



FAN VAULTS



Citation:

K. Bagi (2024): **Mechanics of Masonry Structures**. Course handouts, Department of Structural Mechanics, Budapest University of Technology and Economics

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In case of any question or problem, do not hesitate to contact Prof. K. Bagi, kbagi.bme@gmail.com .

THIS LECTURE

Definition

Preliminaries to fan vaulting

Reminder on the membrane solution

Beginning of fan vaulting

Constructional issues

- jointed masonry versus rib-and-panel system

- pendants

- roofing

- variations to the groundplan

- variations to the spandrel geometry

- the generator curve geometry

Decline of fan vaulting; Fan vaults after the English Gothic

Questions

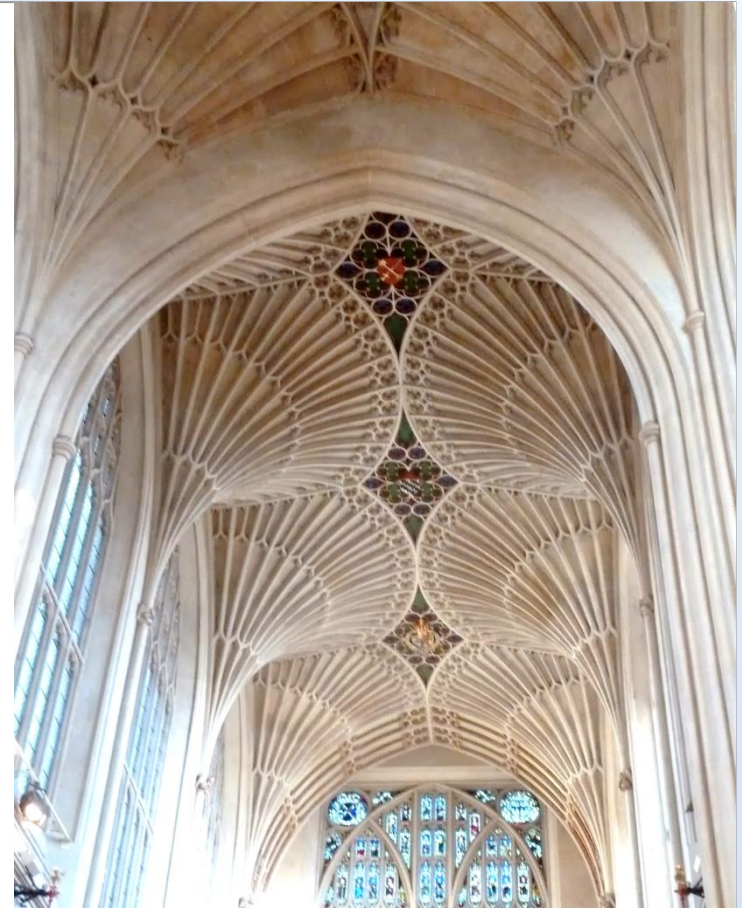
DEFINITION

When: Late Gothic;
between the middle of
XIVth – middle of XVIth century

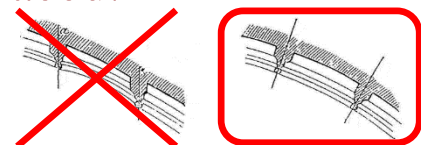
Where: in England only!
(?? baltic examples ??)

Definition:

- The shell is a surface of revolution:
a smooth arc, concave from below,
is rotated about a vertical axis being
on the outer side.
- Vertical main ribs have identical shape,
and are arranged at equal angles.
- Between the conoids, a distinct spandrel panel is placed.
- The ribs are perpendicular to the surface.

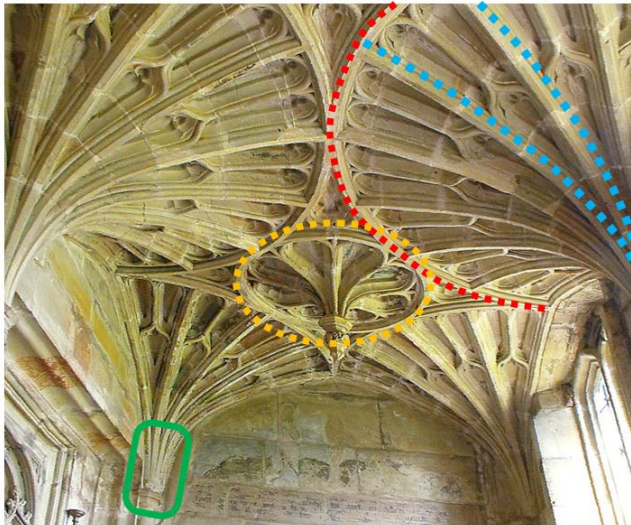


Bath Abbey,
devizesdays.blogspot.com/2013/11/st-andrews-school-sings-in-bath-abbey.html



DEFINITION

Main components:



vertical ribs arranged at equal angles

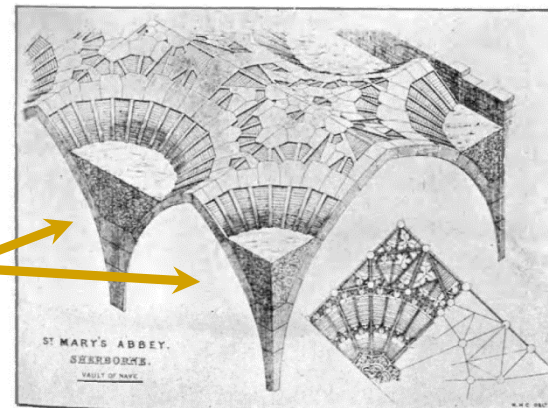
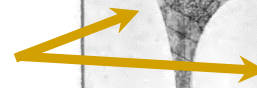
horizontal circular rib

central spandrel

tas-de-charge

+ often applied:

upfill in the vaulting pockets:



Howard, 1911

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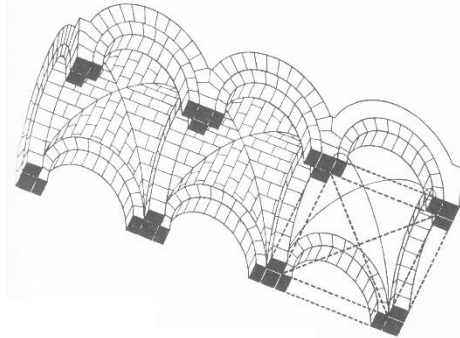
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PRELIMINARIES TO FAN VAULTING

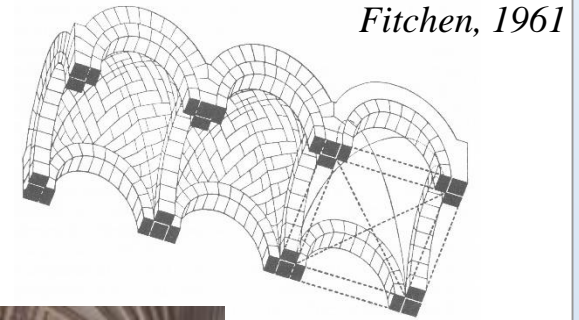
Leedy (1980): Basic characteristics of the English Gothic architecture,
to allow for the idea of fan vaulting:

(1) Stone coursing pattern:

French:



English:



(2) Existence of intermediate vertical ribs:



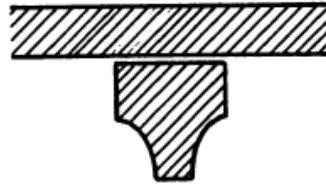
Exeter Cathedral, demolition-exeter.blogspot.hu/2011/07/above-cathedral-vault-i.html

PRELIMINARIES TO FAN VAULTING

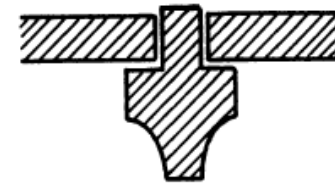
Leedy (1980): Basic characteristics of the English Gothic architecture,
to allow for the idea of fan vaulting:

(3) The practice how to join rib and web voussoirs:

French:



English:

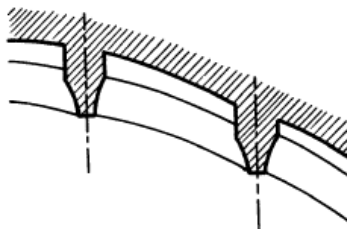


⇒ the idea of flat plates between ribs ⇒ spandrel

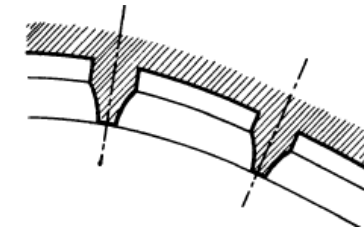
⇒ the idea of having masonry panels with ribs „sculptured” on them

(4) The ribs are perpendicular to the shell: [from early XIVth century]

French:



English:



(...)

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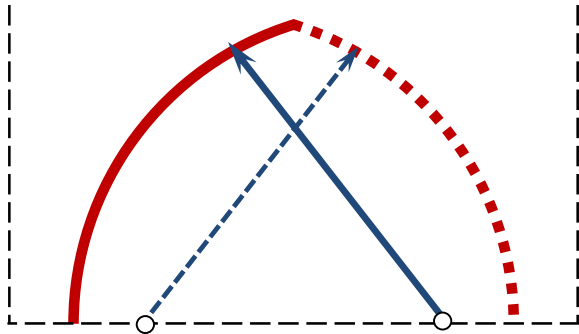
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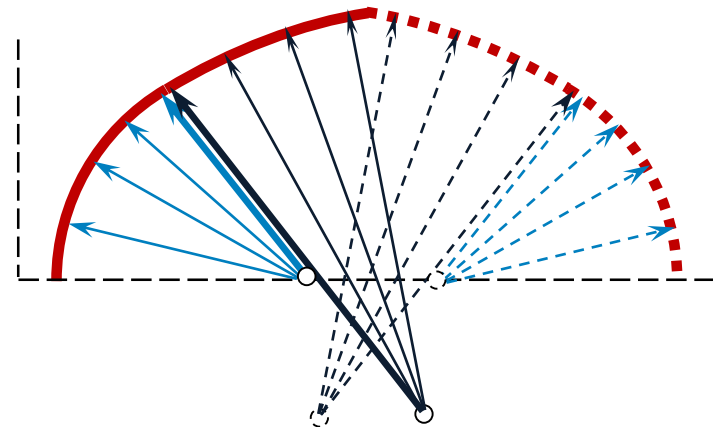
Questions

REMEMBER: MEMBRANE SOLUTION

The generator curve:



uniform curvature
„two-centered arch”



two different curvatures („Tudor arch”)
„four-centered arch”

membrane solution was seen for this

REMEMBER: MEMBRANE SOLUTION

Remember:

1) if $\varphi_0 = 0$:

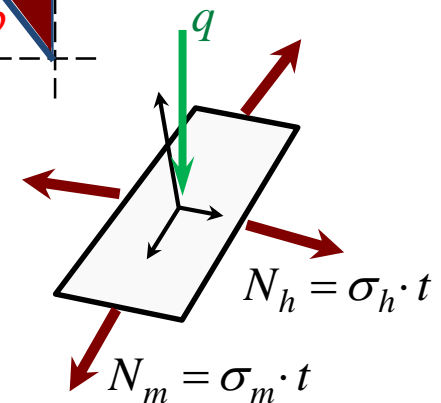
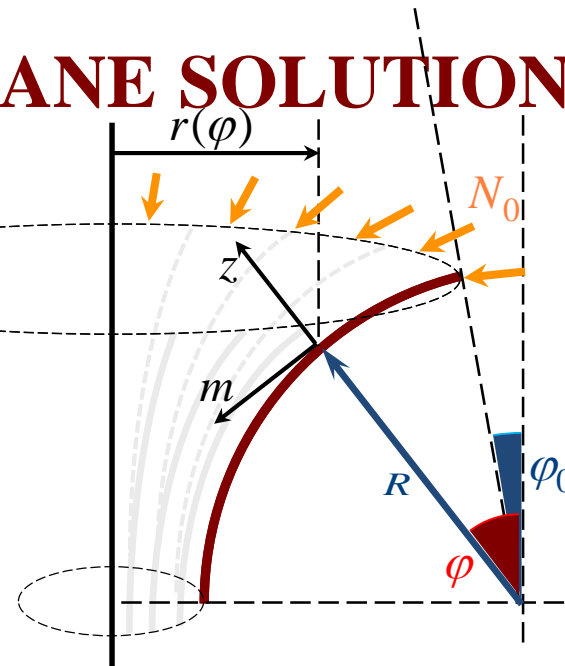
$$\sin \varphi_0 = 0 \Rightarrow N_h(\varphi_0) \rightarrow \infty$$

\Rightarrow the conoid must be truncated !

2) if $\varphi_0 \neq 0$:

$$\text{if } N_0 = 0 : N_m(\varphi_0) = 0 \Rightarrow N_h > 0$$

\Rightarrow spandrel load is needed to avoid hoop tension !



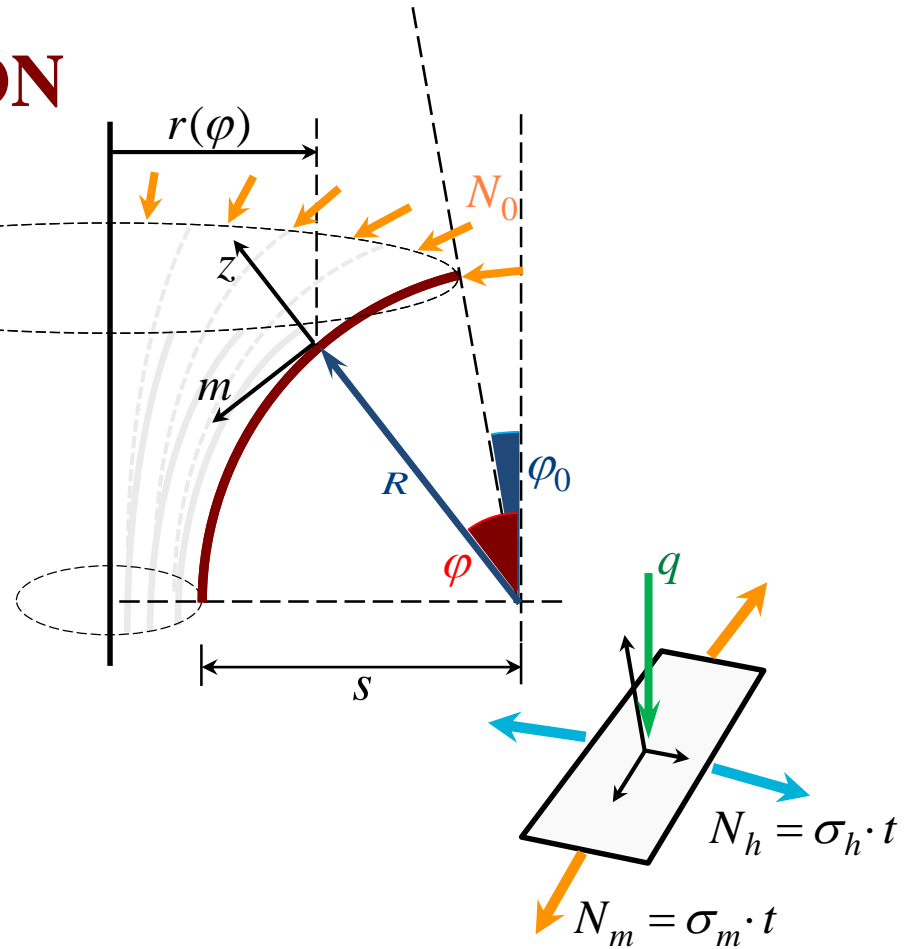
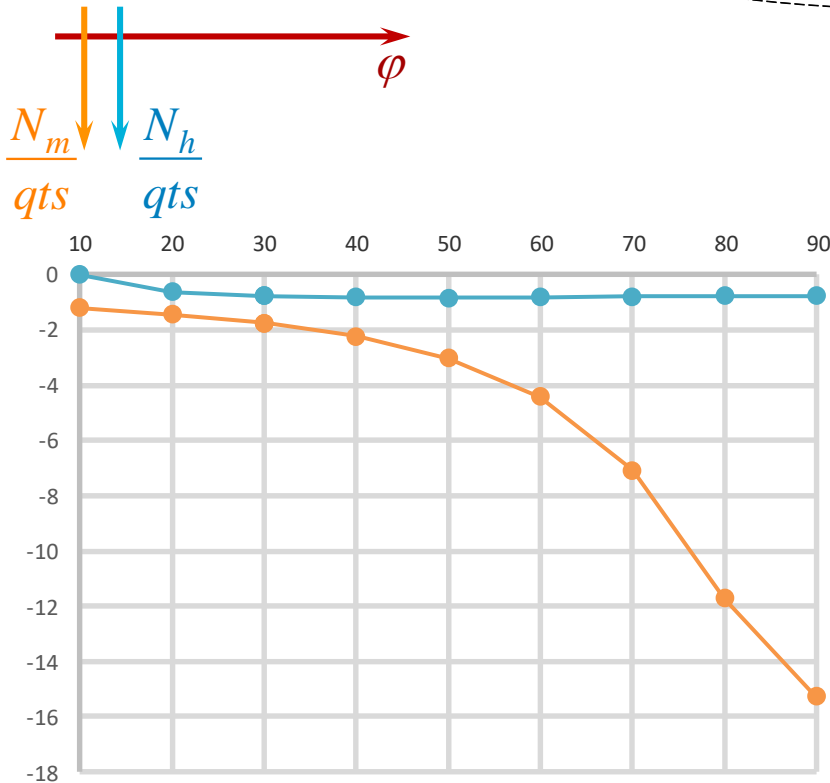
$$\Leftarrow N_0 \geq q \cdot R \cdot \cos \varphi_0$$

$$N_m(\varphi) = \frac{-1}{\sin \varphi \cdot r(\varphi)} \left(N_0 \sin \varphi_0 \cdot r(\varphi_0) + \frac{A_{cap}(\varphi)}{\pi} \cdot q \right)$$

$$N_h(\varphi) = \frac{r(\varphi)}{\sin \varphi} \left(\frac{N_m(\varphi)}{R} + q \cdot \cos \varphi \right)$$

MEMBRANE SOLUTION

for $N_0 = q \cdot R \cdot \cos \varphi_0$:

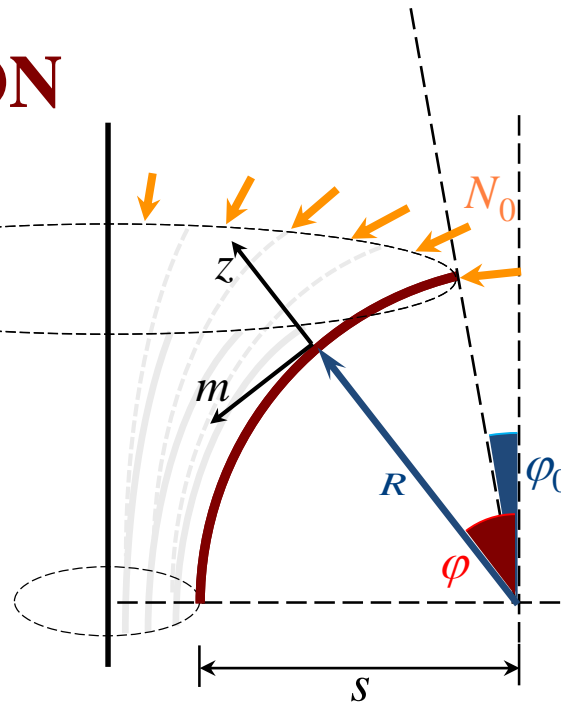
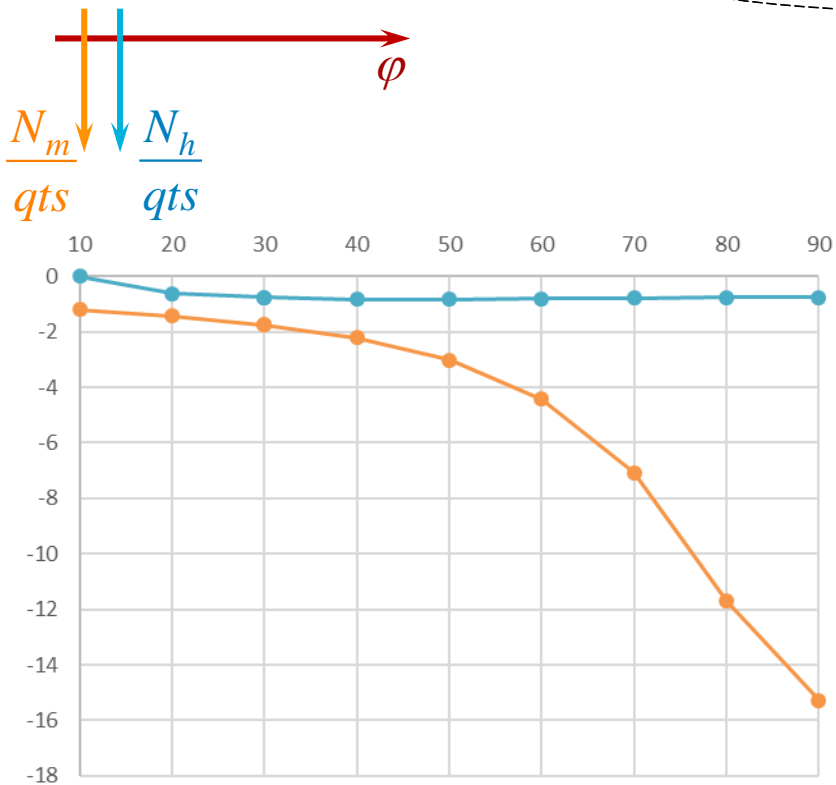


$$N_m(\varphi) = \frac{-1}{\sin \varphi \cdot r(\varphi)} \left(N_0 \cdot \sin \varphi_0 \cdot r(\varphi_0) + \frac{A_{cap}(\varphi)}{\pi} \cdot q \right)$$

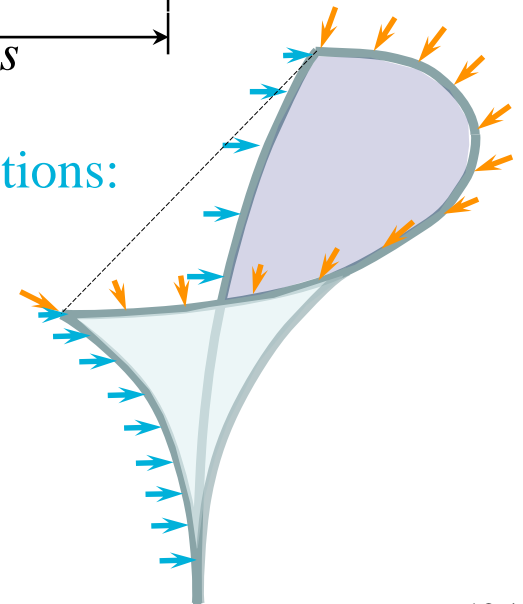
$$N_h(\varphi) = \frac{r(\varphi)}{\sin \varphi} \left(\frac{N_m(\varphi)}{R} + q \cdot \cos \varphi \right)$$

MEMBRANE SOLUTION

for $N_0 = q \cdot R \cdot \cos \varphi_0 :$



Wall reactions:



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BEGINNING OF FAN VAULTING

Origin:

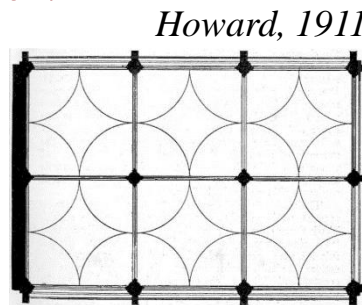
Gloucestershire, England, after 1350



→ Tewkesbury Abbey,
tomb of Sir Hughes Despencer:
„proto-“ fan vault



http://www.picturesofengland.com/user/poe/pictures//England/Gloucestershire/Tewkesbury/Tewkesbury_Abbey



<http://divorceyourtravelagent.com/tewkesbury-abbey/travelogue>

BEGINNING OF FAN VAULTING

Origin:

Gloucestershire, England, after 1350



→ Gloucester Cathedral,
cloister walk:
[jointed masonry;
constant-curvature arc]



<https://www.gloucestercathedral.org.uk/visit/>



<https://www.gloucestercathedral.org.uk/visit/>

BEGINNING OF FAN VAULTING

Origin:

Gloucestershire, England, after 1350

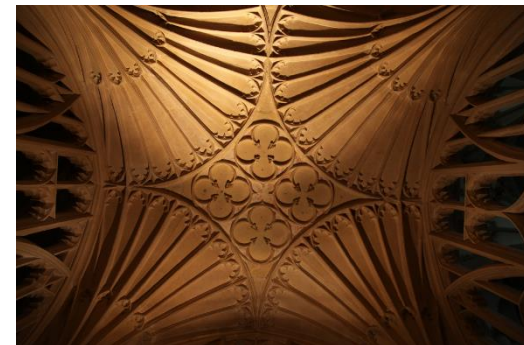
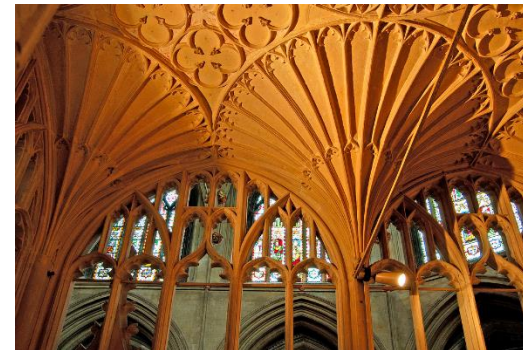


→ Tewkesbury Abbey,
Holy Trinity Chapel:
[jointed masonry;
constant-curvature arc]



<http://raggedrobinsnaturenotes.blogspot.com/2015/04/tewkesbury-part-2-abbey.html>

<https://www.flickr.com/photos/edk7/1407783359>



<https://www.flickr.com/photos/59303791@N00/8369309123/>

BEGINNING OF FAN VAULTING

Origin:

Gloucestershire, England, after 1350

„school” / place for discussions of master masons:

plastered-floor rooms in e.g. Wells Cathedral; Hereford Cathedral; York...



Salter, 2010: How to Build a Cathedral,
<https://www.youtube.com/watch?v=BHqciHh8xb4>

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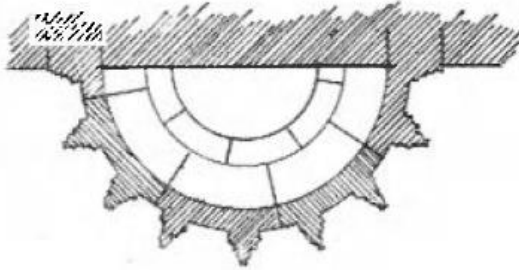
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CONSTRUCTION ISSUES

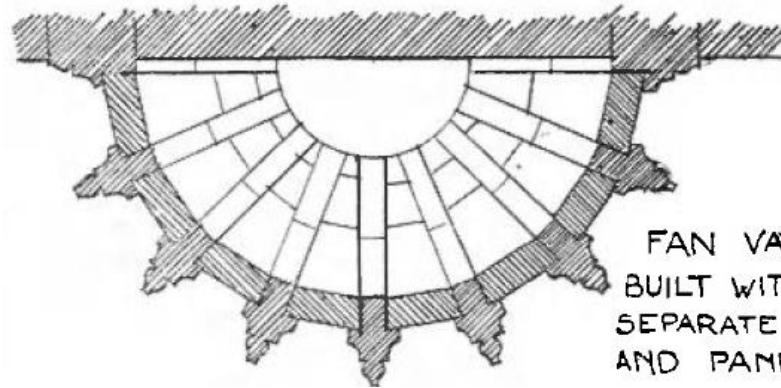
jointed masonry



rib-and-panel



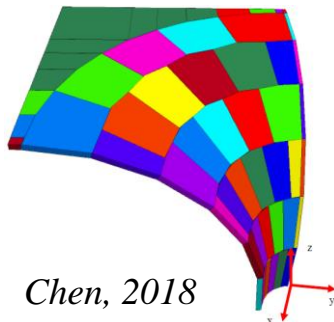
FAN VAULT WITH RIBS
CARVED IN THE SOLID



FAN VAULT
BUILT WITH
SEPARATE RIBS
AND PANELS.

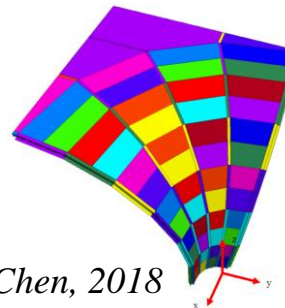
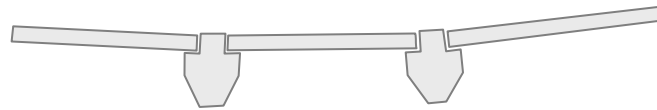
Howard, 1911

early type; or smaller vaults



Chen, 2018

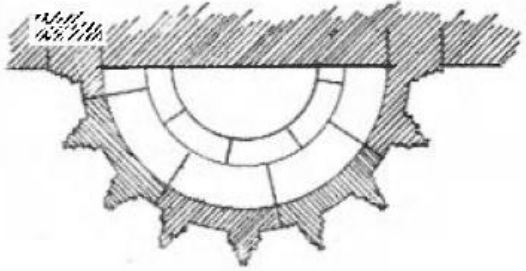
later; larger vaults



Chen, 2018

CONSTRUCTION ISSUES

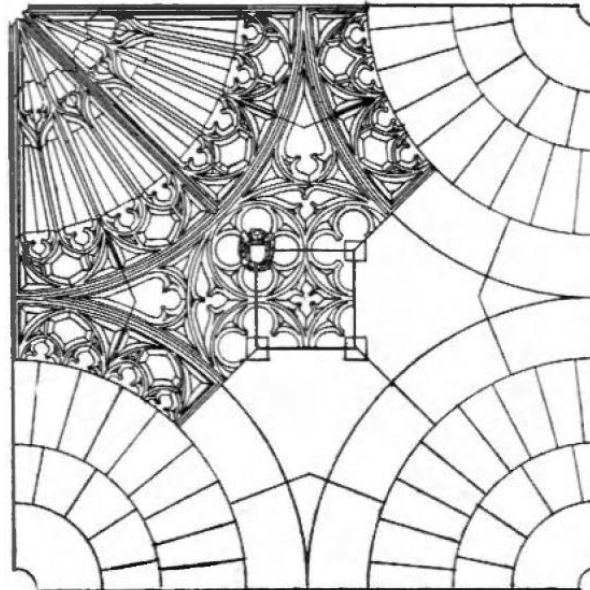
jointed masonry



FAN VAULT WITH RIBS
CARVED IN THE SOLID

early **type**; or smaller vaults

e.g. St Catherine's Chapel,
in Cirencester,
Gloucestershire:
(**end of XVth** ct.)
span: $\approx 3,9$ m



Howard, 1911



[english-church-architecture.
net/gloucestershire/cirencest
er/cirencester.htm](http://english-church-architecture.net/gloucestershire/cirencester/cirencester.htm)



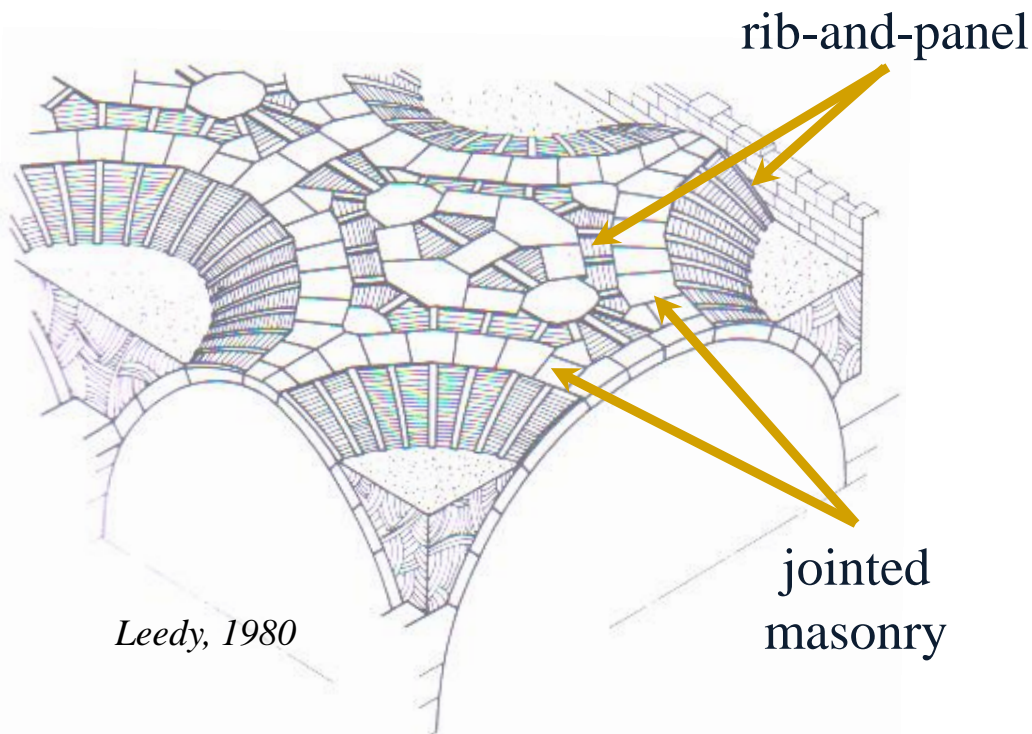
alamy.com

CONSTRUCTION ISSUES

Introduction of the „rib-and-panel” system:

the vault of the chancel of Sherbourne Abbey
(middle of the XVth century):

span: $\approx 7,5$ m



greatenglishchurches.co.uk/html/sherborne_abbey.html



<http://sherborneabbey.com/visit/about-sherborne-abbey/>

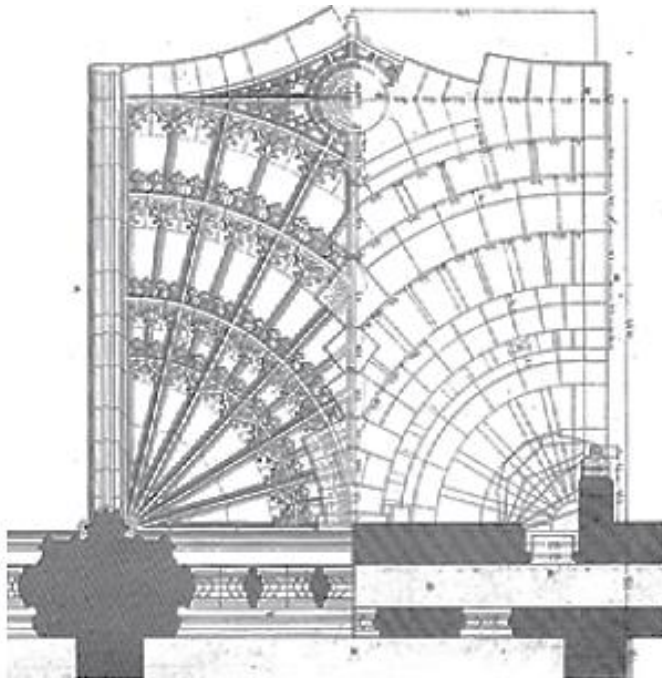
CONSTRUCTION ISSUES

Combination: jointed masonry & „rib-and-panel” system:

↑
tracery

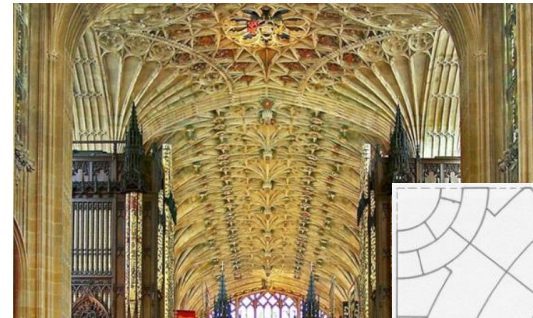
↑
smoother

e.g. King's College Chapel, main vault,
in Cambridge, ≈ 1508-1515:

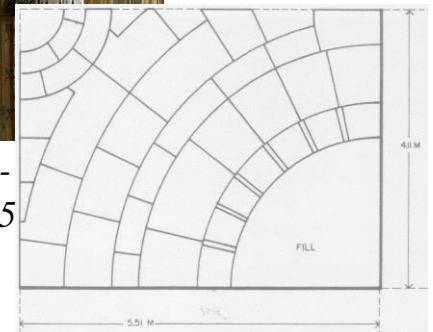


Mackenzie, 1840

e.g. St George's Chapel, crossing,
in Windsor Castle:



windsor.gov.uk/things-to-do/st-georges-chapel-p45



Leedy, 1980

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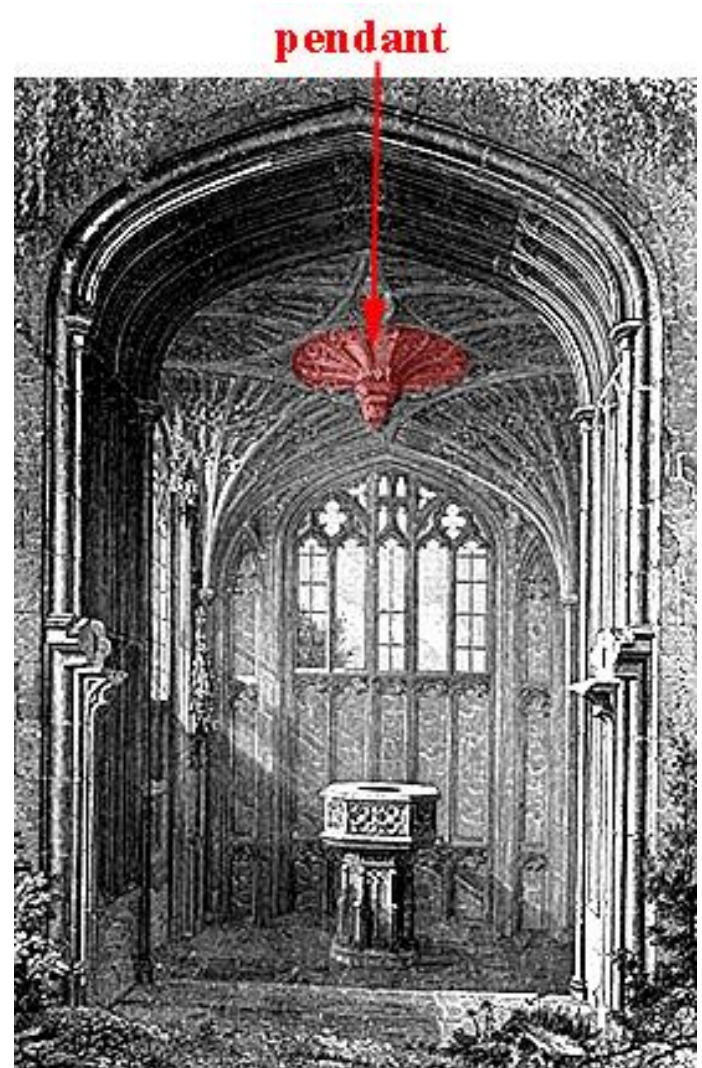
Questions

CONSTRUCTION ISSUES

Pendant: „elongated spandrel”

→ increase weight of the spandrel

→ visual impression



*Church of St. Lawrence, Evesham,
pitt.edu/~medart/menuglossary/pendant.htm*

CONSTRUCTION ISSUES

Pendant: „elongated spandrel”

Ely Cathedral (148?-1500;
≈ 4,7m), Cambridgeshire

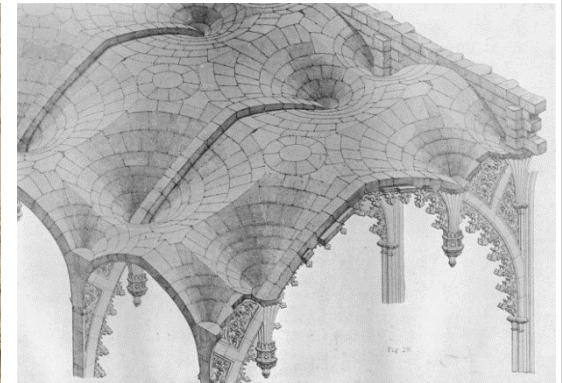


<http://www.ipernity.com/tag/stiffleaf/keyword/1392475>

Henry VII's Chapel (1500-1509, ≈ 10,6 m)
Westminster Abbey, London:



*painting by Canaletto,
[tumblr.com/search/chapel
of henry vii](https://www.tumblr.com/search/chapel%20of%20henry%20vii)*

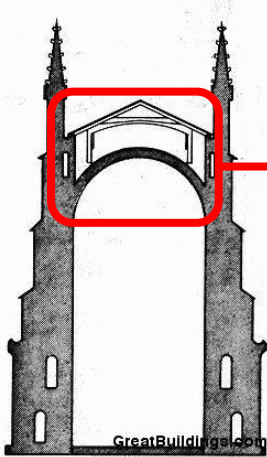


[gutenberg.org/files/47937/
47937-h/47937-h.htm](http://gutenberg.org/files/47937/47937-h/47937-h.htm)

→ to increase the weight of the spandrel
→ visual impression

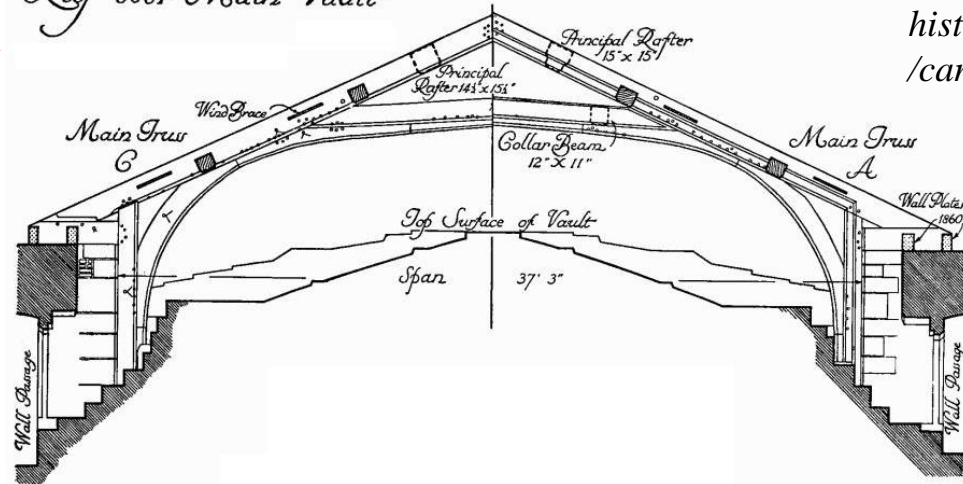
CONSTRUCTION ISSUES

Roofing:



KING'S COLLEGE CHAPEL

Roof over Main Vault



British History Online,
<https://www.british-history.ac.uk/rchme/cambs/pp98-136>

typical in Gothic structures: double roofing

→ a stone vault below [resistant to fire];

→ a wooden roof above [protect the stone vault from weather]

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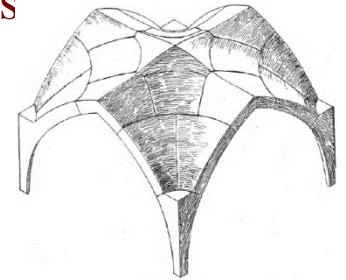
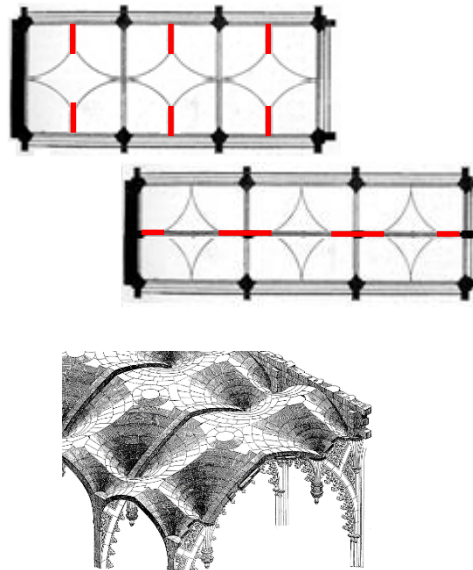
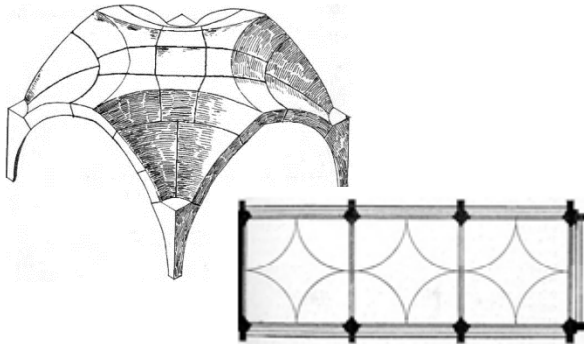
fully
separate
conoids



partly
intersecting
conoids



fully
intersecting
conoids



CONSTRUCTION ISSUES

Variations to the groundplan:

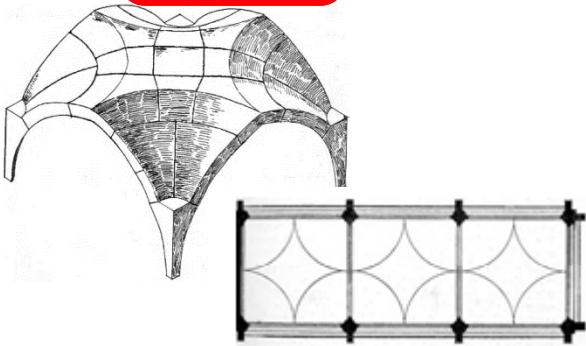
fully
separate
conoids



partly
intersecting
conoids



fully
intersecting
conoids



Examples:

early fan vaults usually belong to this type:



Tewkesbury Abbey,

[https://www.flickr.com/photos/](https://www.flickr.com/photos/59303791@N00/8369309123/)

[59303791@N00/8369309123/](https://www.flickr.com/photos/59303791@N00/8369309123/)

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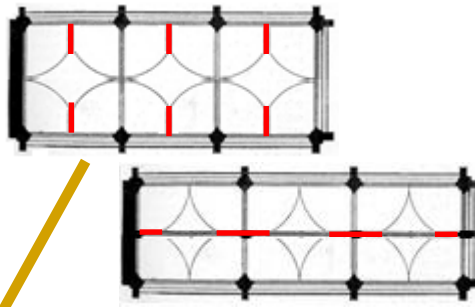
fully
separate
conoids



partly
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Example:

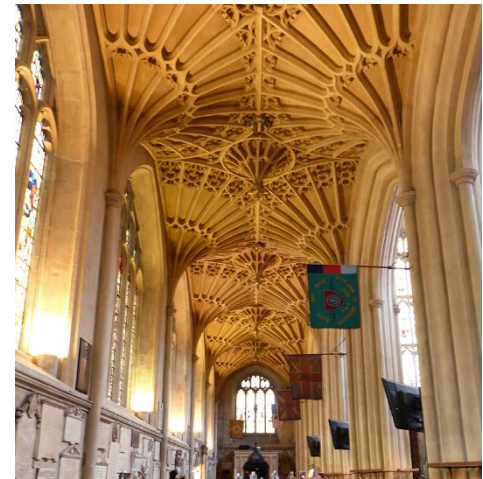
Bath Abbey (early XVIth):

Main nave:



[bbc.co.uk/ahistoryoftheworld/objects/
PoKvTpMlQn22qZc37RXPpg](http://bbc.co.uk/ahistoryoftheworld/objects/PoKvTpMlQn22qZc37RXPpg)

South aisle:



[uksouthwest.net/somerset/bath-abbey/
bath-abbey-south-aisle.html](http://uksouthwest.net/somerset/bath-abbey/bath-abbey-south-aisle.html)

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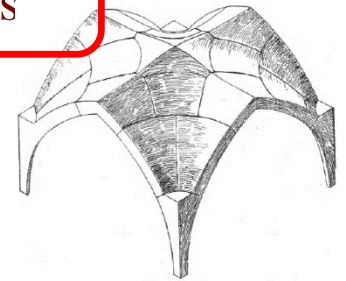
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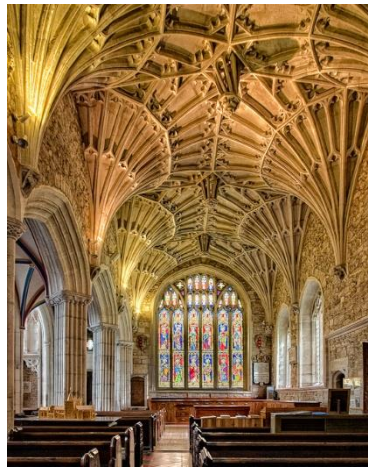


Example:

St Mary's Church, in Ottery St Mary, England,
≈1520



http://greatenglishchurches.co.uk/html/ottery_st_mary.html



<https://hebrideslight.aminus3.com/image/2012-10-20.html>



<https://www.geograph.org.uk/photo/4920056>

CONSTRUCTION ISSUES

Variations to the groundplan:

fully
separate
conoids



partly
intersecting
conoids



fully
intersecting
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Mechanical difference: **OPEN ISSUE!**

Variations to the spandrel geometry:

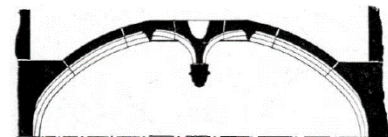
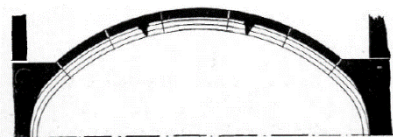
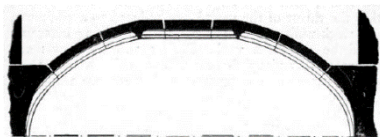
flat spandrel



domed spandrel



spandrel with pendant



[no mechanical difference to the shell]

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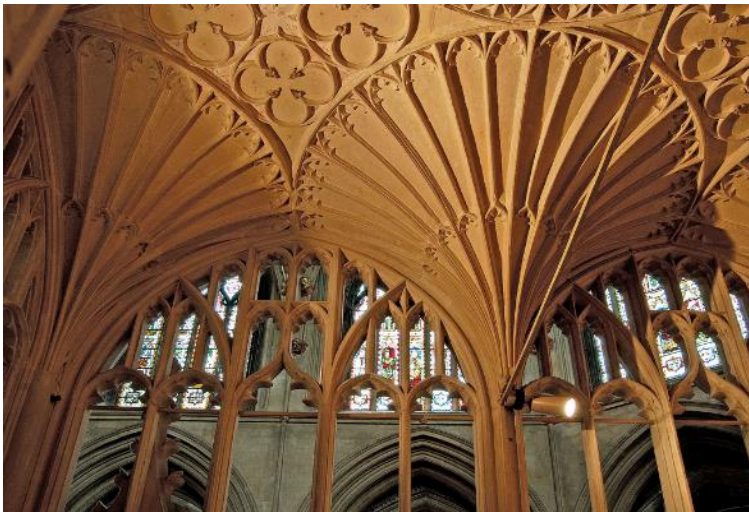
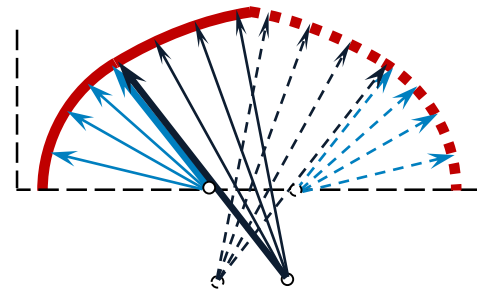
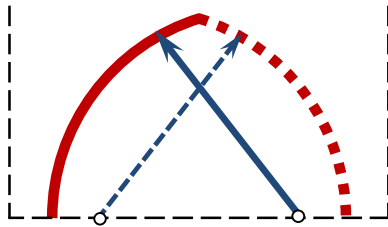
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CONSTRUCTION ISSUES

The generator curve geometry:

„two-centered” (uniform curvature) $\rightarrow \leftarrow$ „four-centered” (Tudor arc)



*Tewkesbury Abbey, \approx 1421-1439,
[flickr.com/photos/edk7/1407783359](https://www.flickr.com/photos/edk7/1407783359)*

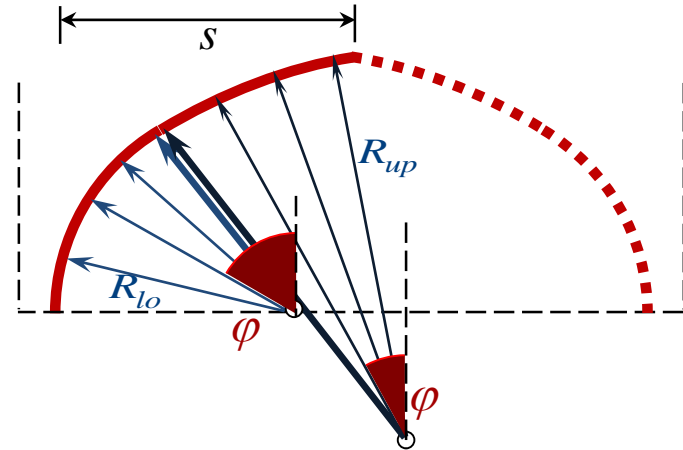
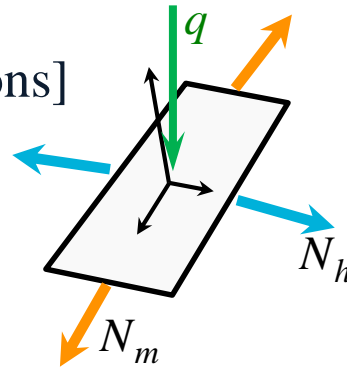
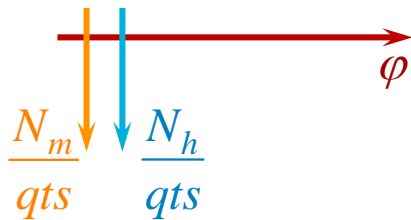


*Peterborough Cathedral, \approx 1518, photorefect.
blogspot.com/2008/05/fan-vault.html*

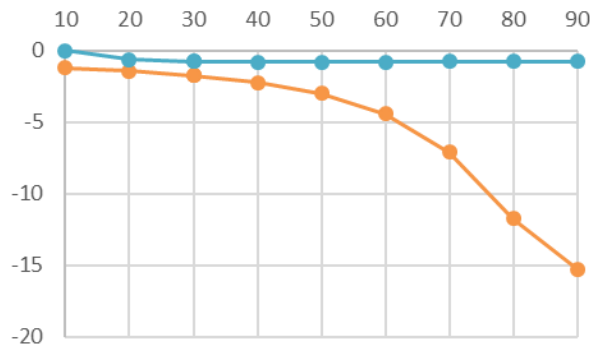
MEMBRANE SOLUTION

With Tudor geometry:

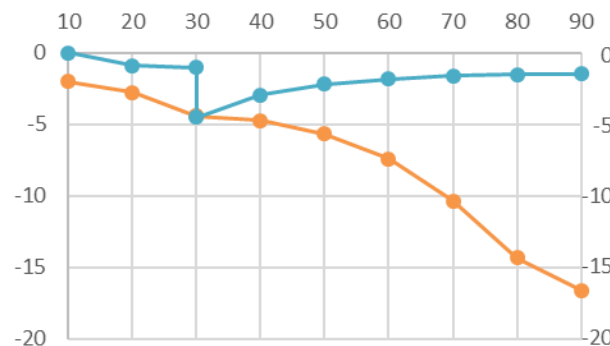
[after lengthy calculations]



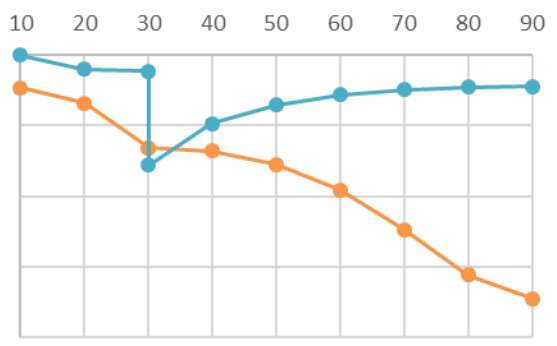
$$\frac{N_m}{R_m} + \frac{N_h}{R_h} = p_z$$



$$R_{lo} : R_{up} = 1 : 1$$



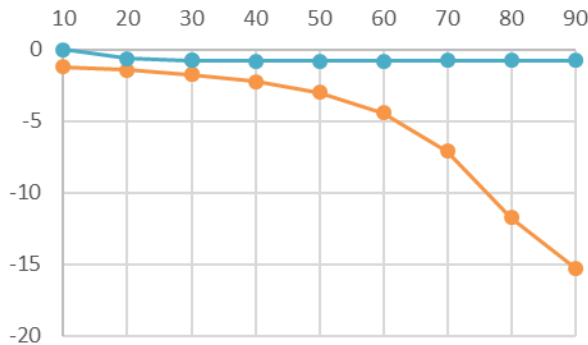
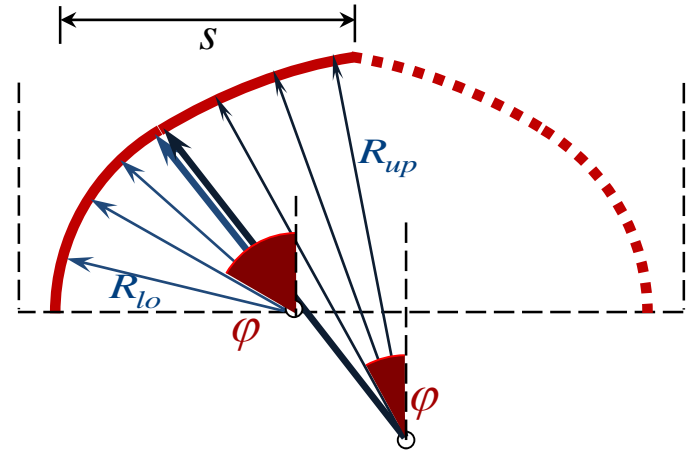
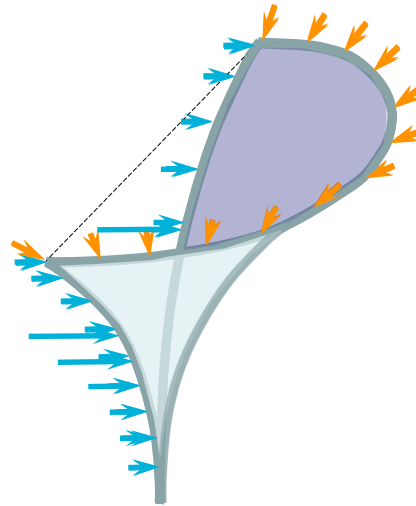
$$R_{lo} : R_{up} = 1 : 3$$



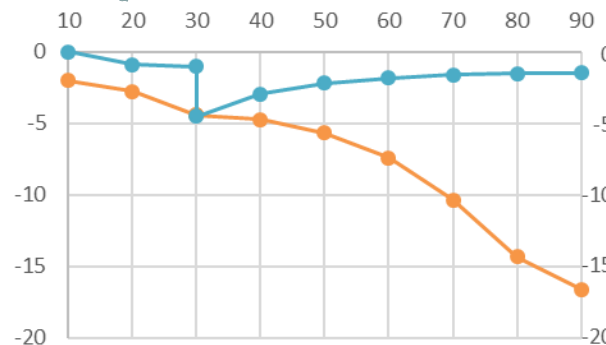
$$R_{lo} : R_{up} = 1 : 5$$

MEMBRANE SOLUTION

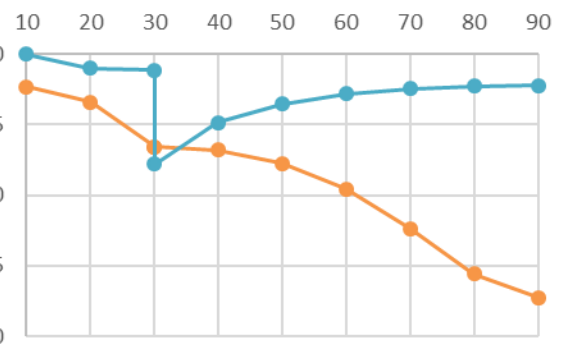
With Tudor geometry:



$$R_{lo} : R_{up} = 1 : 1$$



$$R_{lo} : R_{up} = 1 : 3$$



$$R_{lo} : R_{up} = 1 : 5$$

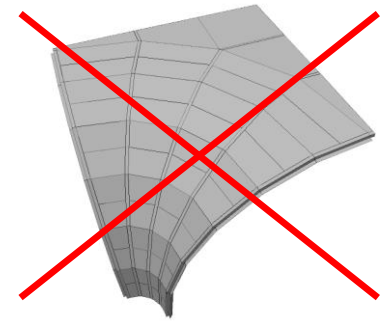
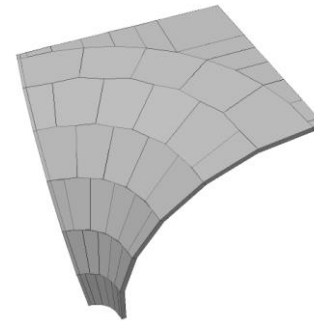
Conclusions: Tudor arch means stronger compression in hoop direction

\Rightarrow improved stability of the shell, but increasing wall reactions

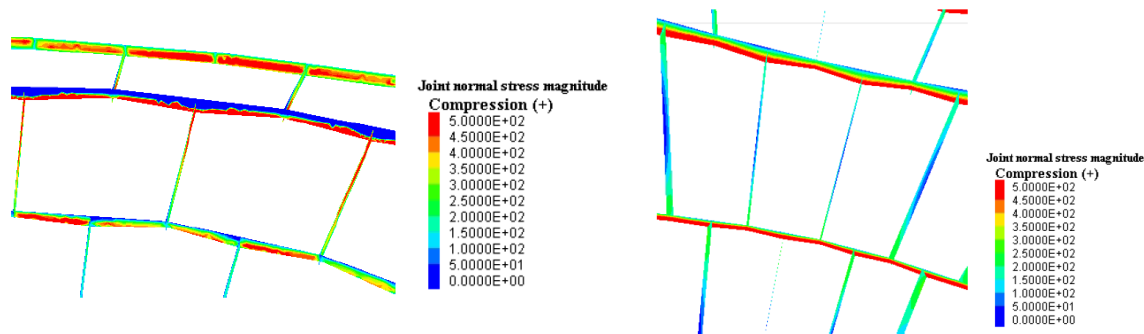
BUT: abrupt change in the wall reaction; + strain incompatibility

MEMBRANE SOLUTION

Criticism of membrane model: Chen, 2018



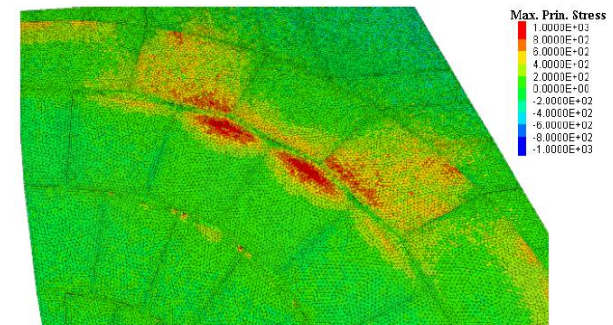
→ Are the vaults indeed in membrane state?



Results: bending is considerable;
rubble fill in the vaulting pocket may be necessary

→ Is there indeed a rotational symmetry?

Result: far from perfect! \Rightarrow shear



Use of membrane solution: e.g. for reactions

THIS LECTURE

Definition

Preliminaries to fan vaulting

Reminder on the membrane solution

Beginning of fan vaulting

Constructional issues

- jointed masonry versus rib-and-panel system

- pendants

- roofing

- variations to the groundplan

- variations to the spandrel geometry

- the generator curve geometry

Decline of fan vaulting; Fan vaults after the English Gothic

Questions

DECLINE OF FAN VAULTING

≈ first half of the XVIth century

[economic decline in England]



*slideplayer.com/
slide/8958879/*

Shell surface as a membrane:

all points are hyperbolic:

⇒ unlike for domes, centring is needed until the structure is ready



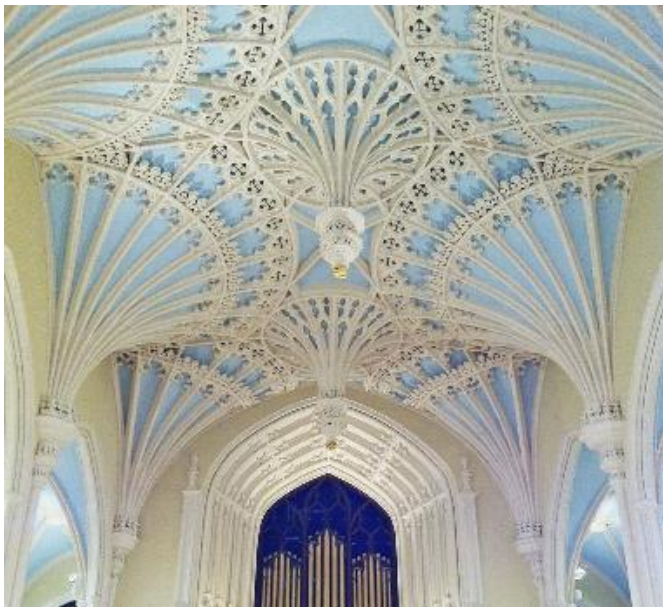
decline of fan vaulting is presumably due mostly the huge centring costs

FAN VAULTS AFTER THE ENGLISH GOTHIC

Fan vaulting inspired later architects:

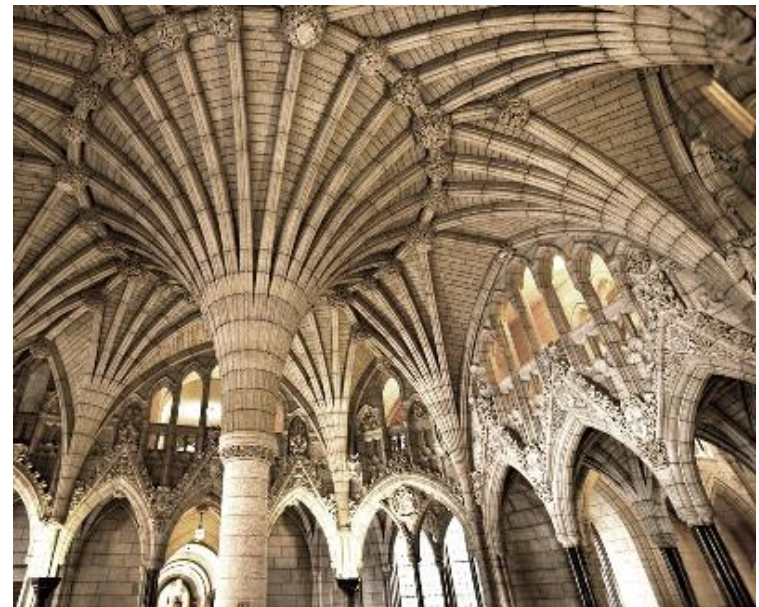
can be found in XIX-XXth century buildings, also in variations

Unitarian Church, Charleston (1887):



brokeincharleston.com/2017/03/17/a-guide-to-historic-churches-in-charleston/

Canadian Parliament (1922):



pinterest.co.uk/pin/64880050853351915/?lp=true

SUGGESTED VIDEOS

Henry VIIth Chapel, Westminster Abbey:

<https://www.khanacademy.org/humanities/medieval-world/gothic1/v/henry-vii-chapel>

Jacques Heyman's lecture: The membrane analysis of thin masonry shells

<https://www.youtube.com/watch?v=DI-leSI68dM>

John Ochsendorf: Form and Forces [about thin shell structures]

<https://www.youtube.com/watch?v=r-tG68WvNDM&t=185s>

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Questions

QUESTIONS

1. **Define** what is a fan vault. **Where** and **when** was fan vaulting born? What were its most important **preliminaries**?
2. Why is it necessary to have a **nonzero inclination at the top** of the vault conoid? Why is it necessary to have a **spandrel**?
3. Explain the following terms: spandrel; pendant; jointed masonry construction; rib-and-panel construction; Tudor arc.
4. According to the **intersection of the conoids**, what are the main types of fan vaulted structures?
5. Compare the **distribution of wall reactions** of a **uniform-curvature** fan vault and of a **Tudor geometry** fan vault: sketch diagrams how the wall reaction varies with location.