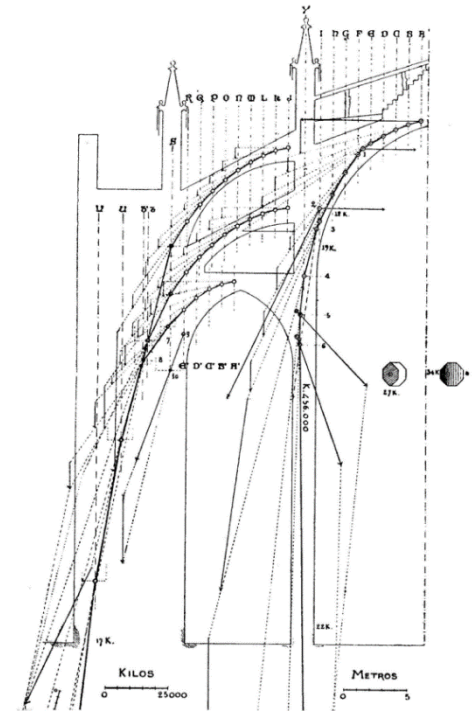
Problem:

extremely tall slender pillars of the main nave

→ is it safe?

Rubio Bellver, 1912: graphical statics analysis

⇒ weights needed over the crown!



Rubio Bellver, 1912



THIS LECTURE:

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Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads

Are there any admissible (P, e) pairs?

Consider a contact j ,

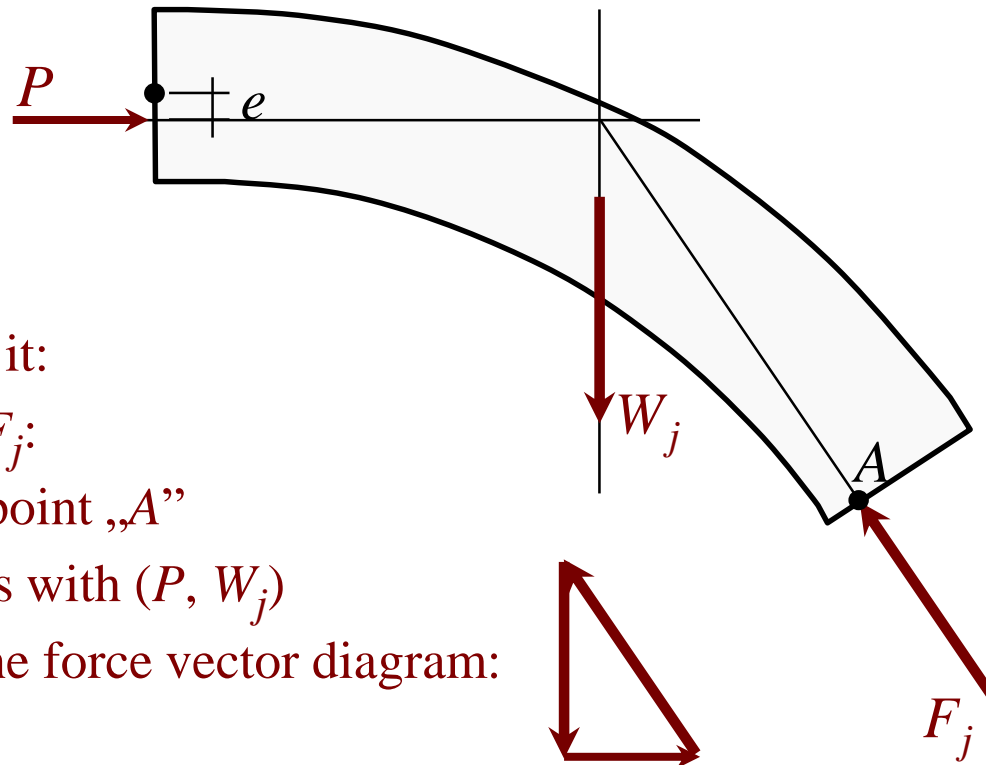
and a point „A” on it:

Contact force resultant, F_j :

acts at the chosen point „A”

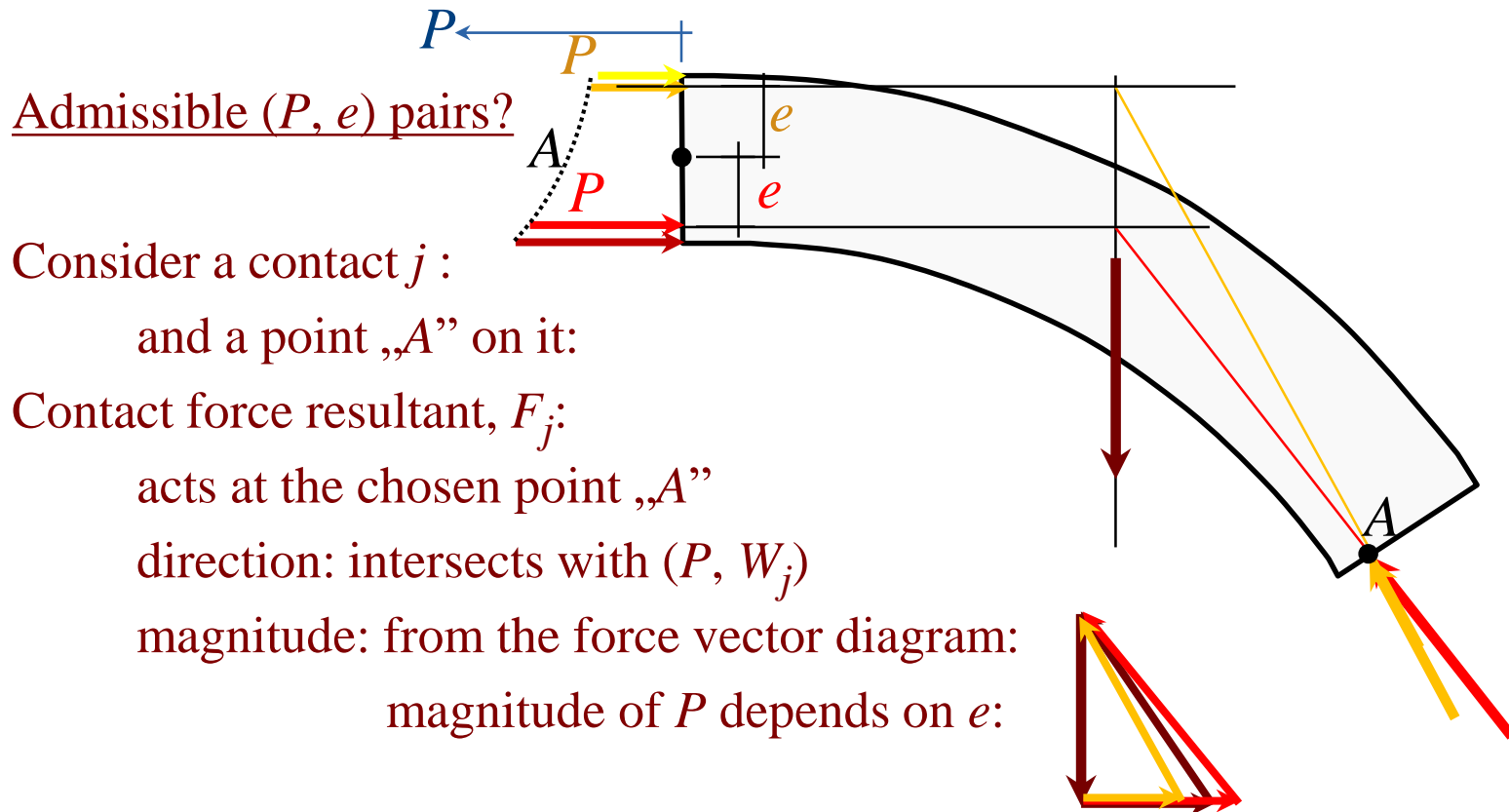
direction: intersects with (P, W_j)

magnitude: from the force vector diagram:



Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads

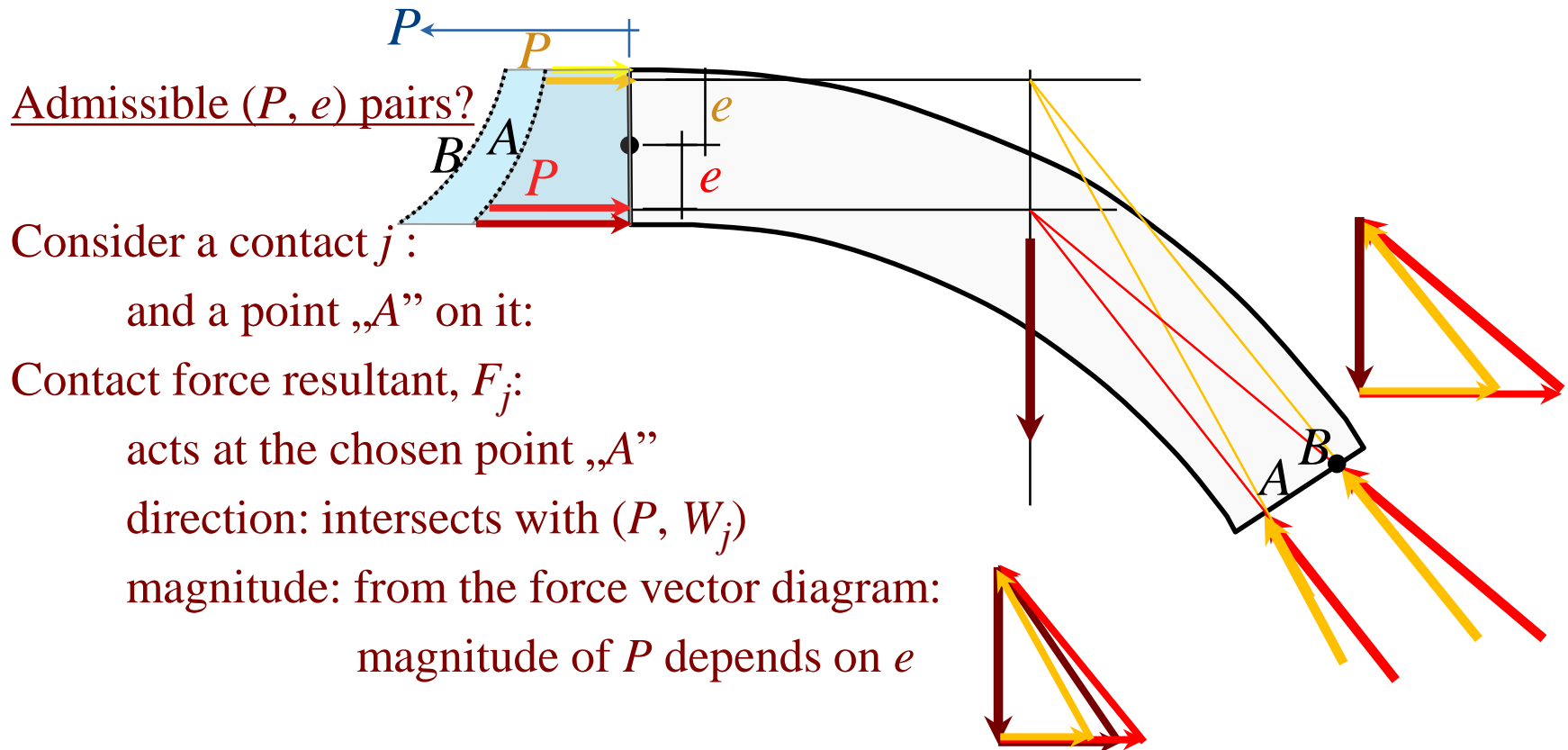


Possible magnitudes of P belonging to „A”:

[see dotted line above]

Durand-Claye's stability area method

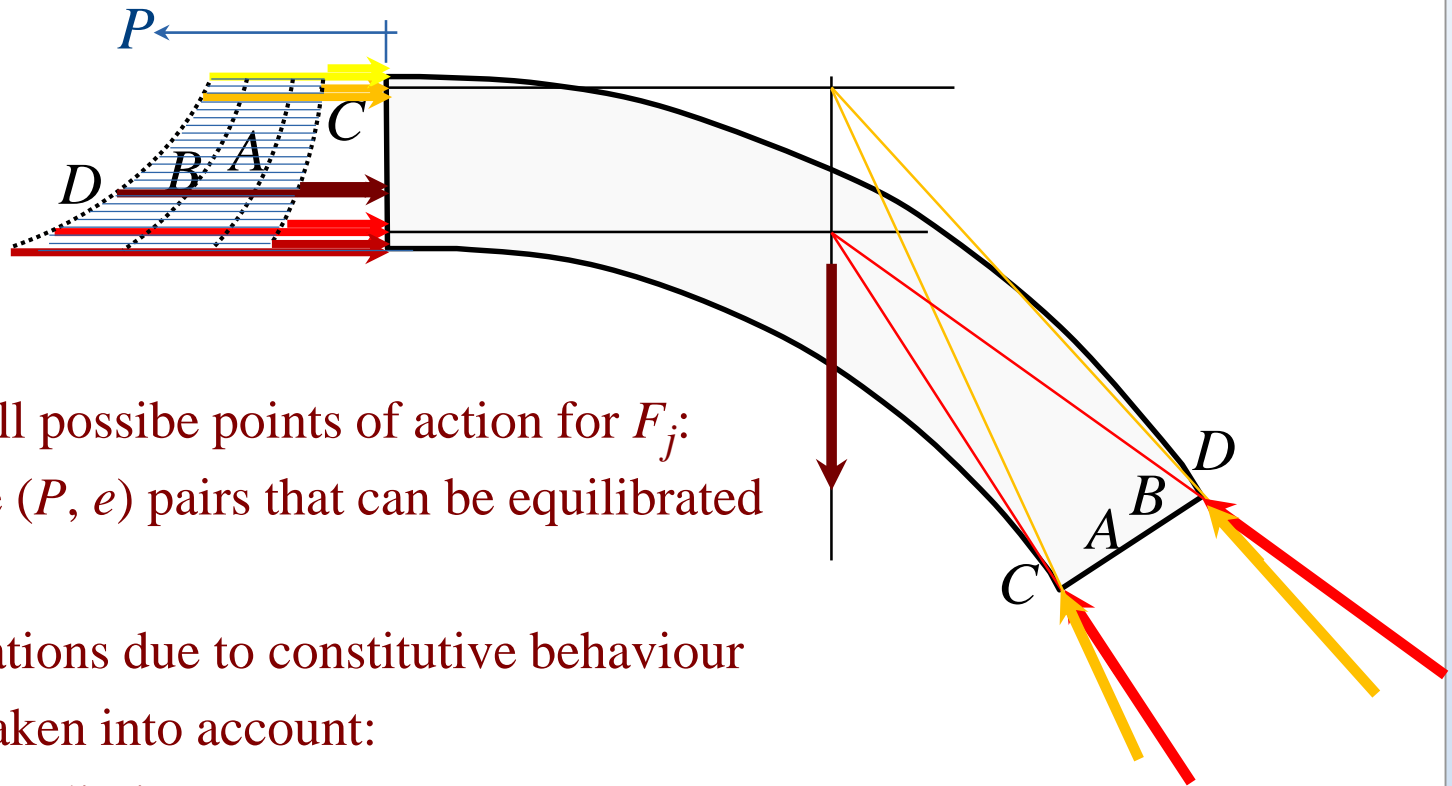
Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Possible magnitudes of P belonging to „A”: [see grey domain horizontal sizes]
 similarly to any „B”: [see cyan domain horizontal sizes]

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



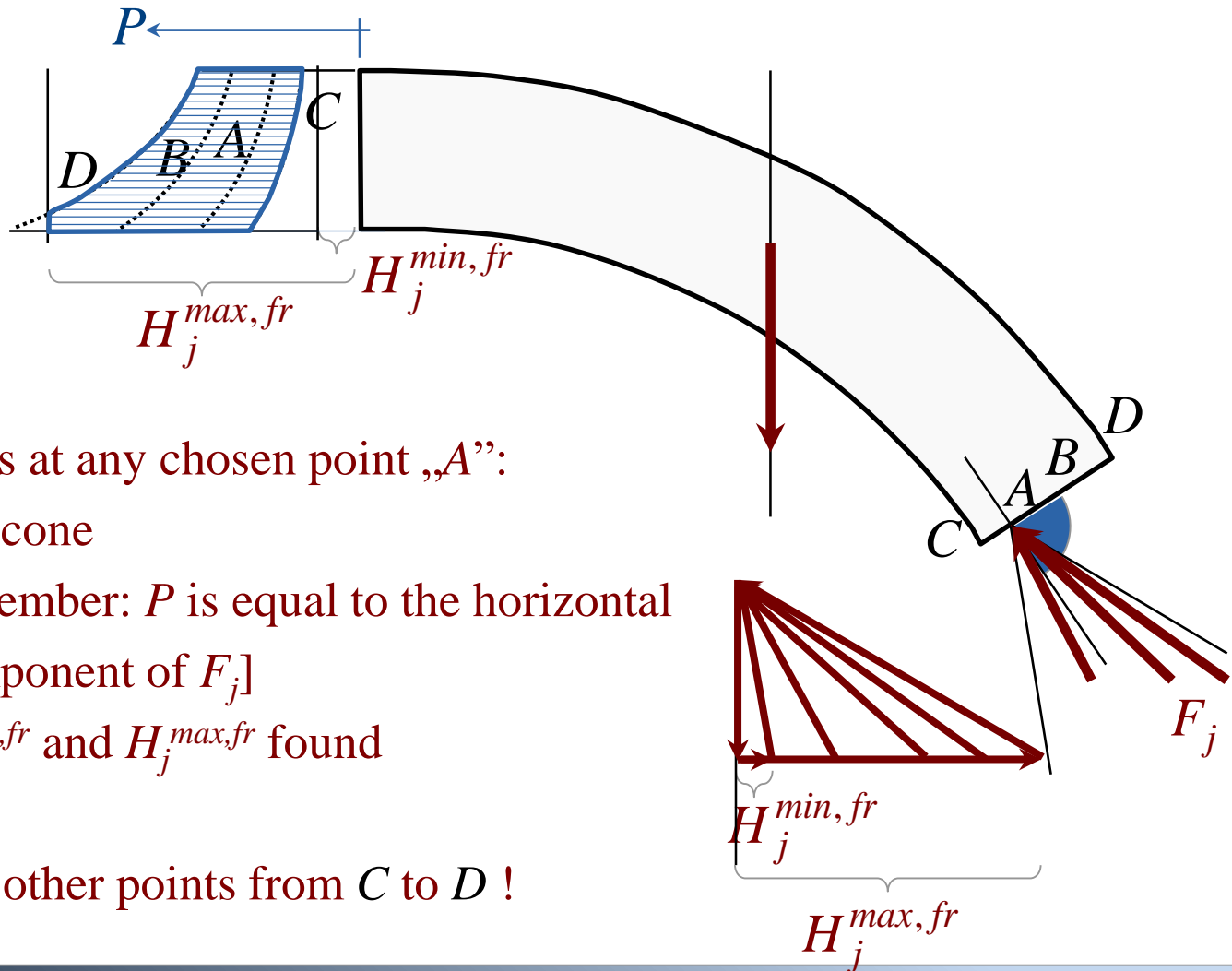
Considering all possible points of action for F_j :
found those (P, e) pairs that can be equilibrated

now the limitations due to constitutive behaviour
have to be taken into account:

- friction limit
- compression strength
- [in new versions: tension strength – will not be shown here]

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Friction limit:

if F_j acts at any chosen point „A“:

friction cone

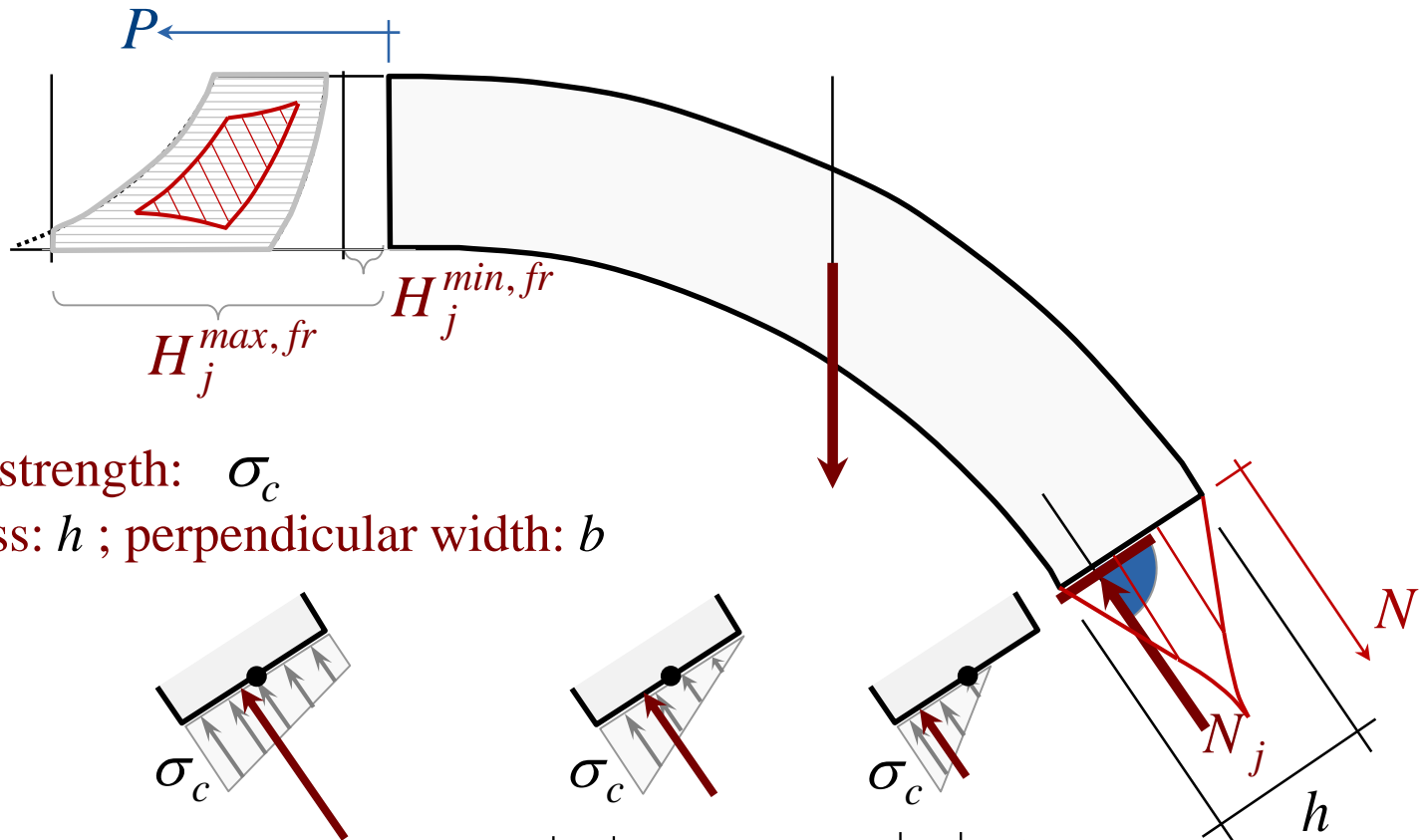
[remember: P is equal to the horizontal component of F_j]

$\Rightarrow H_j^{min,fr}$ and $H_j^{max,fr}$ found

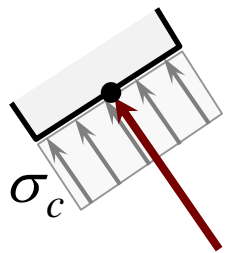
Equals for all other points from C to D !

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads

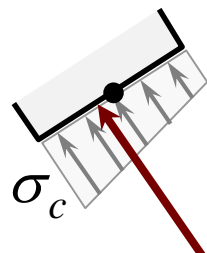


Compression strength: σ_c
 thickness: h ; perpendicular width: b



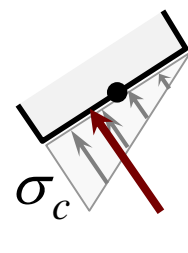
$$e_N = 0:$$

$$N \leq b \cdot h \cdot \sigma_c$$



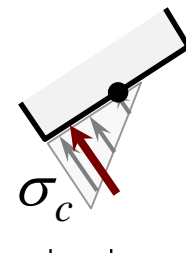
$$|e_N| \leq h/6:$$

$$N \leq \frac{h}{h + 6 \cdot |e_N|} \cdot b \cdot h \cdot \sigma_c$$



$$|e_N| = h/6:$$

$$N \leq \frac{1}{2} b \cdot h \cdot \sigma_c$$

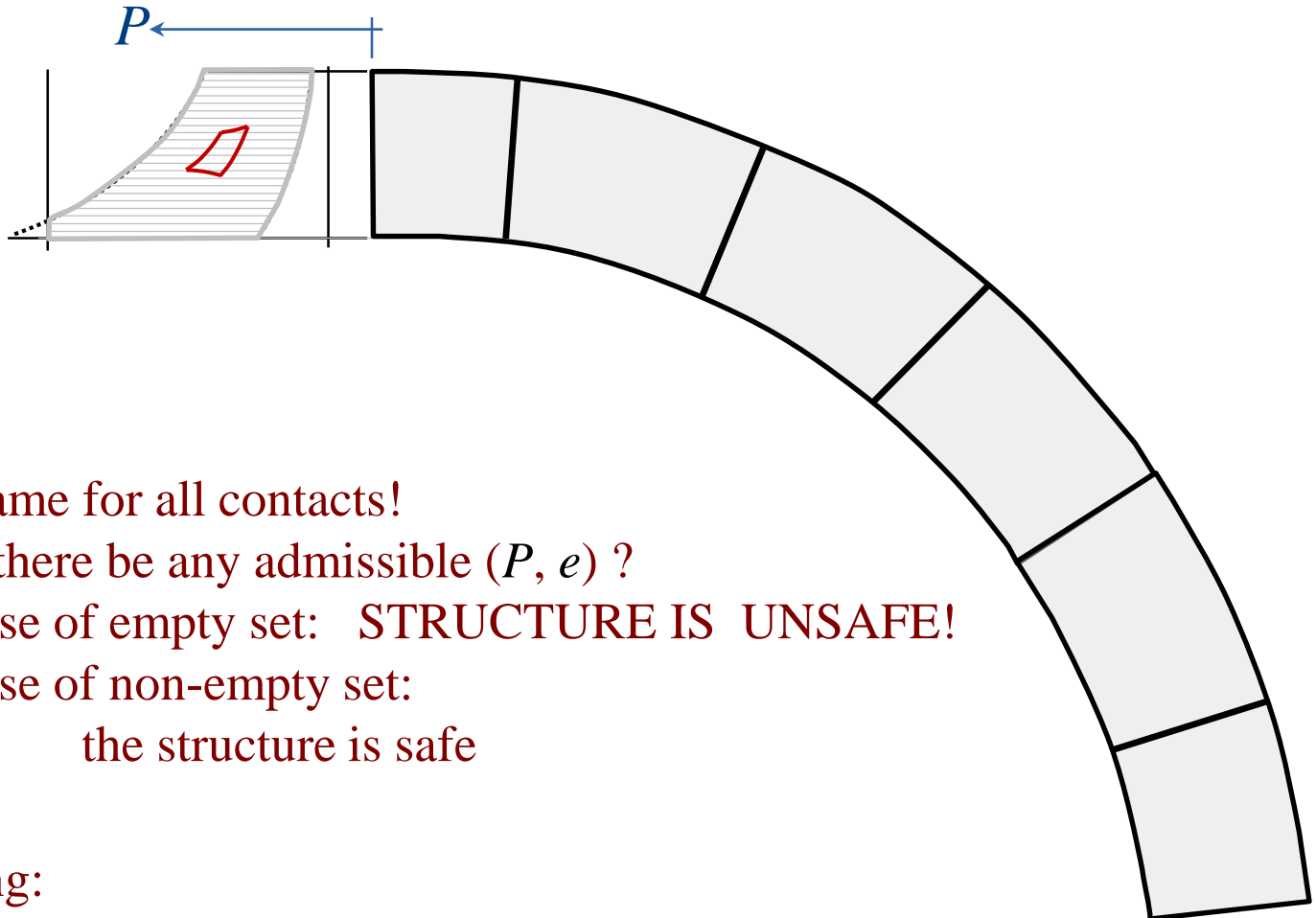


$$|e_N| \geq h/6:$$

$$N \leq \frac{3}{2} \left(\frac{h}{2} - |e_N| \right) \cdot b \cdot \sigma_c$$

Durand-Claye's stability area method

Durand-Claye, 1867: Symmetric arches & symmetric vertical loads



Now do the same for all contacts!

→ will there be any admissible (P, e) ?

In case of empty set: **STRUCTURE IS UNSAFE!**

In case of non-empty set:
the structure is safe

Further reading:

Foce & Aita (2003); Aita et al (2017)

Durand-Claye's stability area method

Applications:

e.g. Barsotti et al (2017):

comparison of different arch types

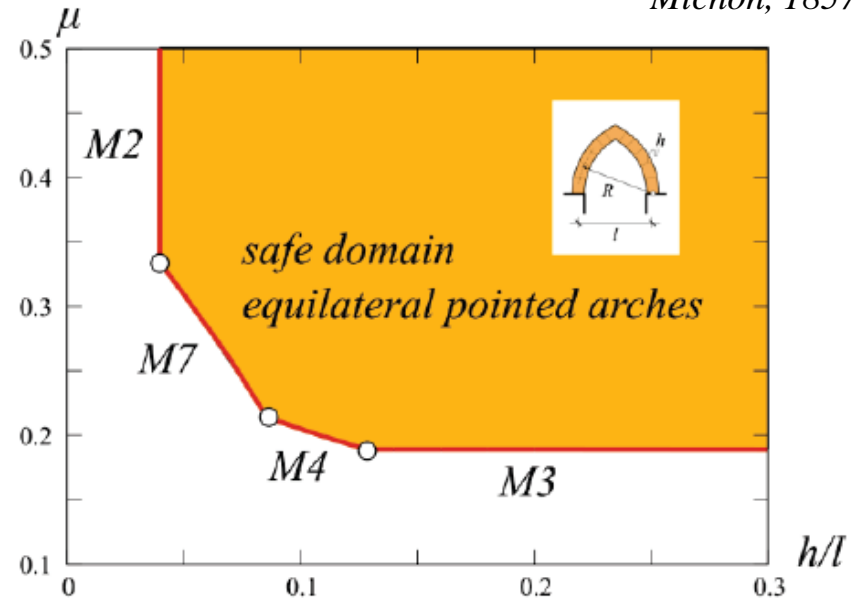
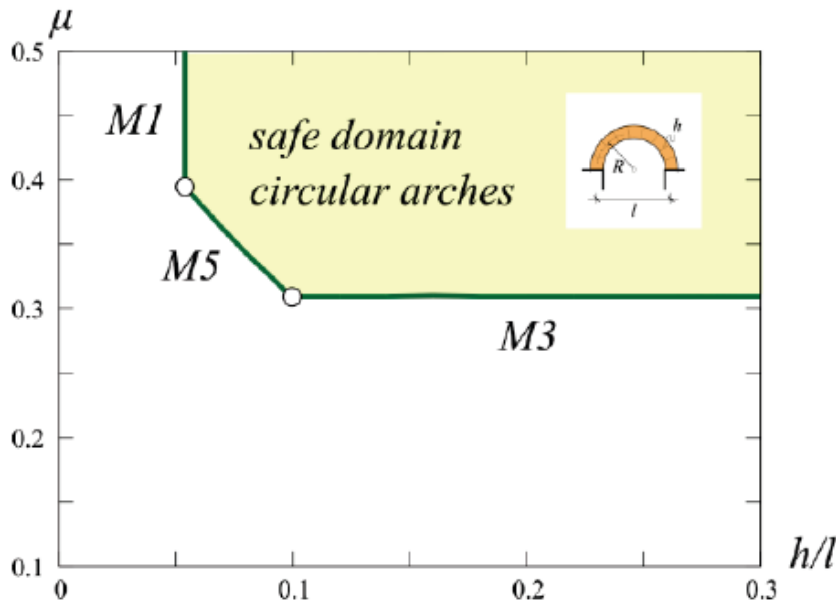
and their possible collapse modes

μ : friction coefficient

h : arch thickness; l : span



Michon, 1857



Durand-Claye's stability area method

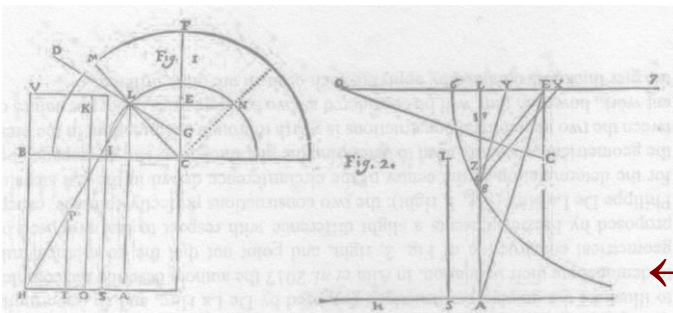
Applications:

e.g. Aita et al (2018a):

geometrical factor of safety for historic design rules:

- find necessary minimum value of a certain size with Durand-Claye's;
- find that size according to historic rule;
- compare!

Comparison of different historical rules for pier thickness
for the same arch-wall-pier system:

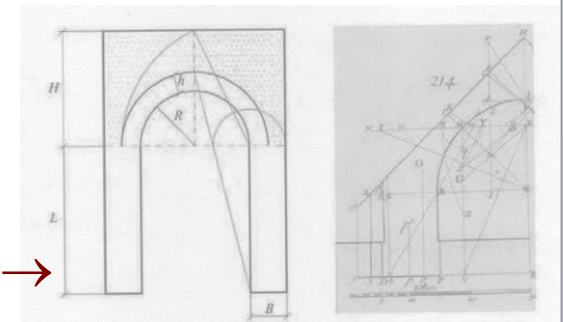


La Hire, 1731

limit width according
to the Durand-Claye's
method: 2,54 m

← 2,81 m

3,61 m →



Frézier, 1737

Durand-Claye's stability area method

Applications:

e.g. Aita et al (2018b): Safety assessment of the dome of Pisa Cathedral



tripadvisor.co.za

constructed: XIth century
dome: oval groundplan,
≈ circular meridians



restoration of the dome going on recently

„On the north side... at about eye level, is an original piece of Roman marble, on which are a series of small black marks. Legend says that these marks were left by the Devil when he climbed up to the dome attempting to stop its construction, and so they are referred to as the scratches of the devil. The legend also says that out of spite the number of scratches always changes when counted.” (Wikipedia)

Durand-Claye's stability area method

Applications:

e.g. Aita et al (2018b):

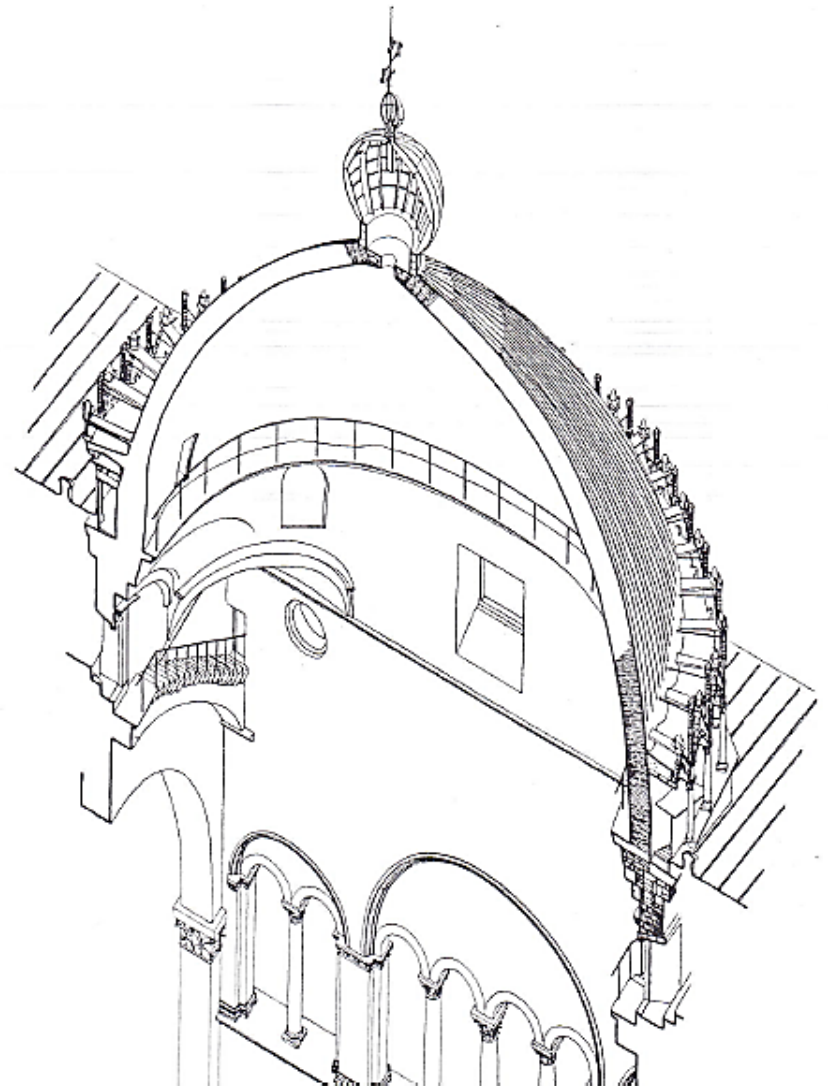
- D-C method extended for domes
with membrane forces
(Durand-Claye, 1880)
- analysis of the dome

Result:

geometrical factor of safety ≈ 2

Further reading:

Aita (2018b)



THIS LECTURE:

2. GRAPHICAL METHODS

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 ... 2018

Wolfe's method for membrane forces in domes

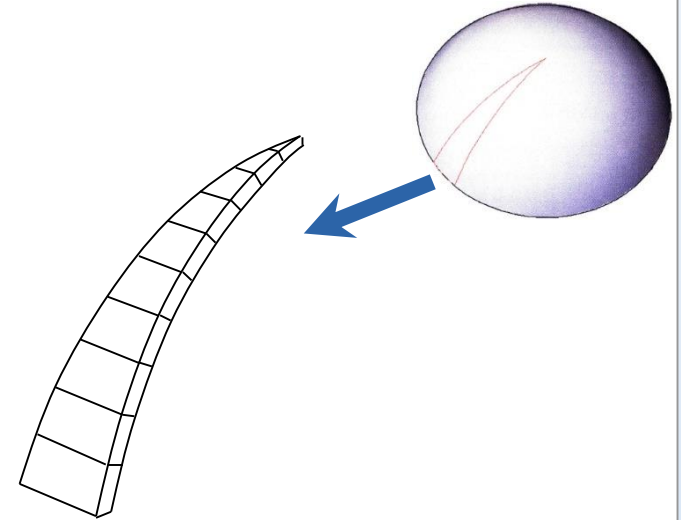
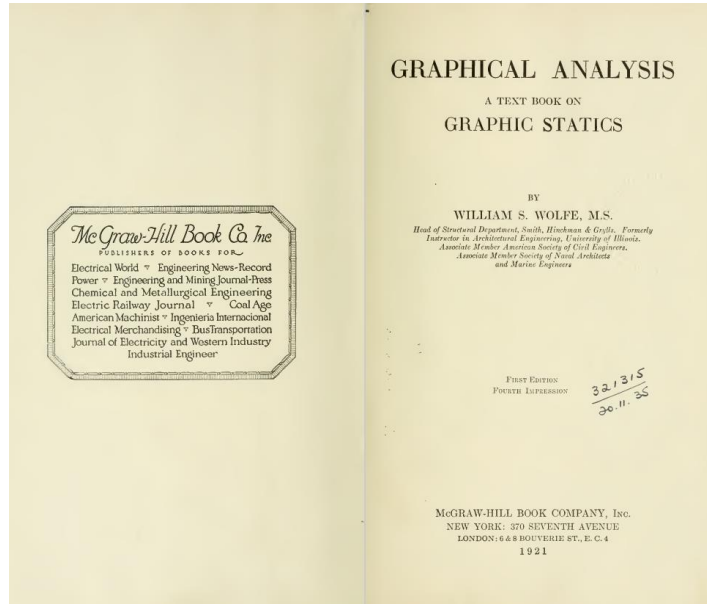
O'Dwyer's funicular analysis \Rightarrow

Thrust Network Analysis (TNA)

Questions

Wolfe's method

Wolfe (1921);



→ Version 1.: domes with tension resistance

→ Version 2.: domes without tension resistance

Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*

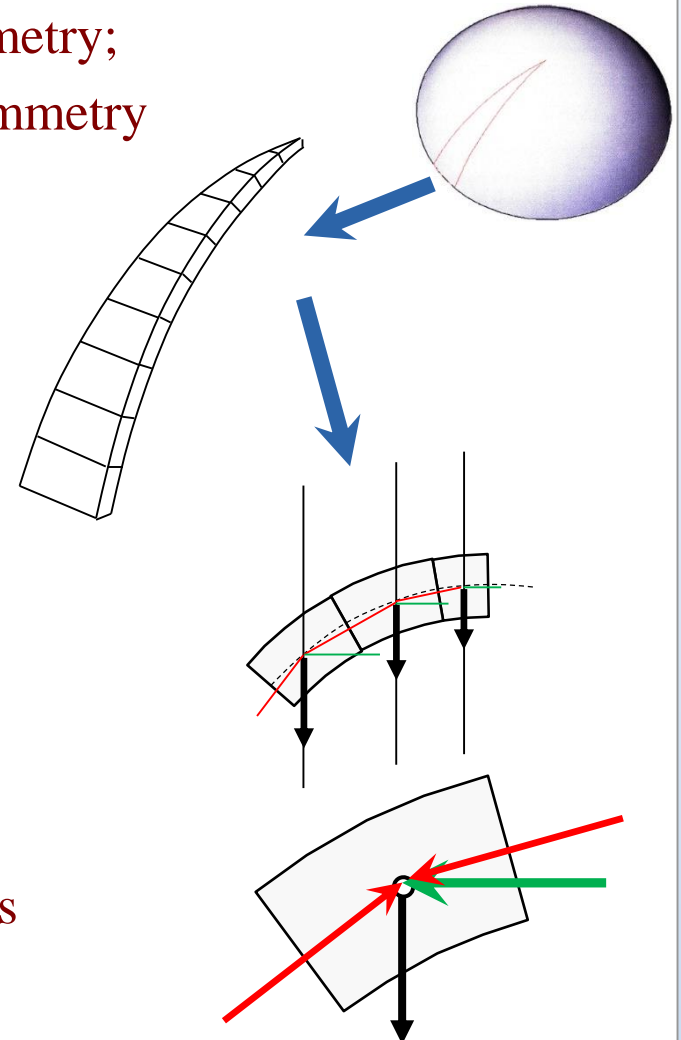
- Version 1.: domes with tension resistance
- Version 2.: domes without tension resistance

Starting step:

lunes; weights of lune voussoirs: at centroids

Assumption:

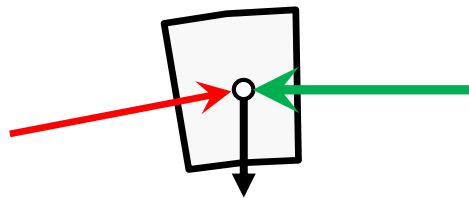
contact force: line of action joins the two neighbouring centroids



Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*

1. Analysis of the top segment:

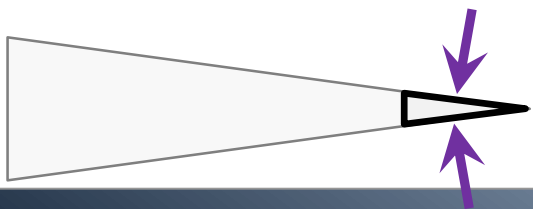


funicular diagram

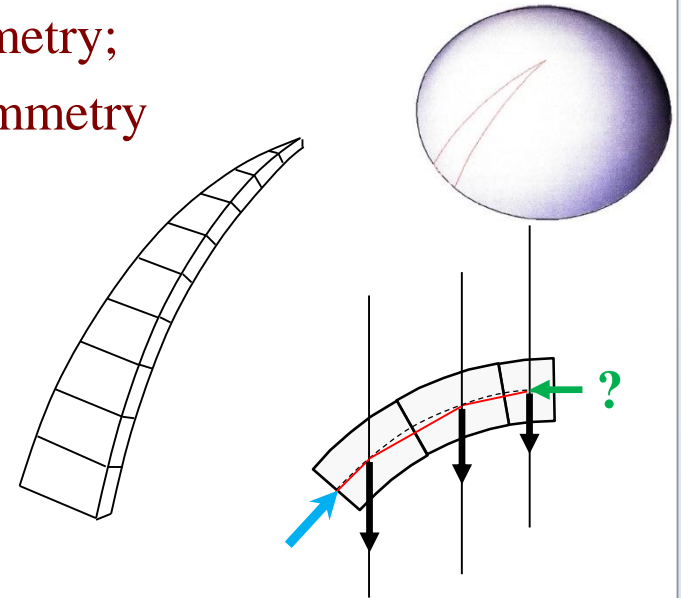


force diagram, front view

top view:



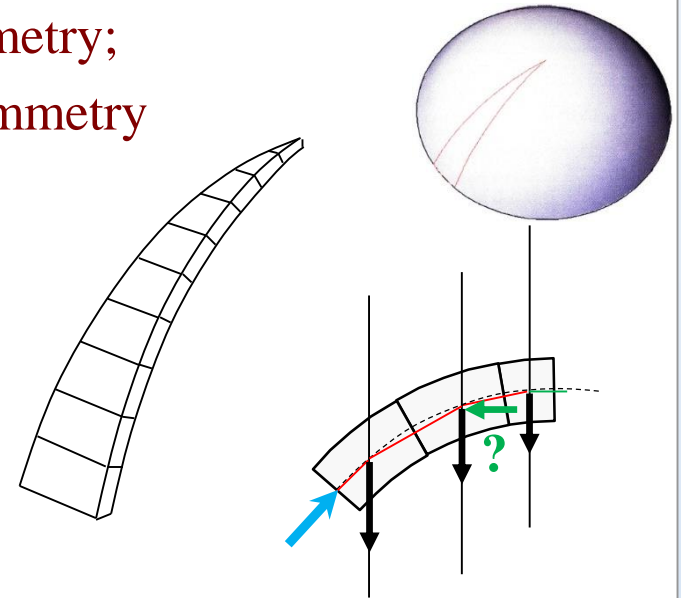
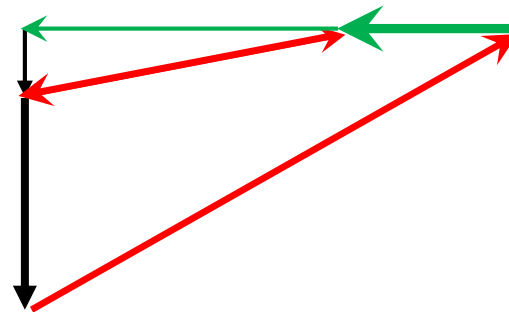
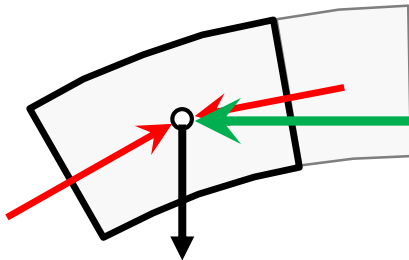
[later]



Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*
⇒ contact force intersect with weight
along the *middle line*

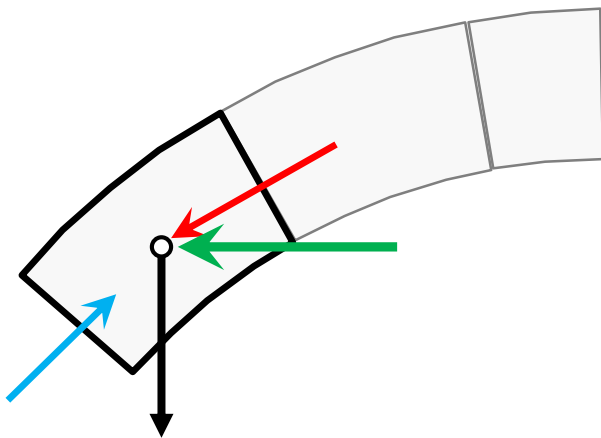
2. Analysis of the 2nd segment:



Wolfe's method

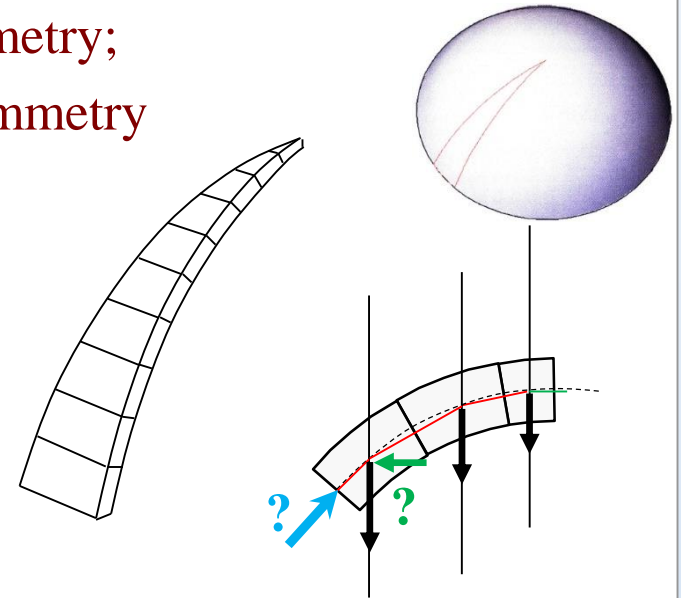
- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*
 - ⇒ contact force intersect with weight
along the *middle line*

3. Analysis of the bottom segment:



Assumption:

Reaction goes through the centroid of the last segment,
perpendicular to lowest contact (\perp to the radial direction)

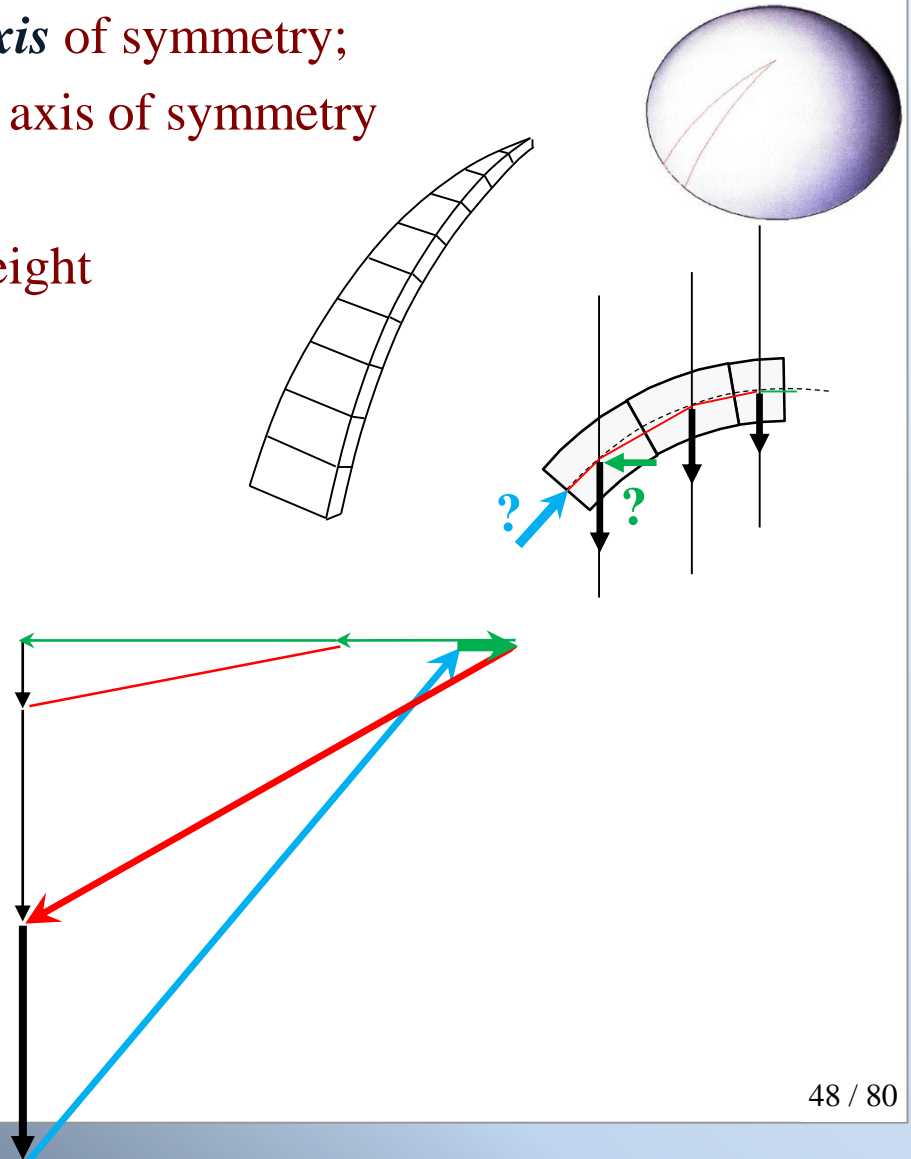
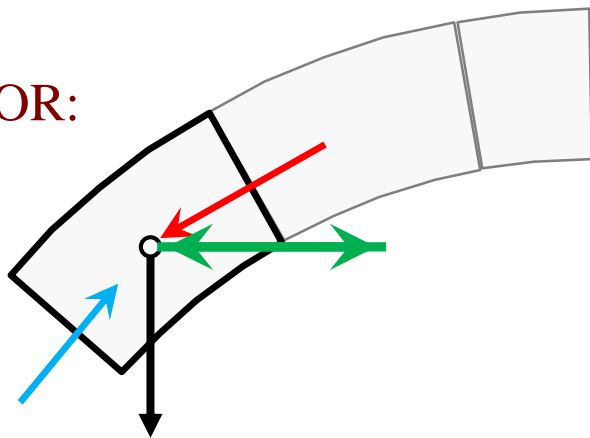


Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
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- basic assumption: *membrane state*
⇒ contact force intersect with weight
along the *middle line*

3. Analysis of the bottom segment:

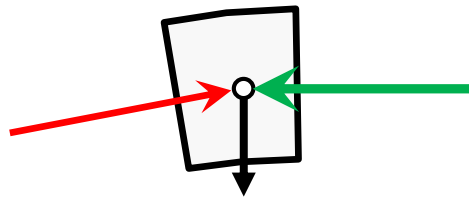
OR:



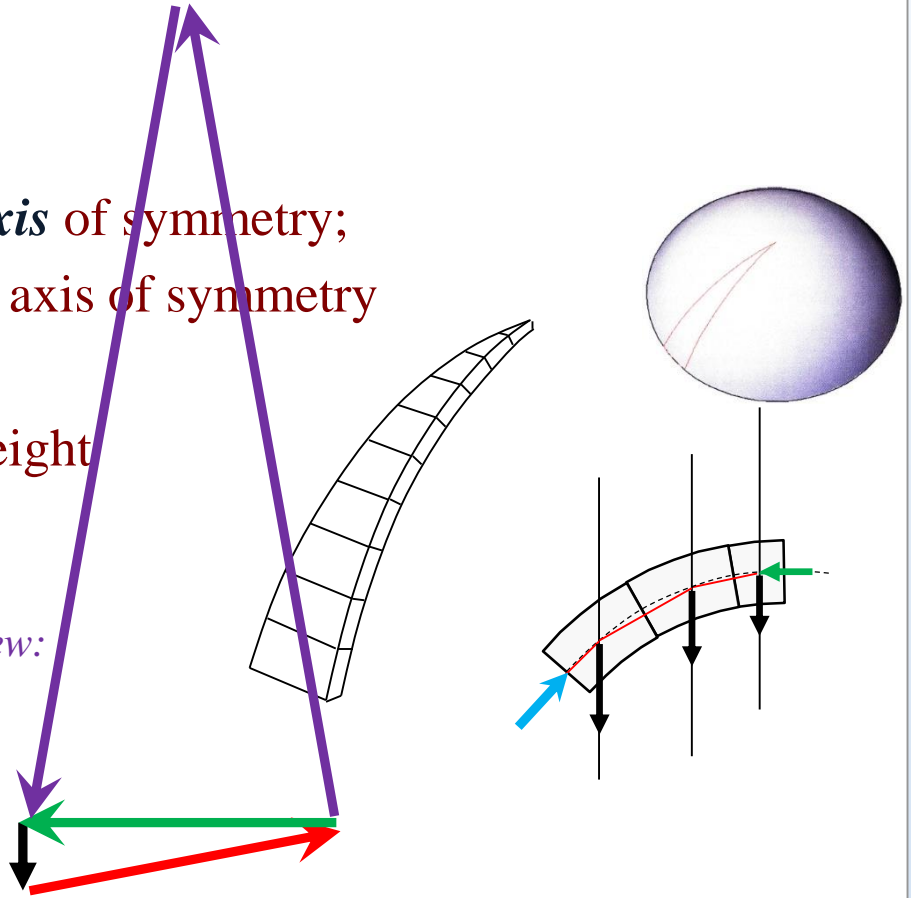
Wolfe's method

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- basic assumption: *membrane state*
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1. Analysis of the top segment:

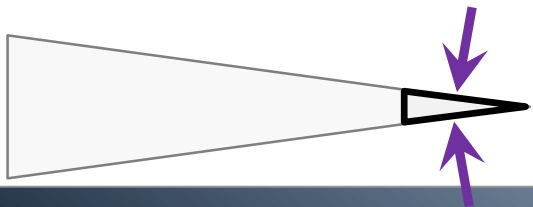


top view:



Hoop forces:

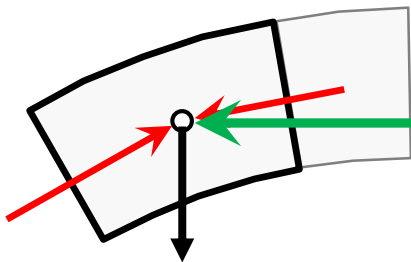
top view:



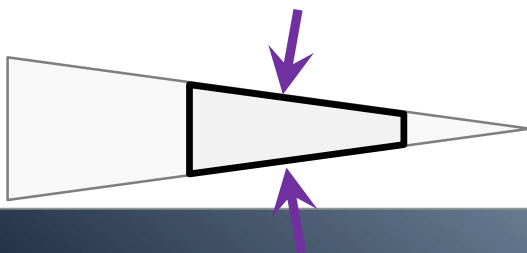
Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*
⇒ contact force intersect with weight
along the *middle line*

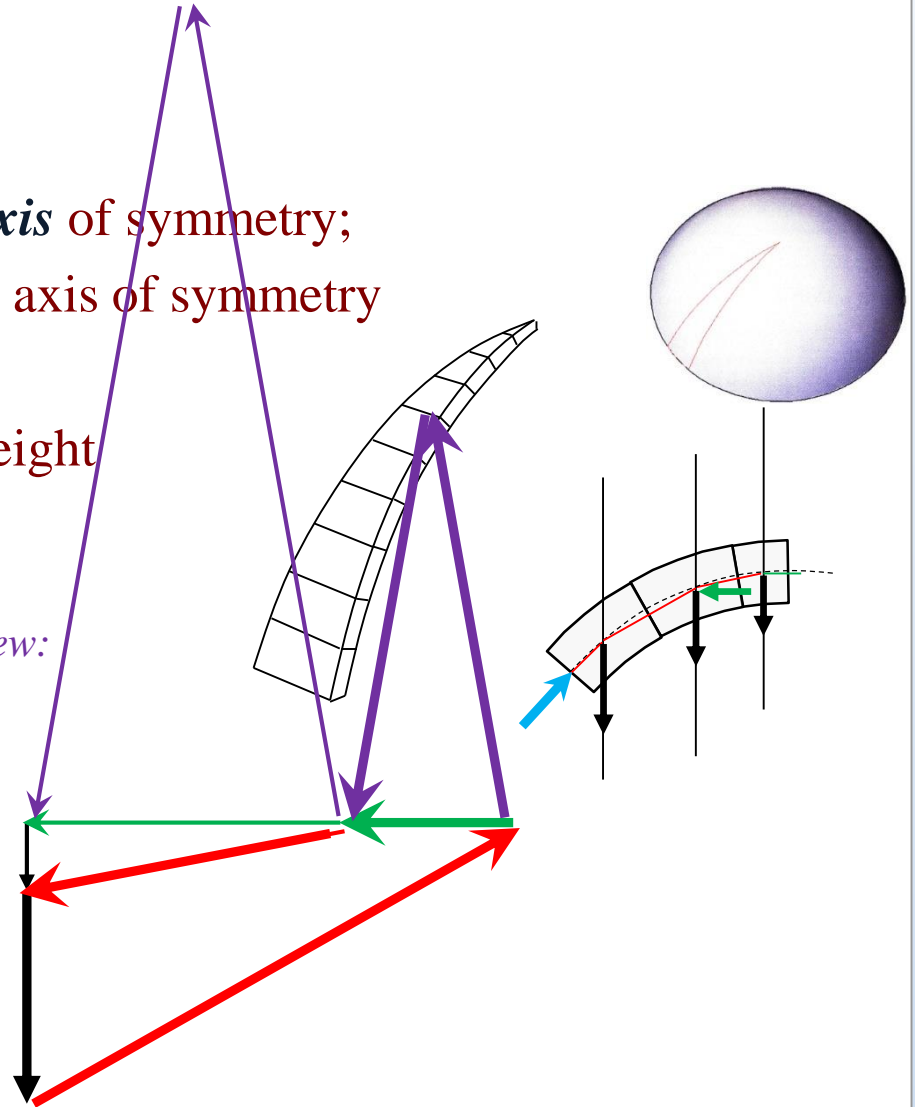
2. Analysis of the 2nd segment:



top view:



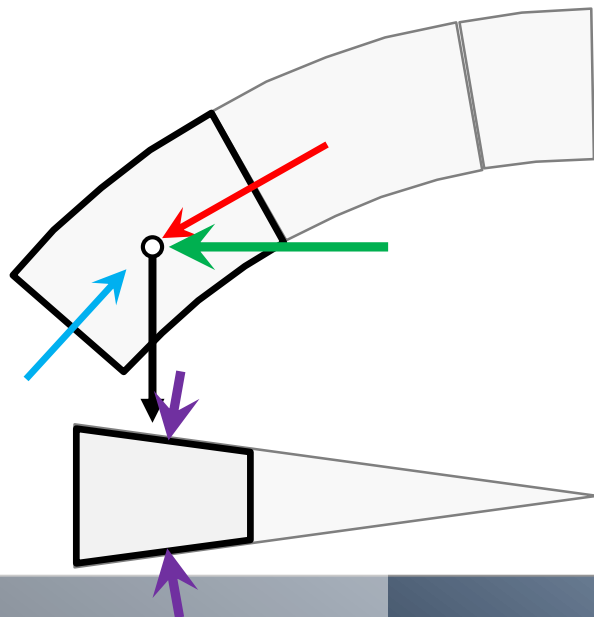
top view:



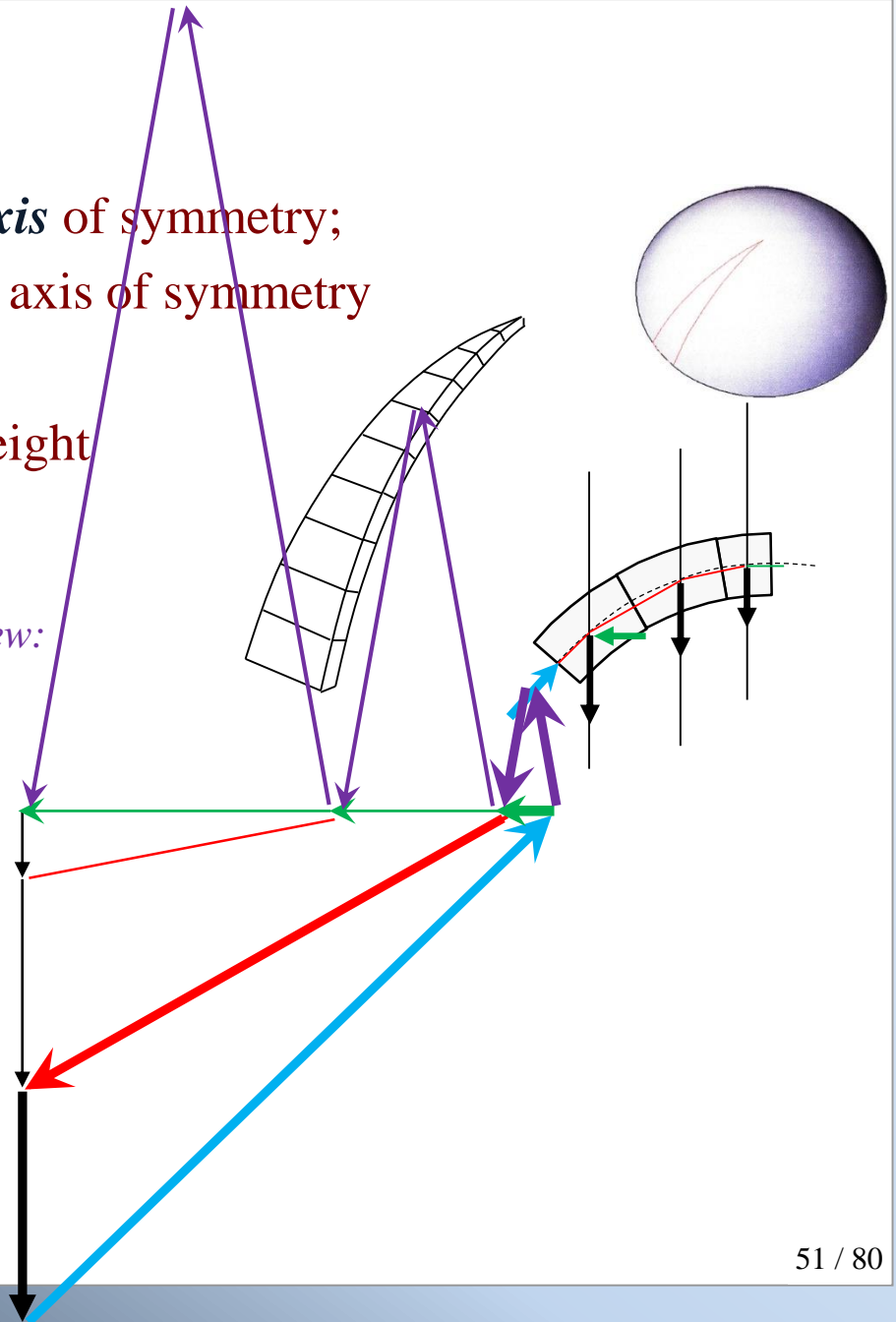
Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*
⇒ contact force intersect with weight
along the *middle line*

3. Analysis of the bottom segment:



top view:

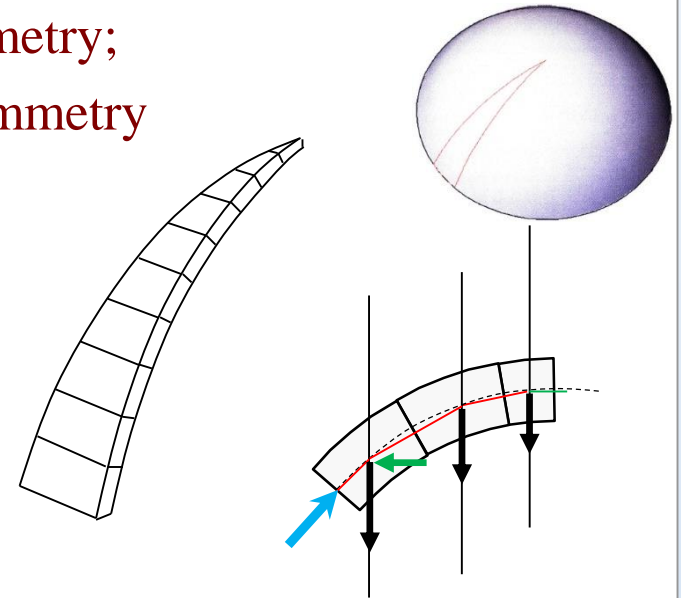
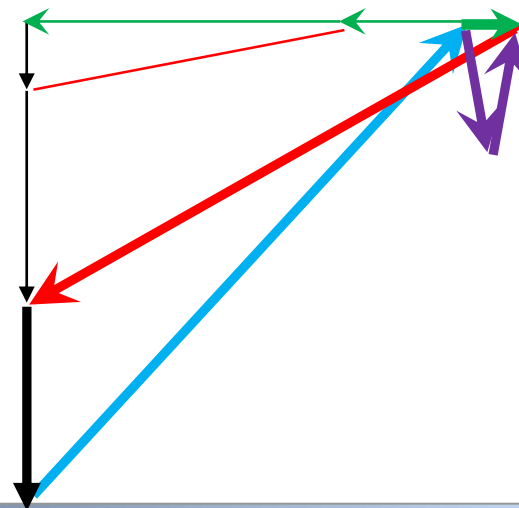
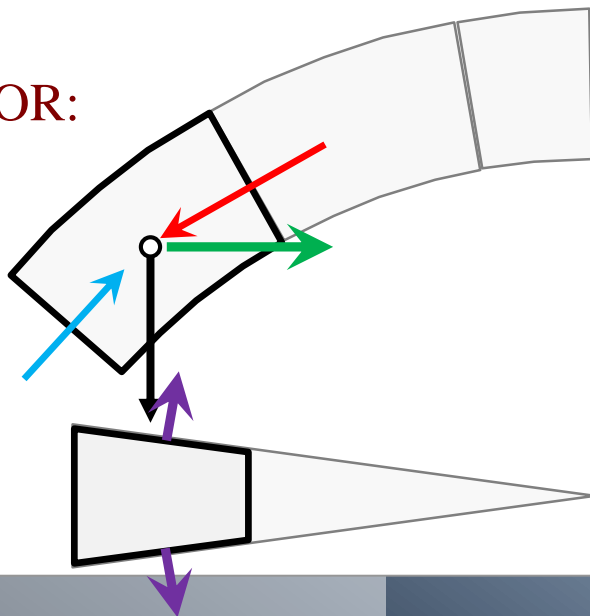


Wolfe's method

- Restricted to: domes with *vertical axis* of symmetry;
under *vertical loads* with vertical axis of symmetry
- basic assumption: *membrane state*
⇒ contact force intersect with weight
along the *middle line*

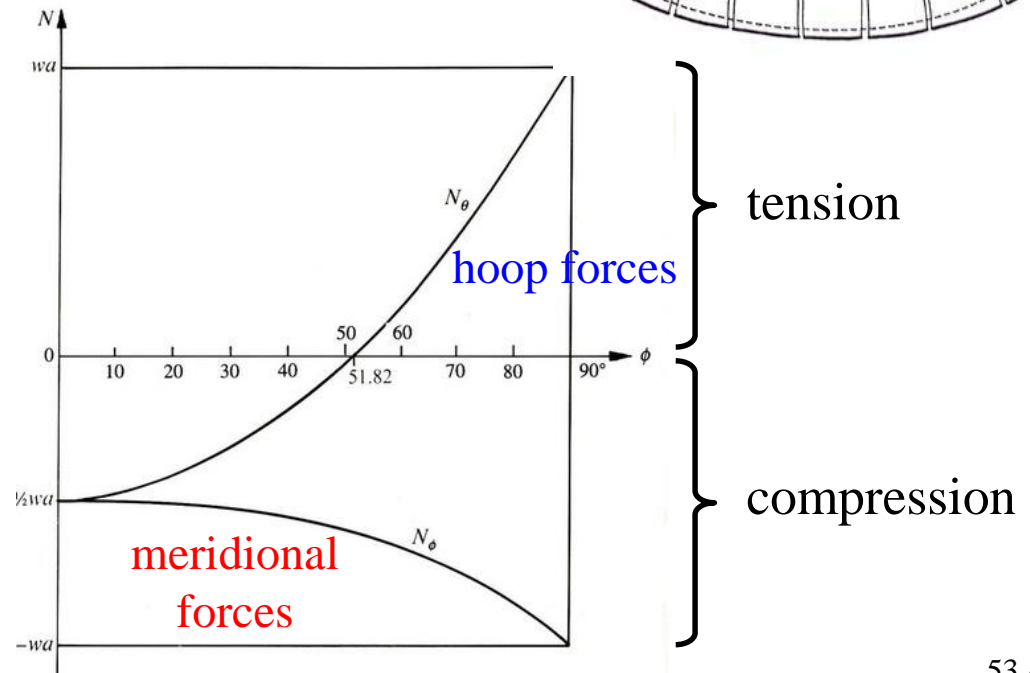
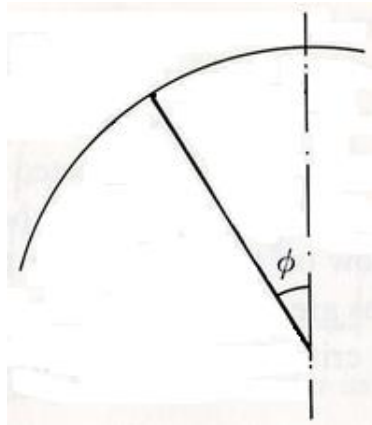
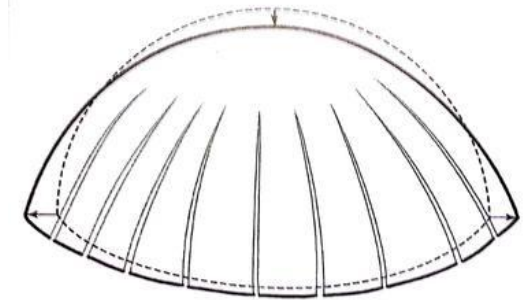
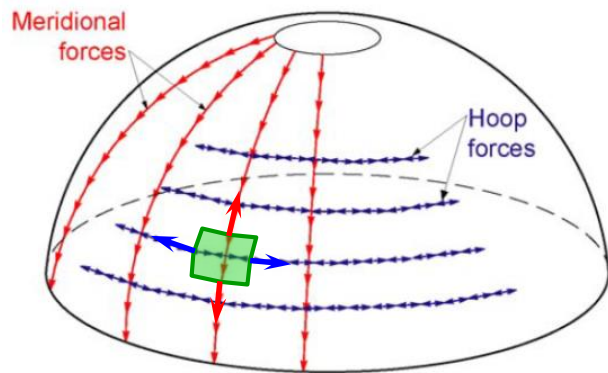
3. Analysis of the bottom segment:

OR:

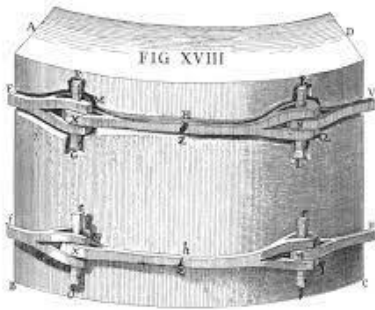


Remark: Membrane solution for spherical cap

Details: next lecture!



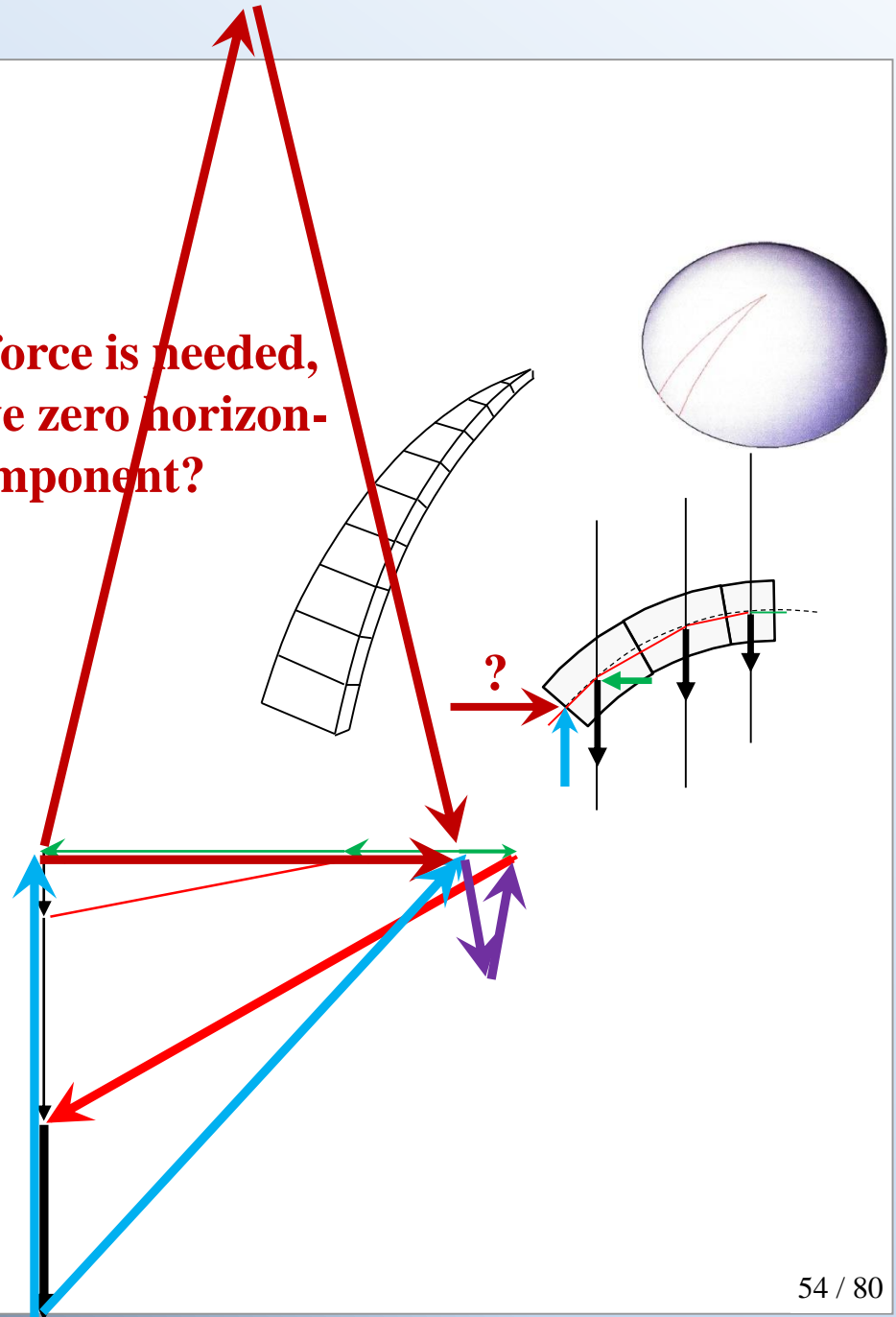
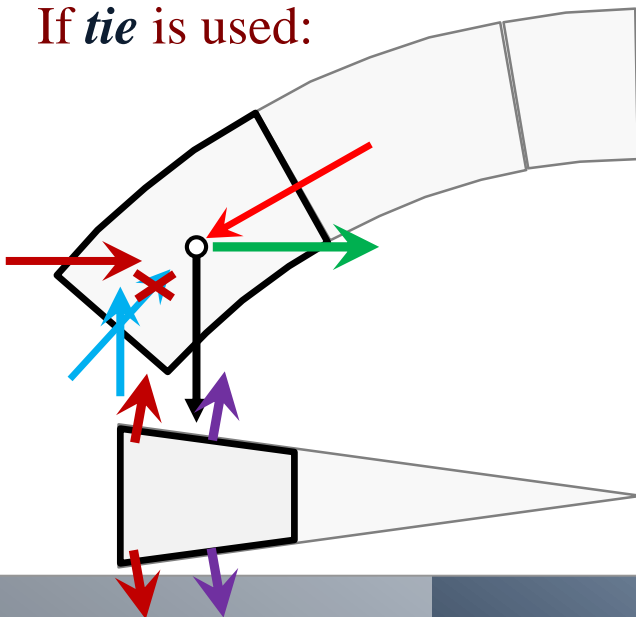
Wolfe's method



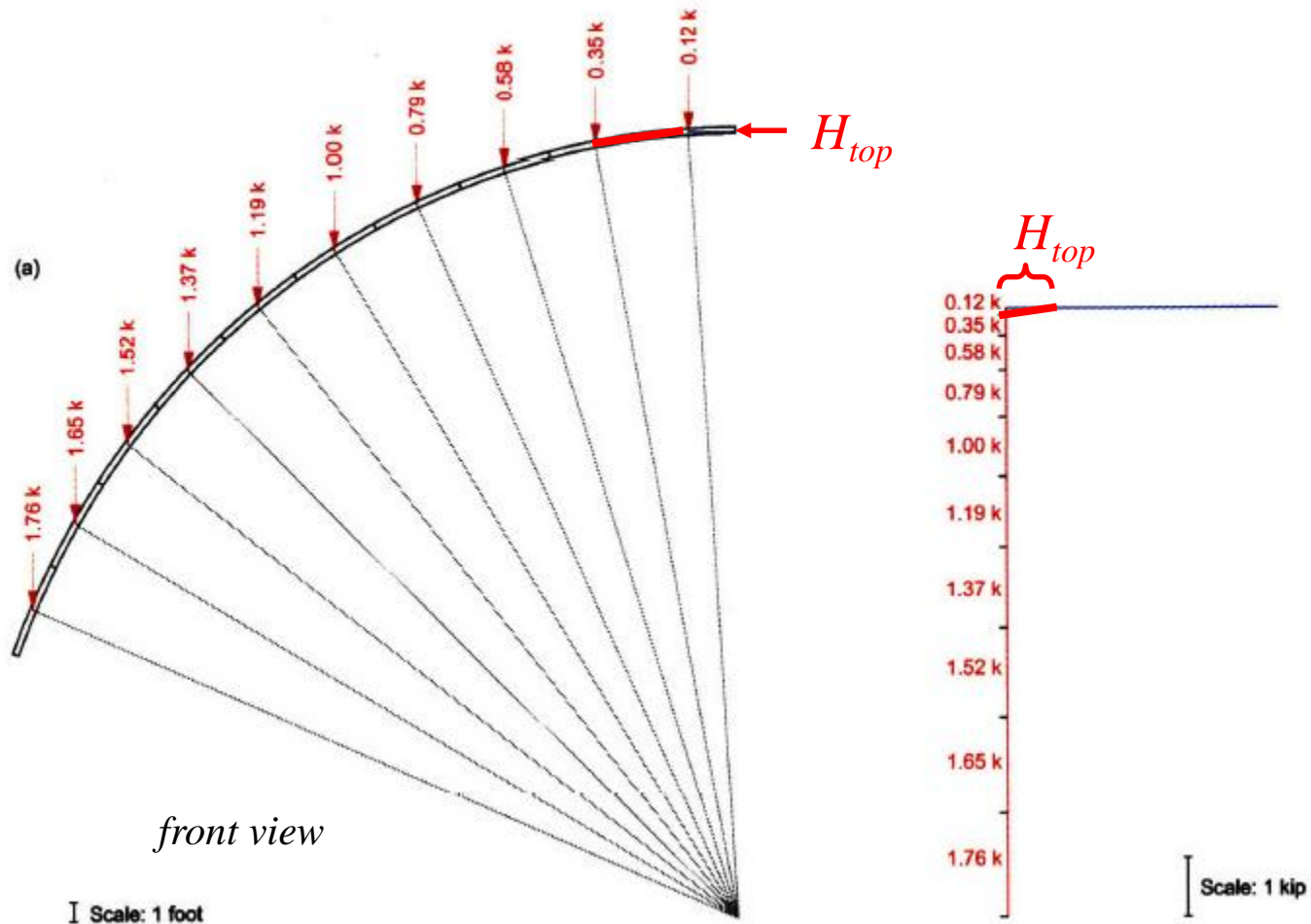
How large tie force is needed, in order to have zero horizontal reaction component?

3. Analysis of the 3rd segment:

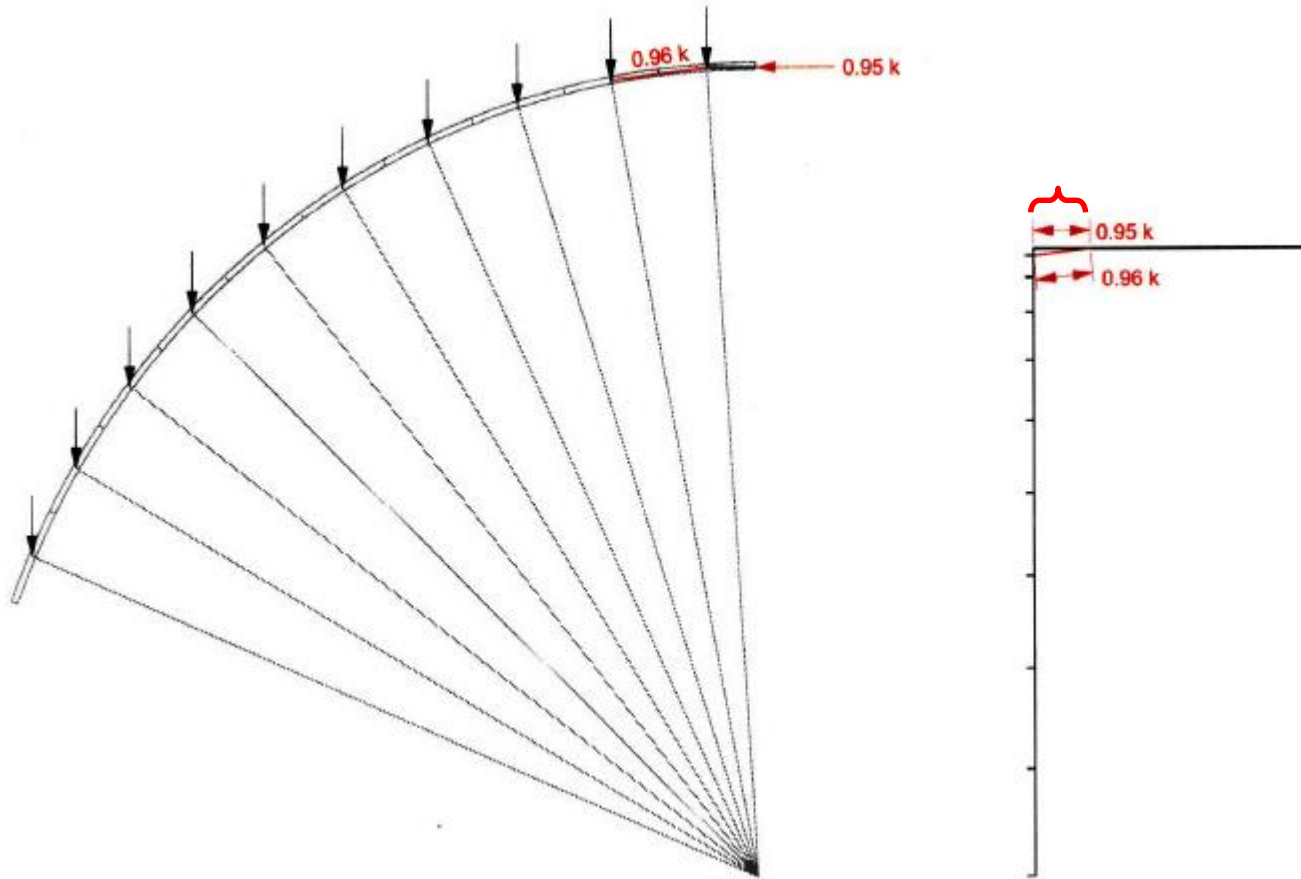
If *tie* is used:



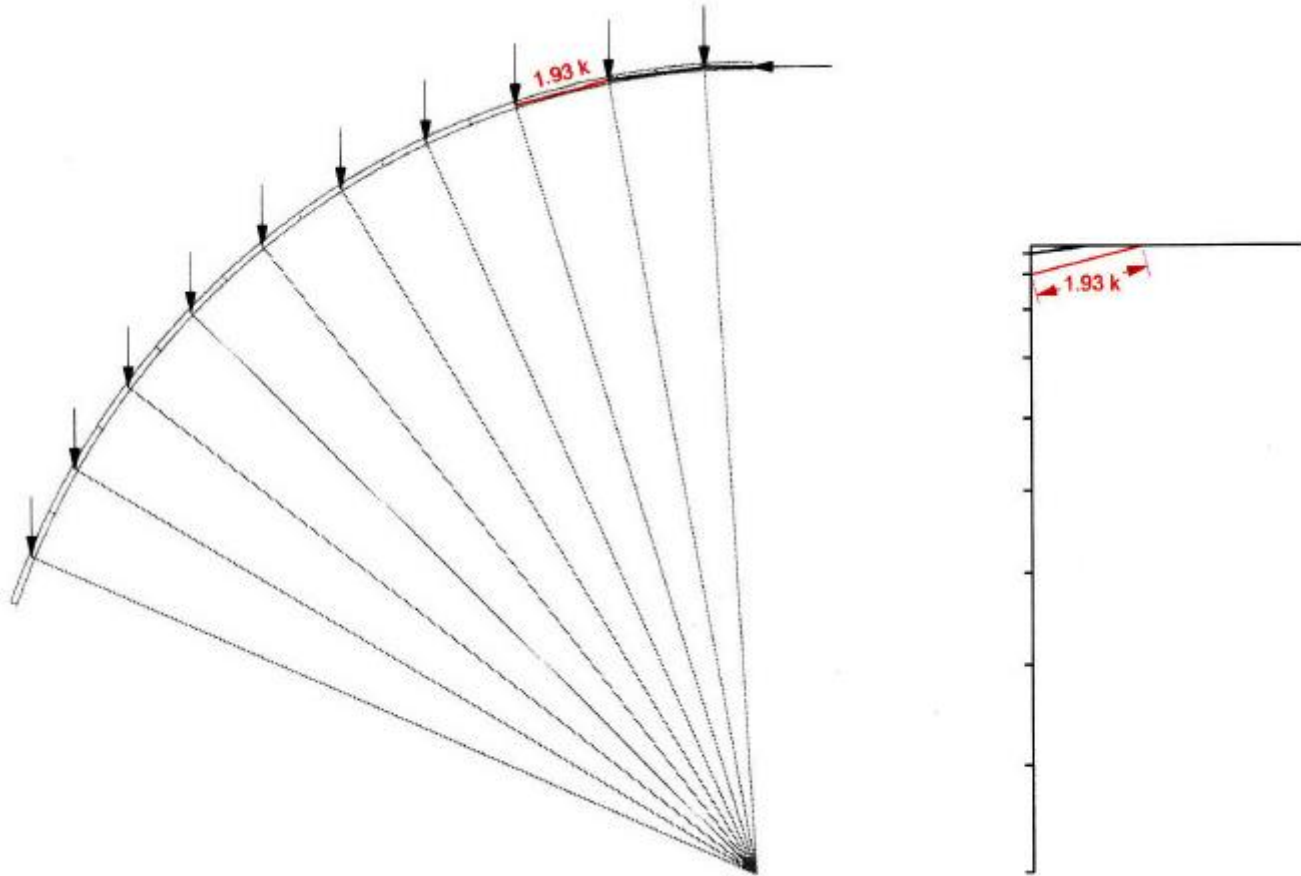
Wolfe's method, for tension-resisting domes



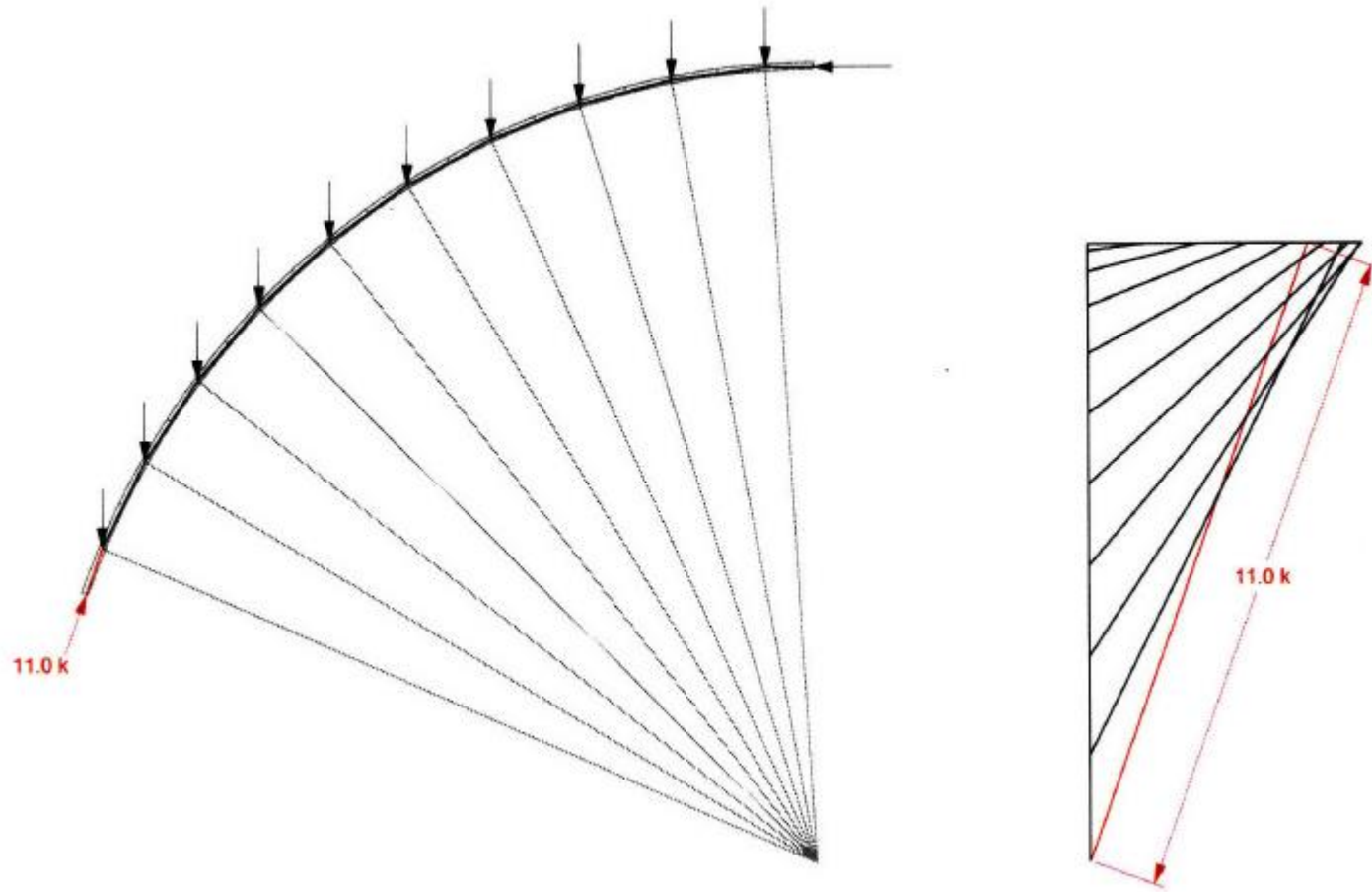
Wolfe's method, for tension-resisting domes



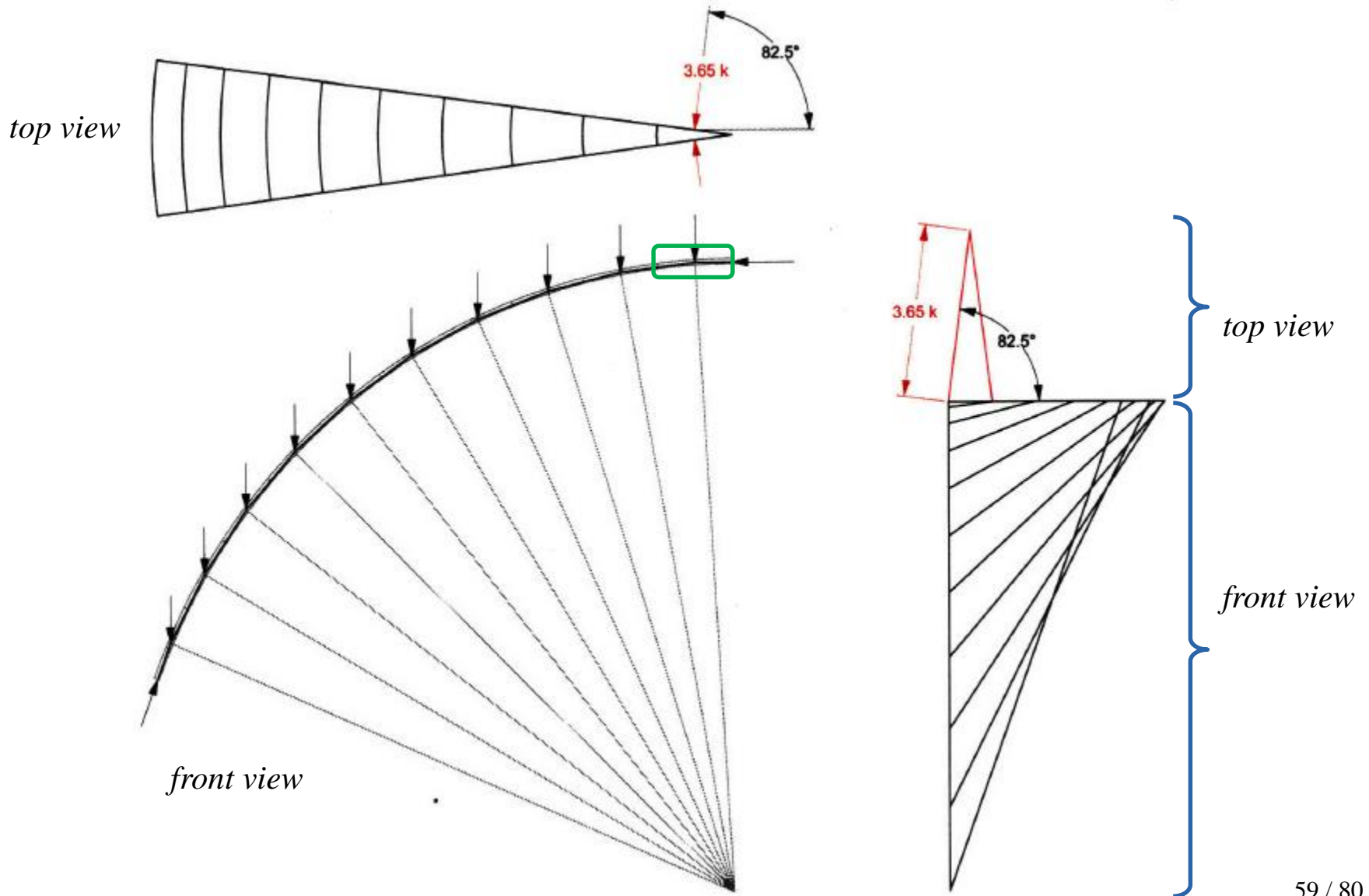
Wolfe's method, for tension-resisting domes



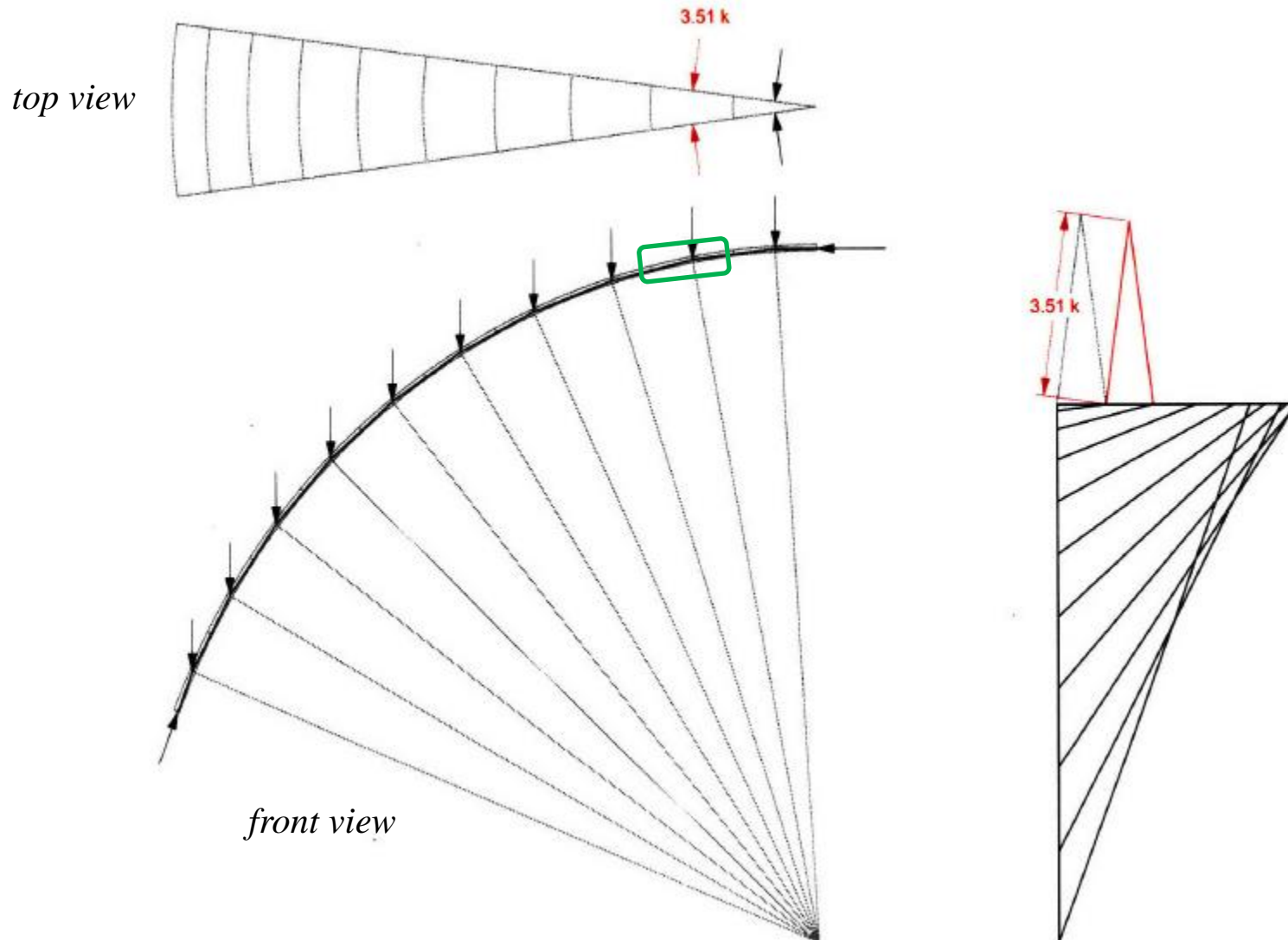
Wolfe's method, for tension-resisting domes



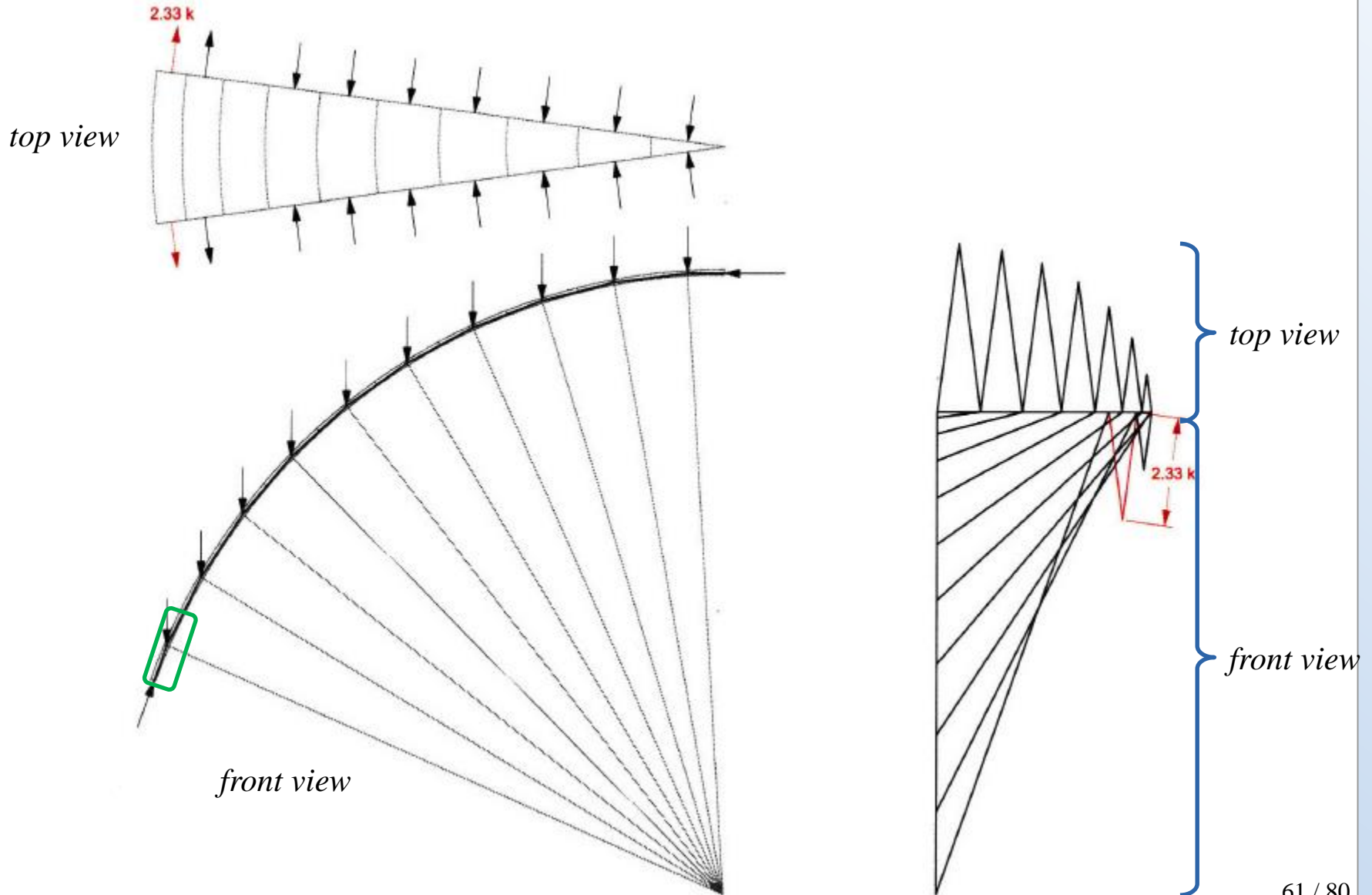
Wolfe's method, for tension-resisting domes



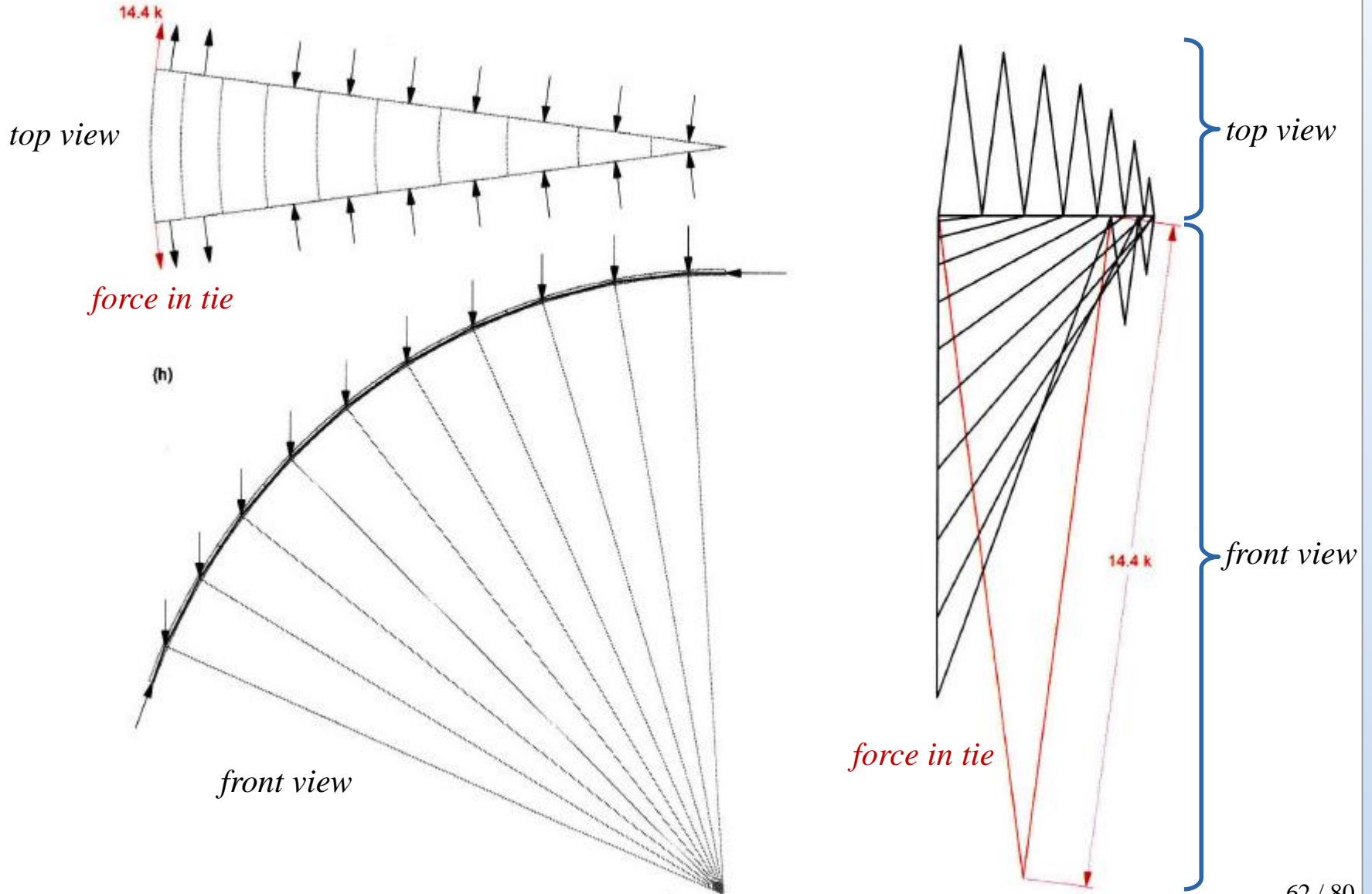
Wolfe's method, for tension-resisting domes



Wolfe's method. for tension-resisting domes

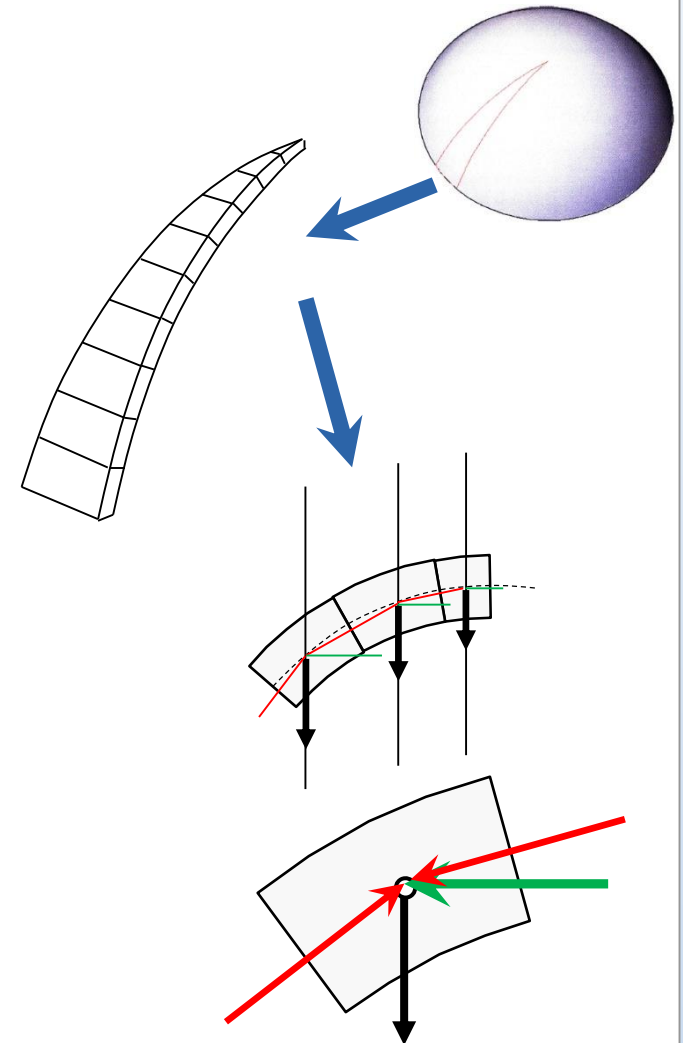


Wolfe's method, for tension-resisting domes



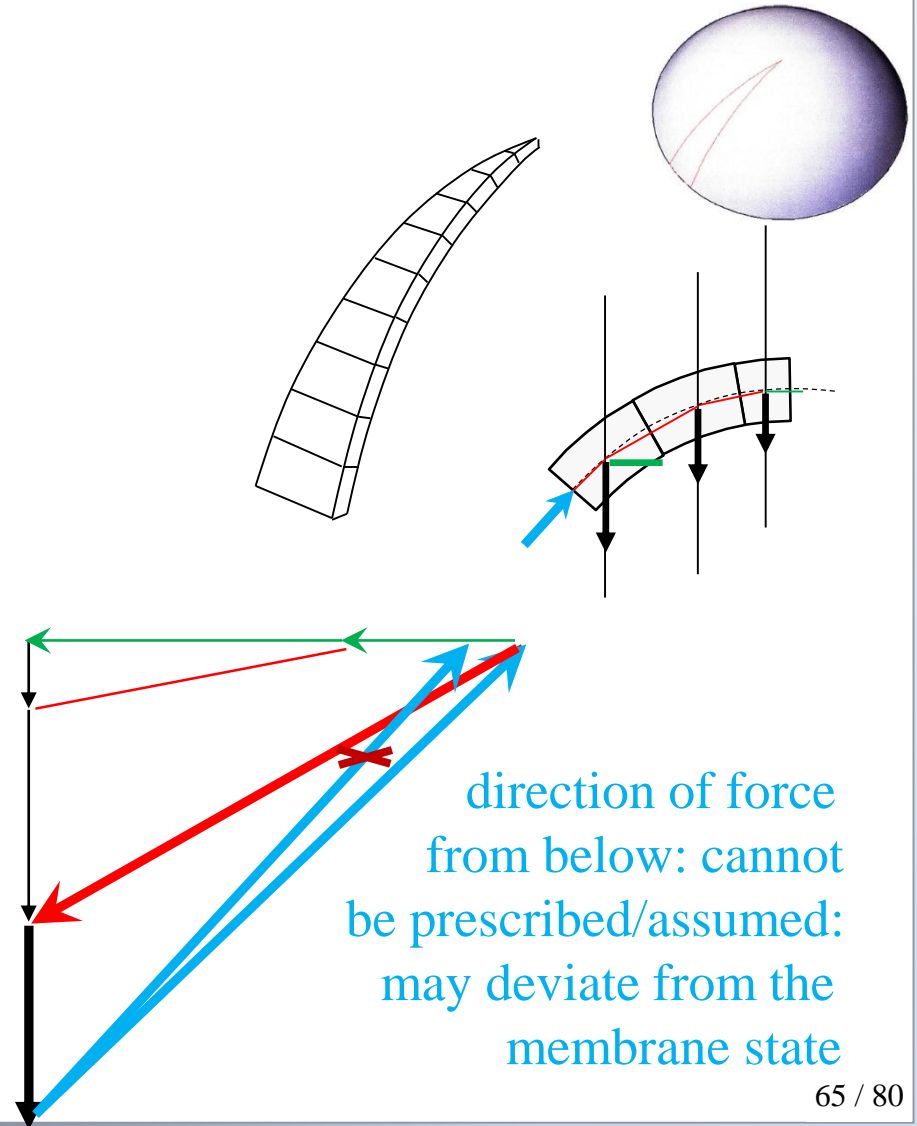
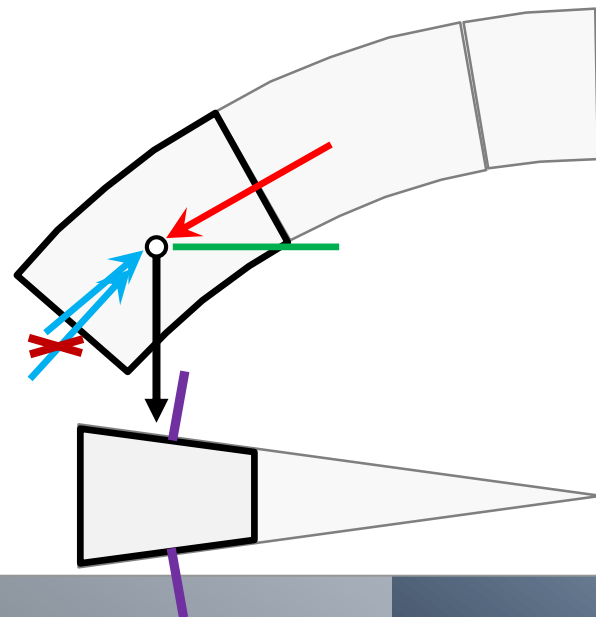
Wolfe's method

- Version 1.: domes with tension resistance
- Version 2.: domes without tension resistance

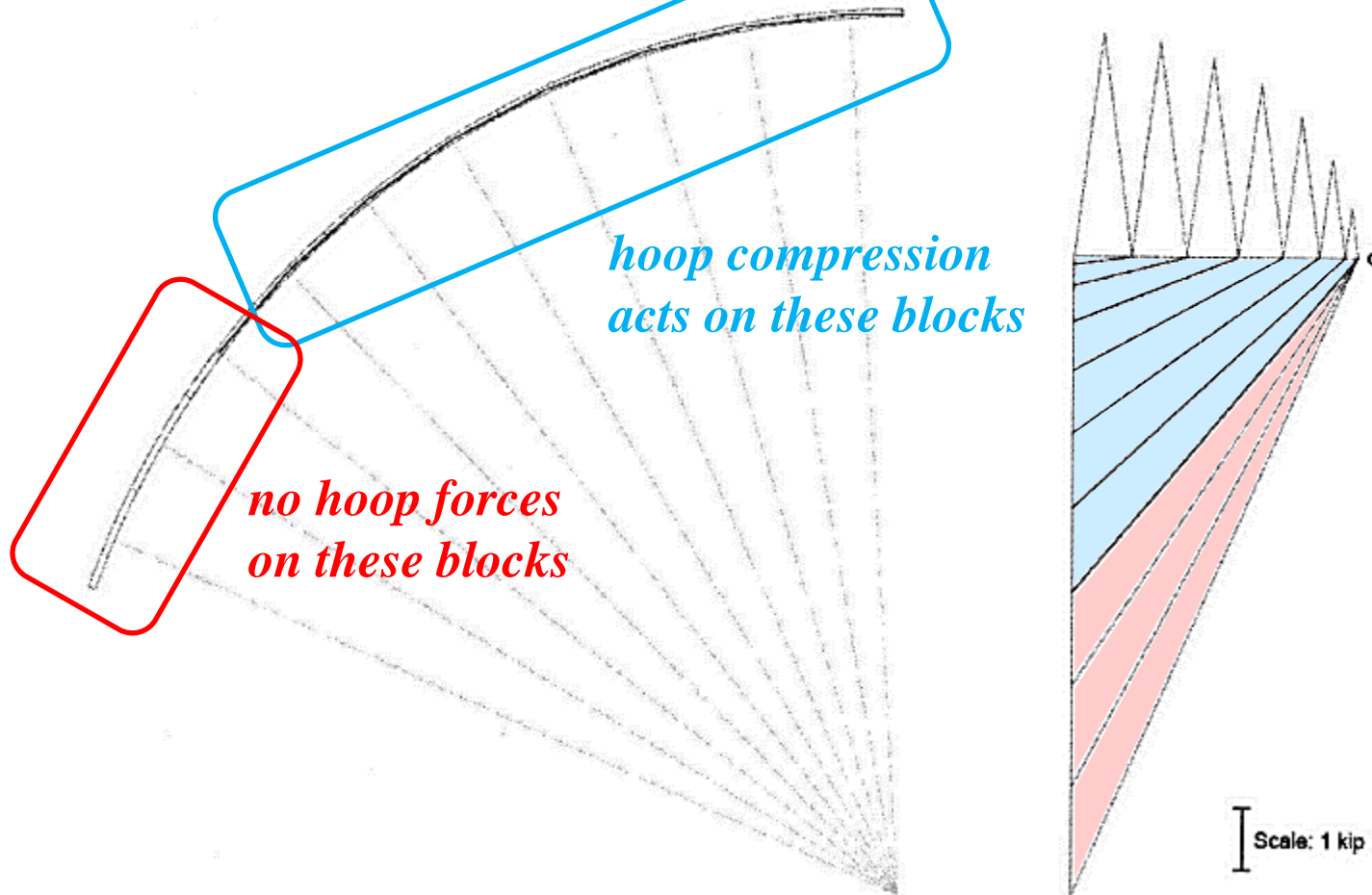


Wolfe's method, for **no-tension domes**

3. Analysis of the bottom segment:



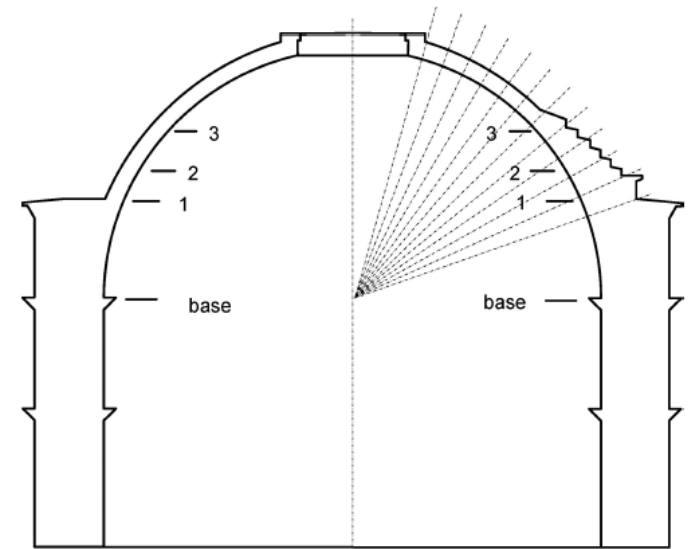
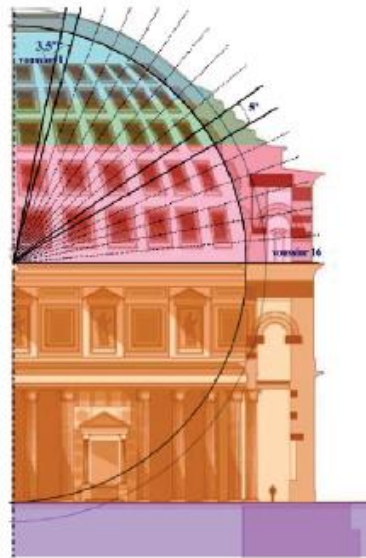
Wolfe's method, for **no-tension domes**



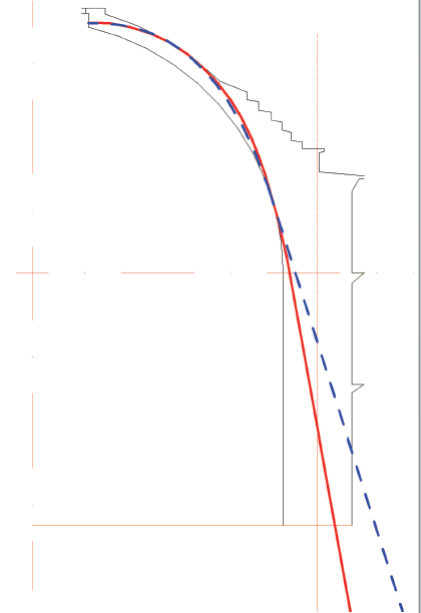
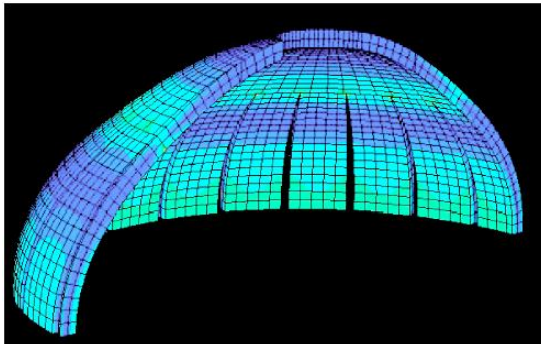
Wolfe's method

Application:

Morer & Goni (2008):
Pantheon in Rome, Italy
[not masonry!]



agreement with ABAQUS



method extended to find line of thrust: Lau (2006)

Wolfe's method

Application:



Cavalagli & Gusella (2015)



Cavalagli et al (2016)

Dome of the „Santa Maria Degli Angeli”

Basilica, Assisi, Italy

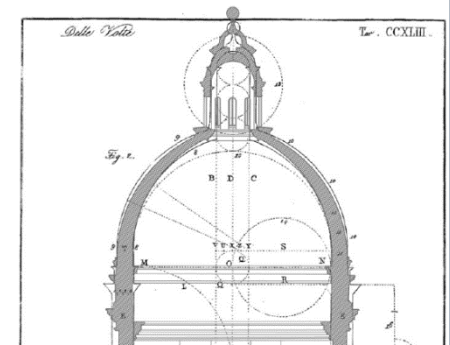
→ construction: 1569-1679; dome completed in 1677

→ dome diameter: ≈ 20 m; thickness: $\approx 180 \dots 90$ cm

perimeter: inside circular, outside octagonal

→ several earthquakes; e.g. 1832

after that: iron rings were added



Cavalagli et al (2016)

Wolfe's method

Application:



Cavalagli & Gusella (2015)

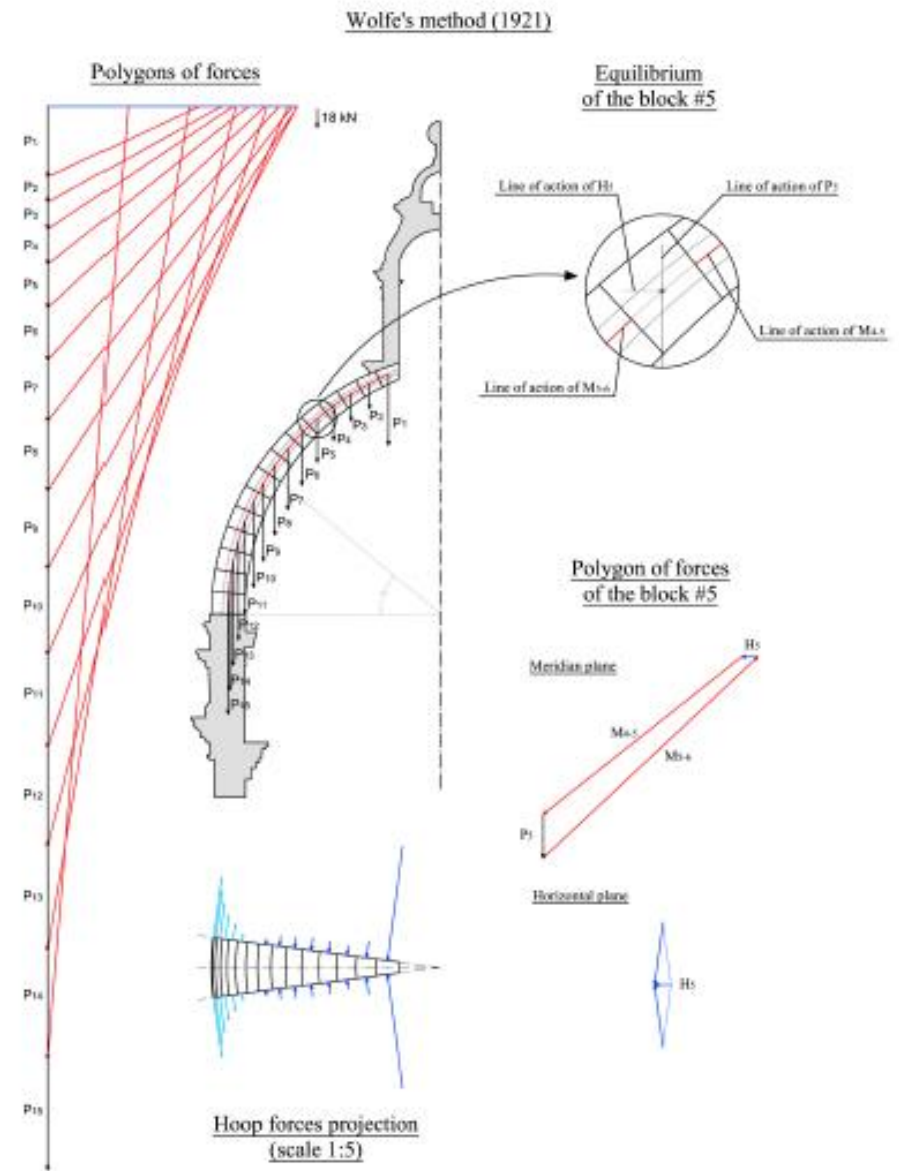
Cavalagli & Gusella (2015):

Wolfe's method compared to:

→ the Italian architect manual

→ another old graphical method:

Guidi (1928)

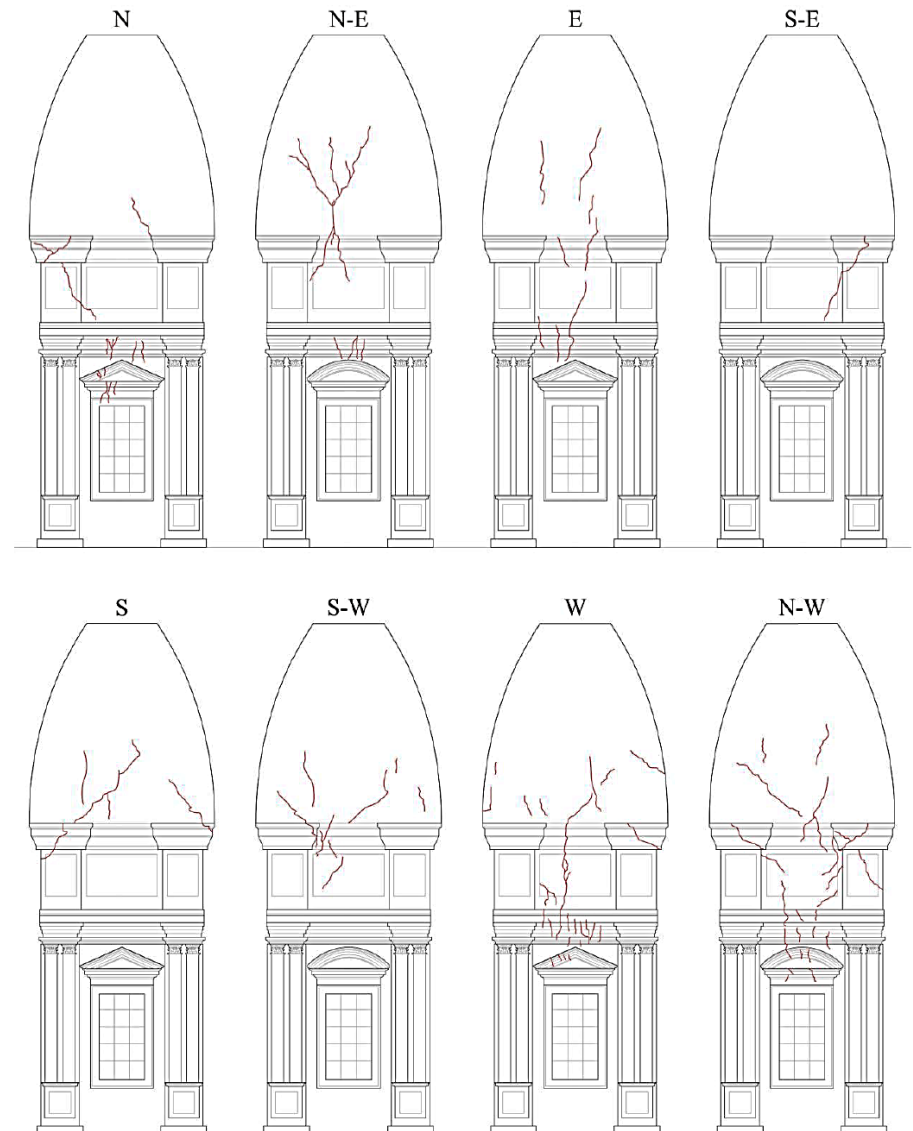


Wolfe's method

Application:



Cavalagli & Gusella (2015)



Cavalagli & Gusella (2015):

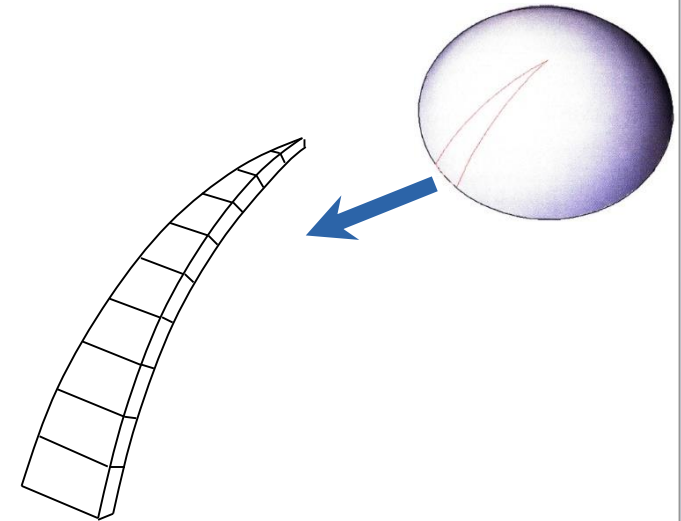
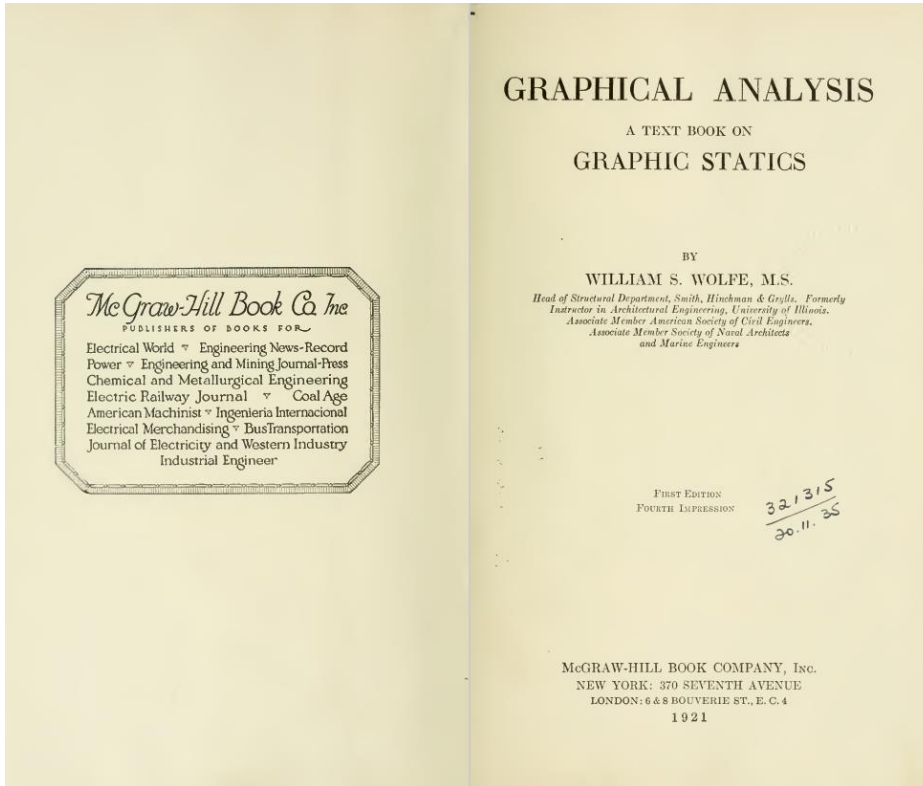
Wolfe's method compared to:

- the Italian architect manual
- another old graphical method

Conclusion:

only the graphical methods predict
crackings near the base

Wolfe's method



Further reading:

Wolfe (1921); Reese (2008); Lau (2006); Aita (2023);
Cavalagli, N., Gusella, V. (2015); Morer & Goni (2008)

THIS LECTURE:

GRAPHICAL METHODS

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 ... 2018

Wolfe's method for membrane forces in domes

O'Dwyer's funicular analysis \Rightarrow

Thrust Network Analysis (TNA)

Questions

Thrust Network Analysis

Preliminary: „Funicular Analysis”, O’Dwyer (1999)

~~masonry vault~~ → 3D truss: nodes \approx stone block inner points
bars \approx contacts between blocks
bar forces \approx contact forces

Vertical loads only!

Approximative because:

- all forces acting on a stone block intersect in the same point
- the lines of action in top view must be assumed at the beginning

Given: geometry of the vault; loading forces (dead & live)

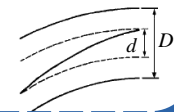
Unknowns: → vertical coordinates (z_i) of the nodes
 → some of the horizontal force magnitudes

*linear
optimization
problem*

Equalities: equilibrium of the nodes

Inequalities: nodes fall inside the material: $z_i^{\text{intrados}} \leq z_i \leq z_i^{\text{extrados}}$

Objective function: either: live load multiplier → max!
 or : deviation from middle surface → min!



Thrust Network Analysis

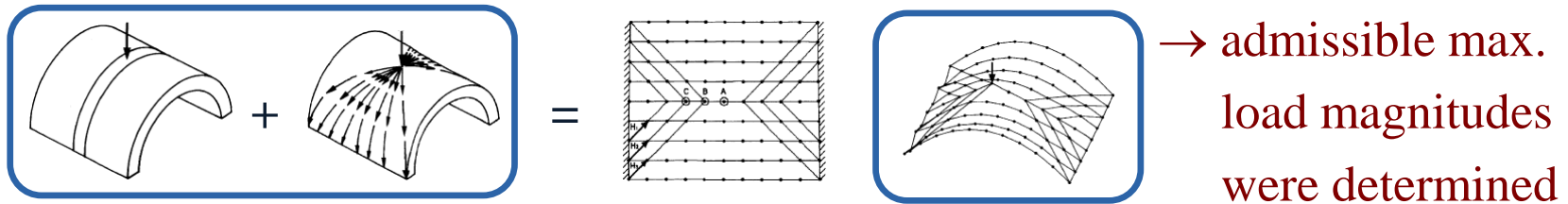
Preliminary: „Funicular Analysis”, O’Dwyer (1999)

~~masonry vault~~ → 3D truss: nodes \approx stone block inner points
bars \approx contacts between blocks
bar forces \approx contact forces

Applications:

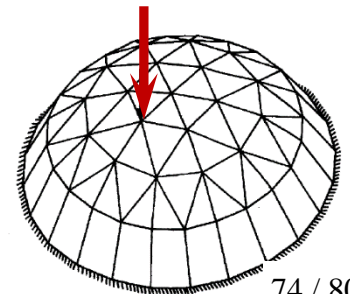
Problem Type 1:

Find maximum admissible live load on a given vault:



Problem Type 2:

Find optimum network shape of a vault under a given load: minimize the vertical deviation of force lines of action from the vault middle surface



Thrust Network Analysis

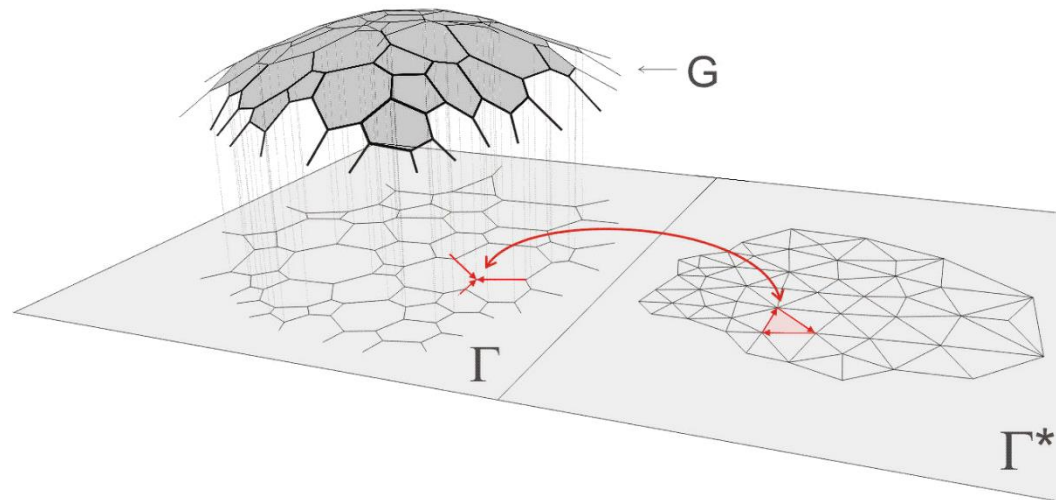
Block & Ochsendorf (2007), (2008); Block & Lachauer (2014):

→ based on O'Dwyer's „Funicular Analysis”

→ sophisticated computer coding; nice graphic representations

objective functions can be:

- (1) minimize deviation from middle surface (max geometrical factor of safety)
- (2) minimal / maximal horizontal thrust (deepest / shallowest force systems)
- (3) maximize live load multiplier which can be added to the given selfweight

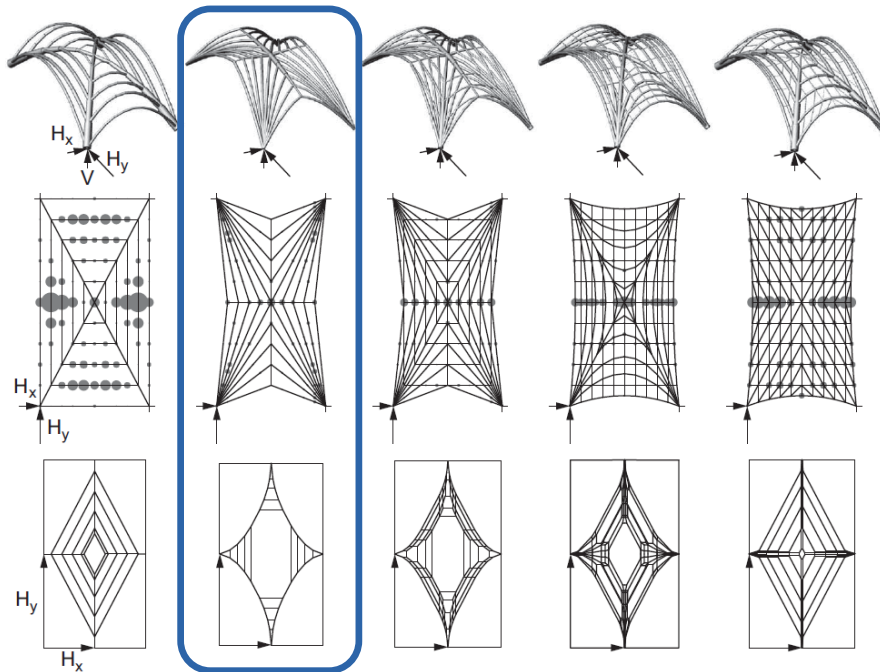


Thrust Network Analysis

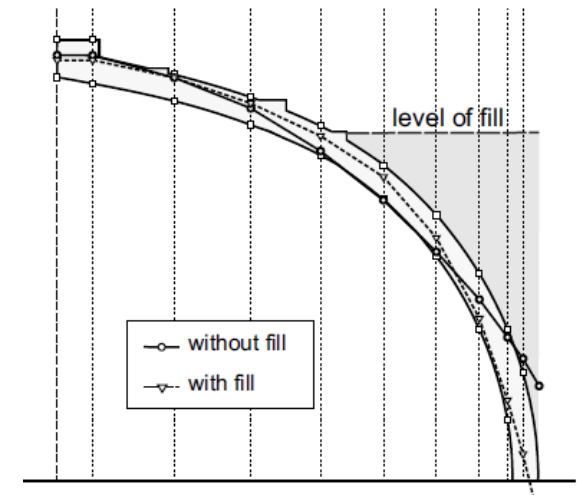
Block & Ochsendorf (2007), (2008); Block & Lachauer (2014):

- based on O'Dwyer's „Funicular Analysis”
- sophisticated computer coding; nice graphic representations
- analysis of several Gothic structures

cross vaults:



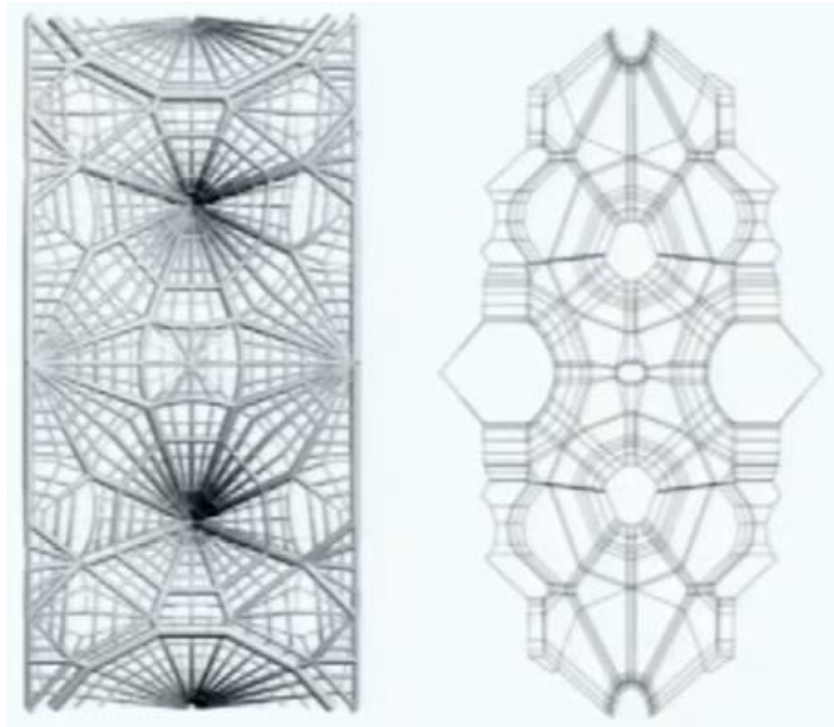
fan vaults:



Thrust Network Analysis

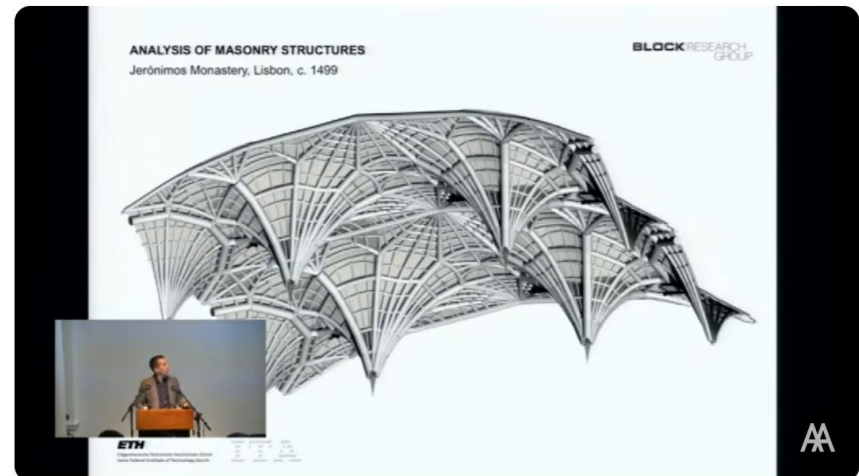
Block (2013):

Jeronimos Monastery, Lisboa, ≈1499:
10 cm thin, how can it stand?



Philippe Block - Stone Skins: New Masonry Shells

⇒ a „compression-only” force system
was found with LSA that fits into:



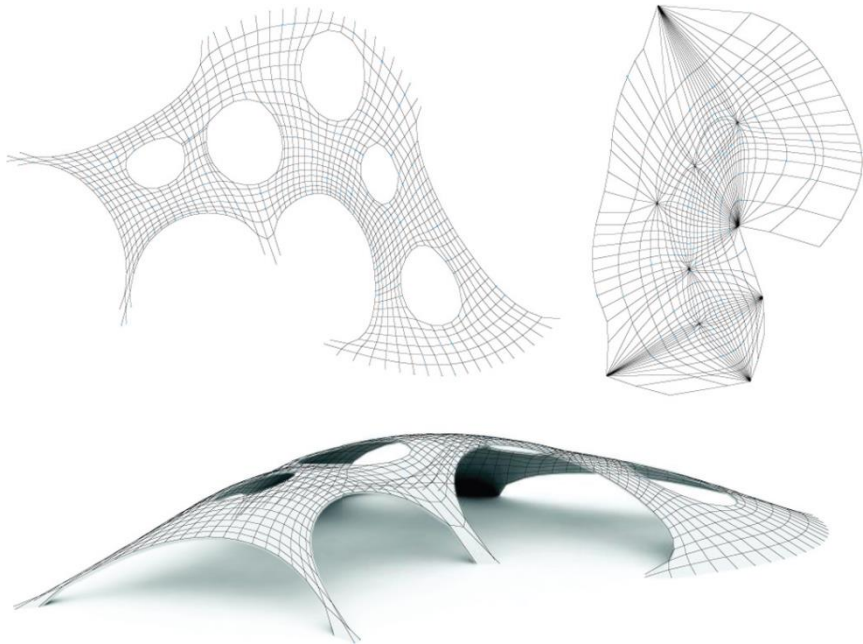
<https://www.youtube.com/watch?v=1Lk8wihM22s>

Philippe Block - Stone Skins: New Masonry Shells

Thrust Network Analysis

Block & Ochsendorf (2007), (2008); Block & Lachauer (2014):

- based on O'Dwyer's „Funicular Analysis”
- sophisticated computer coding; nice graphic representations
- analysis of several Gothic structures
- design optimal shapes for vaults



Thrust Network Analysis

Block Research Group:

e.g. The Red Line project, Rwanda:

drone port:

tile-vaulted (very thin) structures,
easy and cheap to construct

„Durabric” (earth + 8% cement, not burnt)

<https://www.youtube.com/watch?v=mZwIIndTUow>



block.arch.ethz.ch/brg/project/venice-biennale-2016_droneport

THIS LECTURE:

2. GRAPHICAL METHODS

Historical times: Practical geometrical rules

e.g. Vitruvius

e.g. Gothic rules

Graphical statics

The basic problem: Stability of an arch

Durand-Claye's stability area method for arches

computerized & extended for domes: Aita et al 2003 ... 2018

Wolfe's method for membrane forces in domes

O'Dwyer's funicular analysis \Rightarrow

Thrust Network Analysis (TNA)

Questions

QUESTIONS

1. Introduce a chosen historic geometrical design rule. What is the background for this design rule?
2. How to determine the possible minimal and maximal horizontal thrust for an arch under selfweight, using graphical statics?
3. What is the geometrical factor of safety of an arch or vault?
4. Introduce Durand-Claye's stability area method.
5. Introduce Wolfe's method for domes. How is it used for no-tension material, and for determining the tie force?
6. Introduce the Thrust Network Analysis method. What objective functions can be used?