LINEÁRIS, NEMLINEÁRIS EGYENLETRENDSZEREK

PROBLEM A – 12 POINTS

We want to define the parameters of a circular chimney (center, radius). To do this, we measured 4 points at a given height on the chimney (12 points):

point number	x	y
1	637.049	269.486
2	638.084	270.998
3	633.502	270.437
4	635.900	269.091

The following system of linear equations can be written from the point coordinates, and the center and radius of the circle can be determined from the parameters 'c':

$$x_i \cdot c_1 + y_i \cdot c_2 + c_3 = -({x_i}^2 + {y_i}^2)$$

Circle parameters (x_0, y_0 - coordinates of center, r-radius):

$$x_0 = -0.5 \cdot c_1$$
$$y_0 = -0.5 \cdot c_2$$
$$r = \sqrt{{x_0}^2 + {y_0}^2 - c_3}$$

a) Plot the points of the circle in a figure! (1p)

b) Write the overdetermined system of linear equations of 4 equations and solve them! To check the solution, calculate the norm of the residuals also! (3 points)

c) Calculate the parameters of the circle (center, radius) based on the formulas and write the equation of the circle:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Draw the circle in the figure as well! When adjusting the axis, make sure that the entire circle is visible without distortion. (3 points)

d) A power line is designed for the area, which can be given by the following equation:

$$2 \cdot (x - 632) + 5 \cdot (y - 273) = 0$$

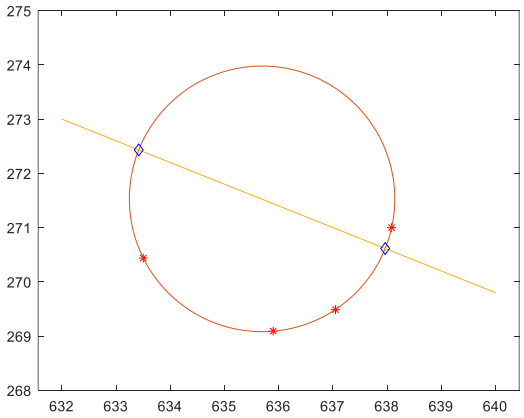
e) Draw this line into the figure as well. (1 point)

f) Unfortunately, the planned line passes through the chimney, determine the intersection points, and draw them into the figure! (4 points)

SOLUTION - A

```
%% Circle 1
%  plot - 1 point
clc; clear all; close all; format shortG;
x = [637.049;638.084;633.502;635.900];
y = [269.486;270.998;270.437;269.091];
figure(1); plot(x,y,'r*')

% solving system of linear equations - 3 points
A = [x y ones(size(x))]
b = -(x.^2+y.^2)
c = A\b %       -1271.4,       -543.06,    4.7783e+05
norm(A*c-b) %  0.01686

% parameter of the circle, plot - 3 points
x0 = -0.5*c(1) %  635.69
y0 = -0.5*c(2) %  271.53
r = sqrt(x0^2+y0^2-c(3)) %  2.4488
kor = @(x,y) (x-x0).^2+(y-y0).^2-r^2
```

```
hold on; fimplicit(kor,[632 640 268 275])
axis equal

% line, plot - 1 points
f = @(x,y) 2*(x-632)+5*(y-273)
fimplicit(f,[632 640 268 275])

% intersections - 4 points
syms x y
kors = (x-x0).^2+(y-y0).^2-r^2
fs = 2*(x-632)+5*(y-273)
sol = solve(kors,fs)
xm = double(sol.x);
ym = double(sol.y);
[xm ym]
%      637.97      270.61
%      633.42      272.43
plot(xm,ym,'bd')
```



PROBLEM – B

A power line was surveyed with 4 points (12 p):

point number	x	y
1	637.049	270.486
2	638.084	270.998
3	633.502	269.437
4	635.900	270.191

a) Draw the measured points in a figure. (1 p)

b) Fit a line to the 4 points by solving the following overdetermined system of equations! To check the solution, calculate the norm of the residuals also!

$$m \cdot x_i + b = y_i$$

Define the equation of the line and draw it in the figure! (4 p)

c) An elliptical chimney is planned for the area, given by the following parametric equations ($t \in [0,2\pi]$):

$$x(t) = 2.5 \cdot \cos(t) + 635.214$$
$$y(t) = 1.5 \cdot \sin(t) + 270.725$$

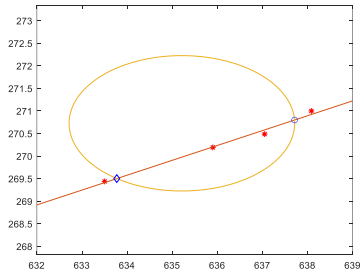
Draw the planned chimney into the figure! (2 p)

d) Unfortunately, the planned chimney crosses the power line, determine the intersection points, and draw them into the figure! (5 points)

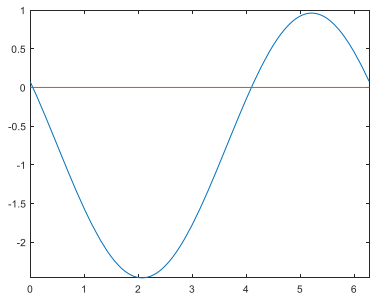
SOLUTION - B

```
%% Line1
% plot - 1 point
clc; clear all; close all; format shortG
x = [637.049;638.084;633.502;635.900];
y = [270.486;270.998;269.437;270.191];
figure(1); plot(x,y,'r*')

% line fitting, solving system of linear equations,plot - 4 points
A = [x ones(size(x))]
b = y
```

```
c = A\b % 0.32939       60.739
norm(A*c-b) % 0.12462
f = @(x) c(1)*x+c(2)
hold on; fplot(f,[632 639])

% ellipse - plot - 2 point
xe = @(t) 2.5*cos(t) + 635.214
ye = @(t) 1.5*sin(t) + 270.725
fplot(xe,ye,[0 2*pi])
axis equal
```



```
% intersection points - 5 points
% substituing xe,ye to f equation
% c(1)*(2.5*cos(t) + 635.214)+c(2) = (1.5*sin(t) + 270.725)
h = @(t) c(1)*(2.5*cos(t) + 635.214)+c(2) - (1.5*sin(t) + 270.725)
figure(2); fplot(h,[0 2*pi]); hold on; plot(xlim,[0,0])
t1 = fzero(h,0) % 0.048393
t2 = fzero(h,4) % 4.0973
x1 = xe(t1), y1 = ye(t1) % 637.71, 270.8
x2 = xe(t2), y2 = ye(t2) %  633.77, 269.5
figure(1); plot(x1,y1,'bo',x2,y2,'bd')
```



PROBLEM C – 12 POINTS

We want to define the parameters of a circular chimney (center, radius). To do this, we measured 4 points at a given height on the chimney (12 points):

point number	x	y
1	537.423	370.357
2	537.446	372.655
3	535.060	369.611
4	536.157	369.527

The following system of linear equations can be written from the point coordinates, and the center and radius of the circle can be determined from the parameters 'c':

$$x_i \cdot c_1 + y_i \cdot c_2 + c_3 = -({x_i}^2 + {y_i}^2)$$

Circle parameters (x_0, y_0 - coordinates of center, r-radius):

$$x_0 = -0.5 \cdot c_1$$
$$y_0 = -0.5 \cdot c_2$$
$$r = \sqrt{{x_0}^2 + {y_0}^2 - c_3}$$

a) Plot the points of the circle in a figure! (1p)

b) Write the overdetermined system of linear equations of 4 equations and solve them! To check the solution, calculate the norm of the residuals also! (3 points)

c) Calculate the parameters of the circle (center, radius) based on the formulas and write the equation of the circle:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Draw the circle in the figure as well! When adjusting the axis, make sure that the entire circle is visible without distortion. (3 points)

d) A parabolic shaped road is designed for the area, which can be given by the following equation:

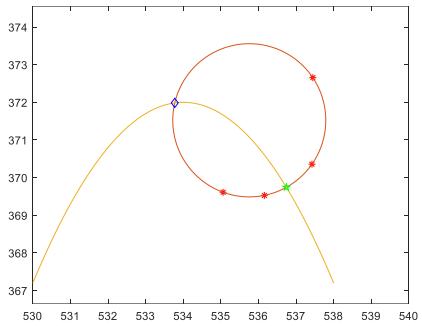
$$y = -0.3 \cdot (x - 534)^2 + 372$$

Draw this parabola in the figure as well. (1 point)

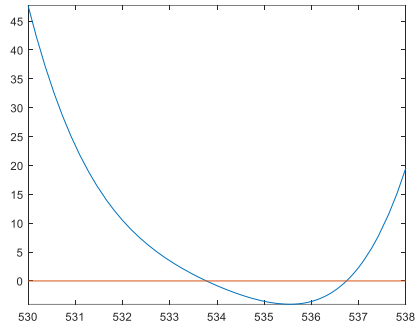
e) Unfortunately, the planned road passes through the chimney, determine the intersection points, and draw them into the figure! (4 points)

SOLUTION – C

```
%% Circle 2, plot - 1 point
clc; clear all; close all; format shortG
x = [537.423;537.446;535.060;536.157];
y = [370.357;372.655;369.611;369.527];
figure(1); plot(x,y,'r*')

% solving system of linear equations - 3 p
A = [x y ones(size(x))]
b = -(x.^2+y.^2)
c = A\b %      -1071.5      -743.04   4.2506e+05
norm(A*c-b) %  0.0039961

% parameter of the circle, plot - 3 p
x0 = -0.5*c(1) %  535.76
y0 = -0.5*c(2) %   371.52
r = sqrt(x0^2+y0^2-c(3)) %  2.0344
kor = @(x,y) (x-x0).^2+(y-y0).^2-r^2
hold on; fimplicit(kor,[533 540 368 374])
axis equal
```




```
% parabola - 1 p
p = @(x) -0.3*(x-534).^2+372
hold on; fplot(p,[530 538])

% intersections - 4 points
% substituting parabola equation into circle
kp = @(x) (x-x0).^2+(-0.3*(x-534).^2+372-y0).^2-r^2
figure(2); fplot(kp,[530 538])
hold on; plot(xlim,[0,0])
x1 = fzero(kp,534) % 533.77
x2 = fzero(kp,[536 538]) % 536.74
y1 = p(x1) %  371.98
y2 = p(x2) % 369.74
figure(1); plot(x1,y1,'bd',x2,y2,'gp')
```



PROBLEM – D

A road was surveyed with 4 points (12 p):

point number	x	y
1	537.123	373.057
2	537.446	372.655
3	535.060	369.611
4	536.157	373.527

a) Draw the measured points in a figure. (1 p)

b) Fit a parabola to the 4 points by solving the following overdetermined system of equations! To check the solution, calculate the norm of the residuals also!

$$c_1 \cdot x_i^2 + c_2 \cdot x_i + c_3 = y_i$$

Define the equation of the parabola and draw it in the figure! (4 p)

c) An elliptical chimney is planned for the area, given by the following equations:

$$\frac{(x-535.6)^2}{1.5^2} + \frac{(y-372.5)^2}{1.2^2} = 1$$

Draw the planned chimney on the figure! (2 p)

d) Unfortunately, the planned chimney crosses the road, determine the intersection points, and draw them into the figure! (5 points)

SOLUTION -D

```
%% parabola - D
% plot - 1 p
clc; clear all; close all; format shortG
x = [537.123;537.446;535.060;536.157];
y = [373.057;372.655;369.611;373.527];
figure(1); plot(x,y,'r*')

% parabola fitting, norm, plot - 4 p
A = [x.^2 x ones(size(x))]; b = y;
c = A\b %   -1.7419       1869.4  -5.0119e+05
p = @(x) c(1)*x.^2 + c(2)*x + c(3)
hold on; fplot(p,xlim)
norm(A*c-b) % 0.2889

% ellipse plot - 2 p
e = @(x,y) (x-535.6).^2/(1.5^2) + (y-372.5).^2/(1.2^2)-1
fimplicit(e,[534 538 370 374])
axis equal

% intersections - 5 points
syms x y
pars = c(1)*x.^2 + c(2)*x + c(3) - y
es = (x-535.6).^2/(1.5^2) + (y-372.5).^2/(1.2^2)-1
sol = solve(pars,es)
xm = double(sol.x);
ym = double(sol.y);
sol = [xm ym]
%       535.41 +         0i      371.31 +         0i
%       537.36 +    0.30657i      372.93 -    0.8086i
%       537.36 -    0.30657i      372.93 +    0.8086i
%       536.27 +         0i      373.57 +         0i
realsol = imag(xm)==0;
plot(xm(realsol),ym(realsol),'bd')
```