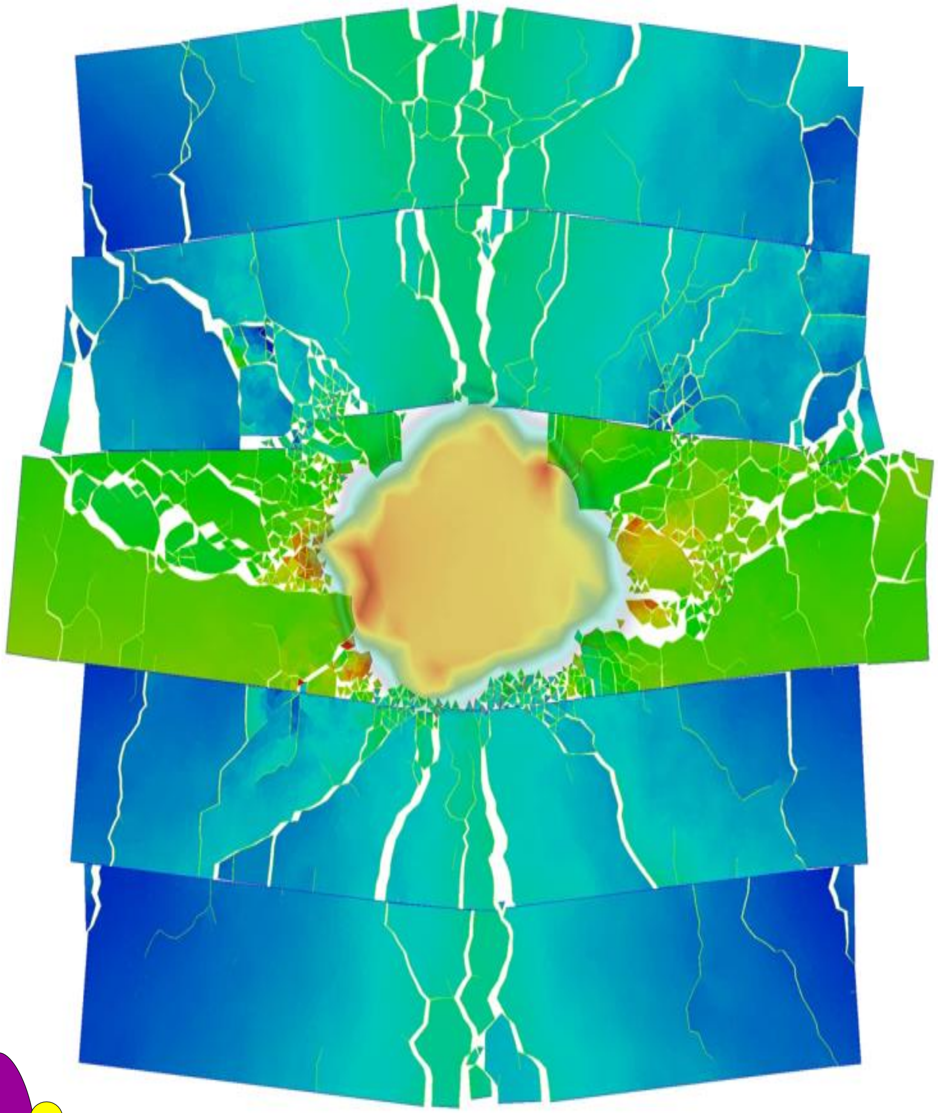


MUNJIZA'S FEM/DEM METHOD

OVERVIEW OF DEM SOFTWARES

Quasi-static methods ← ***equilibrium states** are searched for*

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\mathbf{f}=\mathbf{0}"$$

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← *a **process in time** is searched for*

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ Polyhedral elements, e.g. UDEC *rigid / deformable elements; deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements; deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ Contact Dynamics models *rigid elements, non-deformable contacts*

AIM & THE MAIN IDEA

Ante Munjiza (1999), (2004), …: (2D, 3D)

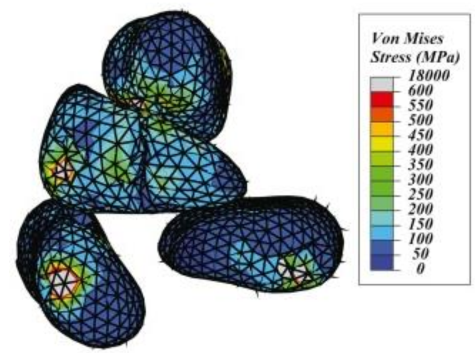
→ to simulate fracture and fragmentation of discrete elements

Recent years:

→ further development of several algorithmic details – e.g. large strains

→ applications to historic masonry

Main features:

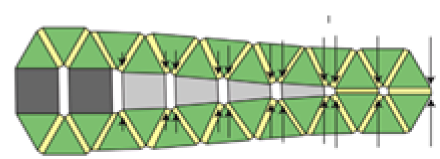


→ deformable, polyhedral discrete elements ;
deformable contacts between them

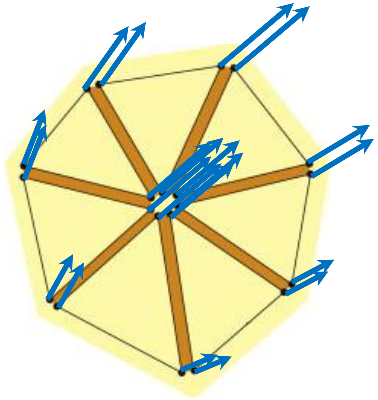
→ discrete elements are subdivided into uniform-strain FEM tetrahedra

→ „joint elements”: „joints” ≠ „contacts”

inside the discrete elements,
between the FEM tetrahedra:
able to soften and crack



THE ELEMENTS



Degrees of freedom:

translations of the **nodes**

$\rightarrow$ similar to 3DEC, **BUT**:

multiplicity of nodes!

Strain in the finite element tetrahedra:

different possibilities available:

small strain tensor; right or left Cauchy-Green strain tensor

Stress options: Cauchy stress tensor; Ist or IInd Piola-Kirchhoff stress tensor

$\rightarrow$ more options than in 3DEC

Constitutive model of the elements:

Hooke law, no plasticity of the finite elements [very simple]

$\rightarrow$ in 3DEC: plastic yield and user-defined constitutive relations can be used

THE EQUATIONS OF MOTION

stress field is reduced to FE nodes

forces transmitted through joint elements reduced to FE nodes

Diagonal mass matrix (Voronoi cells of nodes)

$$\mathbf{Ma} + \mathbf{Cv} + \mathbf{F}_{\mathbf{int}} - \mathbf{F}_{\mathbf{ext}} - \mathbf{F}_{\mathbf{joint}} = \mathbf{0}$$

Diagonal damping matrix (no artificial damping)

Reduced vector of external forces

mass matrix: masses of the Voronoi cells of the nodes $\rightarrow$ like in 3DEC

stress field inside the tetrahedra: reduced to the nodes $\rightarrow$ like in 3DEC

Solution: Central Difference Method $\rightarrow$ like in 3DEC

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code: 8139 5415

A few features of 3DEC are listed here. Which features are valid for Munjiza's FEM/DEM method too?

✓ → The elements are made deformable by using tetrahedral subdivision.

✓ → The strain inside a tetrahedron is uniform.

✘ → The elements are unbreakable.

✘ → The tetrahedra can yield.

✓ → The time integration is done by the Central Difference Method.

THE EQUATIONS OF MOTION

stress field is reduced to FE nodes

forces transmitted through joint elements reduced to FE nodes

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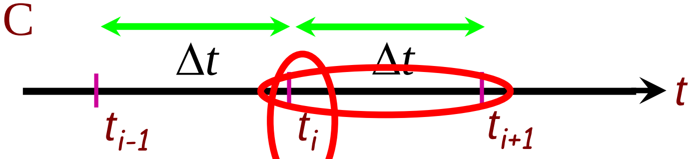
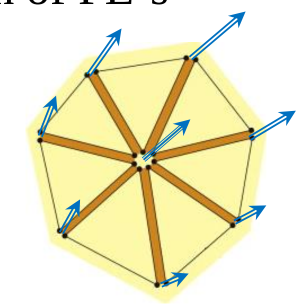


Analysis of a time step:

1. Evaluation of stresses $\rightarrow$ the internal forces, based on the deformation of FE-s
2. Evaluation of joint forces based on the deformation of joint elements
3. Fracture of joints: open the cracks and upgrade forces if needed
4. Run the contact detection algorithm: find candidate pairs for contact
5. Run the contact interaction algorithm: evaluate the contact forces
6. Apply the external forces; finalize the reduced force vector
7. Solution of the equation of motion: apply the Central Difference Method



OVERVIEW

stress field is reduced to FE nodes

forces transmitted through joint elements reduced to FE nodes

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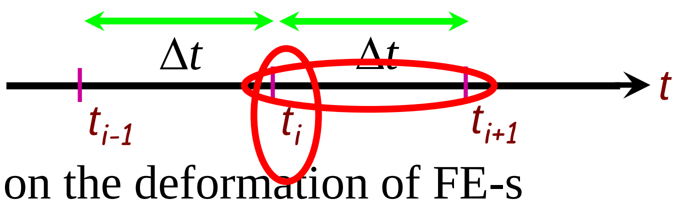

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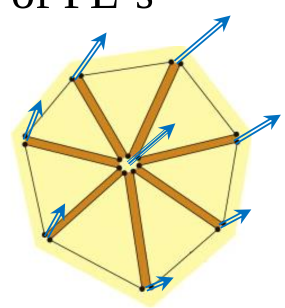
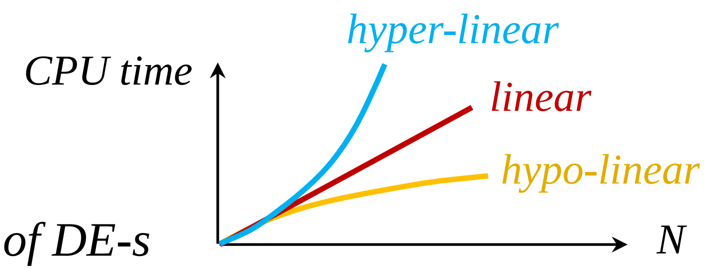


CONTACT DETECTION ALGORITHM

<u>Aim:</u> to detect pairs close to each other $\rightarrow$ possible overlap for detailed analysis

contact detection algorithms in general:



N := the number of DE-s

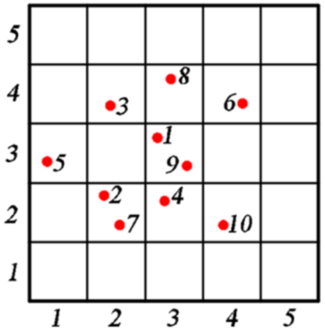
Different techniques are available in FEM/DEM:

e.g. Munjiza's NBS algorithm (1995) : time $\propto N$

e.g. „screening array algorithm" : time $\propto N$ but too large RAM requirement

e.g. „sorting algorithm" : very small RAM but time $\propto N \log_2 N$ (hyper-linear)

Munjiza's NBS algorithm:



$\rightarrow$ grid assigned to the system (!!! correct choice of grid size !!!)

$\rightarrow$ scan DE-s: assign to the cell containing its centroid ($\propto N$)

$\rightarrow$ scan DE-s: check neighbouring cells only ($\propto N$)

important: no loop over cells!

OVERVIEW

stress field is reduced to FE nodes

forces transmitted through joint elements reduced to FE nodes

Diagonal mass matrix (Voronoi cells of nodes)

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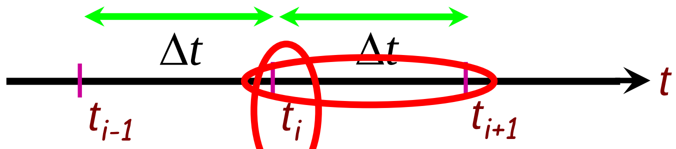
Reduced vector of external forces



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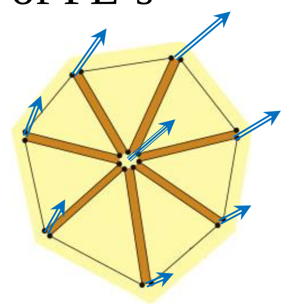


CONTACT INTERACTION ALGORITHM

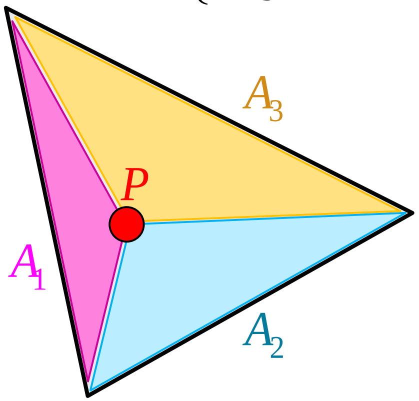
aim: to produce the contact forces transmitted between partly-overlapping DE-s

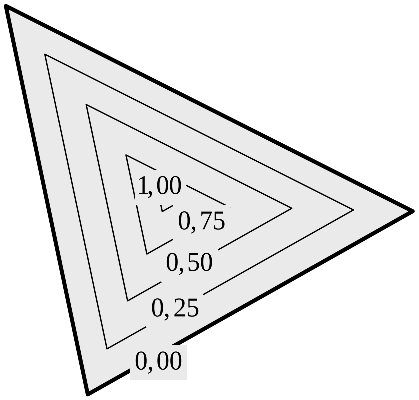
How it works in 2D: [$\rightarrow$ similar in 3D]

„potential function" inside every FE:

$$\varphi(P) = \min \begin{cases} 3A_1 / A \\ 3A_2 / A \\ 3A_3 / A \end{cases}$$

(„P": arbitrary point in the FE)





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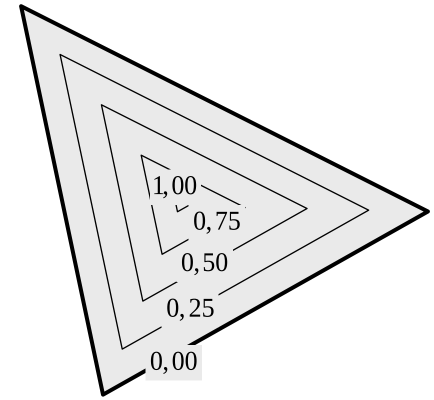
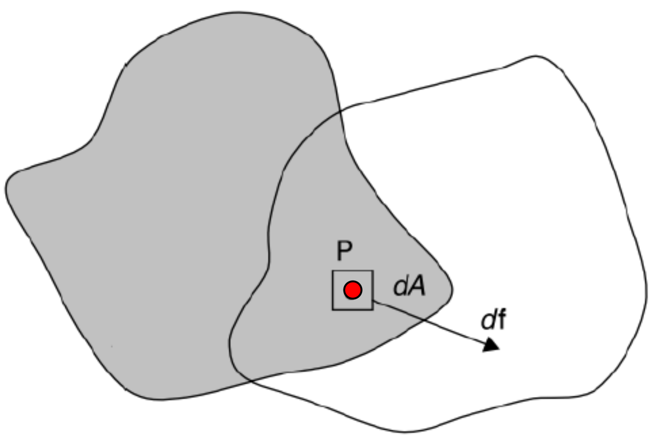


Case of two overlapping discrete elements:

P scans over the total overlap !!! only max one layer of FE-s allowed !!!



Potential functions of the two FE-s

$$\mathrm{d}\boldsymbol{f} = \left[\mathrm{grad}\,\varphi_1(P) - \mathrm{grad}\,\varphi_2(P)\right]\mathrm{d}A$$

⇒ distributed force along the overlap: then reduced to the nodes

CONTACT INTERACTION ALGORITHM

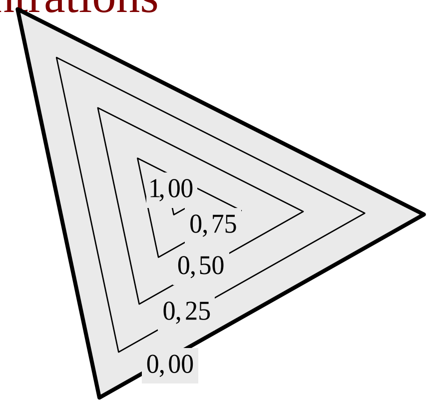
<u>Advantageous features:</u>

$\rightarrow$ distributed contact forces: no unrealistic stress concentrations

$\rightarrow$ complicated contact behaviour (sliding, plasticity, cohesion etc): easy to incorporate

$\rightarrow$ energy conservation satisfied!

$\rightarrow$ computationally relatively efficient



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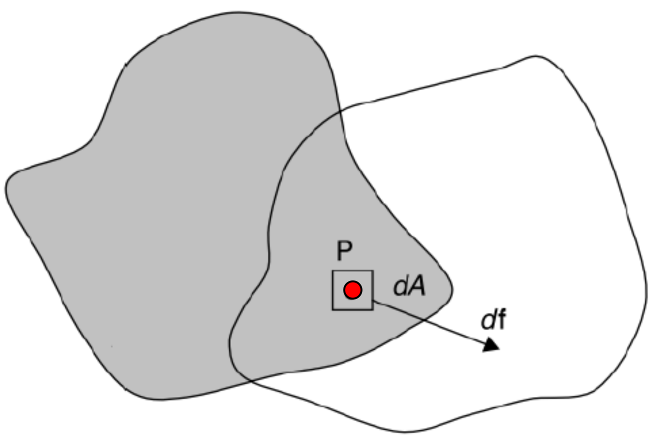

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$\Rightarrow$ distributed force along the overlap: then reduced to the nodes

OVERVIEW

stress field is reduced to FE nodes

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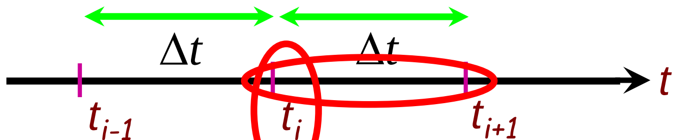

Analysis of a time step:

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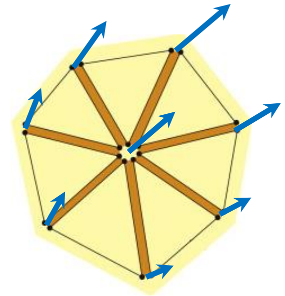


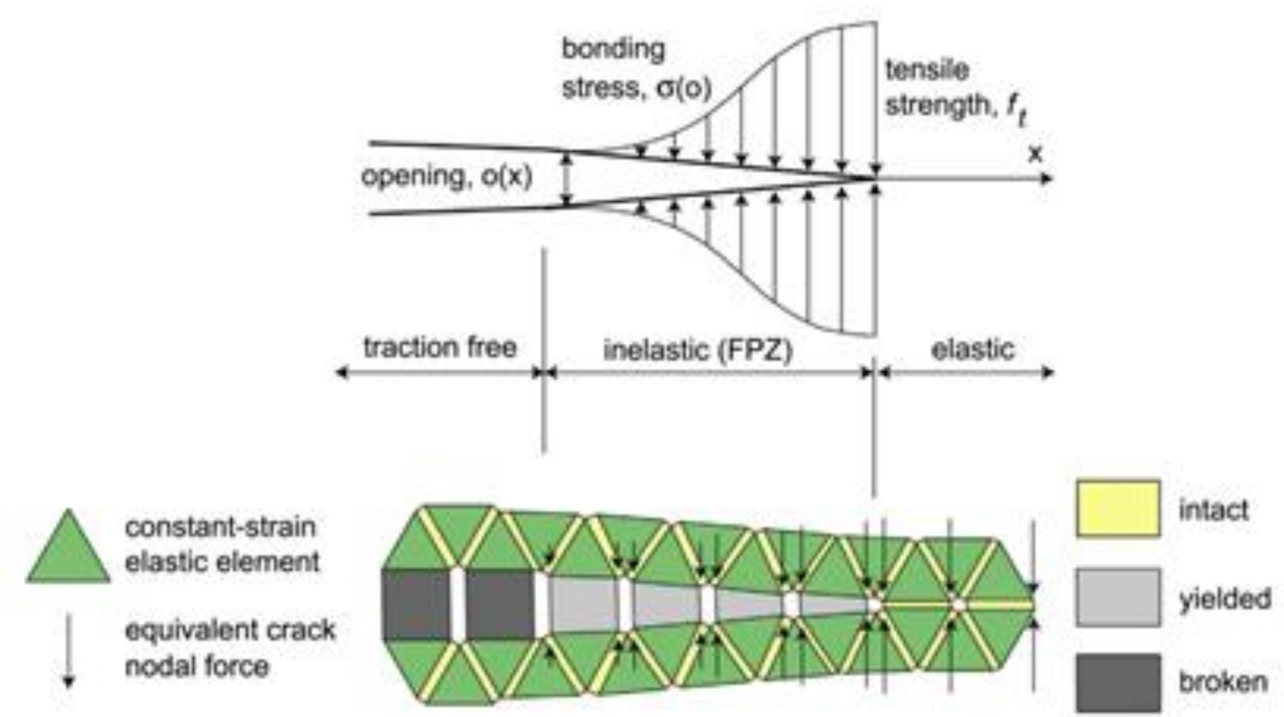
FRACTURE & FRAGMENTATION ALGORITHM

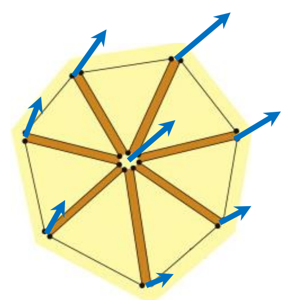
<u>aims:</u> → to define crack initiation

→ to describe how cracks propagate,

→ to replace the released internal forces with new forces

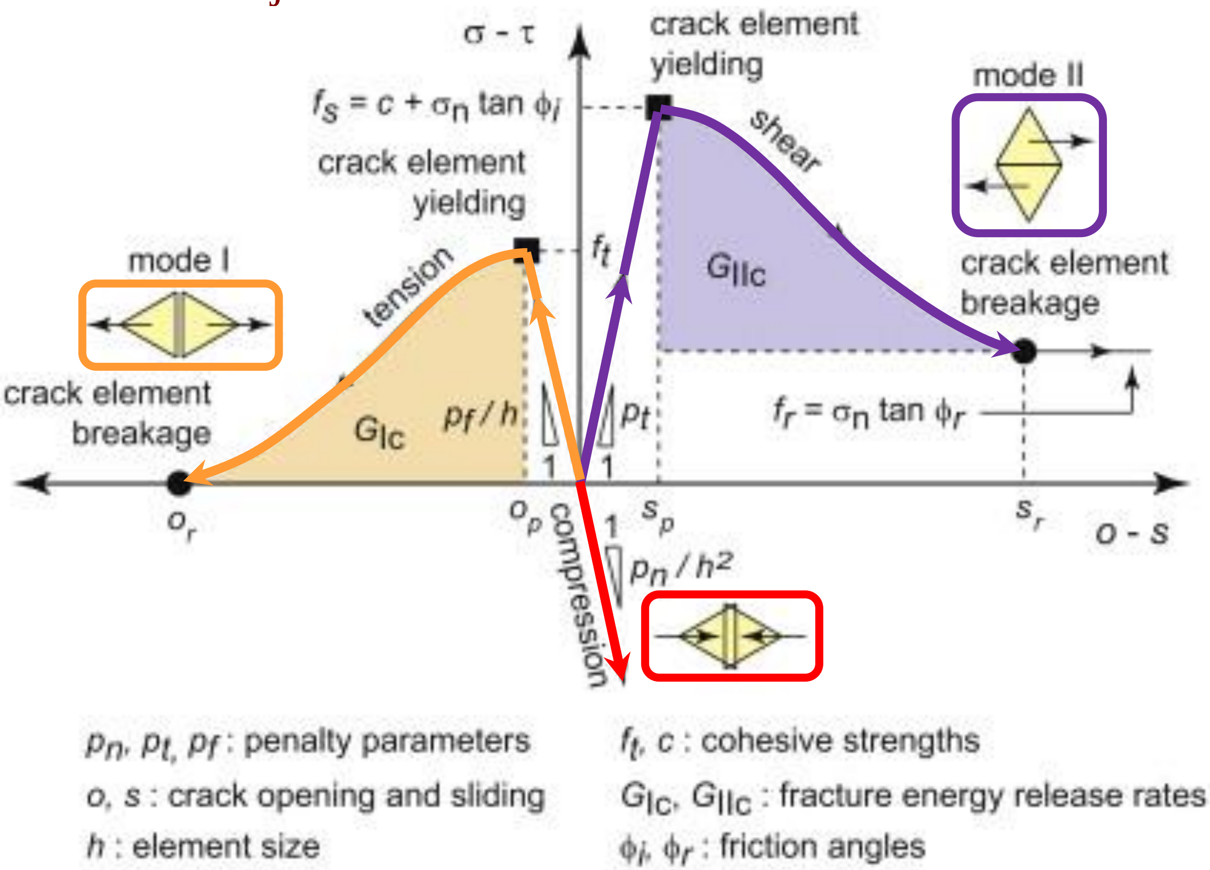
„joint elements”: the common surfaces between FE-s ! in the <u>interior</u> of DE-s !





THE JOINT ELEMENTS

Mechanical behaviour of joints:



Disadvantage:

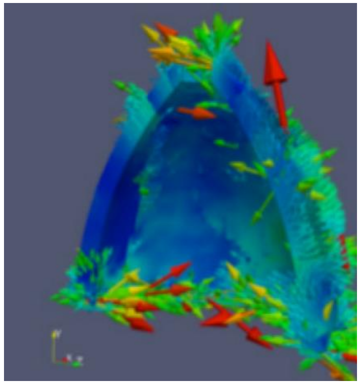
simulated fracture behaviour is very sensitive to mesh density & orientation

⇒ very dense subdivision of the DE-s is needed

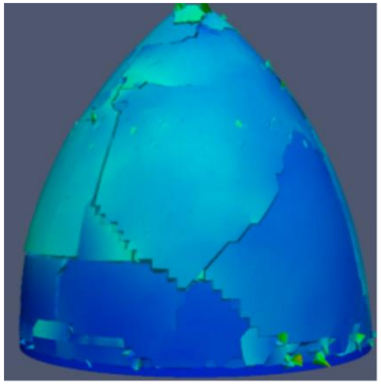
APPLICATIONS

e.g. Rougier et al (2014):

Seismic analysis of the Dome of the Santa Maria del Fiore cathedral (Florence, Italy)



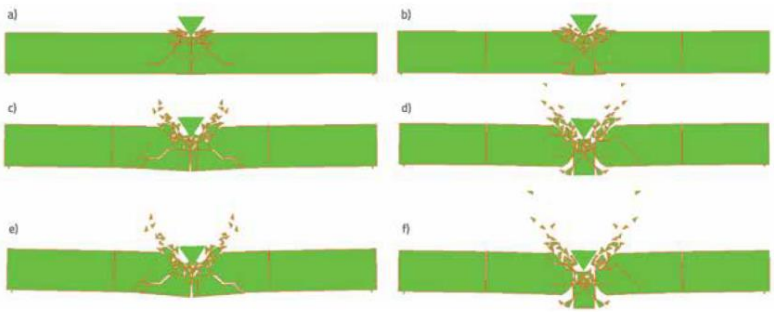
stress vawe propagation



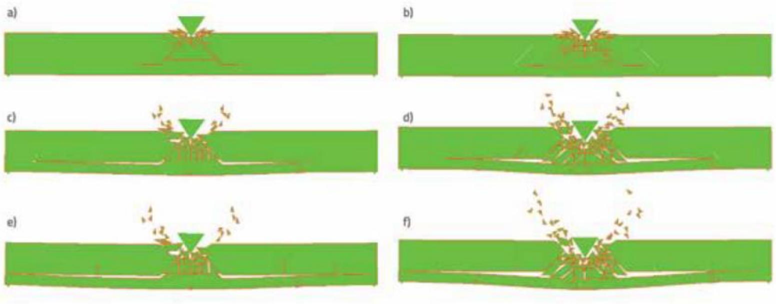
cracked final state

e.g. Zivaljic et al (2014):

Impact loading of a concrete beam



unreinforced

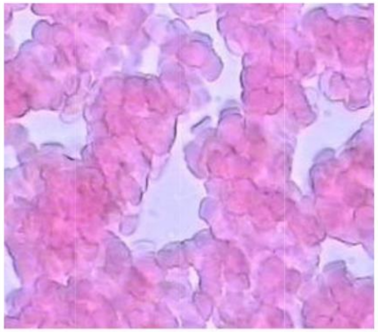


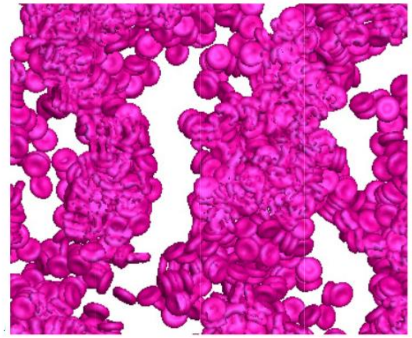
reinforced

APPLICATIONS

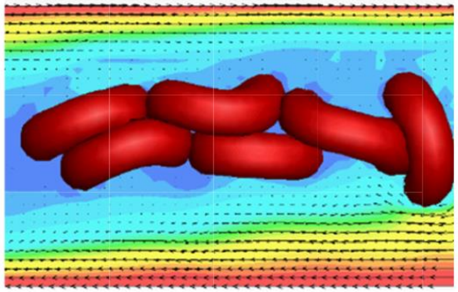
e.g. Munjiza et al (2019):

simulation of red blood cells

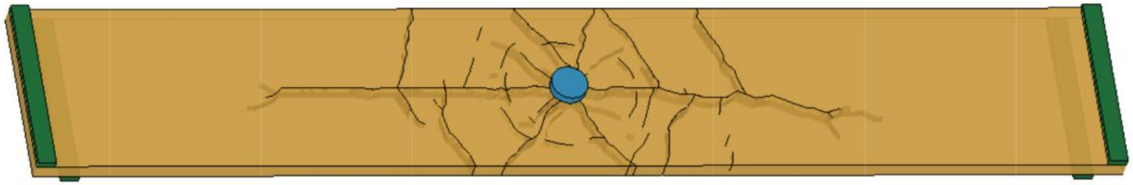




blood flow in an artery:



impact into car windscreen:



QUESTIONS

1. How are the discrete elements made deformable in FEM/DEM? What are the degrees of freedom in the FEM/DEM model? (*Hint: Slides 3-4*)

2. List four similarities and two differences between 3DEC and FEM/DEM. List two similarities and three differences between FEM/DEM and DDA.

3. The 7 main steps of the analysis of a time step are listed below. Put them after one another according to the correct order: (*Hint: Slide 7*)

A) Considering each joint in the interior of the discrete elements, from the relative translations of the nodes belonging to the actual joint the algorithm determines the joint forces.

B) External effects (like e.g. gravity) are included now finally and added to the previously determined nodal forces.

C) From the translations of the nodes, the element strains are determined, from which the stresses are calculated. Then the effect of the stresses are reduced to the nodes.

D) Calculate (using an explicit time integration technique) the incremental translations of the nodes during the analysed time step, based on Newton's force-acceleration law.

E) Using a potential function approach, the candidate pairs are analysed in order to figure out the contact forces between them.

F) Considering each joint in the interior of the discrete elements, from the joint forces the algorithm decides whether the joint opens up or not, and if necessary, it modifies the joint forces.

G) The elements are scanned through whether they are close to each other hence deserving an accurate, detailed analysis as a possible contact.

QUESTIONS

4. Introduce each of the 7 main steps of the analysis of a time step (tell 1-2 sentences about each step).

5. What are the advantages of using the potential approach in the contact interaction algorithm? (*Hint: Slide 13*)

6. Explain the overall meaning and every detail of the following diagram: