BALL-type Models

OVERVIEW OF DEM SOFTWARES

Quasi-static methods ← ***equilibrium states** are searched for*

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K}\cdot\Delta\mathbf{u}+\mathbf{f}=\mathbf{0}"$$

circular, rigid elements	Kishino, 1988 Bagi-Bojtár, 1991	Baraldi & Cecchi (2017) Galassi & Tempesta (2019)	*not DEM:* → *no new contacts;* → *no large displs*

Time-stepping methods $"\mathbf{M}\cdot\mathbf{a}(t)=\mathbf{f}\,(t,\mathbf{u}(t),\mathbf{v}(t))"$ ← *a **process in time** is searched for*

simulate the motion of the system along small, but finite Δt timesteps

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simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ Polyhedral elements, e.g. UDEC *rigid / deformable elements; deformable contacts*

→ BALL-type models e.g. PFC *rigid elements; deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ Contact Dynamics models *rigid elements, non-deformable contacts*

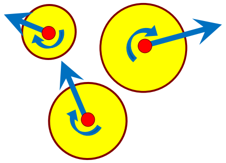
THIS PRESENTATION

BALL-type models:

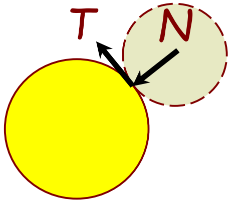
- History and definition
- Most important codes:
  - → PFC (BALL-type version)
  - → EDEM
  - → YADE
  - → Rocky-DEM
  - → OVAL
- Applications

HISTORY

BALL: P.A. Cundall, 1979

elements:  2D, perfectly rigid; $\mathbf{u}^p(t) = \begin{bmatrix} u_x^p(t) \\ u_y^p(t) \\ \varphi_z^p(t) \end{bmatrix}$

contacts: point-like;
Coulomb friction



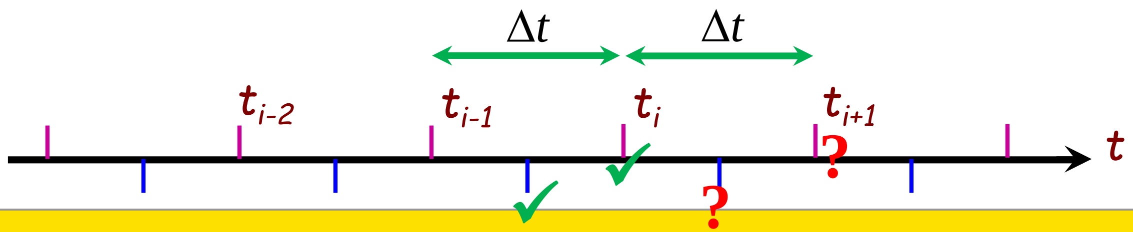
eqs of motion: $\boxed{\mathbf{M}^p(t)\mathbf{a}^p(t) = \mathbf{f}^p(t, \mathbf{u}(t), \mathbf{v}(t))}$

$$\left\{ \begin{aligned} m^p a_x^p &= f_x^p \\ m^p a_y^p &= f_y^p \\ I^p \beta_z^p &= m_z^p \end{aligned} \right.$$

$$\mathbf{M}^p = \begin{bmatrix} m^p & & \\ & m^p & \\ & & I^p \end{bmatrix}$$

numerical integration:
method of central differences

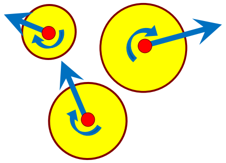
$$\boxed{\mathbf{v}^p(t_i + \Delta t/2) = \mathbf{v}^p(t_i - \Delta t/2) + \left(M^p\right)^{-1} \mathbf{f}^p(t_i)\Delta t}$$



positions
forces, stresses
accelerations
velocities

HISTORY

BALL: P.A. Cundall, 1979

elements:  2D, perfectly rigid; $\mathbf{u}^p(t) = \begin{bmatrix} u_x^p(t) \\ u_y^p(t) \\ \varphi_z^p(t) \end{bmatrix}$

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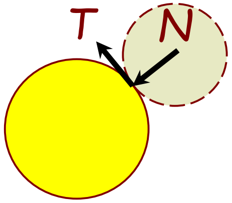


eqs of motion: $\boxed{\mathbf{M}^p(t)\mathbf{a}^p(t) = \mathbf{f}^p(t, \mathbf{u}(t), \dot{\mathbf{u}}(t))}$

numerical integration technique: method of central differences

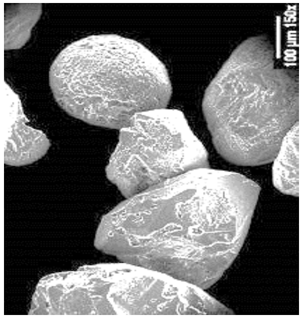
$$v_x^p(t_i + \Delta t/2) = v_x^p(t_i - \Delta t/2) + \frac{f_x^p(t_i)}{m^p}\Delta t \qquad u_x^p(t_i + \Delta t) = u_x^p(t_i) + v_x^p(t_i + \Delta t/2)\cdot\Delta t$$

$$v_y^p(t_i + \Delta t/2) = v_y^p(t_i - \Delta t/2) + \frac{f_y^p(t_i)}{m^p}\Delta t \qquad u_y^p(t_i + \Delta t) = u_y^p(t_i) + v_y^p(t_i + \Delta t/2)\cdot\Delta t$$

$$\omega_z^p(t_i + \Delta t/2) = \omega_z^p(t_i - \Delta t/2) + \frac{m_z^p(t_i)}{I^p}\Delta t \qquad \varphi_z^p(t_i + \Delta t) = \varphi_z^p(t_i) + \omega_z^p(t_i + \Delta t/2)\cdot\Delta t$$

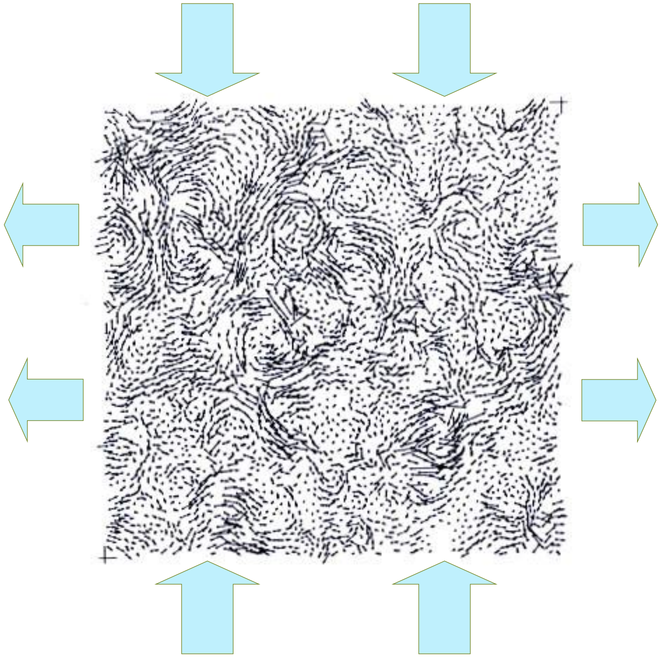
$\Rightarrow$ upgrade **contact states**; compile $\mathbf{f}^p(t_{i+1})$ and calculate the next timestep

GRANULAR ASSEMBLIES

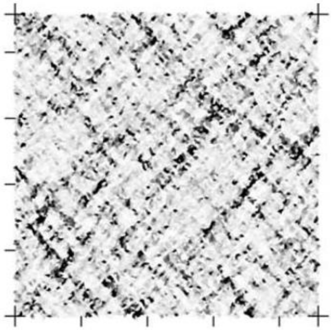


Thanks to BALL:

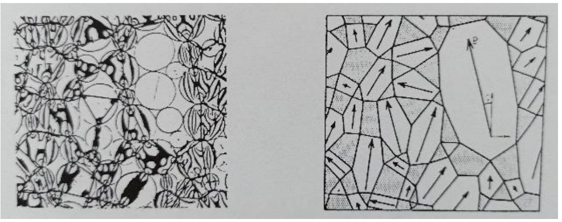
Translations of the particles
in a biaxial shear test



Quasi-rigid ***subdomains***:



Development of voids:

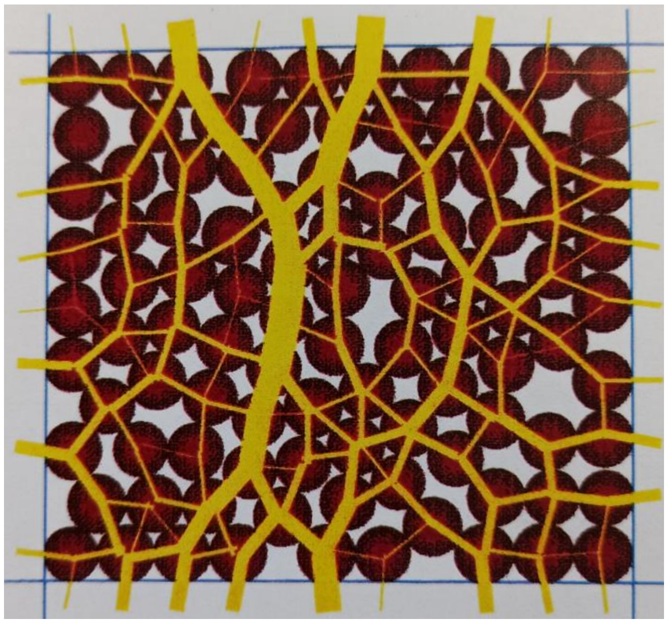


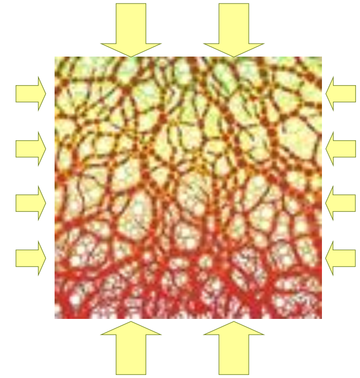
ORGANIZED NATURE OF THE KINEMATICS

GRANULAR ASSEMBLIES

<u>Thanks to BALL:</u>

Force chains discovered:





With increasing loading:

→ force chains become more and more ***vawy***

→ finally ***buckle***, but neighbours get involved

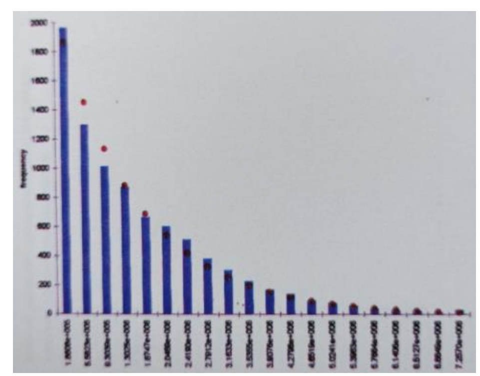
→ failure: ***no more neighbours*** to involve

ORGANIZED NATURE OF THE STATICS

Frequency diagrams of contact forces:

importance of ***exponential distribution***

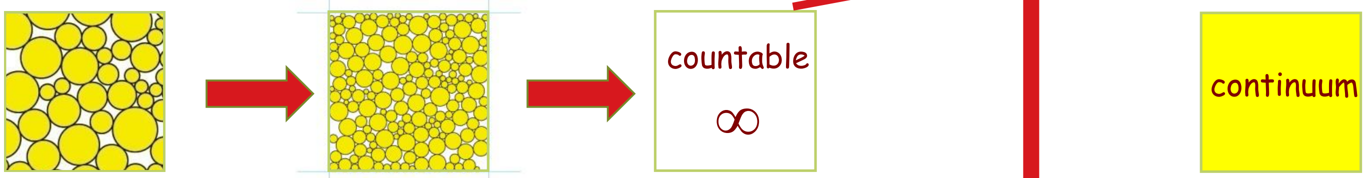
↑↑↑ entropy max.



GRANULAR ASSEMBLIES

Get rid of the common misbelief about granular assemblies:

„ number of grains tend to infinity ” → ~~„ a continuum is reached ”~~ ???



# of elements:				
	finite	finite	countably infinite	continuum-infinite
# of neighbours of the elements:				
	finite	finite	finite	continuum-infinite
Degrees of freedom:				
	translations, rotations, …	translations, rotations, …	translations; rotations, …	[different]
The translation field:				
	very strongly heterogeneous	very strongly heterogeneous	very strongly heterogeneous	continuous & differentiable

HISTORY

BALL: P.A. Cundall, 1979

3D version: 1983, TRUBAL

NSF grant, it was free for anyone who asked for

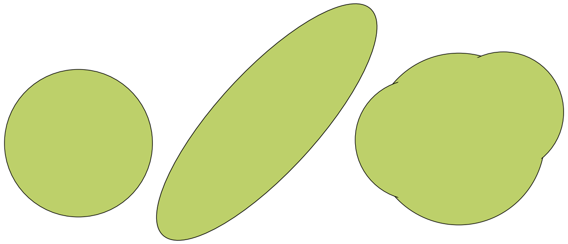
$\Rightarrow$ huge effect on granular mechanics researches !!!

validate the experiences for ***3D*** and ***non-spherical*** elements

countless similar codes were developed around the world

„BALL-type" models:

$\rightarrow$ (1) perfectly ***rigid*** elements



$\rightarrow$ (2) shape: ***smooth*** convex surface „nearly-everywhere"

$\Rightarrow$ ***point-like contacts***

$\rightarrow$ (3) eqs of motion ***separately*** for each element: $\mathbf{M}^p(t)\mathbf{a}^p(t) = \mathbf{f}^p(t, \mathbf{u}(t), \dot{\mathbf{u}}(t))$

$\rightarrow$ (4) numerical solution of the eqs of motion: ***explicit*** time integration,

(mostly: central difference method)

THIS PRESENTATION

BALL-type models:

History and definition

Most important codes:

$\rightarrow$ PFC (BALL-type version)

$\rightarrow$ EDEM

$\rightarrow$ YADE

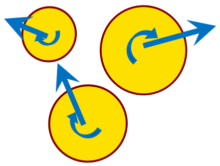
$\rightarrow$ Rocky-DEM

$\rightarrow$ OVAL

Applications

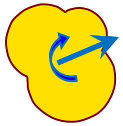
PFC, BALL-TYPE VERSION

<u>Elements:</u>



perfectly rigid cylinders (2D) or spheres (3D)

m: mass, *I*: inertia



„clump”: rigid group of elements, fixed together

m: sum of masses, *I*: sum of inertia about the „centre”

Degrees of freedom:

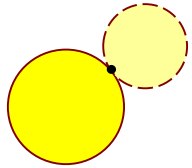
translation of the „centre” (i.e. reference point);

rotation about the „centre”

PFC, BALL-TYPE VERSION

Contact types:

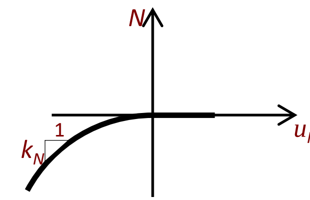
frictional



$$\Delta N^c = k_N \Delta u_N^c \qquad N^c \le 0$$

$$\Delta T^c = k_T \Delta u_T^c \qquad \left|T^c\right| \le -\boldsymbol{\nu} \cdot N^c$$

$\rightarrow$ linear: $k_N = \text{const.};\; k_T = \text{const.}$ $\leftarrow$ *default*

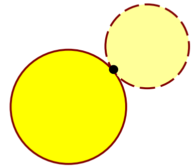


$\rightarrow$ Hertz-Mindlin:

$$k_N = \frac{\langle G\rangle \sqrt{2\langle R\rangle}}{1-\langle \boldsymbol{\mu}\rangle}\sqrt{u_N^c}$$

$$k_T = \frac{2\sqrt[3]{3\langle G\rangle^2 (1-\langle \boldsymbol{\mu}\rangle)\langle R\rangle}}{2-\langle \boldsymbol{\mu}\rangle}\sqrt[3]{-N^c}$$

cemented $\rightarrow$ point-like:



$$N^c \le N_{\max}$$

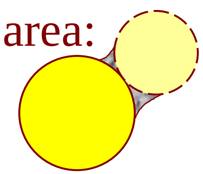
$$\left|T^c\right| \le T_{\max}$$

+ rolling resistance: $M_{bend}^c = k_M \cdot \Delta\varphi_{bend}^c \;;\quad M_{bend}^c \le \mu \cdot (-N^c)$

acts in parallel to the point-like contact: „added”

$\rightarrow$ extended contact area:

„parallel bond”



$$\Delta\boldsymbol{\sigma}_N = \tilde{k}_N \Delta u_N \quad (lin.) \qquad \boldsymbol{\sigma}_N \le \boldsymbol{\sigma}_N^{\max}$$

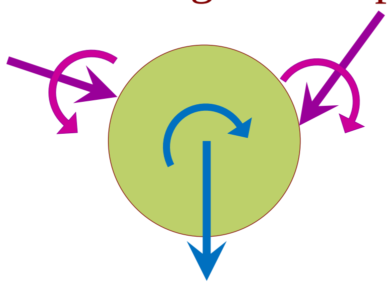
$$\Delta\boldsymbol{\sigma}_T^c = \tilde{k}_T \Delta u_T^c \qquad \left|\boldsymbol{\sigma}_T\right| \le \boldsymbol{\sigma}_T^{\max}$$

Viscoelastic contacts, Softening models, other possibilities , C/C++

PFC, BALL-TYPE VERSION

<u>Displacement calculations</u> Newton II.: „ $m\,a = f$ ”

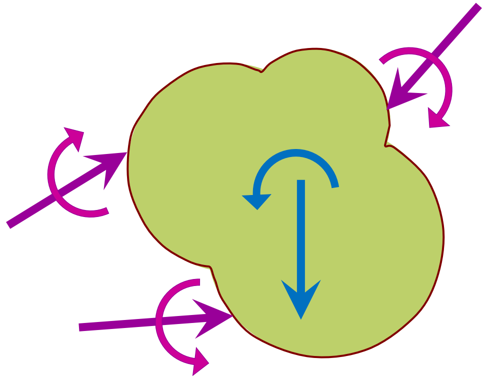
– forces acting on the spherical elements:



← from the contacts of the element
← from the external loads (weight, drag force)
← from damping

→ f, M

– forces acting on the clumps:



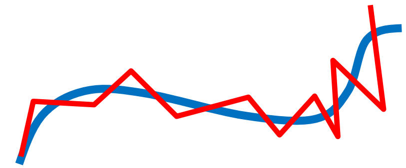
← from the contacts of the element
← from the external loads (weight, drag force)
← from damping

→ f, M

PFC, BALL-TYPE VERSION

Displacement calculations

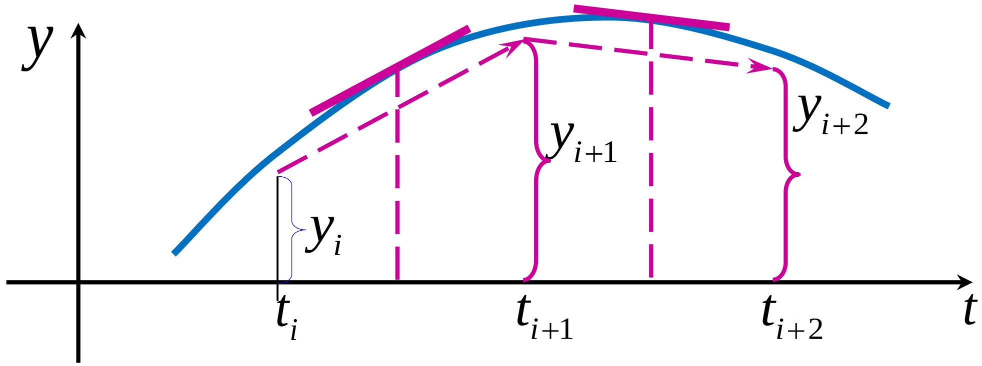
Newton II.: „ $m\,a = f$ ”



Method of Central Differences

– the eqs of motion, discretized: $\mathbf{v}^p(t_i + \Delta t/2) = \mathbf{v}^p(t_i - \Delta t/2) + \left(\mathbf{M}^p\right)^{-1}\mathbf{f}^p(t_i)\Delta t$

– from this: $\mathbf{u}^p(t_i + \Delta t) = \mathbf{u}^p(t_i) + \mathbf{v}^p(t_i + \Delta t/2)\Delta t$



– to ensure numerical stability, and to help fast convergence:

1. estimation for the longest allowed Δt : $\Delta t_{crit} = \min \begin{cases} \sqrt{m/k^{transl}} \\ \sqrt{I/k^{rotat}} \end{cases}$
2. density scaling: to modify masses/inertia

   for time-dependent problems: never use it!
3. damping

PFC, BALL-TYPE VERSION

3. Damping:

the eqs. of motion would be, if including velocity-proportional damping:

$$\xcancel{\mathbf{v}(t_i + \Delta t/2) = \mathbf{v}(t_i - \Delta t/2) + \mathbf{M}^{-1}\left(\mathbf{f}(t_i) - c_u \mathbf{v}(t_i - \Delta t/2)\right) \cdot \Delta t}$$

$\rightarrow$ a.) **Local damping** $\leftarrow$ *default*

$\rightarrow$ b.) **Contact viscous damping**

PFC, BALL-TYPE VERSION

3. Damping:

a.) **Local damping**: e.g. for a sphere:

$$v_x(t_i + \Delta t/2) = v_x(t_i - \Delta t/2) + \left(f_x(t_i) - \alpha \cdot \left| f_x(t_i) \right| \mathrm{sign}(v_x(t_i - \Delta t/2)) \right) \frac{\Delta t}{m}$$

$$\omega_z(t_i + \Delta t/2) = \omega_z(t_i - \Delta t/2) + \left(M_z(t_i) - \alpha \cdot \left| M_z(t_i) \right| \mathrm{sign}(\omega_z(t_i - \Delta t/2)) \right) \frac{\Delta t}{I_z}$$

default: $\alpha := 0.70$

- *unequilibrated motions are damped only;*
- *does not depend on the magnitude of the velocities;*
- *very good for systems in which:*
  parts are already in equilibrium, while parts are still far from the eq.

b.) **Contact viscous damping**:

viscous force added to the contact force:

its direction: opposite to the relative velocity

$$\begin{cases} \left| D_N \right| = c_N \left| \dfrac{\Delta u_N}{\Delta t} \right| \\ \left| D_T \right| = c_T \left| \dfrac{\Delta u_T}{\Delta t} \right| \end{cases}$$

- *realistic*

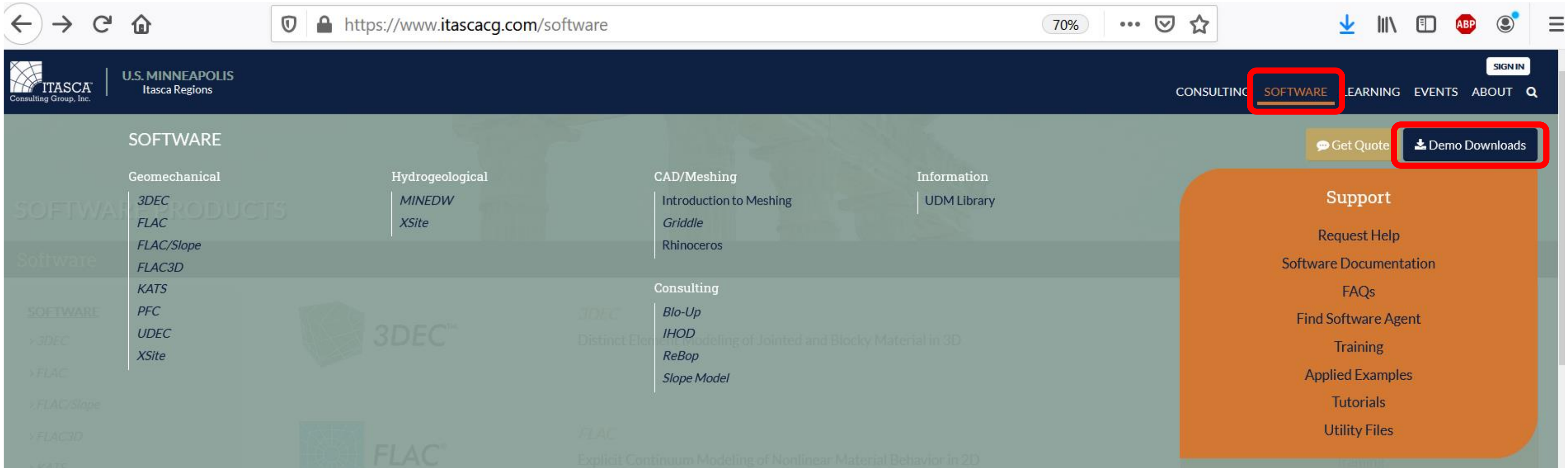
PFC, BALL-TYPE VERSION

Further details:

e.g. application results, publications, courses, conferences, …

www.itascacg.com

→ Software → Demo Downloads



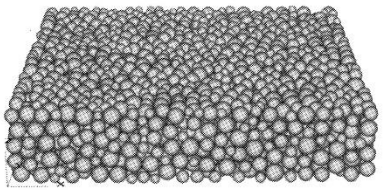
New in PFC: polyhedral elements as well (not BALL-type)

PFC PRACTICAL APPLICATIONS

Weathering of cemented granular rock

Nova et al, 2004

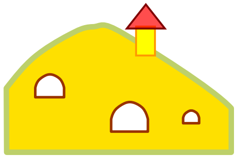
PFC-3D



3500 elements

size range: 1:2

triaxial loading

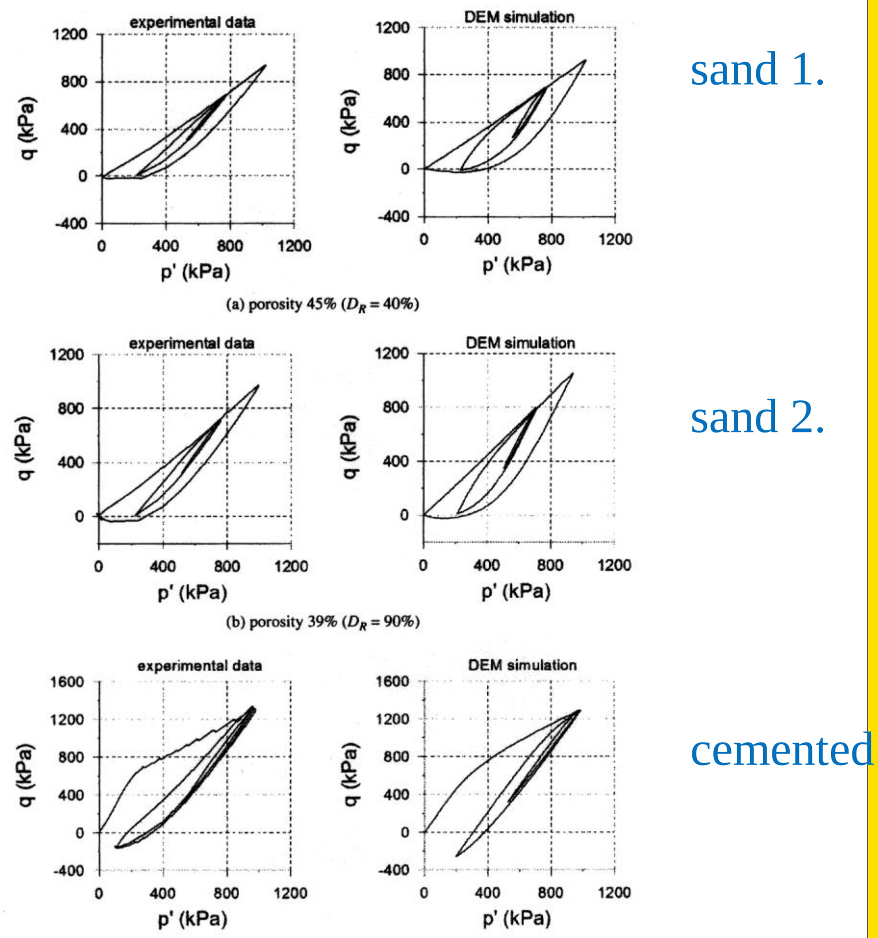


The analyzed problem:

weathering of rock under buildings

The main outcome:

well-calibrated model



(a) porosity 45% ($D_R = 40\%$)

(b) porosity 39% ($D_R = 90\%$)

(c) porosity 49% (cemented)

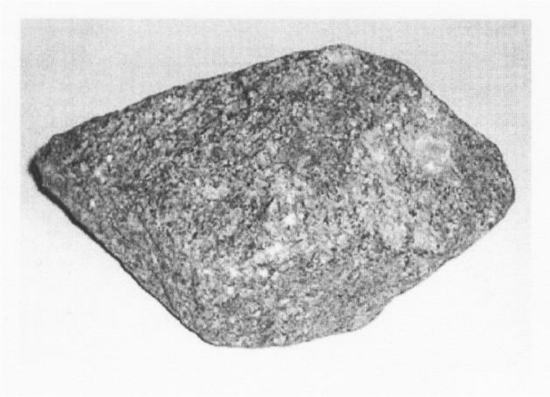
k_n	k_s	μ	c_b	σ_b^t	χ	ΔU^p_{lim}
225 KN/m	135 KN/m	0.2	1500 kPa	250 kPa	0.6	0.2

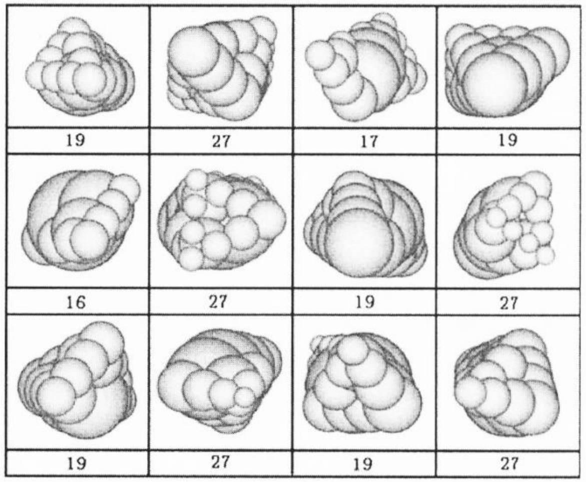
(d) DEM parameters

PFC PRACTICAL APPLICATIONS

Railway ballast modelling

e.g. effect of stone shape: Lu & McDowell, 2007, PFC-3D





PFC PRACTICAL APPLICATIONS

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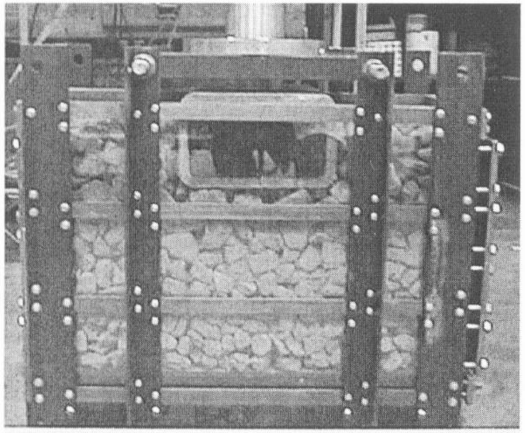

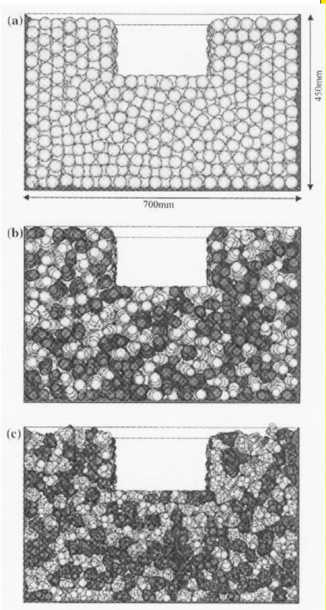

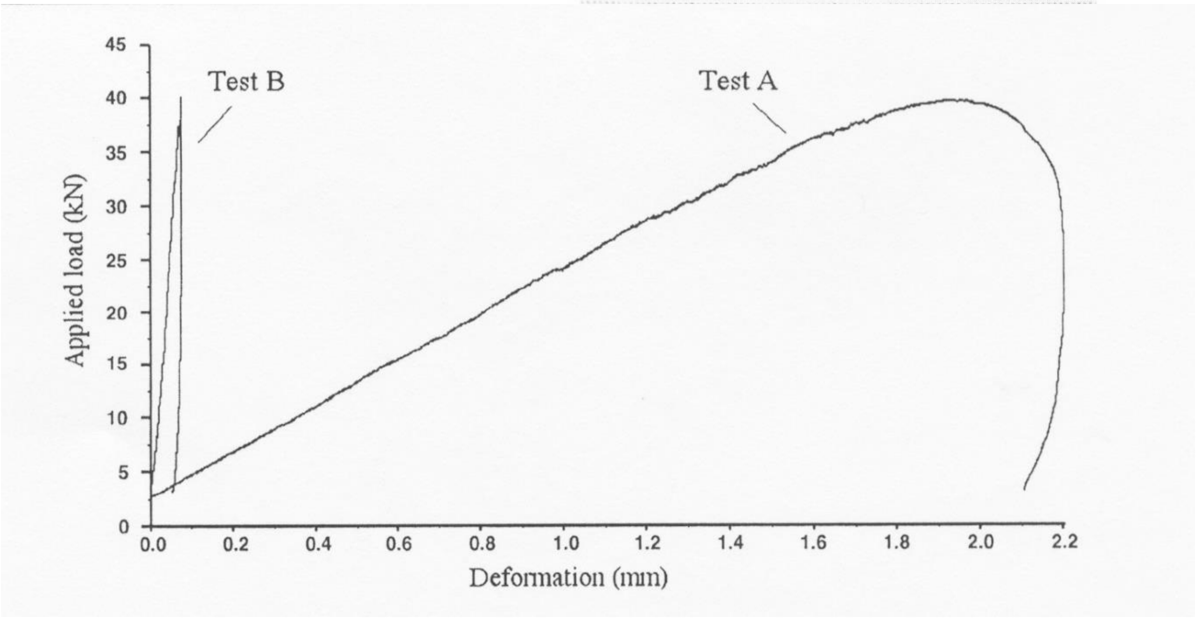

Main outcome:
do NOT use spherical elements for assemblies

THIS PRESENTATION

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  - → PFC (BALL-type version)
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  - → YADE
  - → Rocky-DEM
  - → OVAL
- Applications

EDEM *www.edemsimulation.com*

J. Favier, Edinburgh

→ elements: spheres / composed of spheres

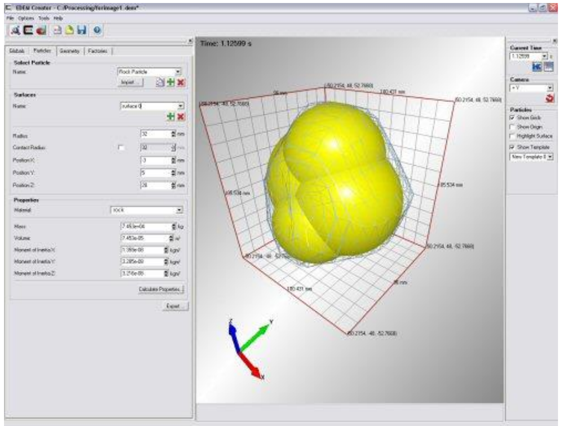
→ contacts: frictional, linearly elastic
Hertz-Mindlin
cohesional
cemented
individually coded

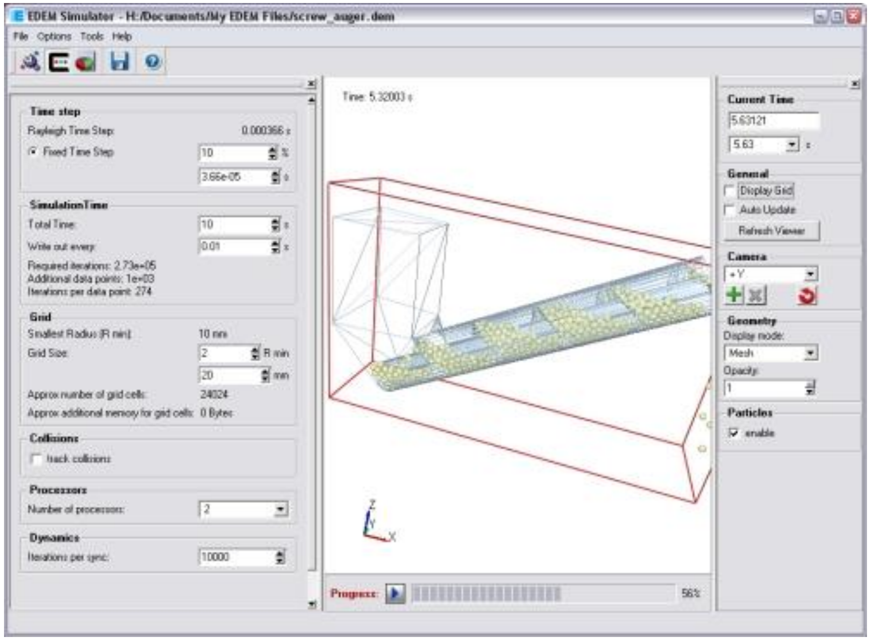
→ boundaries: several types!

→ special features of the
displacement calcultions:

- easy to connect to
  FEM or CFD
  (solids, fluids)
- fast, \\

→ output: rich (videos, pictures, etc.)



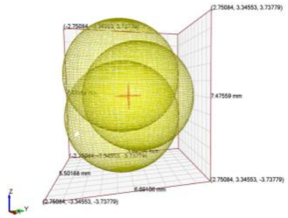


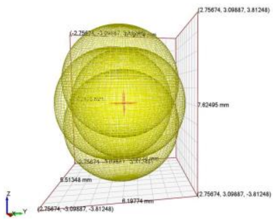
EDEM *www.edemsimulation.com*

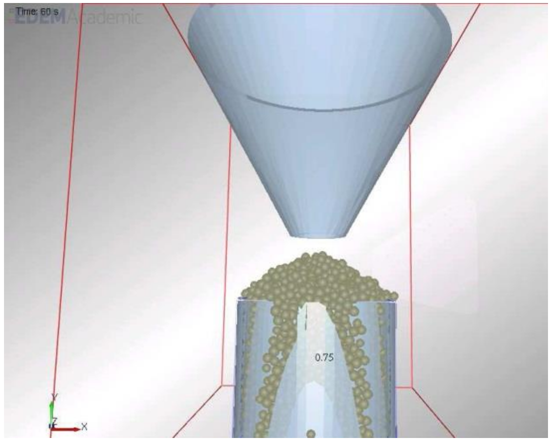
<u>applications:</u>

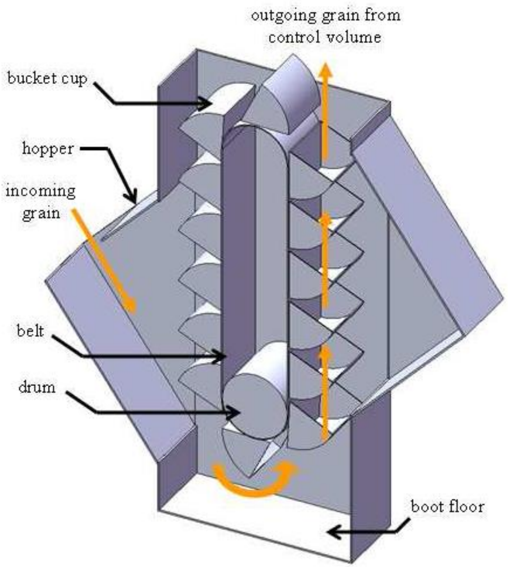
e.g. J. Boac, 2010:
assemblies of soy bean & corn

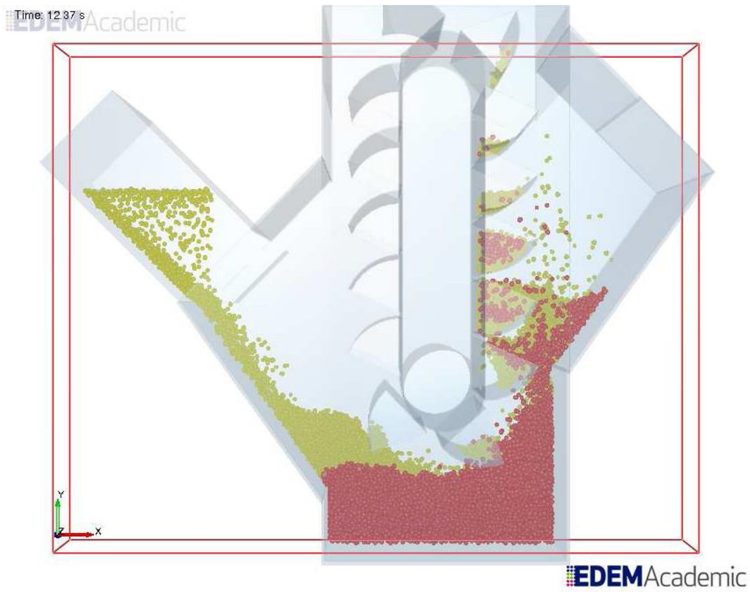
result:
well-calibrated model
for machine design







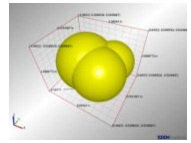


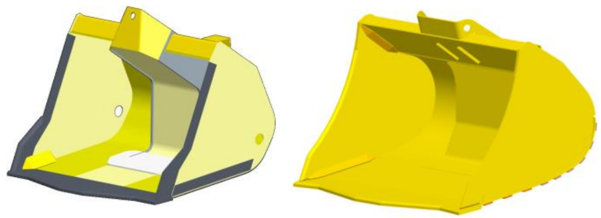


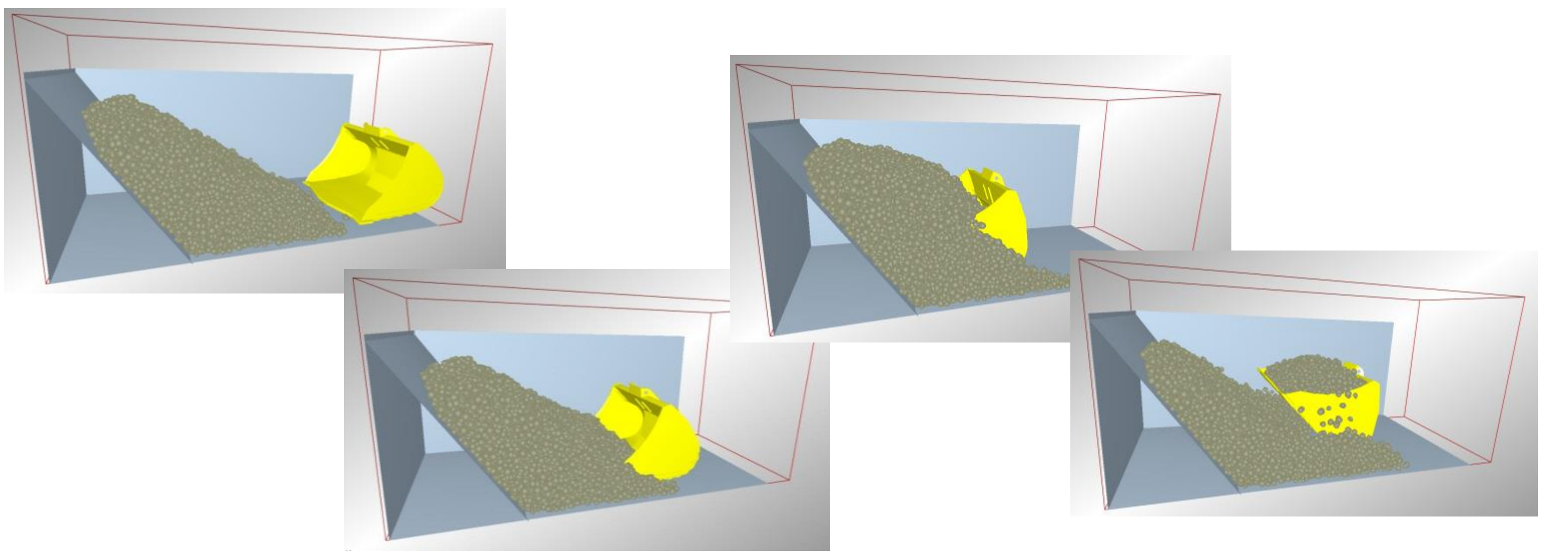
EDEM *www.edemsimulation.com*

<u>applications:</u>

e.g. J. Helgesson, 2010:







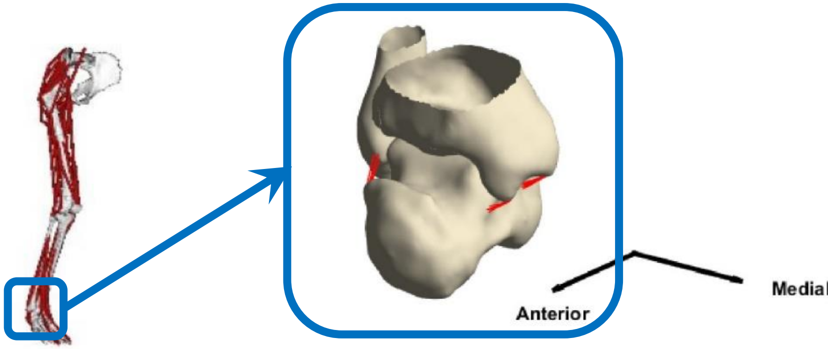
result:

optimize the geometry of the spoon

EDEM *www.edemsimulation.com*

applications:

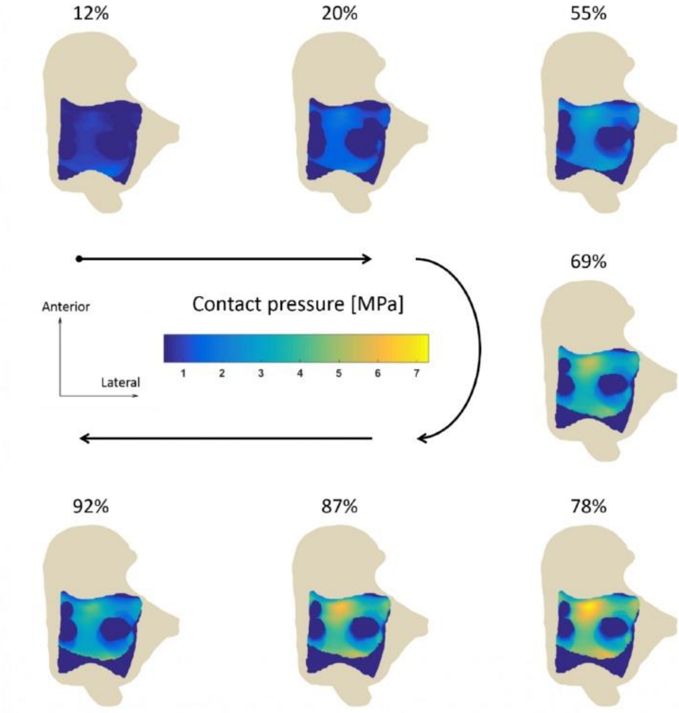
e.g. Benemerito et al (2020):



shapes: from MRI scan of a patient

result:

pressure distribution on the surface



THIS PRESENTATION

BALL-type models:

History and definition

Most important codes:

→ PFC (BALL-type version)

→ EDEM

→ YADE

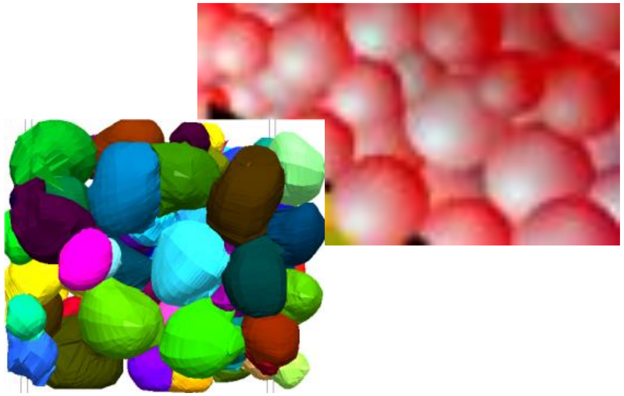
→ Rocky-DEM

→ OVAL

Applications

YADE

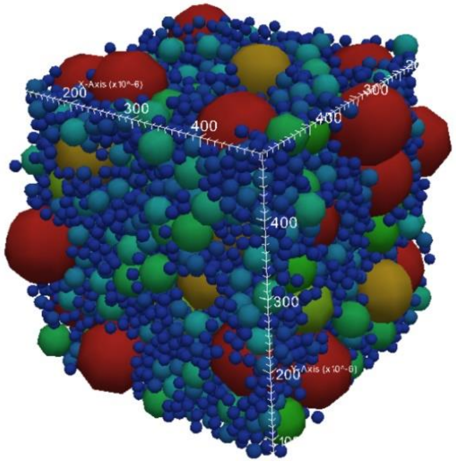
not just a **software**, rather an international **community**



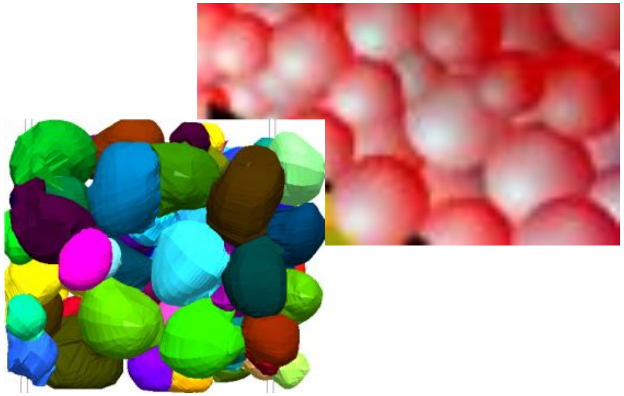
elements: spheres;
complex shapes consisting of spheres;
polyhedra;
[anything personal can be coded]

contacts:
[several models, individual codes shared]

applications:
mostly researchers!
e.g. simulation of lunar regolith: Modenese (2013)



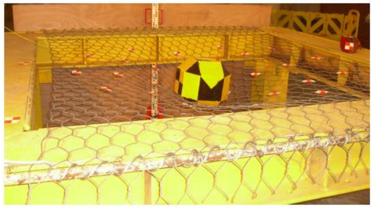
YADE

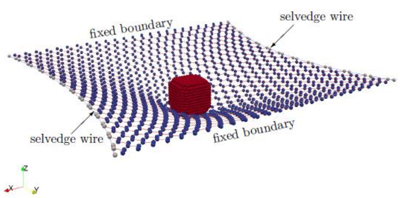


elements: spheres;
complex shapes consisting of spheres;
polyhedra;
[anything personal can be coded]

contacts:
[several models, individual codes shared]

applications:
in engineering: e.g. design of rockfall protective system: Thoeni et al (2011)



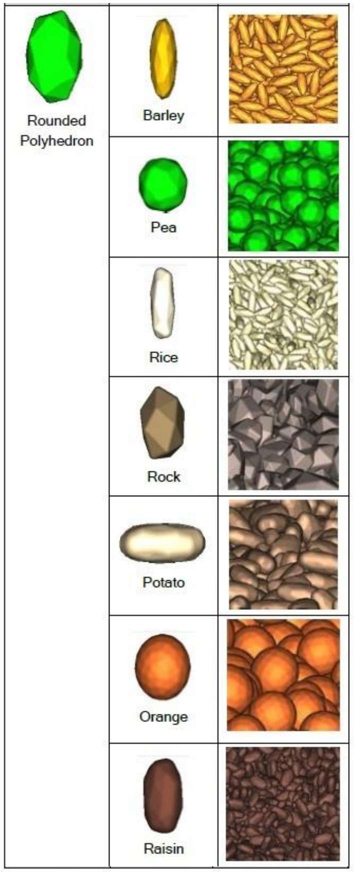


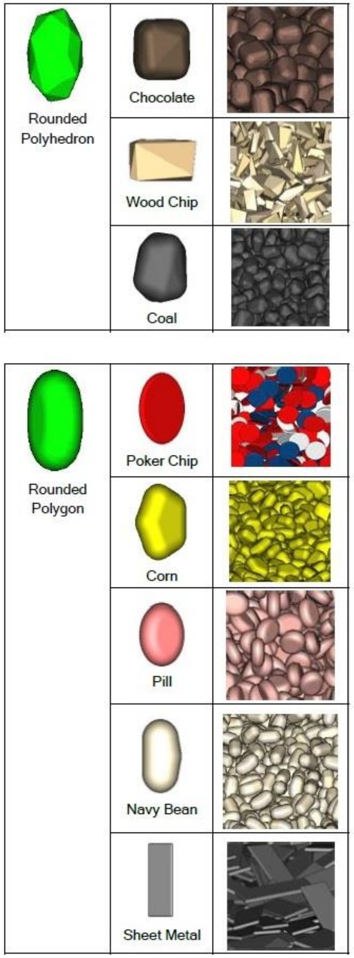
https://yade-dem.org

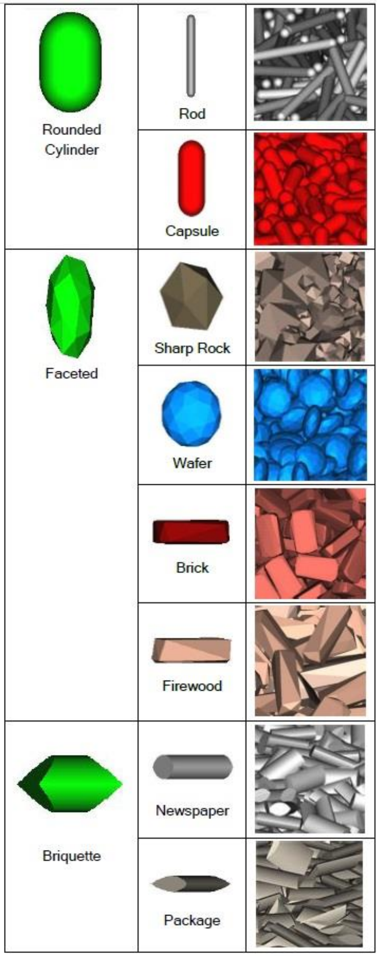
Rocky-DEM

since **2015**; originally: transport of agricultural products

<u>elements:</u> spheres; complex shapes consisting of spheres; polyhedra;
huge variety of other shapes



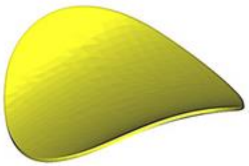
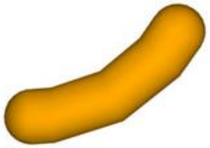




Rocky-DEM

since **2015**; originally: transport of agricultural products

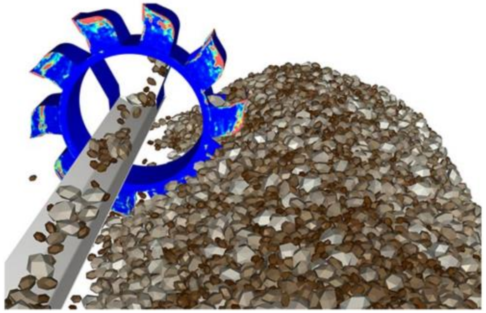
<u>elements:</u> spheres; complex shapes consisting of spheres; polyhedra; huge variety of other shapes

	Real-World Concave Shape	Physical Representation in Rocky DEM
Thin Potato Chip		
Cheetos		

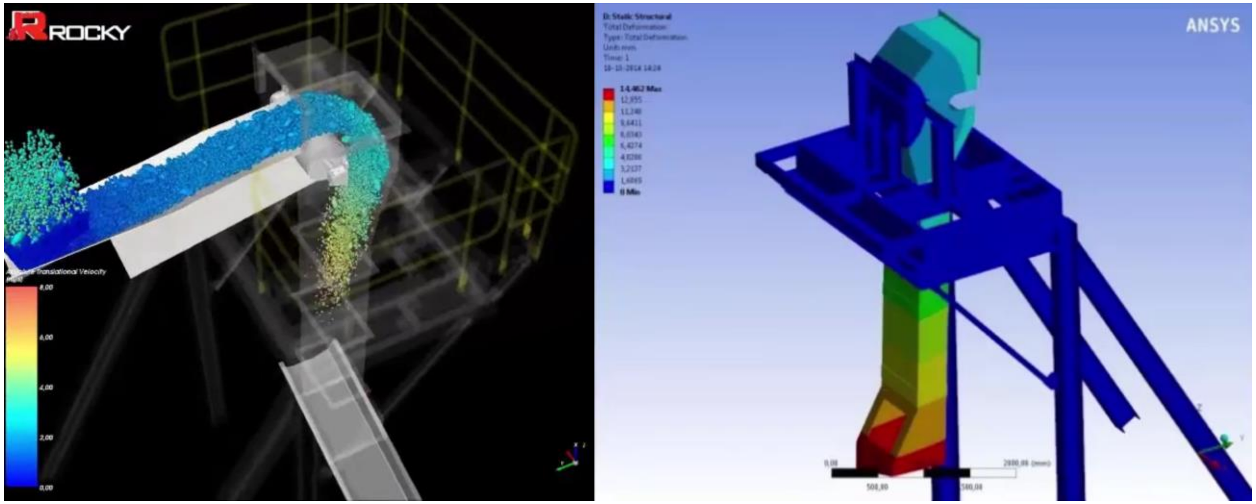
Rocky-DEM

since **2015**; originally: transport of agricultural products

elements: spheres; complex shapes consisting of spheres; polyhedra;
huge variety of other shapes



coupled with ANSYS (also with fluid analysis): DEM $\rightarrow$ FEM

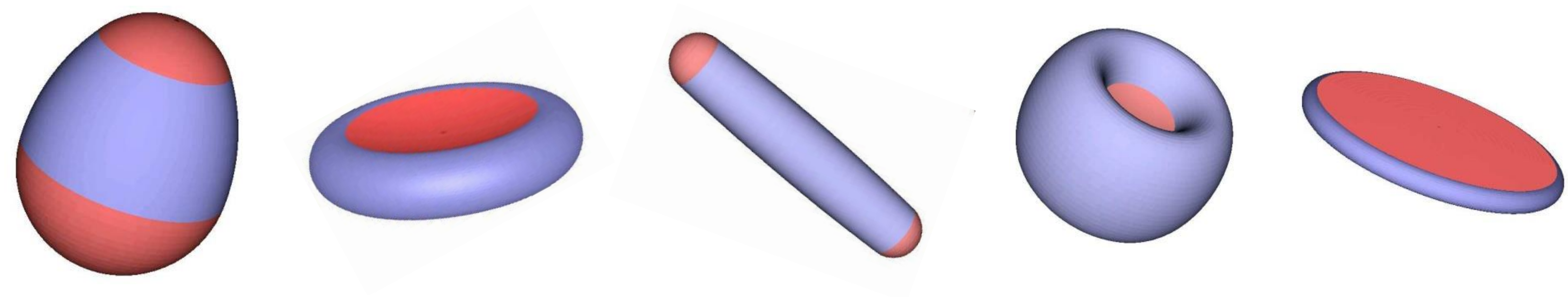


https://rocky.esss.co/software/

OVAL

Matthew R. Kuhn, USA, research code! ***kuhn@up.edu***

→ elements: surface composed of cylindical/spherical/toroidal etc. surfaces

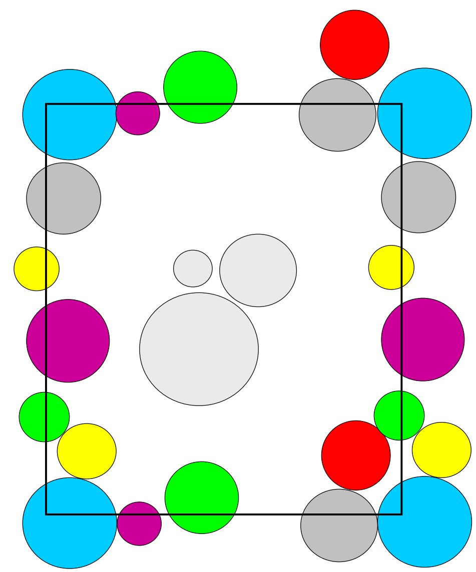


→ contacts:

frictional

→ boundaries: walls; or:

periodic boundaries



[positions & velocities]

→ input: command files,
output: data files, (figures)

THIS PRESENTATION

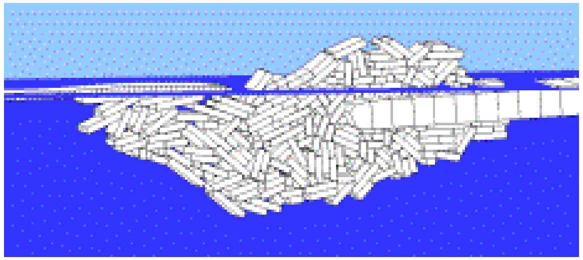
BALL-type models:

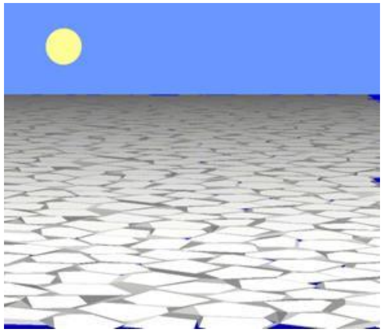
- History and definition
- Most important codes:
  - → PFC (BALL-type version)
  - → EDEM
  - → YADE
  - → Rocky-DEM
  - → OVAL
- Applications

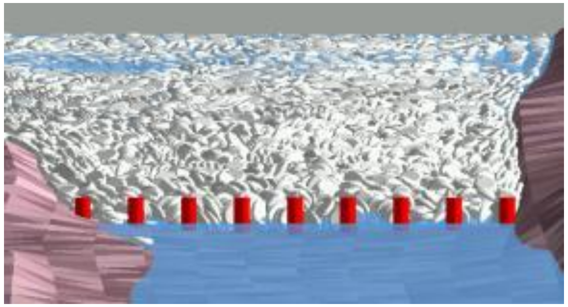
BALL-TYPE MODELS: OTHER APPLICATIONS

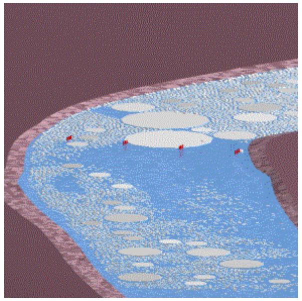
Floating ice blocks: effect on military vessels and structures

US Army Research Institute,
Cold Regions Department:

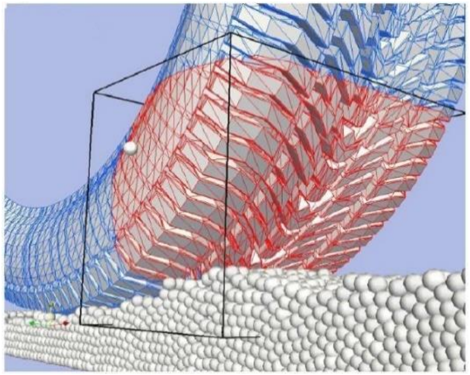








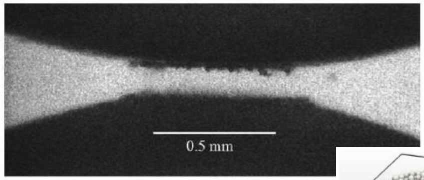
BALL-TYPE MODELS: OTHER APPLICATIONS



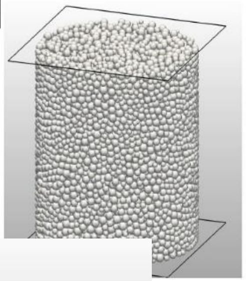
Tyres on snow

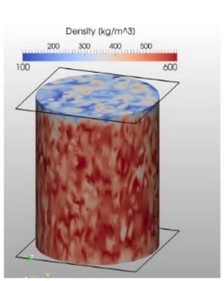
Michael (2014), „XDEM”:

→ Contact properties:

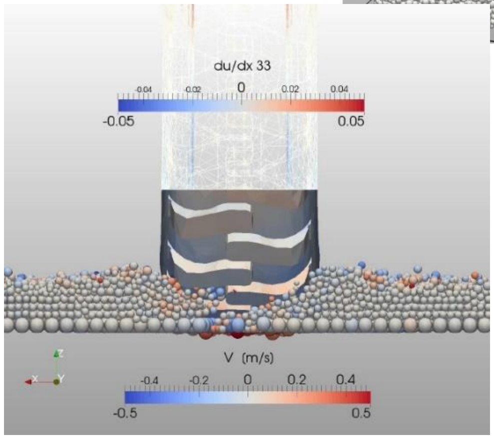


(1) direct analysis of the contacts



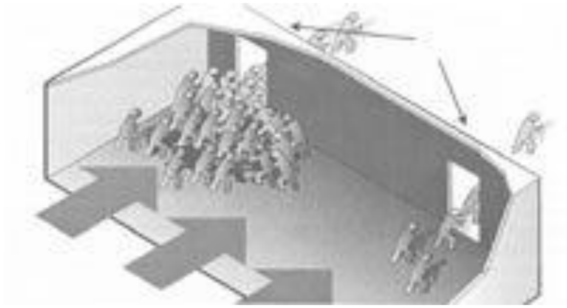


(2) macro-level („bulk”) behaviour

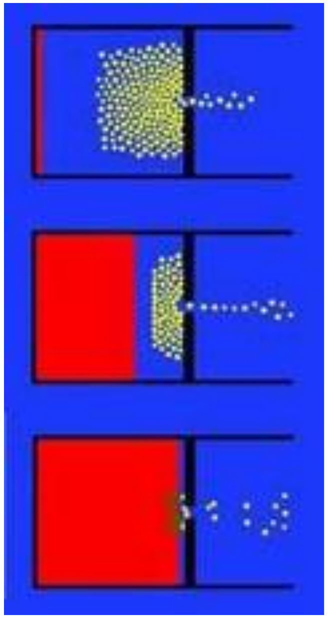


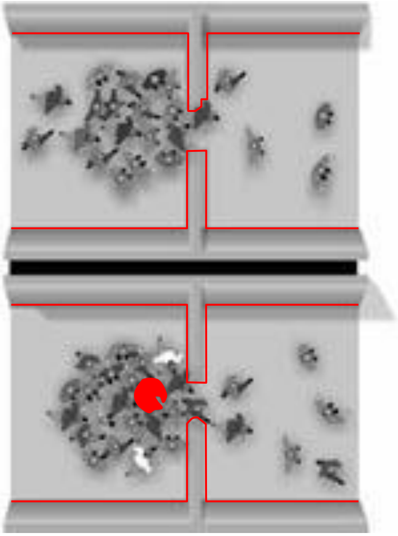
→ simulation of different profiles

BALL-TYPE MODELS:
OTHER APPLICATIONS



Architectural design: Simulation of a crowd in panic





https://www.youtube.com/watch?v=fPXr0HEwoD8 0:03 … 0:27

QUESTIONS

1. What are those **four** characteristics that define the BALL-type models?

2. In PFC, for **frictional** contacts, what is the difference between **linear** contact behaviour and **Hertz-Mindlin** contact behaviour? (Answer in maximum 2 sentences and with an explanatory drawing.)

3. In PFC, for **cemented** contacts, why is the extended contact called "**parallel bond**"?

4. Explain how "**local damping**" works in PFC. Explain how "**contact viscous damping**" works in PFC.

5. Summarize the main steps of the analysis of a timestep in PFC.

6. Explain what does „**periodic boundary**” mean.