UDEC / 3DEC

OVERVIEW OF DEM SOFTWARES

Quasi-static methods ← ***equilibrium states*** ***are searched for***

From an initial approximation of the equilibrium state searched for, the displacements **u** are to be determined taking the system to the equilibrium (assumption: time-independent behaviour, zero accelerations!!!)

$$"\mathbf{K} \cdot \Delta \mathbf{u} + \mathbf{f} = \mathbf{0}"$$

Time-stepping methods $"\mathbf{M} \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))"$ ← ***a process in time is searched for***

simulate the motion of the system along small, but finite Δt timesteps

Explicit timestepping methods:

→ Polyhedral elements, e.g. UDEC *rigid / deformable elements; deformable contacts*

→ BALL-type models, e.g. PFC *rigid elements; deformable contacts*

Implicit timestepping methods:

→ DDA („Discontinuous Deformation Analysis”) *deformable polyhedral elements*

→ Contact Dynamics models *rigid elements, non-deformable contacts*

THIS PRESENTATION

3DEC:

- Origins of UDEC / 3DEC
- Elements
- Contacts in 3D
- Time integration
  - → The "mass of the node" and the reduced force vector
  - → How to calculate the displacement increments during Δt
  - → Methods to help numerical stability
  - → Summary: Main steps of the analysis of a timestep
- UDEC practical applications
- 3DEC practical applications
- Questions

UDEC / 3DEC

<u>Origins:</u> „Universal Distinct Element Code”

P.A. Cundall, PhD thesis, 1971;

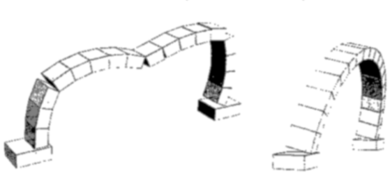
Development through decades

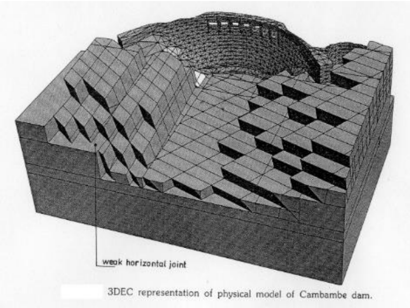
3D: early 1990ies

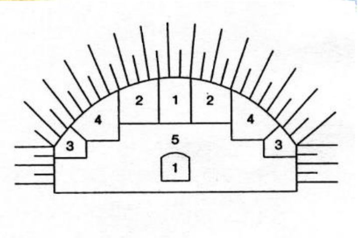
Itasca Consulting Group: graphics, I/O system, manuals, sample applications (www.itascacg.com)

from the 1990ies:

MOST WIDESPREAD DEM CODE IN CIVIL ENGINEERING







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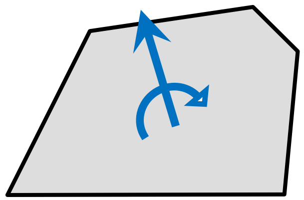
3DEC BASICS – THE ELEMENTS

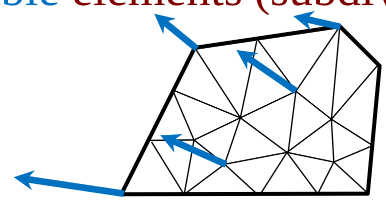
Elements: polygons / polyhedra (planar faces!);

– rigid elements



degrees of freedom:
translation of and rotation about the centroid

– deformable elements (subdivided into simplex zones)



„uniform strain” tetrahedral zones
((*10-node tetrahedra – not worth*))

degrees of freedom: translations of the nodes

e.g. marble: K = 37,2 GPa; G = 22,3 GPa
e.g. granite: K = 43,9 GPa; G = 30,9 GPa
e.g. sandstone: K = 26,8 GPa; G = 7 GPa; Mohr-Coulomb, fric = 28°; coh = 27,2 MPa; tens = 1,17 MPa

$$K = \frac{E}{3(1-2\nu)}; G = \frac{E}{2(1+\nu)}$$

where $\sigma_0 = K(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

Material models for the elements:

(rigid) ↔ deformable:

– „null element” (no material in the element)

default → – linearly elastic, isotropic *(e.g. intact rock; metal)*

– lin. elast., with: Mohr-Coulomb / Prager-Drucker failure crit.
(e.g. soils, concrete) *(e.g. clay)*
+ tensile strengh + cohesion + dilation angle

– …

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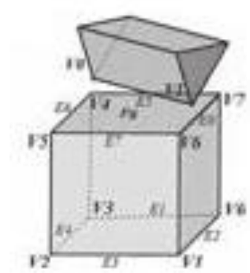
3DEC BASICS – CONTACTS

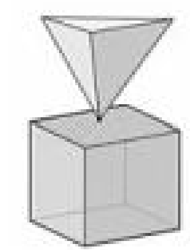
Contacts in 3D:

Types:

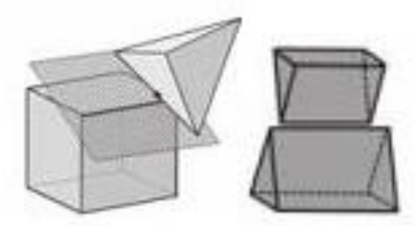
face-to-face



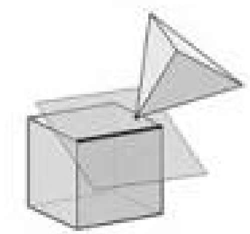
edge-to-face



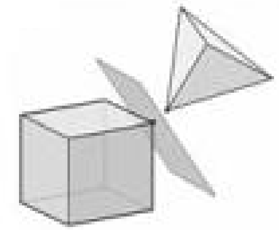
corner-to-face



edge-to-edge



corner-to-edge



corner-to-corner

The aim: ← **„common-plane" technique**

→ **recognize** the contacts;

→ produce their AREA ($A^{(k)}$) and their CONTACT NORMAL ($\mathbf{n}^{(k)}$) in such a way that **abrupt changes** during block motions are **avoided**

→ mechanical model: $\Delta\boldsymbol{\sigma}_n^{(k)} = -k_n \Delta u_n^{(k)}$ $\Delta\boldsymbol{\sigma}_t^{(k)} = k_s \Delta u_t^{(k)}$

3DEC BASICS – CONTACTS

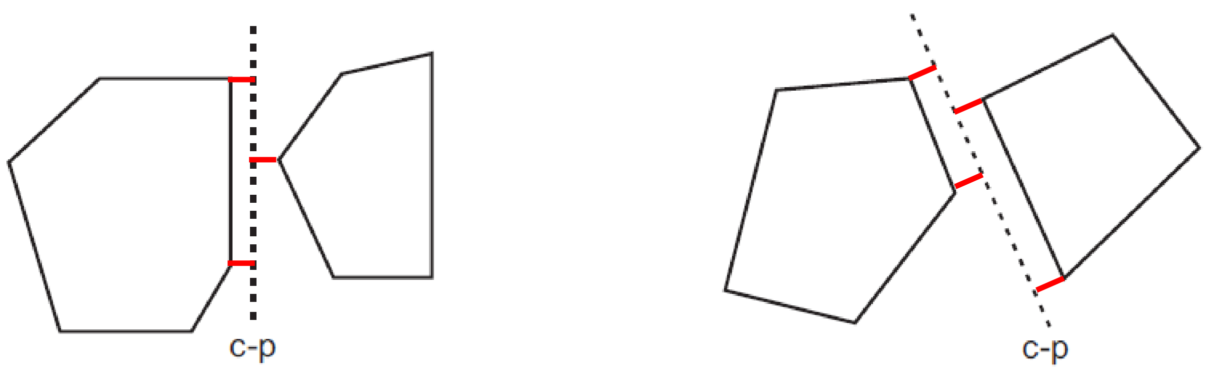
Contacts in 3D:

To recognize if there is a contact:

„common-plane” concept:

„***Maximize*** the gap between the common-plane and the node with the smallest gap.” or, equivalently:

„Locate the plane to have the ***largest*** minimal distance between any nodes of the two polyhedra and the common-plane.”



the common-plane is found with a small OPTIMIZATION SUBROUTINE

⇒ if this gap turns out to be **NEGATIVE**: ⇒ means that a **contact** is found

3DEC BASICS – CONTACTS

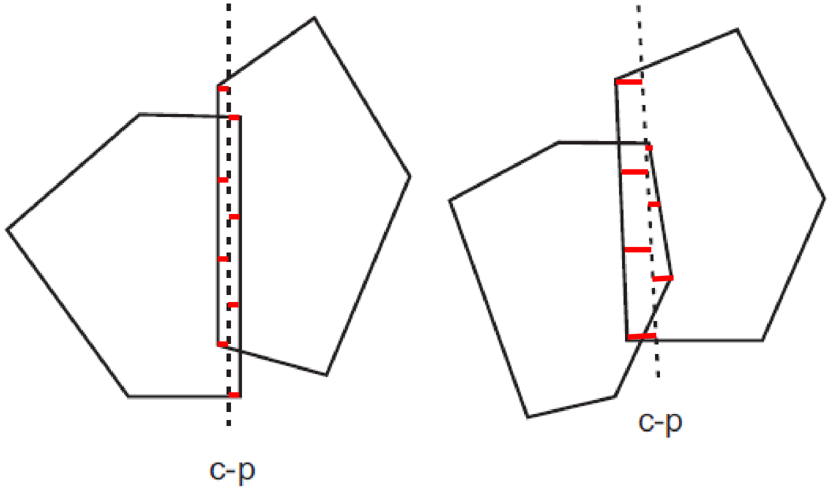
Contacts in 3D:

if the **gap** is **negative**, i.e. a **positive overlap**:

„common-plane" concept:

„***Minimize*** the overlap between the common-plane and the node with the greatest overlap." or, equivalently:

„Locate the plane to have the ***smallest*** maximal distance between nodes on the other side and the common-plane."



⇒ contact normal ✓

⇒ then **separately for the two elements**:

assign a **sub-contact** to every node

which is on the other side of the c-p

result: two sets of sub-contacts

[see on next slide how]

3DEC BASICS – CONTACTS

Contacts in 3D:

What to mean by AREA ($A^{(k)}$) and CONTACT NORMAL ($\mathbf{n}^{(k)}$) of a subcontact?

The definition of the **sub-contact system**:

[prepared twice, independently from both sides]

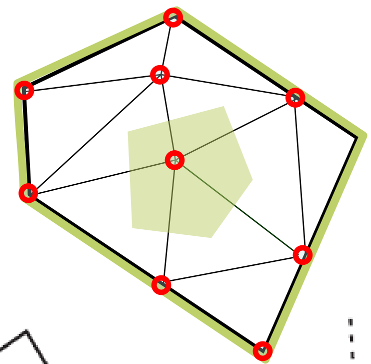
→ draw ⊥ from the nodes to the c-p

→ area assigned to a sub-contact: **1/3** [≈ !]

$\Rightarrow A^{(k)}$ ✓ (≈ area of Voronoi-cell)

→ special treatment at the edges

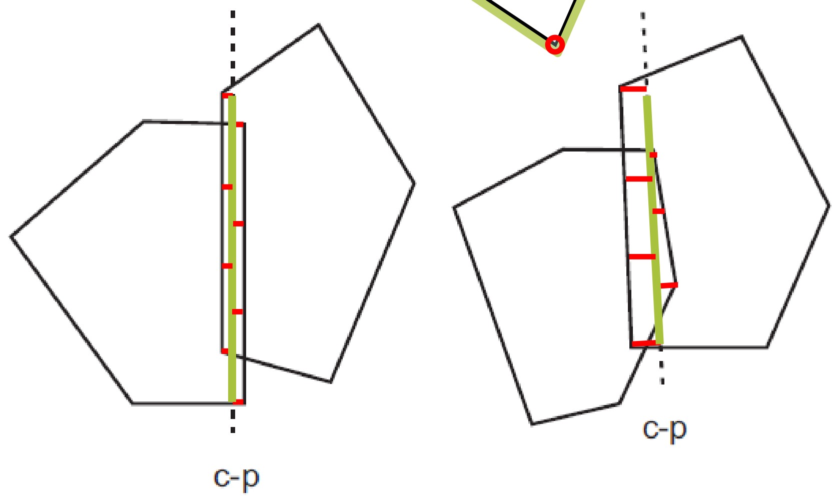
(not detailed here)



this view: ⊥ to the common plane

(Rigid blocks:

surface discretized; then similarly)



Sub-contact deformation increment during Δt:

relative vel. of a node and its projection on the **other face**: $\mathbf{v}_{rel}^{(k)} = \mathbf{v}_{node}^{(k)} - \mathbf{v}_{opposite\ face}^{(k)}$

during Δt : $\mathbf{v}_{rel}^{(k)} \cdot \Delta t \quad \Rightarrow \Delta u_n^{(k)}; \Delta u_s^{(k)}$

3DEC BASICS – CONTACTS

Distributed forces along the sub-contacts:

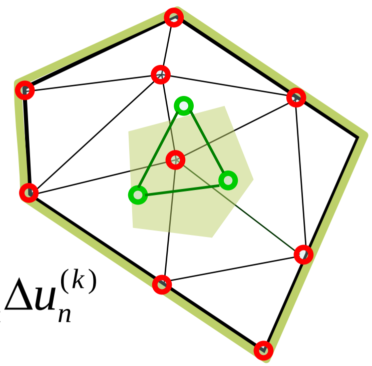
sub-contact area: $A^{(k)}$; normal direction: $\mathbf{n}^{(k)}$

Sub-contact deformation increment during Δt:

$$\mathbf{v}_{rel}^{(k)} = \mathbf{v}_{node}^{(k)} - \mathbf{v}_{opposite\ face}^{(k)}$$

by linear interpolation

$$\mathbf{v}_{rel}^{(k)} \cdot \Delta t \quad \Rightarrow \Delta u_n^{(k)}; \Delta u_t^{(k)}$$



this view: ⊥ to the common plane

because of the normal and shear stiffness:

→ increment of contact normal stress: $\Delta \boldsymbol{\sigma}_n^{(k)} = -k_n \Delta u_n^{(k)}$

(**uniformly** distributed contact force)

→ increment of contact shear stress: $\Delta \boldsymbol{\sigma}_t^{(k)} = k_s \Delta u_t^{(k)}$

(**uniformly** distributed contact force)

Resultant force assigned to the node; ← „**subcontact force**” ↓

opposite resultant distributed among the three nodes on the opposite face

After doing the same also for all the face nodes of other block:

two sets of nodal forces („**sub-contact forces**”) are gained for both blocks!

⇒ ½ („averaged”), for every node, on both faces

3DEC BASICS – CONTACTS

Material models for the contacts:

[aim: calculate the increments of distrib. contact forces from the increments of rel.disps]

– if no material in the contacts: → k_n, k_s: numerical parameters, ∞; friction: real value

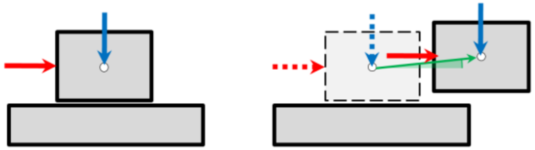
– if material is in the joints: (modelled as length or area, with zero thickness):

default → – linear behaviour for compression and shear, Coulomb-friction,
+ cohesion and tensile strength

– linear behaviour for compression and shear, Coulomb-friction,
+ cohesion & tensile strength + softening + dilation angle

$$\Delta U_n(dil) = \Delta U_s \tan\psi$$

– others …



examples for characteristic values:

normal and shear stiffness: 10 – 100 MPa/m … … 100 GPa/m
(*soft, with clay*) … … (*hard rock, healed*)

friction angle: 10° … … 50°

cohesion and tensile strength: from 0 … … till the strength of intact rock… …

dilation angle: 0° … 10°

THIS PRESENTATION

3DEC:

Origins of UDEC / 3DEC

Elements

Contacts in 3D

Time integration

→ The “mass of the node” and the reduced force vector

→ How to calculate the displacement increments during Δt

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→ Summary: Main steps of the analysis of a timestep

UDEC practical applications

3DEC practical applications

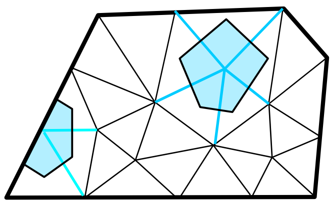
Questions

3DEC BASICS – THE TIME INTEGRATION

Calculation of nodal displacements

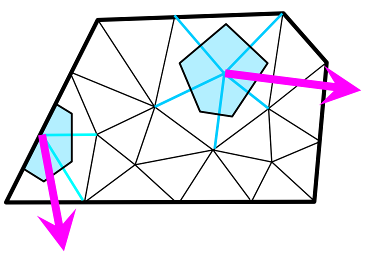
Newton II.: „ $m a = f$ ”

– mass assigned to the node:



Voronoi-cell

– force on the node: resultant of the forces acting on the Voronoi-cell of the node



← from the neighbouring element

← from external forces (e.g. self weight, drag force)

← from the stresses inside the simplexes

- steps to get the force from the stress inside a simplex:
  - nodal translations ⇒ simplex strain ✓
  - from this and material characteristics ⇒ uniform stress in the simplex ✓
  - stress vector acting on the face of the cell: $\sigma_{ij} n_j = p_i$; resultant ✓

3DEC BASICS – THE TIME INTEGRATION

Calculation of nodal displacements

Newton II.: „ $m\,a = f$ ”

– discretized form of the eqs of motion:

$$m\frac{\mathbf{v}(t_i+\Delta t/2)-\mathbf{v}(t_i-\Delta t/2)}{\Delta t}=\mathbf{f}(t_i)$$

or:

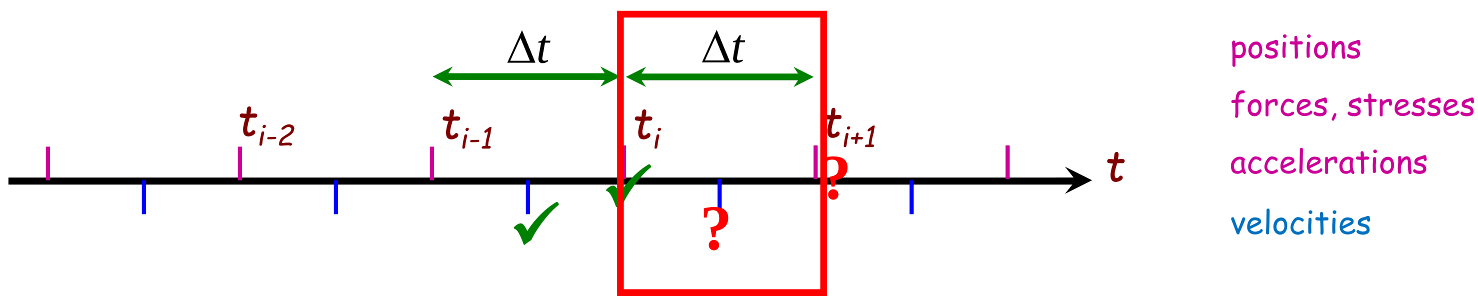
$$\mathbf{v}(t_i+\Delta t/2)=\mathbf{v}(t_i-\Delta t/2)+\frac{\mathbf{f}(t_i)}{m}\Delta t$$

– at t_i : the ***positions of the nodes*** and the ***forces and stresses*** are known;

at $t_i-\Delta t/2$: the ***nodal velocities*** are known;

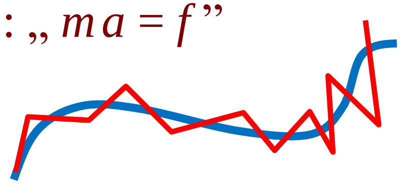
determine the ***nodal velocities*** at $t_{i+1/2}=t_i+\Delta t/2$

and the ***positions of the nodes*** at $t_{i+1}=t_i+\Delta t$



3DEC BASICS – THE TIME INTEGRATION

Calculation of nodal displacements Newton II.: „ $m\,a = f$ ”



– series of small finite time steps:

– main disadvantages: explicit; no stiffness matrix!!!

$\Rightarrow$ numerical instabilities, convergence problems

– to help numerical stability:

1. estimate the longest allowed Δt

ELASTIC SYSTEMS:

requirement for deformation calculations: $\Delta t \le \Delta t_{nodes} = \min\limits_{(nodes)} \left\{ 2\sqrt{\frac{m_{node}}{k_{node}}} \right\}$

requirement for contact deformation calculations:

$$\Delta t \le \Delta t_{blocks} = \gamma \cdot \left\{ 2\sqrt{\frac{\min\limits_{(blocks)}\left(mass_{block}\right)}{\max\limits_{(\mathbf{joints})}\left(k_{\mathbf{joints}}\right)}} \right\}$$

$\uparrow$

default: $\gamma := 0.10$

$$\Delta t := \min_{(p)} \begin{cases} \Delta t_{nodes} \\ \Delta t_{blocks} \end{cases}$$

NOT ELASTIC!!! $\rightarrow$ friction; damping; plastic yield; …

3DEC BASICS – THE TIME INTEGRATION

– to help numerical stability:

2. Artifical dampings

Velocity-proportional damping:

$$m\frac{\mathbf{v}(t_i+\Delta t/2)-\mathbf{v}(t_i-\Delta t/2)}{\Delta t}+c_v\mathbf{v}(t_i-\Delta t/2)=\mathbf{f}(t_i)$$

$$\mathbf{v}(t_i+\Delta t/2)=\mathbf{v}(t_i-\Delta t/2)+\left(\mathbf{f}(t_i)-c_v\mathbf{v}(t_i-\Delta t/2)\right)\frac{\Delta t}{m}$$

„local damping”: ← *default*

$$v_x(t_i+\Delta t/2)=v_x(t_i-\Delta t/2)+\left(f_x(t_i)-\alpha\cdot|f_x(t_i)|\frac{v_x(t_i-\Delta t/2)}{|v_x(t_i-\Delta t/2)|}\right)\frac{\Delta t}{m}$$

↑ *default:* α := 0.80

advantageous if some parts of the system are already equilibrated, others are just collapsing

„adaptive global damping” or „auto damping”:

≅ velocity-proportional damping, with coefficients being adjusted, so that the change of kinetic energy during Δt is decreased (eg 50%)

advantageous if the whole system oscillates around the equilibrium

3DEC BASICS – THE TIME INTEGRATION

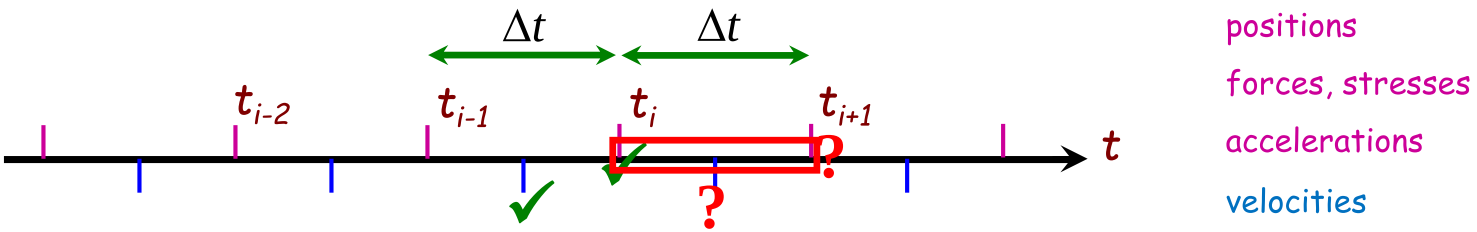
Summary: Main steps of the analysis of a timestep:

→ determine the **reduced force vector** for every node;

→ determine the new **velocities**;   ← [consider damping if exists]

→ determine the **translation increments** and the **new positions**;

→ **check contacts states** (lost? new? sliding/broken?) and **upgrade contact forces**



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UDEC practical applications

3DEC practical applications

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UDEC PRACTICAL APPLICATIONS

Ice hockey cavern, Norway, Gjovick:

The problem:

- ⇒ Fractured rock
- ⇒ Large dimensions

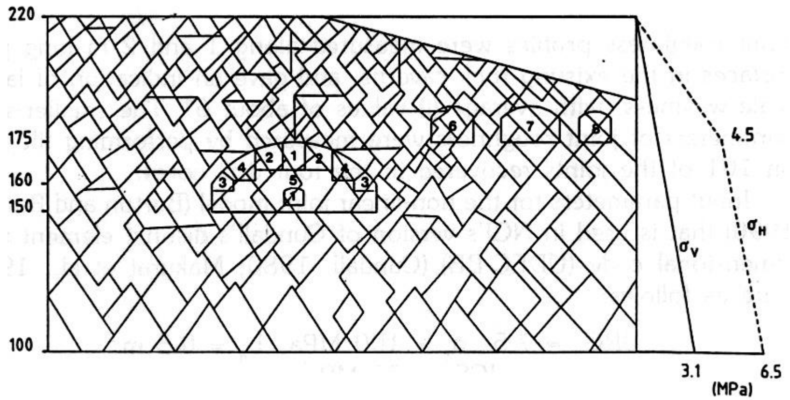
„The ice hockey cavern has a finished span of 62 m, a length 91 m and a height of 24 m. The spectator capacity is currently 5300, making it far the largest cavern for public use in the world. As is typical when one is extending the limits of experience and technology, the initial skepticism that had to be overcome was formidable."

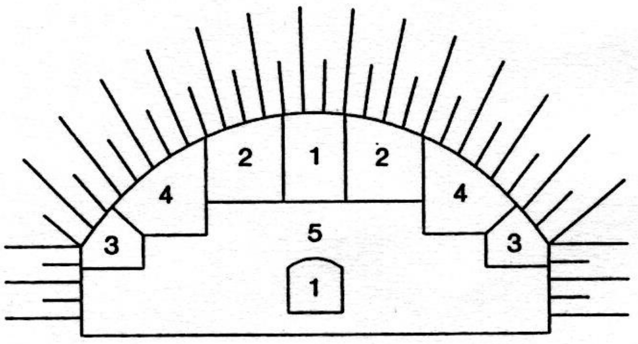
(completion: 1993)

UDEC PRACTICAL APPLICATIONS

Ice hockey cavern, Norway, Gjovick:

- Soil: fractured gneiss
  joint systems: 5
  no clay ☺
- Geological state wellknown:
  existing caves + drill
  ⇒ material characteristics ✓
  ⇒ initial stress state ✓
- the structure:
  cables / bars;
  shotcrete
- Numerical model:
  UDEC (2D)
  deformable elements

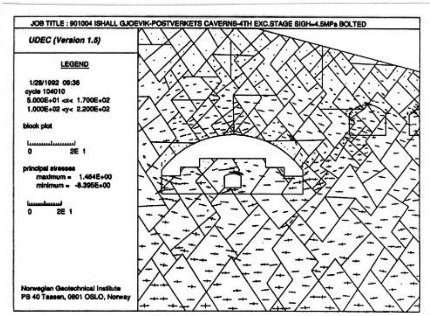


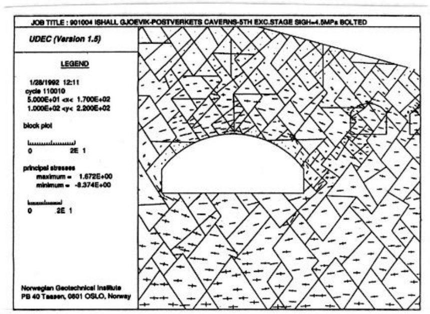


UDEC PRACTICAL APPLICATIONS

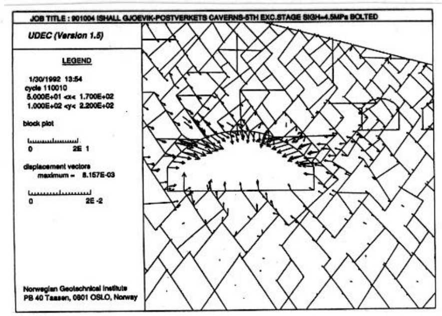
Ice hockey cavern, Norway, Gjovick:

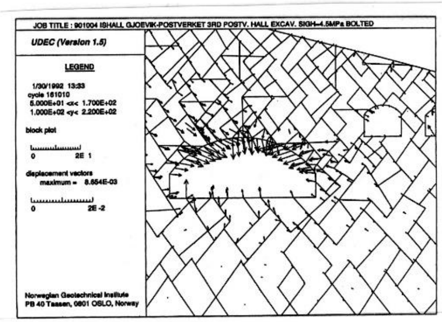
e.g. development of principal stresses:





e.g. translations:





UDEC PRACTICAL APPLICATIONS

Ice hockey cavern, Norway, Gjovick:

UDEC:

Table 1. Summary of Gjøvik Olympic cavern run (with Postal service caverns)

Parameter	Step 1	Step 2	Step 3	Step 4	Step 5	Excav. of 1st cavern	Excav. of 2nd cavern	Excav. of 3rd cavern
Maximum principal stress MPa	9.29	11.49	9.91	8.39	8.37	8.56	8.71	8.83
Maximum displacement (mm)								
total	1.85	1.80	2.63	6.99	8.16	8.28	8.43	8.65
wall	—	—	—	1.33	3.78	3.88	3.92	3.97
crown (vertical component)	0.50	1.08	2.62	4.05	4.33	4.39	4.87	7.01
Maximum shear displacement (mm) along horizontal	1.11	1.54	2.49	3.51	4.67	5.67	5.54	5.56
joint crown	1.11	1.54	2.49	3.51	3.70	3.70	4.10	6.85
Maximum hydraulic aperture (mm) crown	0.69	1.01	1.62	2.64	2.86	3.68	3.72	4.13
Maximum axial forces on bolts (tnf)	7.0	25	25	25	25	25	25	25

Measured:

Table 2. Summary of Gjøvik Olympic cavern *in situ* measurements for Location E4. The number given refer to total deformation. (NGI extensometers (E4) + SINTEF (S2) + surface subsidence).

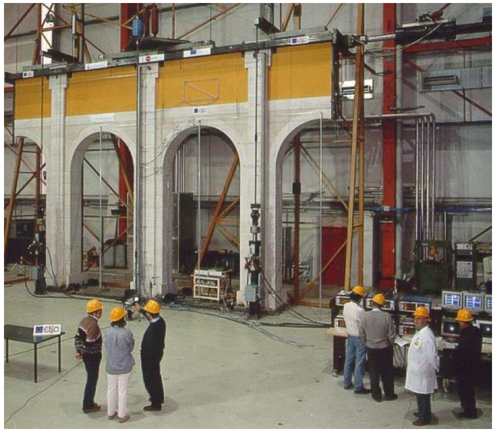
Parameter	Step 1	Step 2	Step 3	Step 4	Step 5
Total deformation (mm)	0.65	1.31	2.86	6.56	8.55

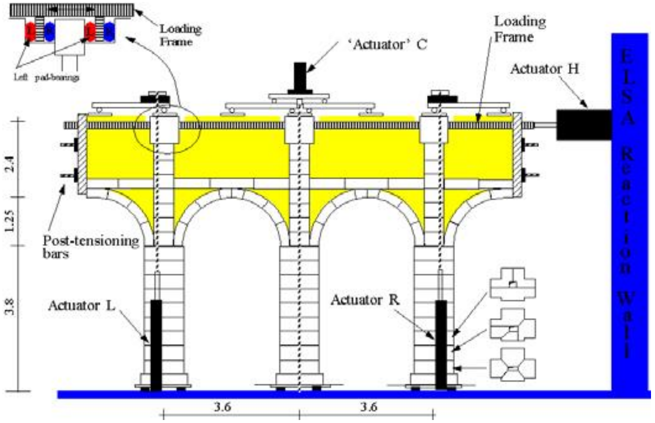
UDEC PRACTICAL APPLICATIONS

<u>Sao Vicente de Fora monastery, Portugal:</u>

previous studies:

- experiments
- FEM simulations





UDEC PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

Giordano et al, 2002: simulations with UDEC and with different FEM models

UDEC model:

→ geometry: 2D

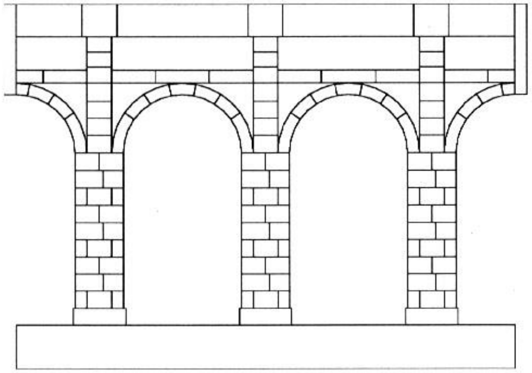


Fig. 18. UDEC discrete element model.

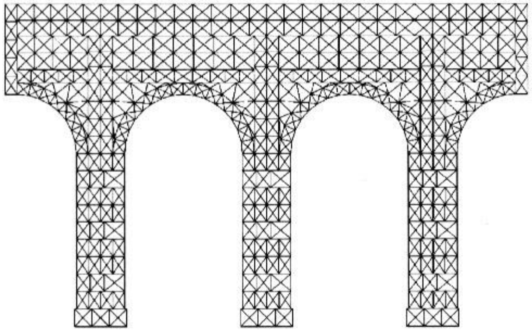


Fig. 19. UDEC internal finite element mesh.

UDEC PRACTICAL APPLICATIONS

Sao Vicente de Fora monastery, Portugal:

→ material parameters:

blocks:

Table 2
Parameter values for the CASTEM model

	Stones	Infill panels
Weight per unit volume (kg/m^3)	2500	2500
Young's modulus (Gpa)	65	6.5
Poisson's ratio	0.2	0.2
k_n: normal stiffness (Gpa)	115	
k_s: shear stiffness (Gpa)	47.9	
N_t: tensile strength	0	
ϕ: friction angle	30	
μ: dilatancy angle	5°	

contacts:

Table 3
Parameter values for the UDEC model

k_n: normal stiffness (Gpa)	115
k_s: shear stiffness (Gpa)	46
N_t: tensile strength	0
ϕ: friction angle	35
μ: dilatancy angle	0
c: cohesion	0

UDEC PRACTICAL APPLICATIONS

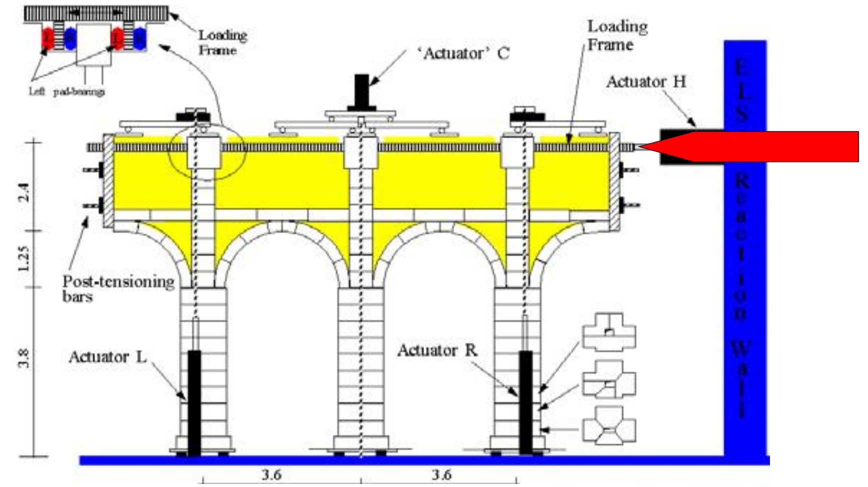
Sao Vicente de Fora monastery, Portugal:

→ loading process:

constant vertical load;

lateral „force”: disp-controlled,

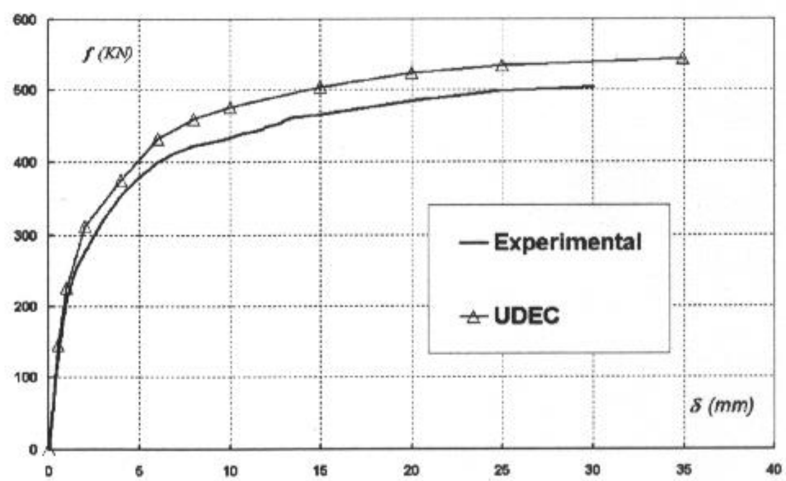
increasing translation



→ force-displacement-diagram:

[filling: linear elastic,

isotropic model]



→ UDEC advantages:

large displacements O.K.,

crack opening O.K.

THIS PRESENTATION

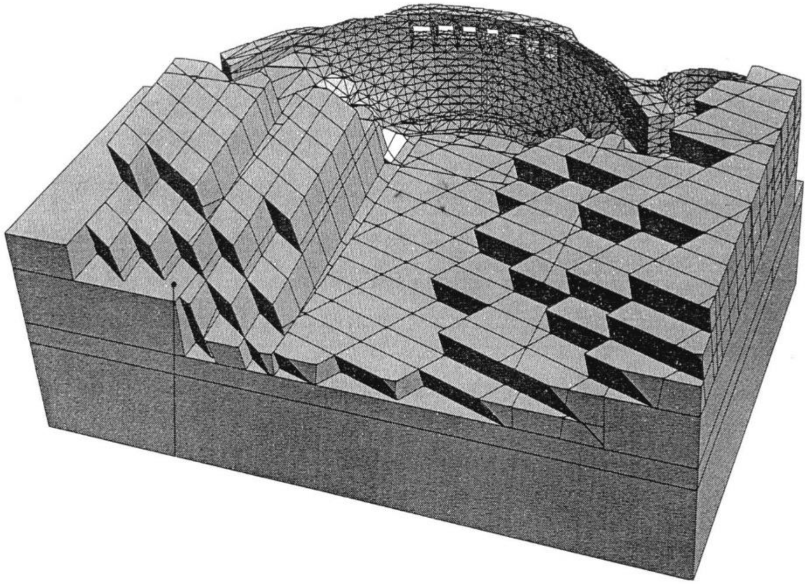
3DEC:

- Origins of UDEC / 3DEC
- Elements
- Contacts in 3D
- Time integration
  - → The "mass of the node" and the reduced force vector
  - → How to calculate the displacement increments during Δt
  - → Methods to help numerical stability
  - → Summary: Main steps of the analysis of a timestep
- UDEC practical applications
- 3DEC practical applications
- Questions

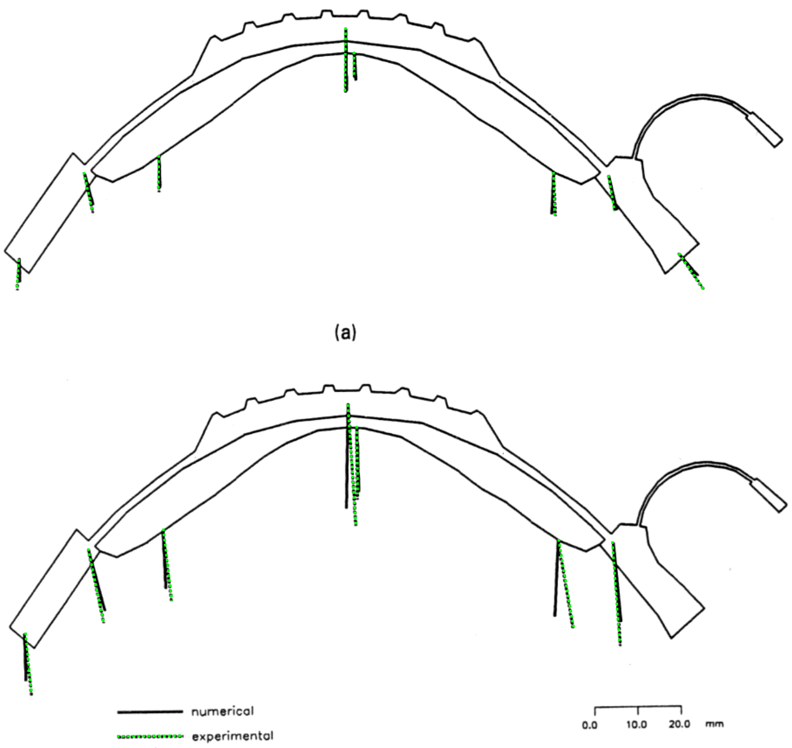
3DEC PRACTICAL APPLICATIONS

Cambambe dam 1995

Discrete element model: 3DEC + FEM



Measurements:
(translations at different water levels)

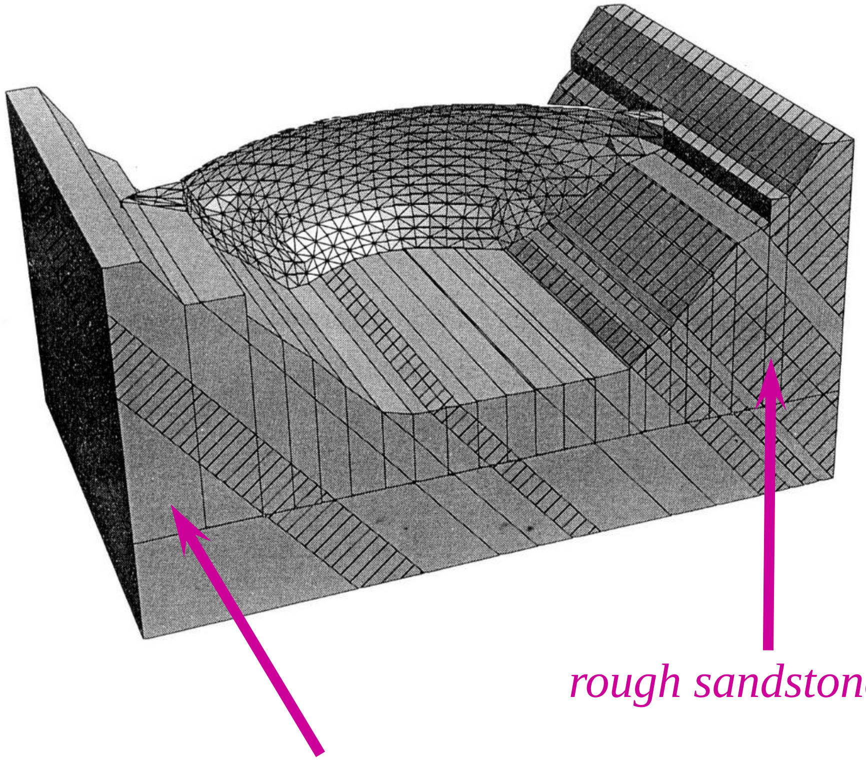


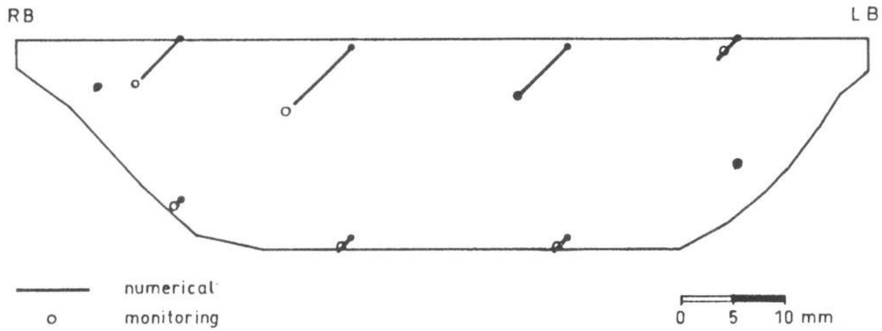
—— 3DEC

—— measured

3DEC PRACTICAL APPLICATIONS

Funcho dam (Heterogeneous rock; + strongly unsymmetric)





rough sandstone, E = 30 GPa

clay slate, E = 3 GPa

OWN EXAMPLES

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QUESTIONS

1. Give two alternative definition of the „**common plane**” between two polyhedra.

2. Explain how to determine the **area of a sub-contact** in 3DEC.

3. In 3DEC, what is understood on the "**relative velocity**" belonging to a **node** in a contact?

4. In 3DEC, how is the **subcontact force** calculated (i.e. when the calculation of the actual timestep has been finished and the displacement increments have been found)?

6. Explain the main steps of how a **time step** is analysed in 3DEC.

7. Introduce the two kinds of **damping** used in 3DEC.