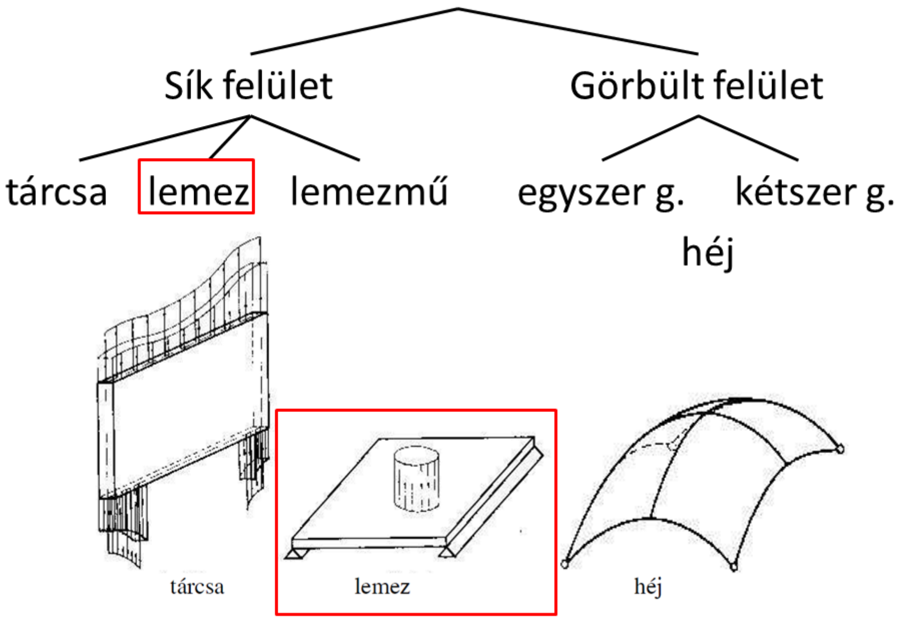
MŰEGYETEM 1782
Tartószerkezetek Mechanikája Tanszék

Tartók statikája II

Lemezek számítása

Dr. Hortobágyi Zsolt

Felületszerkezetek



Lemezelméletek

Klasszikus lemezelmélet (*Kirchhoff-Love*, 1888): Az Euler-Bernoulli gerendaelmélet kiterjesztése lemezekre:

- a középfelületre merőleges egyenesek az alakváltozás után is egyenesek és merőlegesek maradnak a középfelületre (***nyírási alakváltozás elhanyagolása***)
- a lemez vastagsága nem változik a lemez alakváltozása során

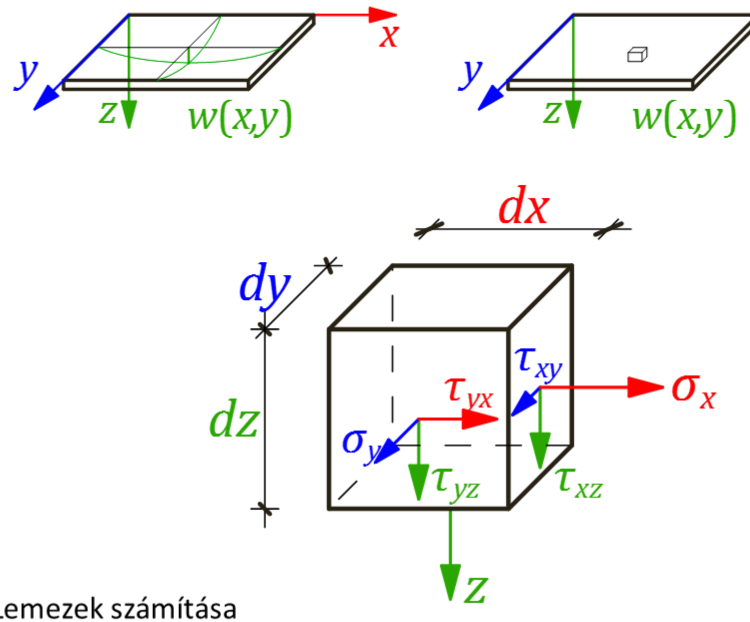
Mindlin-Reissner lemezelmélet (1945)

- a középfelületre merőleges egyenesek az alakváltozás után is egyenesek lesznek, de nem maradnak merőlegesek a középfelületre (***nyírási alakváltozás figyelembe vétele***)

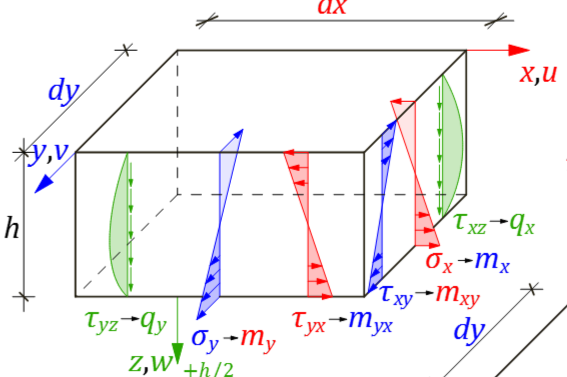
Alapfeltevések

- a lemez anyaga homogén, izotrop, lineárisan rugalmas
- a lemez vastagsága állandó
- a lemez vékony ($h/l_{min}<0.1$)
- a lemez kis elmozdulást végez ($w<0.2h$), a középsík normálisán lévő pontok az alakváltozás után is a normálison maradnak
- elsőrendű elmélet
- nem keletkeznek a lemez síkjában ható reakcióerők

Feszültségek a lemezben



Igénybevételek a lemezben



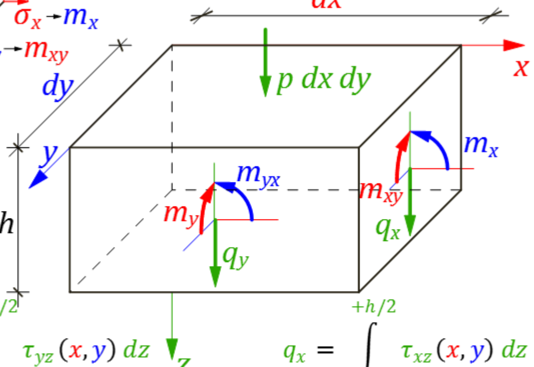
$$m_x = \int_{-h/2}^{+h/2} \sigma_x(x,y)\, z\, dz$$

$$m_{xy} = \int_{-h/2}^{+h/2} \tau_{xy}(x,y)\, z\, dz$$

$$m_{xy} = -m_{yx}$$

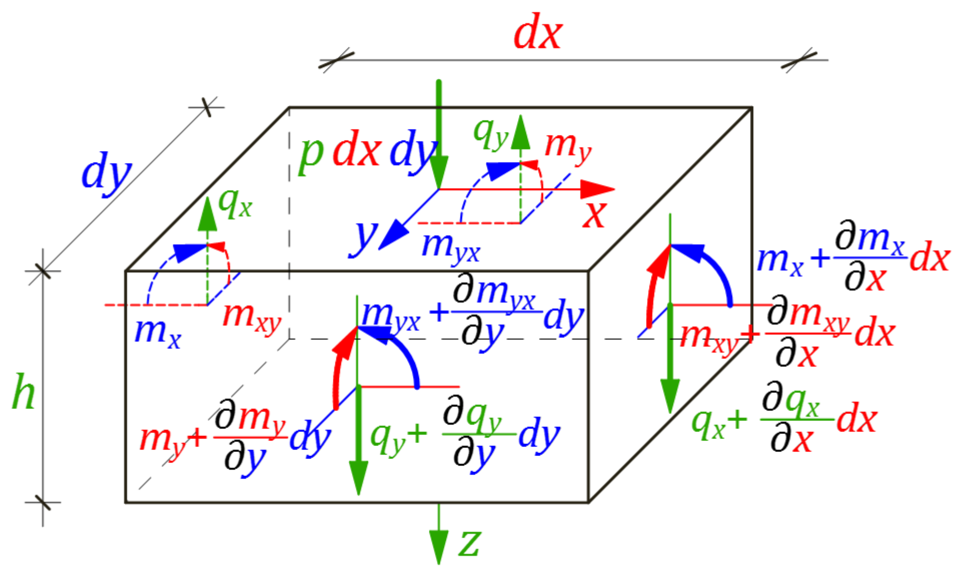
$$m_y = \int_{-h/2}^{+h/2} \sigma_y(x,y)\, z\, dz$$

$$m_{yx} = \int_{-h/2}^{+h/2} \tau_{yx}(x,y)\, z\, dz$$



$$q_y = \int_{-h/2}^{+h/2} \tau_{yz}(x,y)\, dz \qquad q_x = \int_{-h/2}^{+h/2} \tau_{xz}(x,y)\, dz$$

Egyensúlyi egyenlet



Egyensúlyi egyenlet

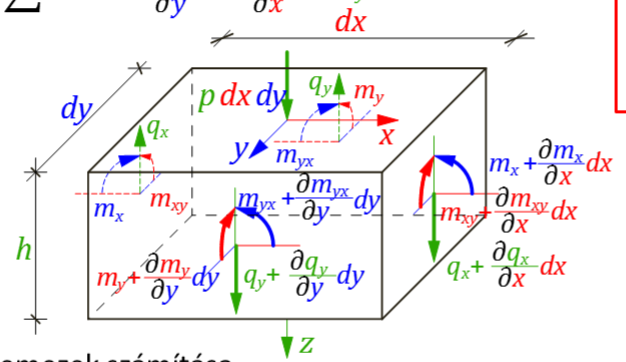
$$\sum F_{iz}: \frac{\partial q_x}{\partial x} dx dy + \frac{\partial q_y}{\partial y} dy dx + p dx dy = 0$$

$$\boxed{\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + p = 0}$$

$$\sum M_{ix}: -\frac{\partial m_y}{\partial y} dy dx - \frac{\partial m_{xy}}{\partial x} dx dy + q_y dx dy + \frac{\partial q_y}{\partial y} dy dx \frac{dy}{2} = 0$$

$$\sum M_{ix}: -\frac{\partial m_y}{\partial y} - \frac{\partial m_{xy}}{\partial x} + q_y = 0$$

$$\boxed{q_y = \frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x} \qquad q_x = \frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y}}$$



Egyensúlyi egyenlet

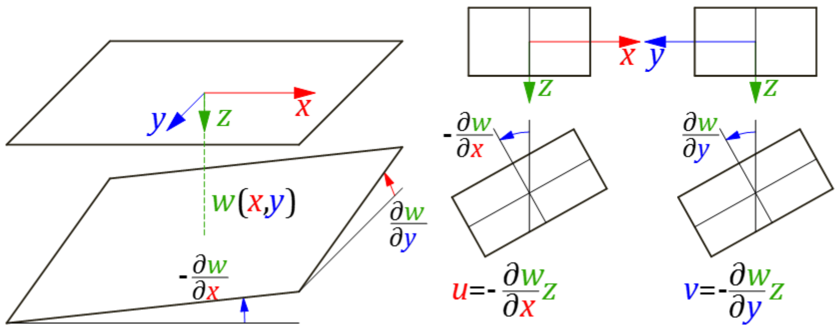
$$q_x = \frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y} \qquad q_y = \frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x}$$

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + p = 0$$

$$\frac{\partial\left(\frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y}\right)}{\partial x} + \frac{\partial\left(\frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x}\right)}{\partial y} + p = 0$$

$$\boxed{\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{yx}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + p = 0}$$

Geometriai egyenletek



$$\varepsilon_x(x,y,z) = \frac{\partial u}{\partial x} = -\frac{\partial^2 w(x,y)}{\partial x^2} z$$

$$\varepsilon_y(x,y,z) = \frac{\partial v}{\partial y} = -\frac{\partial^2 w(x,y)}{\partial y^2} z$$

$$\gamma_{xy}(x,y,z) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2\frac{\partial^2 w(x,y)}{\partial x \partial y} z$$

Anyagegyenletek

$$\sigma_x(x,y,z) = \frac{E}{1-\upsilon^2}(\varepsilon_x + \upsilon\varepsilon_y) = -\frac{E}{1-\upsilon^2}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right) z$$

$$\sigma_y(x,y,z) = \frac{E}{1-\upsilon^2}(\varepsilon_y + \upsilon\varepsilon_x) = -\frac{E}{1-\upsilon^2}\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial x^2}\right) z$$

$$\tau_{xy}(x,y,z) = G\gamma_{xy} = -\frac{E}{2(1+\upsilon)} 2\frac{\partial^2 w(x,y)}{\partial x \partial y} z = -\frac{E}{(1+\upsilon)}\frac{\partial^2 w(x,y)}{\partial x \partial y} z$$

$$m_x = \int_{-h/2}^{+h/2} \sigma_x(x,y)\, z\, dz = -\frac{E}{1-\upsilon^2}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right)\int_{-h/2}^{+h/2} z^2\, dz$$

$$\int_{-h/2}^{+h/2} z^2\, dz = \frac{h^3}{12} \qquad \boxed{K = \frac{Eh^3}{12(1-\upsilon^2)}} \quad \text{Lemezmerevség}$$

$$m_x = -K\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

Lemezegyenlet

$$m_x = \int_{-h/2}^{+h/2} \sigma_x(x,y)\, z\, dz = -K\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

$$m_y = \int_{-h/2}^{+h/2} \sigma_y(x,y)\, z\, dz = -K\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial x^2}\right)$$

$$m_{xy} = -m_{yx} = \int_{-h/2}^{+h/2} \tau_{xy}(x,y)\, z\, dz = -K(1-\upsilon)\frac{\partial^2 w(x,y)}{\partial x \partial y}$$

Lemezegyenlet

$$m_x = -K\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon \frac{\partial^2 w(x,y)}{\partial y^2}\right) \qquad m_y = -K\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \upsilon \frac{\partial^2 w(x,y)}{\partial x^2}\right) \qquad m_{xy} = -m_{yx} = -K(1-\upsilon)\frac{\partial^2 w(x,y)}{\partial x \partial y}$$

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{yx}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + p = 0$$

$$-K\left(\frac{\partial^2\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right)}{\partial x^2} + 2\frac{\partial^2\left((1-\upsilon)\frac{\partial^2 w(x,y)}{\partial x \partial y}\right)}{\partial x \partial y} + \frac{\partial^2\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial x^2}\right)}{\partial y^2}\right) + p = 0$$

$$-K\left(\frac{\partial^4 w(x,y)}{\partial x^4} + 2\frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y)}{\partial y^4}\right) + p = 0$$

$$\boxed{\frac{\partial^4 w(x,y)}{\partial x^4} + 2\frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y)}{\partial y^4} = \frac{p}{K}}$$

$$\boxed{\Delta\Delta w = \frac{p}{K}} \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{Laplace operátor}$$

Nyíróerők

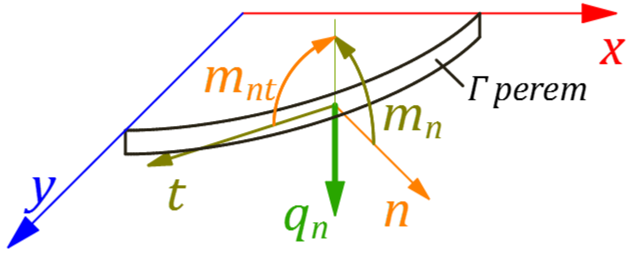
$$q_x = \frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y} = -K\left(\frac{\partial\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial y^2}\right)}{\partial x} + \frac{\partial\left((1-\upsilon)\frac{\partial^2 w(x,y)}{\partial x \partial y}\right)}{\partial y}\right)$$

$$= -K\left(\frac{\partial^3 w(x,y)}{\partial x^3} + \frac{\partial^3 w(x,y)}{\partial x \partial y^2}\right) = -K\frac{\partial}{\partial x}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) = -K\frac{\partial}{\partial x}(\Delta w)$$

$$q_x = \frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y} = -K\frac{\partial}{\partial x}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) = -K\frac{\partial}{\partial x}(\Delta w)$$

$$q_y = \frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x} = -K\frac{\partial}{\partial y}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) = -K\frac{\partial}{\partial y}(\Delta w)$$

Peremfeltételek

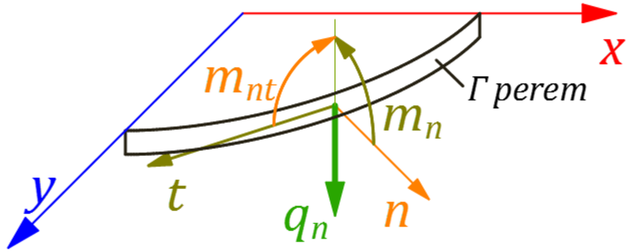


Merev befogás: $w_\Gamma = 0 \qquad \varphi_{t,\Gamma} = \left.\frac{\partial w}{\partial n}\right|_\Gamma = 0$

Csuklós megtámasztás:

$$w_\Gamma = 0 \qquad m_{n,\Gamma} = \left.\left(\frac{\partial^2 w(x,y)}{\partial n^2} + \upsilon\frac{\partial^2 w(x,y)}{\partial t^2}\right)\right|_\Gamma = 0$$

Peremfeltételek



Szabadszél: $m_{n,\Gamma} = 0 \qquad q_{n,\Gamma} = 0 \qquad m_{nt,\Gamma} = 0$

$$\boxed{\left.\left(\frac{\partial^2 w}{\partial n^2} + \upsilon\frac{\partial^2 w}{\partial t^2}\right)\right|_\Gamma = 0}$$

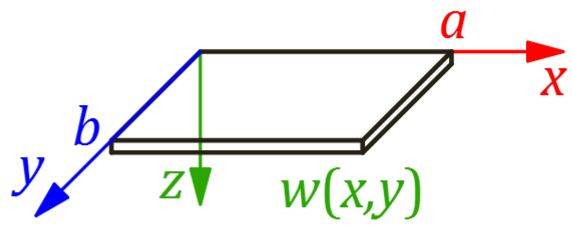
$$q_{n,\Gamma} = \left.\frac{\partial m_{nt}}{\partial t}\right|_\Gamma$$

$$\left.\left(q_{n,\Gamma} - \frac{\partial m_{nt}}{\partial t}\right)\right|_\Gamma = 0$$

$$-K\left.\left(\frac{\partial^3 w}{\partial n^3} + \frac{\partial^3 w}{\partial n \partial t^2}\right)\right|_\Gamma - K(1-\upsilon)\frac{\partial}{\partial t}\left.\left(\frac{\partial^2 w}{\partial n \partial t}\right)\right|_\Gamma = -K\left.\left(\frac{\partial^3 w}{\partial n^3} + (2-\upsilon)\frac{\partial^3 w}{\partial n \partial t^2}\right)\right|_\Gamma = 0$$

$$\boxed{\left.\left(\frac{\partial^3 w}{\partial n^3} + (2-\upsilon)\frac{\partial^3 w}{\partial n \partial t^2}\right)\right|_\Gamma = 0}$$

Peremfeltételek



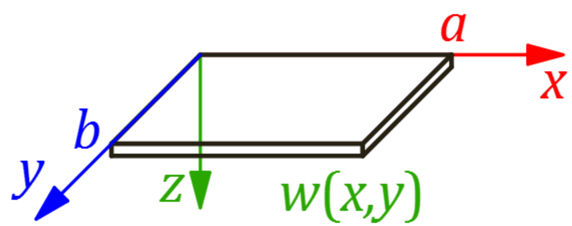
Befogás: pl. x=0, x=a $\quad w=0 \quad \varphi_y=\frac{\partial w}{\partial x}=0$

pl. y=0, y=b $\quad w=0 \quad \varphi_x=\frac{\partial w}{\partial y}=0$

Csukló: pl. x=0, x=a $\quad w=0 \quad \kappa_y=\frac{\partial^2 w}{\partial x^2}=0$ vagy $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=0$

pl. y=0, y=b $\quad w=0 \quad \kappa_x=\frac{\partial^2 w}{\partial y^2}=0$ vagy $\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}=0$

Peremfeltételek



Szabad perem: pl. x=0, x=a

$$m_x=\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}=0$$

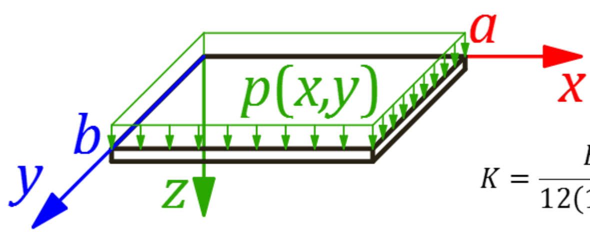
$$q_x=\frac{\partial^3 w}{\partial x^3}+(2-\upsilon)\frac{\partial^3 w}{\partial x\partial y^2}=0$$

pl. y=0, y=b

$$m_y=\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}=0$$

$$q_y=\frac{\partial^3 w}{\partial y^3}+(2-\upsilon)\frac{\partial^3 w}{\partial y\partial x^2}=0$$

Navier-féle megoldás



$$K=\frac{Eh^3}{12(1-\upsilon^2)}$$

$$\frac{\partial^4 w(x,y)}{\partial x^4}+2\frac{\partial^4 w(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y)}{\partial y^4}=\frac{p(x,y)}{K}$$

Csukló: pl. x=0, x=a $\quad w=0 \quad \kappa_y=\frac{\partial^2 w}{\partial x^2}=0$

pl. y=0, y=b $\quad w=0 \quad \kappa_x=\frac{\partial^2 w}{\partial y^2}=0$

Navier-féle megoldás

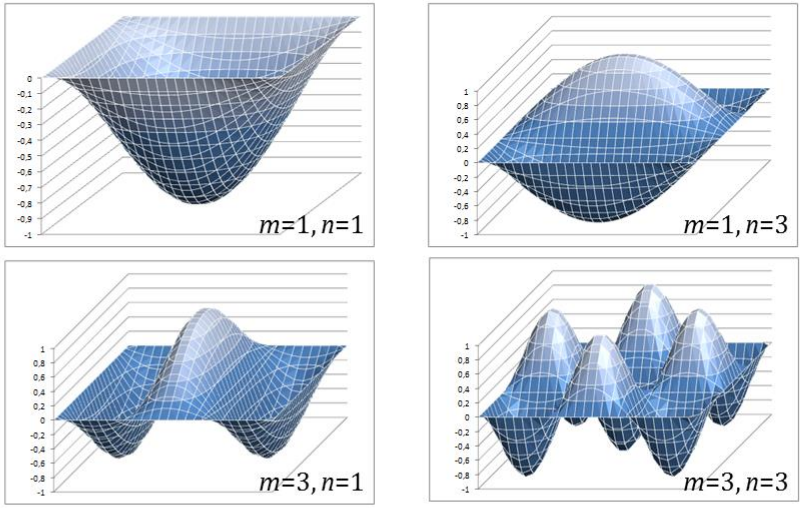
$$\frac{\partial^4 w(x,y)}{\partial x^4}+2\frac{\partial^4 w(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y)}{\partial y^4}=\frac{p(x,y)}{K}$$

$$p(x,y)=\sum_m\sum_n p_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$p_{m,n}=\frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dy\,dx$$

$$w_{mn}(x,y)=\sum_m\sum_n w_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

Bázisfüggvények



$$\varphi_{mn}(x,y)=\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

Navier-féle megoldás

$$\frac{\partial^4 w(x,y)}{\partial x^4}+2\frac{\partial^4 w(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y)}{\partial y^4}=\frac{p(x,y)}{K}$$

$$w_{mn}(x,y)=\sum_m\sum_n w_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\quad \text{Fourier-sor}$$

$$\frac{\partial^4 w_{mn}(x,y)}{\partial x^4}=\sum_m\sum_n\left(\frac{m\pi}{a}\right)^4 w_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\frac{\partial^4 w_{mn}(x,y)}{\partial x^2\partial y^2}=\sum_m\sum_n\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 w_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\frac{\partial^4 w_{mn}(x,y)}{\partial y^4}=\sum_m\sum_n\left(\frac{n\pi}{b}\right)^4 w_{m,n}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$w_{m,n}\left(\left(\frac{m\pi}{a}\right)^4+2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+\left(\frac{n\pi}{b}\right)^4\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=\frac{p_{m,n}}{K}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$w_{m,n}=\frac{p_{m,n}}{K\pi^4\left(\left(\frac{m}{a}\right)^4+2\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2+\left(\frac{n}{b}\right)^4\right)}=\frac{p_{m,n}}{K\pi^4\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}$$

Navier-féle megoldás

$$p(x,y)=p=\underline{\text{állandó}}$$

$$p_{m,n}=\frac{4}{ab}p\int_0^a\int_0^b\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dy\,dx=\frac{4p}{ab}\frac{a}{m\pi}\left[\cos\frac{m\pi x}{a}\right]_0^a\frac{b}{n\pi}\left[\cos\frac{n\pi y}{b}\right]_0^b=$$

$$=\begin{cases}0 & \text{ha } m \text{ vagy } n \text{ páros}\\ \dfrac{16p}{mn\pi^2} & \text{egyébként}\end{cases}$$

$$p_{1,1}=\frac{16p}{\pi^2}=1{,}6211p$$

$$p_{1,3}=p_{3,1}=\frac{16p}{3\pi^2}=0{,}5404p$$

$$p_{3,3}=\frac{16p}{9\pi^2}=0{,}1801p\qquad p_{3,5}=p_{5,3}=\frac{16p}{15\pi^2}=0{,}1081p\qquad p_{5,5}=\frac{16p}{25\pi^2}=0{,}0648p$$

$$w_{mn}(x,y)=\sum_m\sum_n\frac{\frac{16p}{mn\pi^2}}{K\pi^4\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$w_{mn}(x,y)=\frac{16p}{K\pi^6}\sum_m\sum_n\frac{1}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

Navier-féle megoldás

$$w_{mn}(x,y)=\frac{16p}{K\pi^6}\sum_m\sum_n\frac{1}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

Középső pont lehajlása

$$w_{mn}\left(\frac{a}{2},\frac{b}{2}\right)\qquad \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=\sin\frac{m\pi}{2}\sin\frac{n\pi}{2}=(-1)^{\frac{m+n}{2}-1}$$

a=b négyzet alaprajzú lemez esetén:

$$w_{mn}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16p}{K\pi^6}\sum_m\sum_n\frac{(-1)^{\frac{m+n}{2}-1}}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{a}\right)^2\right)^2}=\frac{16pa^4}{K\pi^6}\sum_m\sum_n\frac{(-1)^{\frac{m+n}{2}-1}}{mn((m)^2+(n)^2)^2}$$

Navier-féle megoldás

$$w_{mn}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16p}{K\pi^6}\sum_m\sum_n\frac{(-1)^{\frac{m+n}{2}-1}}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{a}\right)^2\right)^2}=\frac{16pa^4}{K\pi^6}\sum_m\sum_n\frac{(-1)^{\frac{m+n}{2}-1}}{mn((m)^2+(n)^2)^2}$$

$m=1$, $n=1$ $$w_{1,1}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16pa^4}{K\pi^6}\frac{(-1)^{\frac{1+1}{2}-1}}{1(1+1)^2}=\frac{16pa^4}{K\pi^6}\frac{1}{4}$$

$m=1$, $n=3$ $$w_{1,3}\left(\frac{a}{2},\frac{a}{2}\right)=w_{3,1}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16pa^4}{K\pi^6}\left(\frac{(-1)^{\frac{1+1}{2}-1}}{1(1+1)^2}+\frac{(-1)^{\frac{1+3}{2}-1}}{3(1+9)^2}\right)=\frac{16pa^4}{K\pi^6}\left(\frac{1}{4}-\frac{1}{300}\right)$$

$m=3$, $n=3$ $$w_{3,3}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16pa^4}{K\pi^6}\left(\frac{(-1)^{\frac{1+1}{2}-1}}{1(1+1)^2}+\frac{(-1)^{\frac{3+1}{2}-1}}{3(9+1)^2}+\frac{(-1)^{\frac{1+3}{2}-1}}{3(1+9)^2}+\frac{(-1)^{\frac{3+3}{2}-1}}{9(9+9)^2}\right)$$

$$=\frac{16pa^4}{K\pi^6}\left(\frac{1}{4}-\frac{1}{300}-\frac{1}{300}+\frac{1}{2916}\right)=\frac{16pa^4}{K\pi^6}0.2436763$$

$$w_{\infty,\infty}\left(\frac{a}{2},\frac{a}{2}\right)=\frac{16pa^4}{K\pi^6}0.2440939 \qquad \frac{0.2436763}{0.2440939}=99{,}8\%$$

Navier-féle megoldás

a,b oldalú téglalap alaprajzú lemez esetén:

$$w_{mn}(x,y)=\frac{16p}{K\pi^6}\sum_m\sum_n\frac{1}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$m_{x,mn}=-K\left(\frac{\partial^2 w_{mn}(x,y)}{\partial x^2}+\upsilon\frac{\partial^2 w_{mn}(x,y)}{\partial y^2}\right)$$

$$=K\sum_m\sum_n w_{m,n}\left(\left(\frac{m\pi}{a}\right)^2+\upsilon\left(\frac{n\pi}{b}\right)^2\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$=\frac{16p}{\pi^4}\sum_m\sum_n\frac{\left(\frac{m}{a}\right)^2+\upsilon\left(\frac{n}{b}\right)^2}{mn\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$a=b$ esetén a középső pont hajlítónyomatéka:

$$m_{x,mn}=\frac{16pa^2}{\pi^4}\sum_m\sum_n\frac{m^2+n^2\upsilon}{mn(m^2+n^2)^2}(-1)^{\frac{m+n}{2}-1}$$

Navier-féle megoldás

$a=b$ esetén a középső pont hajlítónyomatéka:

$$m_{x,mn}=\frac{16pa^2}{\pi^4}\sum_m\sum_n\frac{m^2+n^2\upsilon}{mn(m^2+n^2)^2}(-1)^{\frac{m+n}{2}-1}$$

$$m_{x,3,3}=\frac{16pa^2}{\pi^4}\left(\frac{(1+\upsilon)(-1)^{\frac{1+1}{2}-1}}{1(1+1)^2}+\frac{(1+9\upsilon)(-1)^{\frac{1+3}{2}-1}}{3(1+9)^2}+\frac{(9+\upsilon)(-1)^{\frac{3+1}{2}-1}}{3(9+1)^2}\right.$$

$$\left.+\frac{(9+9\upsilon)(-1)^{\frac{3+3}{2}-1}}{9(9+9)^2}\right)=\frac{16pa^2}{\pi^4}\left(\frac{(1+\upsilon)}{4}-\frac{(1+9\upsilon)}{300}-\frac{(9+\upsilon)}{300}+\frac{(9+9\upsilon)}{2916}\right)$$

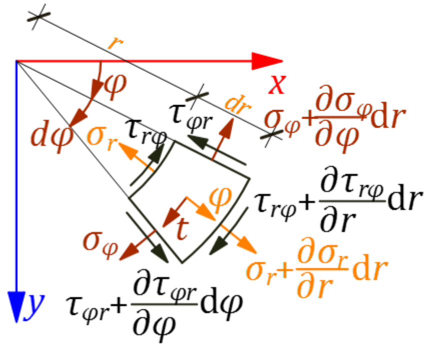
$$\upsilon=0{,}2 \quad m_{x,3,3}=\frac{16pa^2}{\pi^4}(0{,}3000-0{,}0093-0{,}0307+0{,}0037)=\frac{16pa^2}{\pi^4}0{,}2637$$

$$\upsilon=0{,}2 \quad m_{x,\infty,\infty}=\frac{16pa^2}{\pi^4}0{,}2691$$

$$\frac{0{,}2637}{0{,}2691}=98\%$$

Polárkoordinátás alak

$$x=r\cos\varphi \qquad y=r\sin\varphi \qquad r=\sqrt{x^2+y^2} \qquad \varphi=\mathrm{arctg}\frac{y}{x}$$



Laplace operátor:

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}$$

$$\Delta\Delta w(r,\varphi)=\frac{p(r,\varphi)}{K}$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\varphi^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{p(r,\varphi)}{K}$$

Körszimmetrikus alak

$$\Delta\Delta w(r,\varphi) = \frac{p(r,\varphi)}{K} \longrightarrow \Delta\Delta w(r) = \frac{p(r)}{K}$$

Laplace operátor: $\Delta() = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}()\right)$

$$\frac{1}{r}\cdot\left[\frac{d}{dr}\left[r\cdot\left[\frac{d}{dr}(f(r))\right]\right]\right] \text{ simplify } \rightarrow \frac{d^2}{dr^2}f(r) + \frac{\frac{d}{dr}f(r)}{r}$$

$$\Delta\Delta w = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right)\right) = \frac{\partial^4 w}{\partial r^4} + \frac{1}{r^3}\frac{\partial w}{\partial r} - \frac{1}{r^2}\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\frac{\partial^3 w}{\partial r^3} = \frac{p(r)}{K}$$

VÉGE

Köszönöm a figyelmet!

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