IMPLICIT TIMESTEPPING METHODS

→ Discontinuous Deformation Analysis

→ Contact Dynamics

OVERVIEW OF DEM SOFTWARES

Quasi-static methods ← ***equilibrium states** are searched for*

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K} \cdot \Delta \mathbf{u} + \mathbf{f} = \mathbf{0}"$$

Time-stepping methods $"\mathbf{M} \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))"$ ← *a **process in time** is searched for*

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ Polyhedral elements, e.g. UDEC *rigid / deformable elements; deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements; deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis") *deformable polyhedral elements*

→ Contact Dynamics models *rigid elements, non-deformable contacts*

CONTACT DYNAMICS

<u>Jean & Moreau (1992):</u> (2D, 3D) [mostly in physics]

Unger, T. – Kertész, J. (2003): The contact dynamics method for granular media. In: Modeling of Complex Systems, Melville, New York, American Institute of Physics, pp. 116-138

Available software:

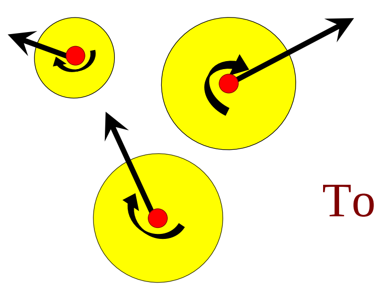
(1) LMGC90 (Dubois & Jean, 2006): **OPEN!**

rigid/deformable; spherical/polyhedral elements

(2) SOLFEC (Koziara & Bicanic, 2008):

rigid/deformable; polyhedral elements

– elements:



Originally: rigid circular/spherical elements

Today: deformable polyhedral elements also

To show in this presentation how the method works:

for RIGID, SPHERICAL elements only

CONTACT DYNAMICS

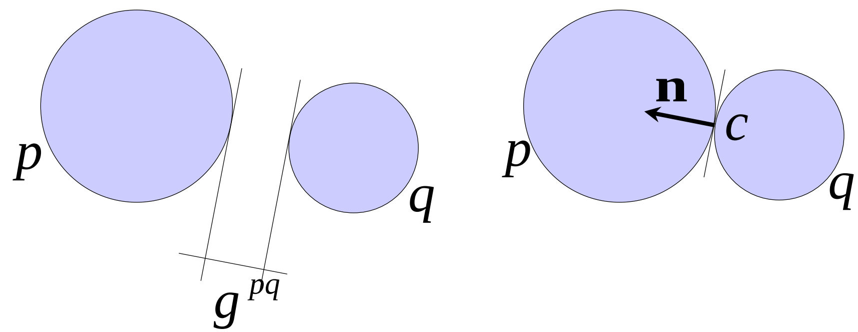
Basic entity of the analysis:

– ~~contacts~~ → „pairs”:

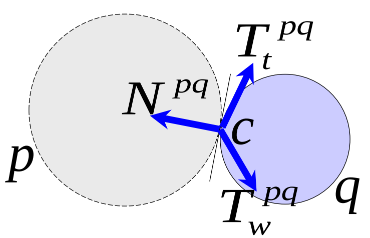
Basic unknowns of the analyis:

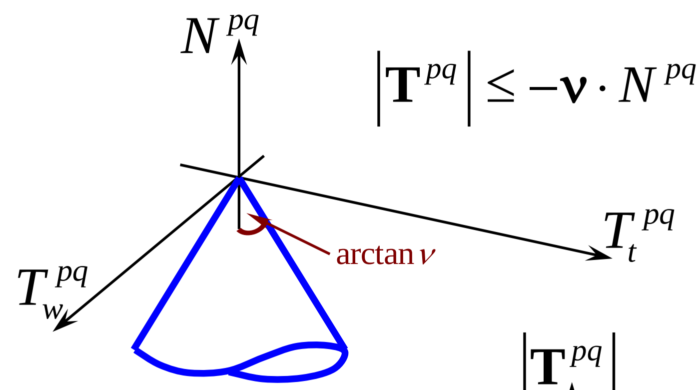
– ~~element displacements~~

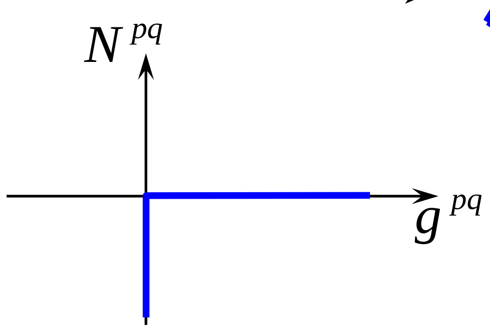
→ contact forces

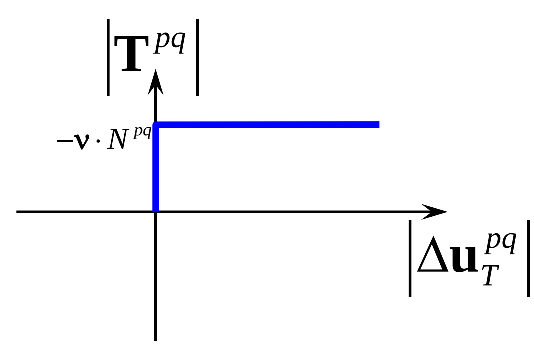


Mechanical conditions for the contact forces:









CONTACT DYNAMICS

Equations of motion for a pair:

$$\begin{bmatrix} \mathbf{M}^p & \\ & \mathbf{M}^q \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \mathbf{v}^p(t) \\ \mathbf{v}^q(t) \end{bmatrix} = \begin{bmatrix} \mathbf{f}^p(t) \\ \mathbf{f}^q(t) \end{bmatrix}$$

forces and moments reduced to the reference points of the elements

translational and rotational velocity of the two elements

for spheres:

$$\mathbf{M}^p = \begin{bmatrix} m^p & & & & & \\ & m^p & & & & \\ & & m^p & & & \\ & & & I^p & & \\ & & & & I^p & \\ & & & & & I^p \end{bmatrix}$$

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In the equations of motion of a pair of spherical elements in CD, how many scalar equations are included?

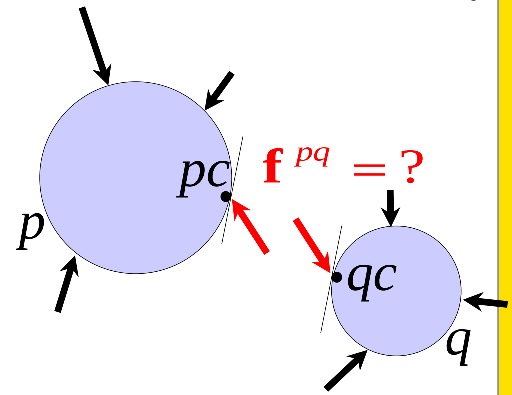
✘ → 3 (*for the three translational accelerations of the element*)

✘ → 6 (*for the three translational and three rotational accelerations of the element*)

✓ → 12 (*for the three translational and three rotational accelerations of each element in the pair*)

✓ → I do not know

CONTACT DYNAMICS



<u>Analysis of the (t_i, t_{i+1}) interval:</u>

with the implicit version of the Euler-method:

$$\begin{bmatrix} \mathbf{v}^p_{i+1} \\ \mathbf{v}^q_{i+1} \end{bmatrix} := \begin{bmatrix} \mathbf{v}^p_{i} \\ \mathbf{v}^q_{i} \end{bmatrix} + \Delta t \cdot \begin{bmatrix} (\mathbf{M}^p)^{-1} & \\ & (\mathbf{M}^q)^{-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{f}^p_{i+1} \\ \mathbf{f}^q_{i+1} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{u}^p_{i+1} \\ \mathbf{u}^q_{i+1} \end{bmatrix} := \begin{bmatrix} \mathbf{u}^p_{i} \\ \mathbf{u}^q_{i} \end{bmatrix} + \Delta t \cdot \begin{bmatrix} \mathbf{v}^p_{i+1} \\ \mathbf{v}^q_{i+1} \end{bmatrix}$$

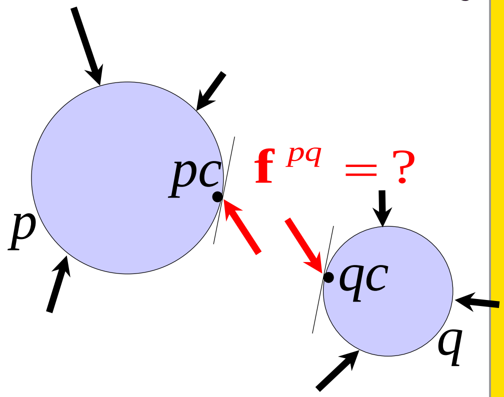
contain the basic unknowns i.e. the contact forces acting between the two elements, $\mathbf{f}^{pq}_{i+1}$

The core of the method:

<u>The „iterative solver”:</u> sweeps along all pairs, one-by-one in random order; repeatedly determine the $\mathbf{f}^{pq}_{i+1}$ **contact forces** in every contact, so that at t_{i+1} the following conditions would be just met:

$$g^{pq}_{i+1} \geq 0\,; \quad N^{pq}_{i+1} \leq 0\,; \quad \left|\mathbf{T}^{pq}_{i+1}\right| \leq -\nu \cdot N^{pq}_{i+1}$$

CONTACT DYNAMICS



How to find the forces belonging to t_{i+1} :

The „iterative solver”:

→ Consider each pair individually, one after the other!

→ Analysis of a (*p*, *q*) pair:

– Compile the reduced forces $\mathbf{f}_{i+1}^{p}$ and $\mathbf{f}_{i+1}^{q}$,

but **WITHOUT** a force $\mathbf{f}_{i+1}^{pq}$ („no contact between *p* and *q*”)

– assume constant acceleration during Δ*t*, and calculate it from the reduced forces

⇒ the predicted position of *p* and *q* can be calculated

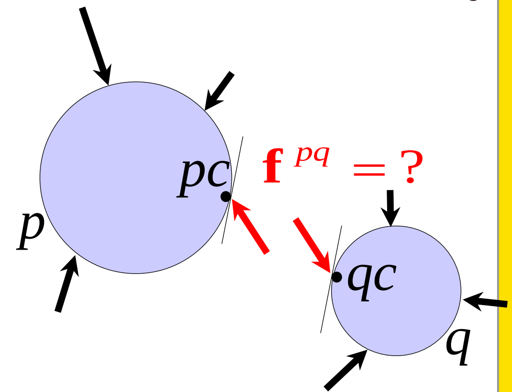
– check whether *p* and *q* are indeed not in contact:

⇒ if $g_{i+1}^{pq} > 0$: (i.e., no contact between *p* and *q*)

the contact force is indeed zero, the analysis of the pair is ready,

take the next pair!

CONTACT DYNAMICS



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but **WITHOUT** a force $\mathbf{f}^{pq}_{i+1}$ („no contact between *p* and *q*”)

– assume constant acceleration during Δt, and calculate it from the reduced forces

⇒ the predicted position of *p* and *q* can be calculated

– check whether *p* and *q* are indeed not in contact:

⇒ if $g^{pq}_{i+1} < 0$: (i.e., contact exists between *p* and *q*)

a non-zero *p*-*q* contact force exists; determine it! (*eqs. of motion*):

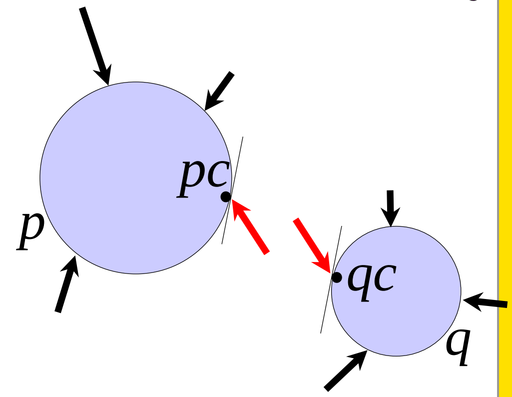
$\mathbf{f}^{pq}_{i+1}$ has to cause reduced forces which just lead to $g^{pq}_{i+1} = 0$ at t_{i+1} .

– Now check its tangential component; is $\left|\mathbf{T}^{pq}_{i+1}\right| \le -\nu \cdot N^{pq}_{i+1}$ satisfied?

⇒ if satisfied: the analysis of the pair is ready, take the next pair!

⇒ if not satisfied, i.e. if $\left|\mathbf{T}^{pq}_{i+1}\right| > -\nu \cdot N^{pq}_{i+1}$: truncate $\mathbf{T}$; take the next pair!

CONTACT DYNAMICS



How to find the forces belonging to t_{i+1} :

The iterative solver, overview:

The analysis of one timestep contains several „sweeps”.

→ One „sweep” means the following:

→ Consider each pair individually, one after the other, in random order.
For the actual pair considered, find the suitable contact force.

→ For a complete sweep: Every pair is taken once. Find a contact force for each.

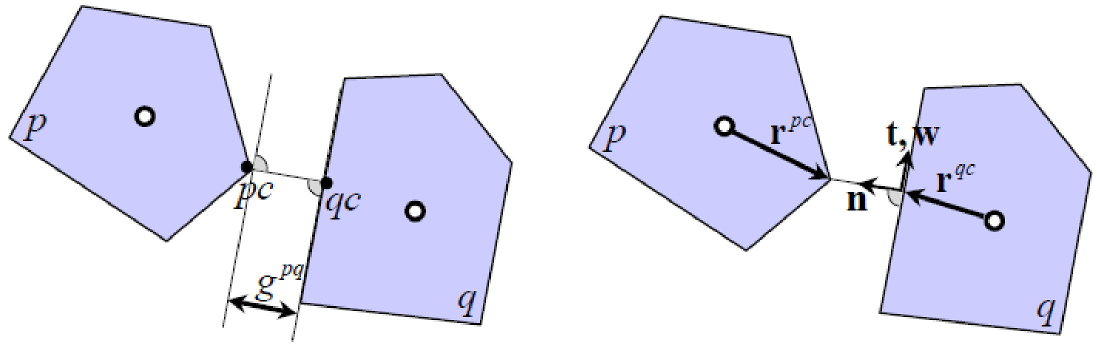
[Remark: the next sweeps will be in different orders …]

→ Sweeps are done along the pairs, repeatedly over and over again,
until it is noticed that the change in the forces becomes negligibly small;
the forces belonging to t_{i+1} are received

if the timestep is done, the next time step can be considered

CONTACT DYNAMICS

Polyhedral elements:

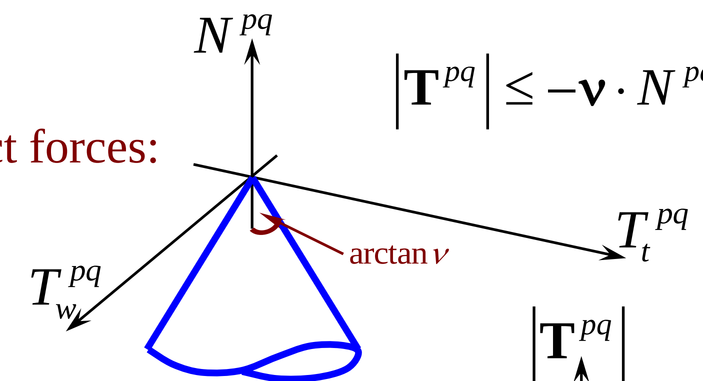


„common plane concept”

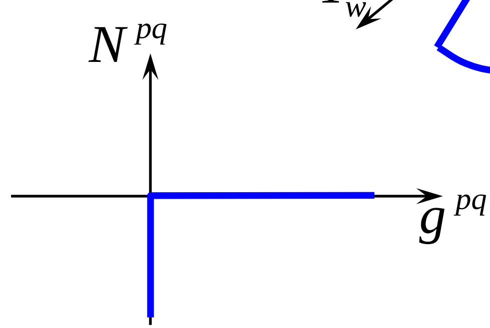
Rigid polyhedral elements:

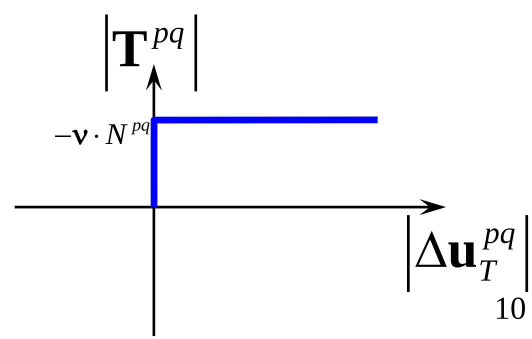
Mechanical conditions for the contact forces:

$$\left|\mathbf{T}^{pq}\right| \leq -\boldsymbol{\nu} \cdot N^{pq}$$



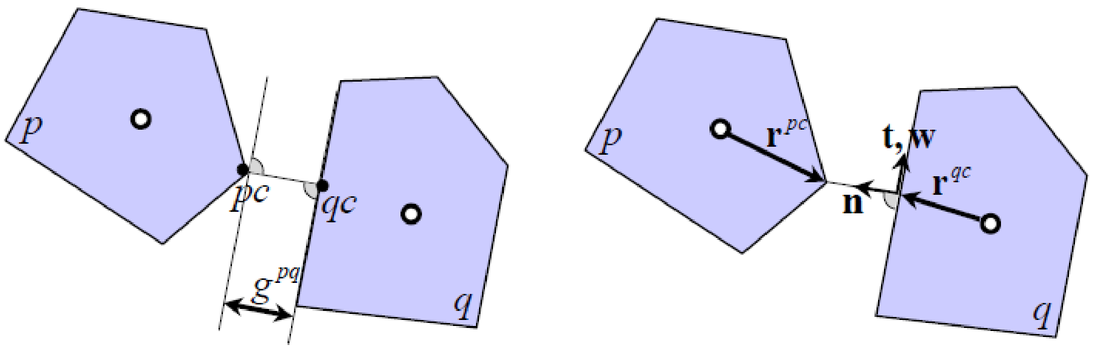
[the same]





CONTACT DYNAMICS

Polyhedral elements:

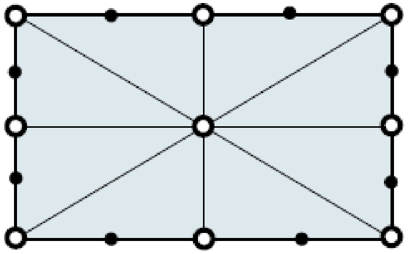


„**common plane concept**”

Deformable polyhedral elements: ~~constant strain~~ → unfavourable experiences

uniform-strain tetrahedral subdivision

The point of action of the contact force:



• : middle point of the face

„**approximated contact point**”

contact: if it touches another face

Masses: distributed to the **nodes** ⭘ (1/3 of the triangle areas, ≈ Voronoi)

Equations of motion: for every pair of **node** ⭘ [no rotations considered]

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How are the elements made deformable in CD?

✘ → Similarly to DDA, the whole element is of uniform strain.

✓ → Similarly to 3DEC, the element is subdivided into uniform-strain tetrahedra.

✘ → The elements cannot be deformable in CD.

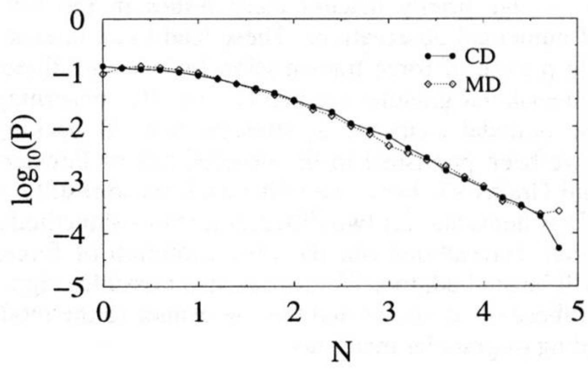
✓ → I do not know

CONTACT DYNAMICS

General remarks: [for rigid or deformable elements; for all shapes]

– advantages: very fast for motions in time

⇒ VERY efficient for dynamic phenomena



– disadvantage: if an equilibrium state is searched for:
(slow convergence);
non-unique solution:

for **rigid** elements & rigid contacts:

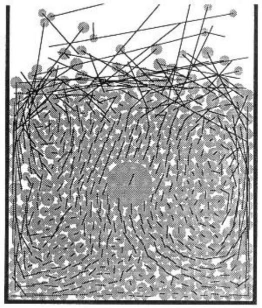
gives just **one** of the **many** statically admissible solutions of the statically indeterminate system!

for **deformable** elements and/or contacts:

history-dependent behaviour →← pairs are scanned in random order

Applications
e.g. granular flows, avalanches
e.g. vibration, mixing
e.g. dynamic, cyclically repeated loads

Simulation of segregation:



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Why the solution produced by CD **is non-unique** for a system of **rigid** elements and **rigid** contacts?

✘ → Because the scanning along the pairs within a sweep is done in random order.

✓ → Because the problem itself is statically indeterminate without having a unique solution.

✓ → I do not know.

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code: 3322 1621

Why the solution produced by CD **is non-unique** for a system of **deformable** elements and/or **deformable** contacts?

✘ → Because the problem itself is statically indeterminate without having a unique solution.

✓ → Because the response of the system is history-dependent, but the scanning along the pairs is done in random order.

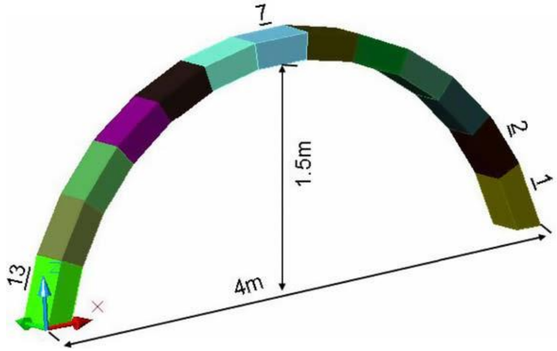
✓ → I do not know.

CONTACT DYNAMICS

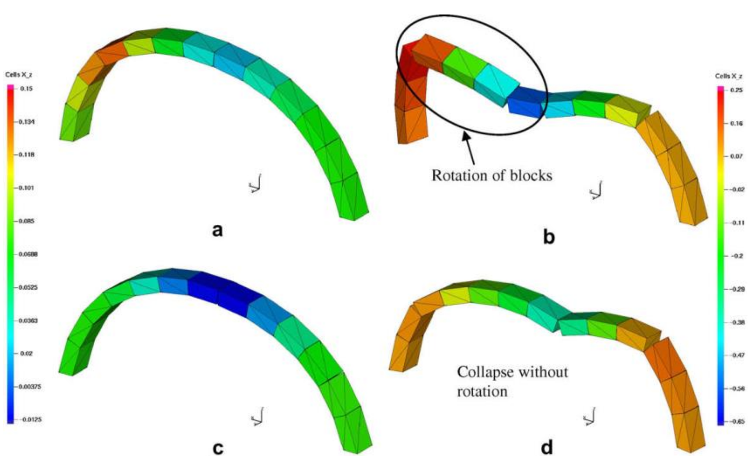
<u>Civil engineering applications</u>

e.g. Rafiee et al (2008):

CD numerical model with deformable elements:



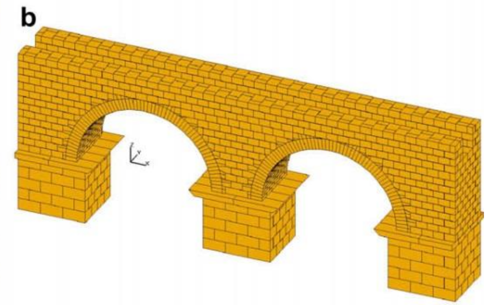
<u>results:</u> e.g. earthquake simulations



CONTACT DYNAMICS

<u>Civil engineering applications</u>

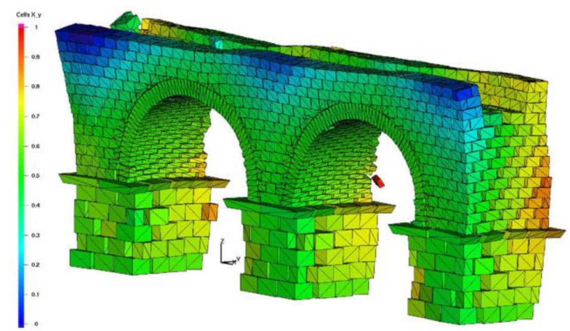
e.g. Rafiee et al (2008):



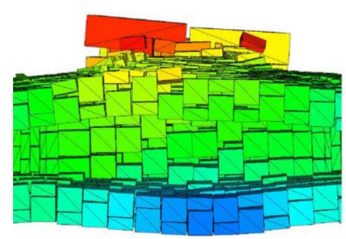
CD numerical model with deformable elements:

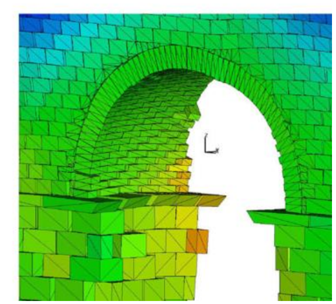
Arles, aqueduct

Earthquake simulations:

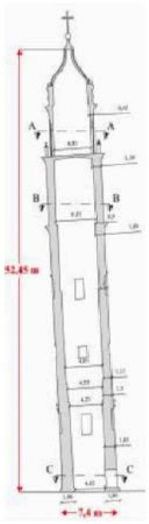


☹ Experimental verification?





CONTACT DYNAMICS



<u>Civil engineering applications</u>

e.g. Clementini et al (2018):

San Benedetto Church, Ferrara

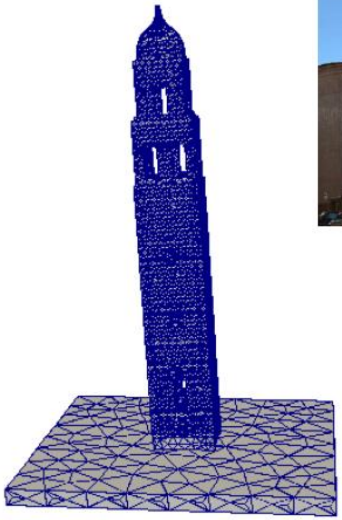
aim: analyse seismic behaviour

Model assumptions: (LMGC90)

rigid blocks

Coulomb-frictional contacts

perfectly plastic impact (no bouncing)

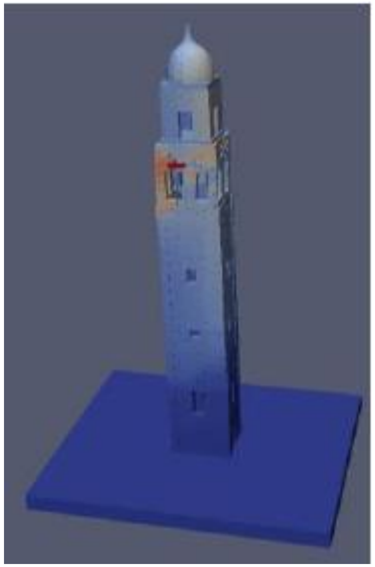


Load: basement oscillations $v(t) = C \sin(2\pi \cdot f \cdot t)$

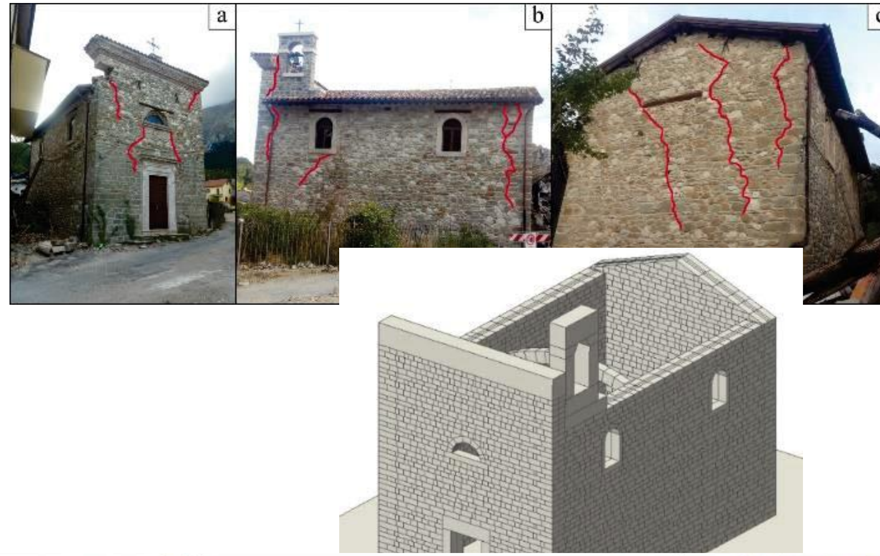
≡ earthquake simulations

Model validation: compare first frequency to reality

Outcome: vulnerable regions of the structure



CONTACT DYNAMICS



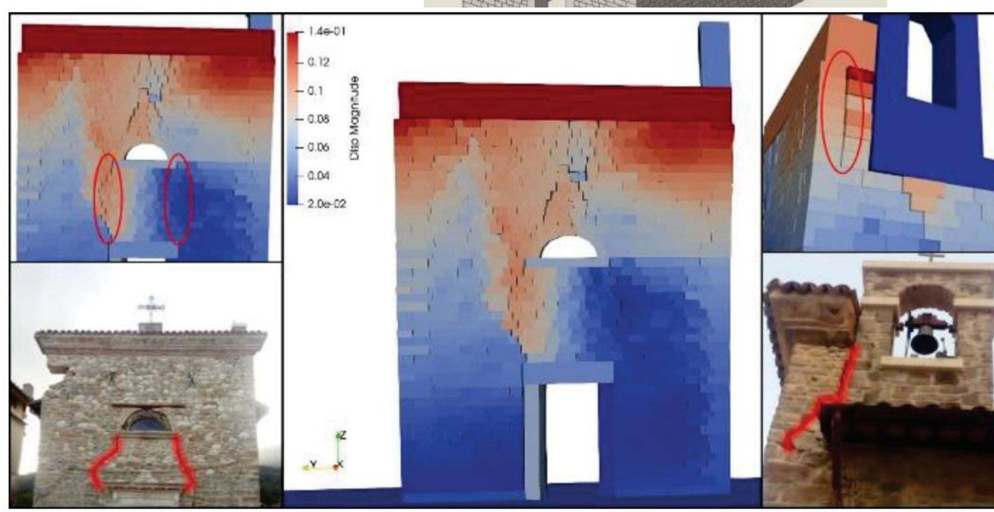
<u>Civil engineering applications</u>

e.g. Ferrante et al et al (2019):
  a cracked church in Italy
  aim: reproduce seismic damage

Model assumptions: (LMGC90)
  rigid blocks; no damping
  Coulomb-frictional contacts

Load:
  three shocks, as recorded

no previous validation;
no data about material parameters



Outcome:
  successfully reproduced the cracked regions of the structure

QUESTIONS

1. Describe the mechanics of the contacts in NSCD: sketch the diagrams about the normal and tangential components of the contact force, and explain what can be seen on these diagrams.

2. Explain how the „iterative solver" works.

3. In case of deformable polyhedral deformable elements in NSCD, what are the kinematic degrees of freedom? how are the elements made deformable? What is the „mass of a node"? On the surface of an element, which material points can form contacts with a neighbouring elements? (*hint: Slide 11*)

4. The solution given by NSCD is non-unique. Why is it non-unique for rigid elements with rigid contacts? Why does the non-uniqueness maintains for deformable elements and/or deformable contacts? (*hint: Slide 13, middle*)