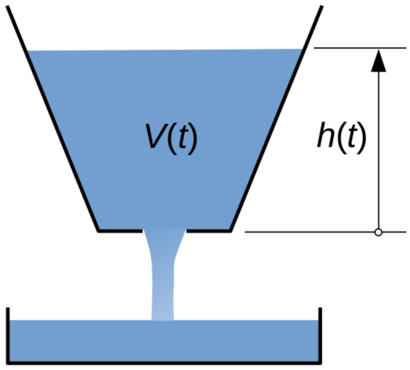
1. Water is discharged from a tub with a truncated conic shape through a bottom circular orifice. Radius of the base circle of the tub is $R = 2$ m, the height of the tub is 2 m, diameter of the orifice is $d = 0.05$ m, inclination angle of the side of the tub with respect to the vertical is $\alpha = 75°$, discharge coefficient is $C_d = 0.65$. At the initial epoch $t_0 = 0$ the water volume in the tub is $V_0 = 95$ m³.



The following tasks are to be completed (20 points):

a) Volume V of water in the tub in terms of water depth h is:

$$V(h) = \frac{\pi}{3\tan(\alpha)}\left[(R + h \cdot \tan(\alpha))^3 - R^3\right]$$

Determine initial water level h_0 from the formula. (Remember that in Matlab, the default angle is in radians!) (4 points)

b) The next differential equation gives the rate of water level decrease in the tub:

$$\frac{dh}{dt} = -C_d d^2 \frac{\sqrt{2gh}}{4[R + h\tan(\alpha)]^2}$$

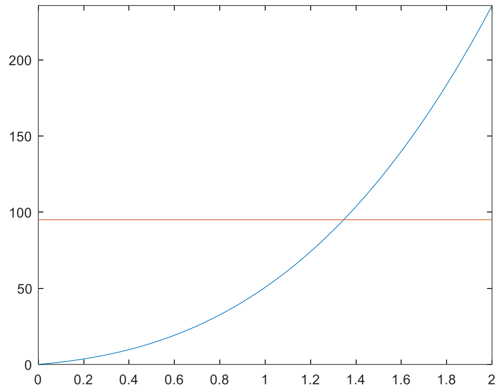
where $g = 9.81$ m/s². Write the Matlab function of the right hand side of the differential equation of water discharge. (4 points)

c) Solve the differential equation for the time interval t = [0, 10000] seconds with Matlab's built-in fourth order Runge-Kutta method with the given initial condition (initial water level: h_0). Specify both relative and absolute errors of the solution as 10^{-4}. Plot the solution. What will be the water level in the tub after 10000 seconds? (6 points)

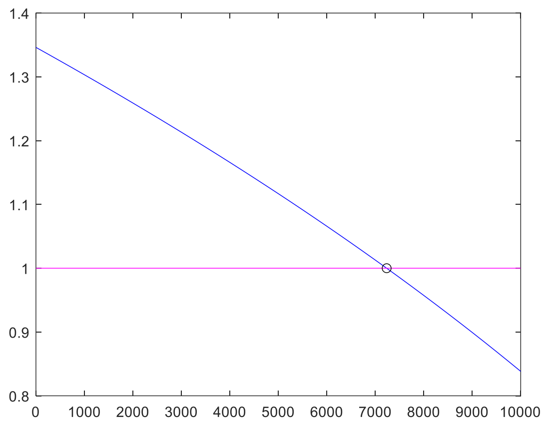
d) Determine the time t_1 to reach water level of 1 m in hours. For this make second-order cubic spline interpolation of the solution. Take initial guess from the figure, and plot the solution in the same figure. (4 points)

e) Determine the change in water level for a period of 10 hours if the tank is full at the initial time ($h_0 = 2$ m)! Plot the solution in a new figure! (2 points)

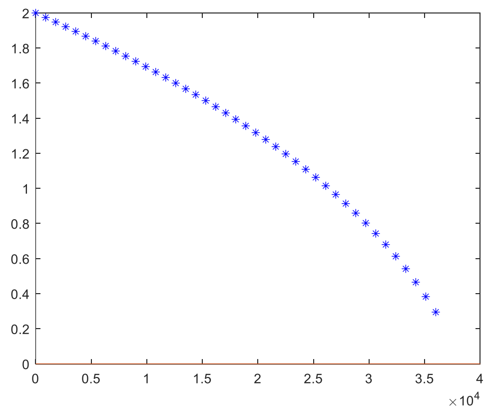
```
%% 1st task
clc; clear all; close all; format shortG;
% tub size
R = 2;  % Radius of the base circle
alfa = 75; % inclination angle of the side of the tub (degree)
ar = pi/180*alfa % inclination angle in radian
d = 0.05;  % diameter of the orifice
Cd = 0.65;  % discharge coefficient
V0 = 95  % m^3 water volume in the tub

% a) initial water level (4 p)
% Volume function
V = @(h) pi/(3*tan(ar))*((R+h*tan(ar)).^3-R^3)
figure(1); fplot(V,[0,2]); hold on; plot(xlim, [95,95])
V95 = @(h) V(h)-95;
h0 = fzero(V95,1.4) %  1.3462
% or reordering the Volume function
hf = @(V) 1/tan(ar)*((3*V*tan(ar)/pi+R^3)^(1/3)-R)
h0 = hf(V0) % 1.3462
```

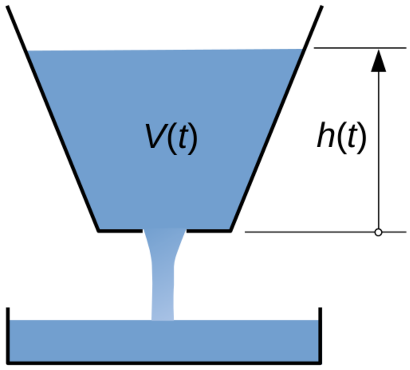



```
% b) rate of water level decrease (4 p)
g = 9.81;
% ODE
dhdt = @(t,h) -Cd*d^2*sqrt(2*g*h)./(4*(R+h*tan(ar)).^2);

% c) ODE solution (6 p)
options = odeset('AbsTol',1e-4, 'RelTol', 1e-4);
[t,h] = ode45(dhdt,[0,10000],h0,options);
figure(2); plot(t,h,'b-')
hv = h(end) % 0.83829

% d) water level of 1 m (4 p)
hS = @(u) spline(t,h,u);
gS = @(h) hS(h) -1
tk = fzero(gS,7000) % tk = 7234.9
hold on; plot(xlim,[1,1],'m-')
% solution
plot(tk,hS(tk),'ko')
% time (hour)
tkh = tk/3600  % 2.0097
```




```
% e) h0=2, 10 hour (2 pont)
h0 = 2;
[t,h] = ode45(dhdt,[0,10*3600],h0,options);
figure(3); plot(t,h,'b*'); hold on; plot(xlim,[0,0])
```





2. Water was discharged from a tub with a truncated conic shape through a bottom circular orifice, while the height *h(t)* of the water in the tank was measured quarterly for 5 hours (‘*waterlevel1.txt*’). The radius of the base circle of the tank is R = 2 m, inclination angle of the side of the tub with respect to the vertical is $\alpha = 75°$. Determine the output flow rate function, the amount of water leaked and how much water is left in the tank after 5 hours.

The following tasks are to be completed (20 points):

a) Read measurements from file `waterlevel1.txt`. In the first column are the measurement times in hours, in the second column the water heights in m. Plot these data in a figure, label the axes and put also the correct units. (2 points)

b) Volume V of water in the tub in terms of water depth h is:

$$V(h) = \frac{\pi}{3\tan(\alpha)}\left[(R + h \cdot \tan(\alpha))^3 - R^3\right]$$

Determine initial water volume in the tub using the formula. (Remember that in Matlab, the default angle is in radians!) (2 points)

c) Define the function (*h(t)*) of the change in water level over time by fitting a third-order global polynomial to the measured heights. Plot the regression polynomial in the figure also. Determine by extrapolation using the fitted polynomial when the tank is expected to be emptied? (4 points)

d) Make a function of the rate of change of water heights dh/dt by calculating the derivative of the global third-degree polynomial you have determined at c). (4 points)

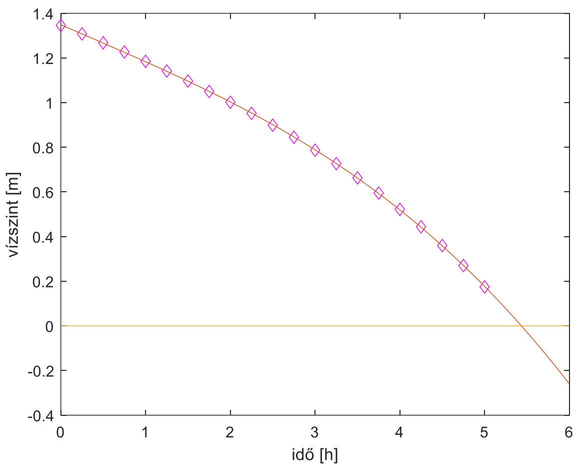
e) Determine and plot in a new figure the output flow rate function $Q(t)$ in the measured interval, use the next formula: (2 points)

$$Q(t) = \pi \cdot [R + h(t) \cdot \tan(\propto)]^2 \cdot \frac{dh}{dt}(t)$$

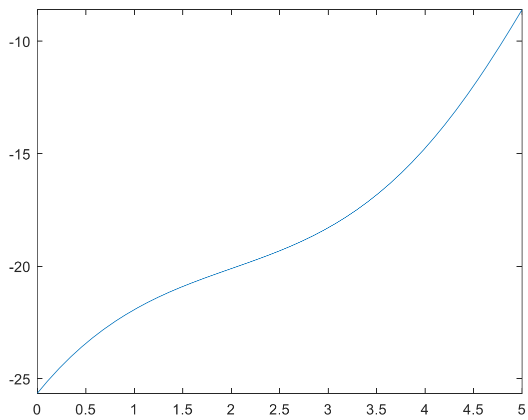
f) Determine the amount of water leaked by integrating the water flow rate function over the measurement interval and determine how much water is left in the tank after 5 hours. Use Simpson's rule! (3 points)

g) Solve the previous problem using the trapezoidal rule! (3 points)

```
%% 2nd task
% a) load data - 2 p
clc; clear all;close all; format shortG;
adat = load('vizszint1.txt');
th = adat(:,1); % time - hours
h = adat(:,2);
figure(); plot(th,h,'md');
xlabel("idő [h]"); ylabel("vízszint [m]")

% b) % Volume function - 2 p
R = 2;  % radius
alfa = 75; ar = pi/180*alfa; % inclination angle in rad
V = @(h) pi/(3*tan(ar))*((R+h*tan(ar)).^3-R^3)
h0 = h(1) % 1.346
V0 = V(h0) % 94.966

% c) regression, extrapolation - 4 p
c = polyfit(th,h,3)
%       -0.0033072    0.0026635     -0.16499       1.3489
fh = @(t) polyval(c,t)
hold on; fplot(fh,[0 6])
plot(xlim,[0,0])
x0 = fzero(fh,5) %  5.4348
```

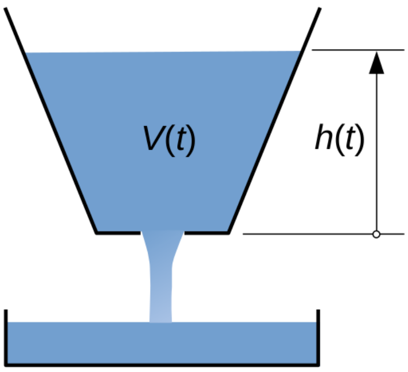



```
% d) rate of change of water heights dh/dt - 4 p
c2 = polyder(c)
%  -0.0099215    0.0053271     -0.1649
dhdt = @(t) polyval(c2,t)

% e) output flow rate function - 2 p
Q = @(t) pi*(R+fh(t)*tan(ar)).^2.*dhdt(t);
figure(2); fplot(Q,[0 max(th)])

% f) how much water is left in the tank? Simpson - 3 p
Vki = quad(Q,0, max(th)) % -92.374
Vm = V0+Vki % 2.5915
```



```
% g) how much water is left in the tank? trapezoidal- 3 pont
Qt = Q(th);
Vki2 = trapz(th,Qt) %  -92.364
Vm = V0+Vki2 %  2.6015
```

3. Water was discharged from a tub with a truncated conic shape through a bottom circular orifice. During this operation at each and every half hour epoch the water level $h(t)$ was measured. Radius of the base circle of the tub is $R = 2$ m, diameter of the orifice is $d = 0.04$ m, inclination angle of the side of the tub with respect to the vertical is α, discharge coefficient is $C_d = 0.75$. Determine the unknown inclination angle α from the measurements.



The following tasks are to be completed (20 points):

a) Read measurements from file `waterlevel2.txt`. In the first column are the measurement times in hours, in the second column the water heights in m. Plot these data in a figure, label the axes and put also the correct units. (2 points)

b) Define the function ($h(t)$) of the change in water level over time by fitting a global fourth-degree polynomial to the measured water heights. Plot the regression polynomial in the previous figure. Make a bar graph of fit residuals in a new figure. (4 points)

c) Make a function of the rate of change of water heights dh/dt by calculating the derivative of the global fourth-degree polynomial you have determined at b). (4 points)

d) Create a function for calculating the area A of the water surface in the tub at an arbitrary epoch t by using the function of the rate of change of water heights dh/dt with the aid of the following formula (4 points)

$$g = 9.81 \cdot (3600)^2 \text{ m/h}^2$$

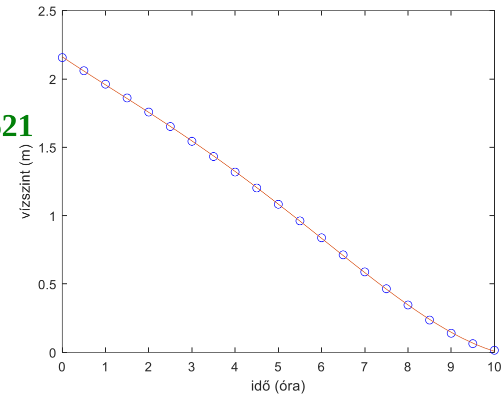
$$A(t) = -C_d \cdot \pi \cdot \frac{d^2}{4} \frac{\sqrt{2 \cdot g \cdot h(t)}}{\frac{dh}{dt}}$$

e) Write a function $r(t)$ for calculating the radius of the circular water surface at water height h in the tub at time t! What will be the radius of the water surface at the beginning and at the end of the measured discharge period? (3 points)

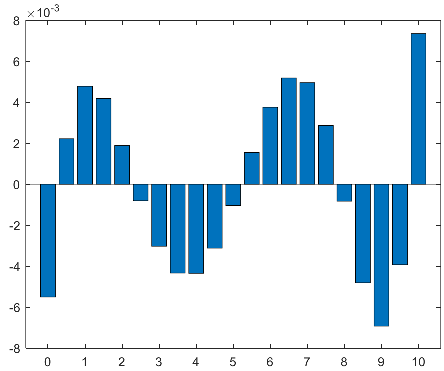
$$r(t) = \sqrt{\frac{A(t)}{\pi}}$$

f) Determine the unknown inclination angle (α) of the side of the tub with respect to the vertical in degrees, using the measured water levels at the beginning and at the end of the discharge period and the radii of the water surface. (Remember that Matlab's default angle is in radians!) (3 points)

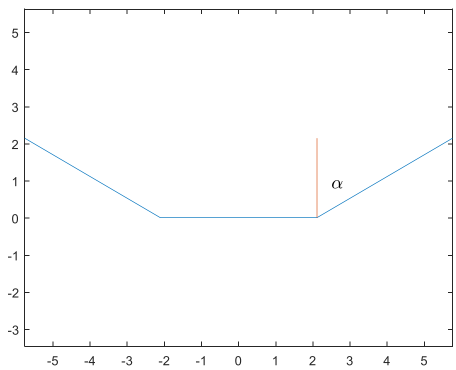
```
%% 3rd task
clc; clear all; close all
% a) load data- 2 p
data = load("vizszint2.txt");
t = data(:,1); h = data(:,2);
% plot
plot(t,h,"bo"); xlabel("idő (óra)"); ylabel("vízszint (m)")
```



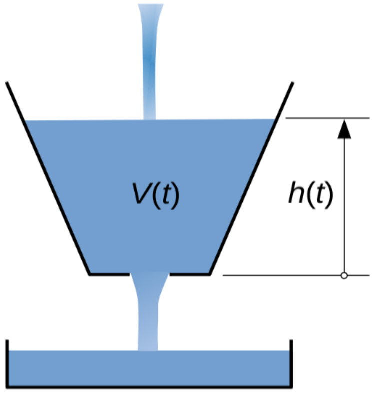

```
% b) global fourth-degree polynomial - 4 p
% data: t, h
c = polyfit(t,h,4)
% 0.00022788   -0.0033463     0.010338     -0.21201       2.1621
hp = @(t) polyval(c,t)
% plot
hold on; fplot(hp,[min(t),max(t)])
% residuals
rh = h - hp(t);
figure(2); bar(t,rh)
```




```
% c) dh/dt with derivation - 4 p
cd = polyder(c)
% 0.0009115    -0.010039     0.020676     -0.21201
dhdt = @(t) polyval(cd,t)

% d) calculating the area - 4 p
d = 0.04;  % diameter of the orifice
Cd = 0.75;  % discharge coefficient
g = 9.81; gh = g*3600^2; % m/h^2
A = @(t) -Cd*pi*d^2/4*sqrt(2*gh*hp(t))./dhdt(t)

% e) calculating the radius of the circular water surface r(t) - 3 p
r = @(t) sqrt(A(t)/pi)
% radius of the water surface at the beginning and at the end
r0 = r(t(1)) % 5.7601
rt = r(t(end)) %  2.1097

% f) unknown inclination angle in degree? - 3 p
h0 = h(1) % 2.1566
ht = h(end) % 0.0156
% tan(alfa) = dr/dh
alfa = atand((r0-rt)/(h0-ht)) %  59.608

% Plot of water tank and alpha
figure(3); plot([r0,rt,-rt,-r0],[h0,ht,ht,h0]); axis equal
hold on; plot([rt, rt],[h0,ht])
text(2.5,1,'\alpha','FontSize',14)
```



4. Water was discharged from a tub with a truncated conic shape for 10 hours through a bottom circular orifice while there was an unknown input flow $Q_{in}(t)$ into the tub from above. During this operation at each and every half hour epoch the water levels $h(t)$ and the outflow rates $Q_{out}(t)$ were measured. Radius of the base circle of the tub is $R = 2.5$ m, inclination angle of the side of the tub with respect to the vertical is $\alpha = 25°$. The task is to determine the total volume of water inflow in the tub for each epoch $V_{in}(t)$.



The following tasks are to be completed (20 points):

a) Read measurements from the file `outflow.txt`. In the first column are the measurement times in hours, in the second column the water heights in m, in the third column the outflow rates $Q_{out}(t)$ in m³/h. Plot these data in two subfigures, label the axes and put also the correct units. (3 points)

b) To calculate the amount of water entering the tank, the actual water volumes at each epoch must also be known. Based on the water level, the current volume of water can be determined by the following relation:

$$V_{\text{víz}}(h) = \frac{\pi}{3\tan(\alpha)}\left[(R + h(t)\tan(\alpha))^3 - R^3\right]$$

Define the above function and then use it to calculate the volume of water in the tank for the measured times based on the water levels (V_{water})! What is the volume of water in the tank at the beginning and end of the measurement (V_0, V_{END})? (4 points)

c) Determine the function of outflow rate as a function of time ($Q_{out}(t)$) by fitting a cubic second-order spline to the measured points. Draw the fitted function in the corresponding part of the first figure. (3 points)

d) Determine outflow water volume function ($V_{out}(t)$)! This can be obtained by integrating the function ($Q_{out}(t)$) from the starting time t_0 to a given arbitrary time t. Based on the defined function, determine the total outflow volume at the end of the measured period (10 hours)! (4 points)

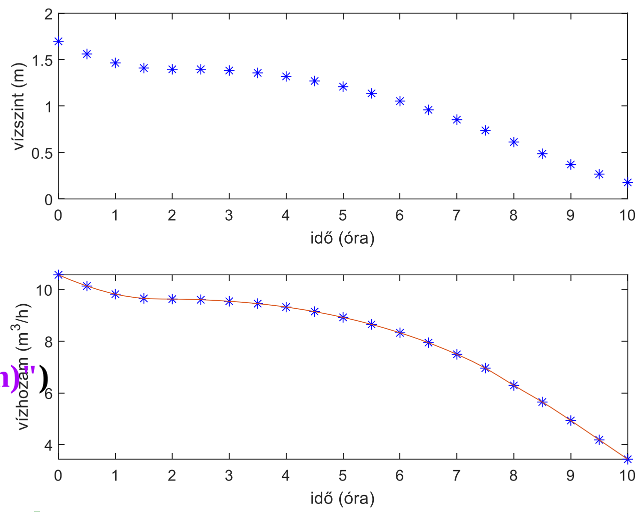
e) Using the defined $V_{out}(t)$ function, calculate the amount of water discharged for all measured t_i times in a for loop! (Store the final result in a column vector!) Plot the volume of leaked water as a function of time in a new figure. (3 points)

f) Calculate and plot the input water volume $V_{in}(t)$. Use the formula

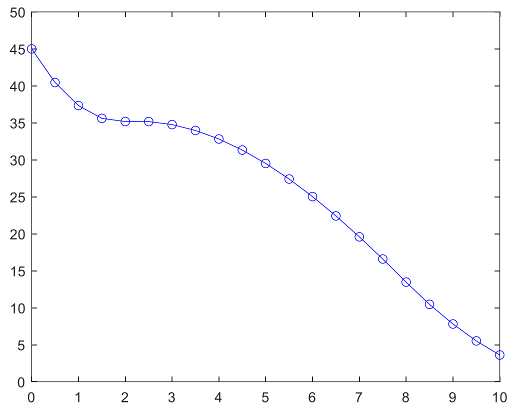
$$V_{in}(t) = V_{water}(t) - V_0 + V_{out}(t)$$

where V_{water} is the volume of water in the tub at every epoch, V_0 is the initial volume of water and V_{out} is the volume of water discharged at every epoch. Plot the amount of water inflow at the measurement times in a new figure! Determine the total volume of water inflow at the end of the measurement period? (3 points)

```
%% 4th task
clc; clear all; close all; format shortG;
% a) load data - 3 p
data = load("vizhozam.txt");
t = data(:,1); h = data(:,2); Q = data(:,3);
% plot
figure(1)
subplot(2,1,1); plot(t,h,"b*");
xlabel("time (hour)"); ylabel("water level (m)")
subplot(2,1,2); plot(t,Q,"b*");
xlabel("time (hour)"); ylabel("outflow rate (m^3/h)")
```




```
% b) volume - 4 p
R = 2.5;  % radius
alfa = 25; ar = pi/180*alfa; % inclination angle in rad
Vh = @(h) pi/(3*tan(ar))*((R+h*tan(ar)).^3-R^3)
Vviz = Vh(h); % volumes
V0 = Vh(h(1)) % initial volume -  45.001
Vv = Vh(h(end)) % volume at the end: 3.6123
figure(2); plot(t,Vviz,'bo-')
```



```
% c) spline interpolation:  h(t) és Q(t) - 3 p
% data: t, h, Q
Qs = @(u) spline(t,Q,u) % outflow rate
%plot
tmin = min(t); tmax = max(t);
figure(1); subplot(2,1,2); hold on;
fplot(Qs,[tmin,tmax])

% d) amount of water discharged - 4 p
Vki = @(t) quad(Qs,0,t)
V10 = Vki(10) % 81.327

% e) amount of water discharged at ti time- 3 p
for i=1:length(t)
   Vkit(i)=Vki(t(i));
end
Vkit = Vkit';

% f) input water volume - 3 p
Vbet = Vviz - V0 + Vkit;
figure(); plot(t,Vbet,'r*-')
Vbet(end) %  39.939
```