MŰEGYETEM 1782
Tartószerkezetek Mechanikája Tanszék

Tartók statikája II

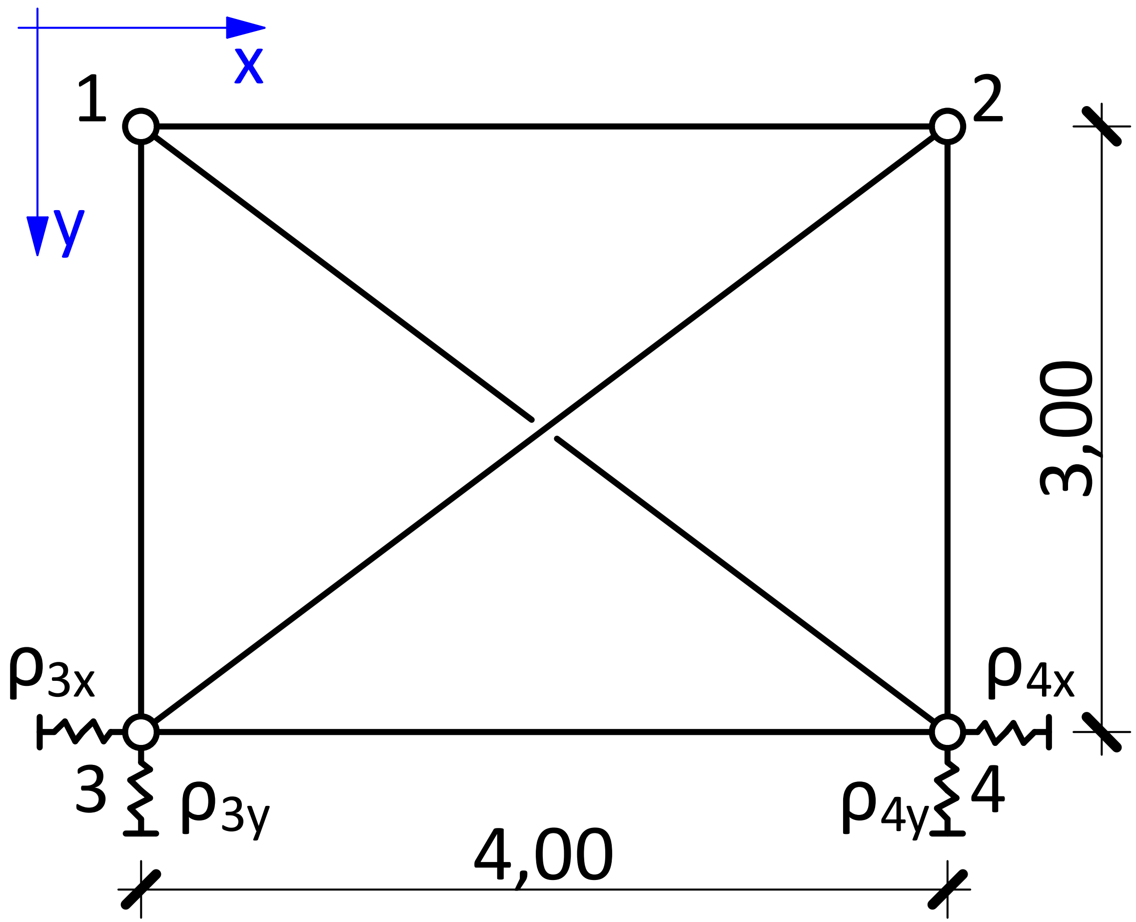
Rácsos tartók

Dr. Hortobágyi Zsolt

Alapelv

- Síkbeli egyenes rudakból álló rúdszerkezet.
- Csak csuklós kapcsolódás lehetősége.
- Prizmatikus rúdelemek (EA a rúd hossza mentén állandó).
- Csak koncentrált erők terhelhetnek a rácsos tartók csomópontjaiban

Modellalkotás



Alapváltozók

Elmozdulás vektor: $\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_i \\ \vdots \\ \mathbf{v}_n \end{bmatrix}$ $\quad \mathbf{v}_i = \begin{bmatrix} u_{ix} \\ v_{iy} \\ \cancel{\varphi_{iz}} \end{bmatrix}$

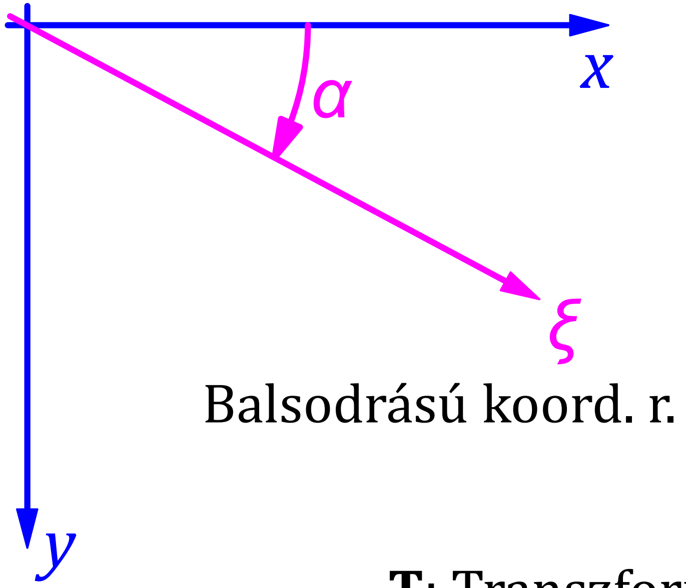
Csomóponti terhek: $\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_i \\ \vdots \\ \mathbf{q}_n \end{bmatrix}$ $\quad \mathbf{q}_i = \begin{bmatrix} F_{ix} \\ F_{iy} \\ \cancel{W_{iz}} \end{bmatrix}$

Kinematikai terhek:

Rúdigénybevételek: $\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_i \\ \vdots \\ \mathbf{s}_r \end{bmatrix}$ $\quad \mathbf{s}_{ij} = \begin{bmatrix} N_{j\xi} \\ \cancel{T_{j\eta}} \\ \cancel{M_{j\zeta}} \end{bmatrix}$

$\mathbf{t} = \begin{bmatrix} \mathbf{t}_1 \\ \mathbf{t}_2 \\ \vdots \\ \mathbf{t}_i \\ \vdots \\ \mathbf{t}_r \end{bmatrix}$ $\quad \mathbf{t}_{ij} = \begin{bmatrix} u_{j\xi} \\ \cancel{v_{j\eta}} \\ \cancel{\varphi_{j\zeta}} \end{bmatrix} = \Delta\ell_{ij}$

Koordináta transzformáció



globális koordinátarendszer: *x,y*
lokális koordinátarendszer: *ξ*

lokális -> globális

$$\mathbf{a}^{xy} = \mathbf{T}\mathbf{a}^{\xi}$$

T: Transzformációs (forgató) mátrix

$$\mathbf{T} = \begin{bmatrix} \cos(x,\xi) \\ \cos(y,\xi) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

Transzformációs mátrixok

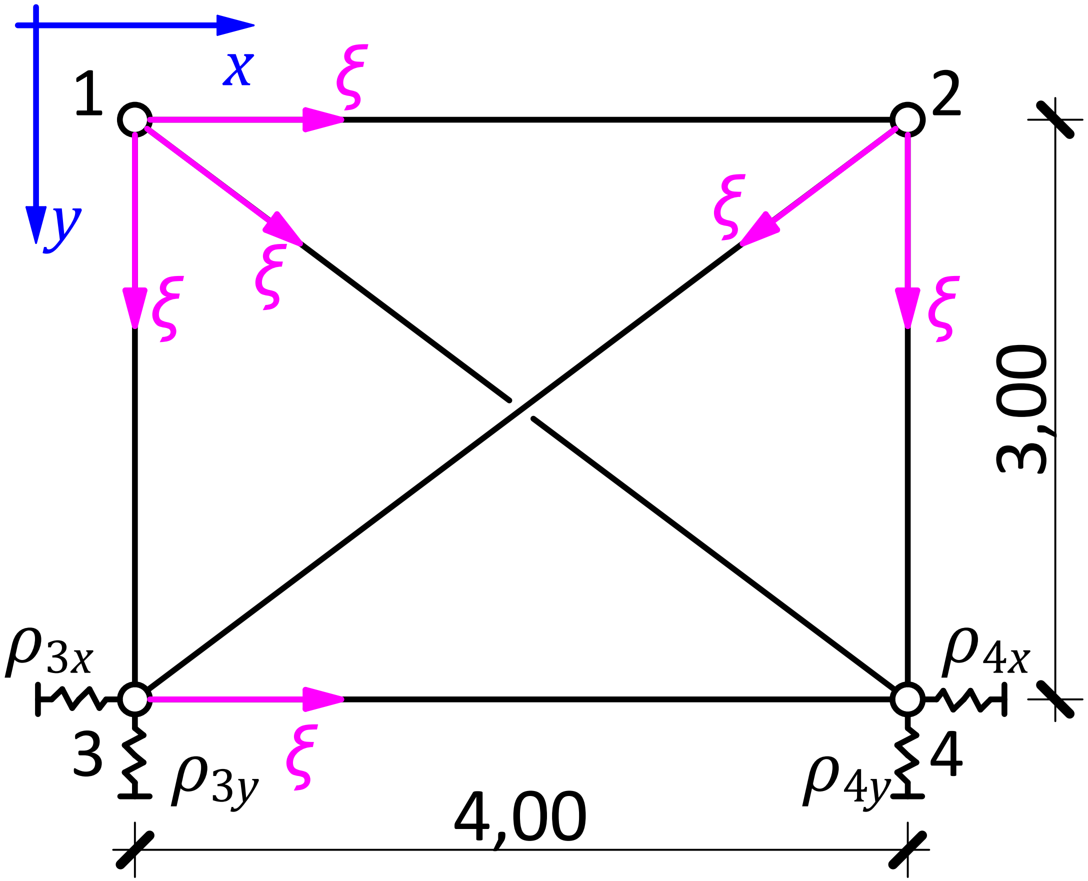
$$\mathbf{T} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

$$\mathbf{T}_{1,2} = \mathbf{T}_{3,4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{T}_{1,3} = \mathbf{T}_{2,4} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{T}_{1,4} = \begin{bmatrix} 0{,}8 \\ 0{,}6 \end{bmatrix}$$

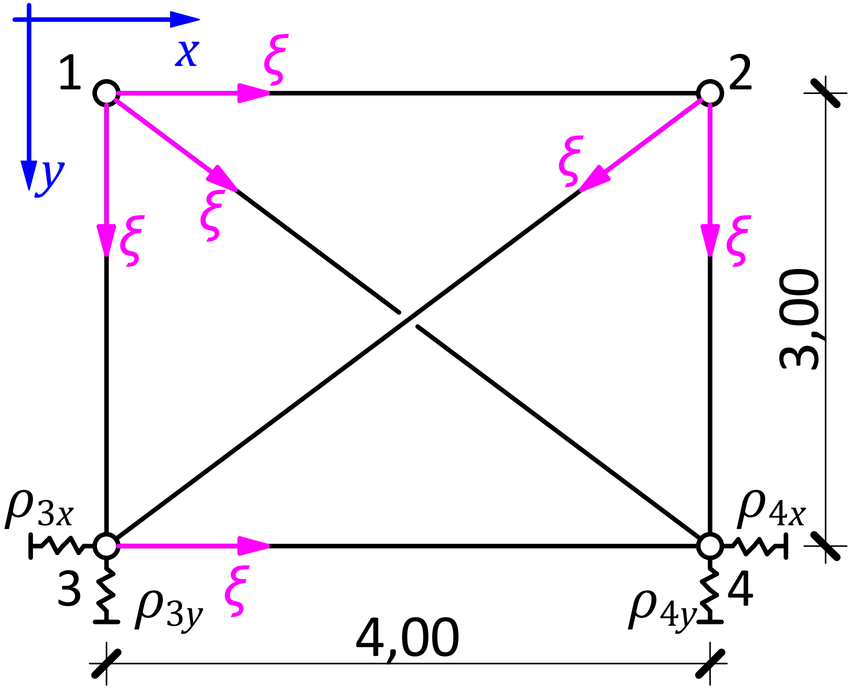
$$\mathbf{T}_{2,3} = \begin{bmatrix} -0{,}8 \\ 0{,}6 \end{bmatrix}$$



Egyensúlyi (geometriai) mátrix

$$\mathbf{G}_e = \begin{bmatrix} \mathbf{TB} \\ -\mathbf{T} \end{bmatrix}; \mathbf{B} = [1];$$

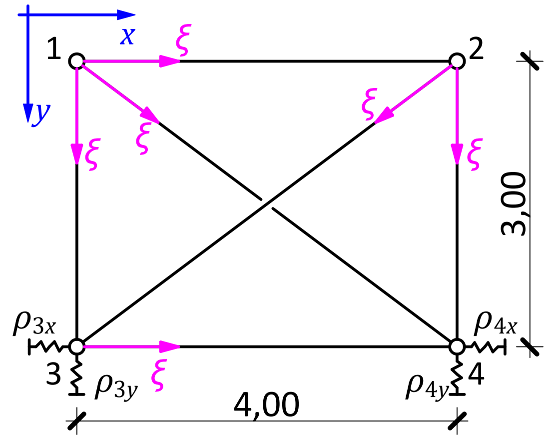
$$\mathbf{G}_e = \begin{bmatrix} \mathbf{T} \\ -\mathbf{T} \end{bmatrix}$$



$$\underset{\text{bl: 4x6}}{\mathbf{G}} = \begin{bmatrix} \mathbf{T}_{1,2} & \mathbf{T}_{1,3} & \mathbf{T}_{1,4} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{T}_{1,2} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{2,3} & \mathbf{T}_{2,4} & \mathbf{0} \\ \mathbf{0} & -\mathbf{T}_{1,3} & \mathbf{0} & -\mathbf{T}_{2,3} & \mathbf{0} & \mathbf{T}_{3,4} \\ \mathbf{0} & \mathbf{0} & -\mathbf{T}_{1,4} & \mathbf{0} & -\mathbf{T}_{2,4} & -\mathbf{T}_{3,4} \end{bmatrix}$$

Egyensúlyi (geometriai) mátrix

$$
\underset{\text{sk: 8x6}}{\mathbf{G}} =
\begin{array}{c}
 \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \text{1-2} & \text{1-3} & \text{1-4} & \text{2-3} & \text{2-4} & \text{3-4} \end{array} \\
\left[\begin{array}{cccccc}
1 & 0 & 0{,}8 & & & \\
0 & 1 & 0{,}6 & & & \\
\hline
-1 & & & -0{,}8 & 0 & \\
0 & & & 0{,}6 & 1 & \\
\hline
 & 0 & & 0{,}8 & & 1 \\
 & -1 & & -0{,}6 & & 0 \\
\hline
 & & -0{,}8 & & 0 & -1 \\
 & & -0{,}6 & & -1 & 0
\end{array}\right]
\end{array}
$$

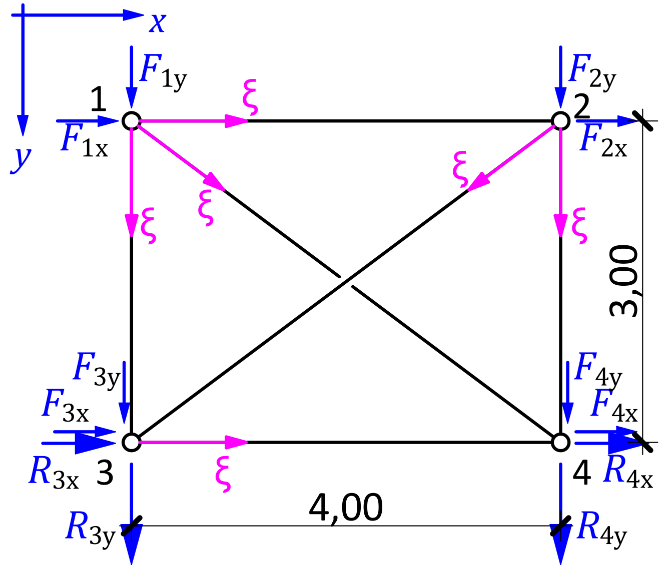


$$
\underset{\text{bl: 4x6}}{\mathbf{G}} =
\begin{bmatrix}
\mathbf{T}_{1,2} & \mathbf{T}_{1,3} & \mathbf{T}_{1,4} & & & \\
-\mathbf{T}_{1,2} & & & \mathbf{T}_{2,3} & \mathbf{T}_{2,4} & \\
 & -\mathbf{T}_{1,3} & & -\mathbf{T}_{2,3} & & \mathbf{T}_{3,4} \\
 & & -\mathbf{T}_{1,4} & & -\mathbf{T}_{2,4} & -\mathbf{T}_{3,4}
\end{bmatrix}
$$

Egyensúlyi egyenlet

$$\mathbf{G}\mathbf{s} + \mathbf{q} = \mathbf{0}$$

bl: 4x6 6 4 4



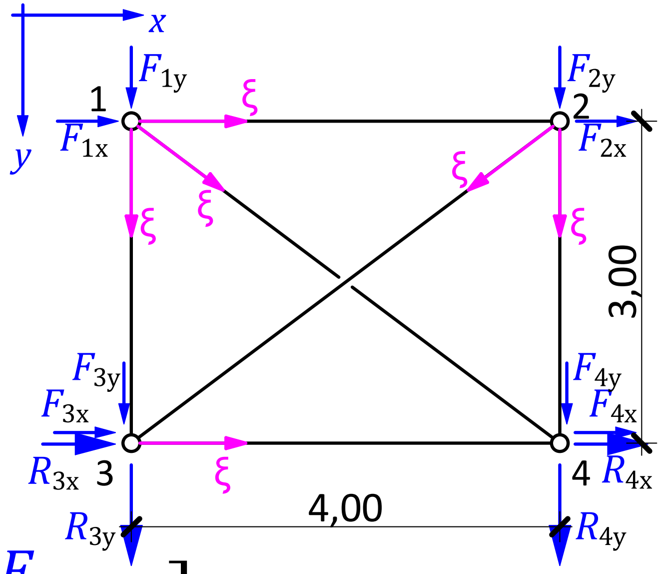
$$\begin{bmatrix} \mathbf{T}_{1,2} & \mathbf{T}_{1,3} & \mathbf{T}_{1,4} & & & \\ -\mathbf{T}_{1,2} & & & \mathbf{T}_{2,3} & \mathbf{T}_{2,4} & \\ & -\mathbf{T}_{1,3} & & -\mathbf{T}_{2,3} & & \mathbf{T}_{3,4} \\ & & -\mathbf{T}_{1,4} & & -\mathbf{T}_{2,4} & -\mathbf{T}_{3,4} \end{bmatrix} \begin{bmatrix} S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix} + \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \\ \mathbf{q}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Egyensúlyi egyenlet

$$\mathbf{G}\mathbf{s} + \mathbf{q} = \mathbf{0}$$

sk: 8x6 6 8 8

$$\begin{bmatrix} \mathbf{T}_{1,2} & \mathbf{T}_{1,3} & \mathbf{T}_{1,4} & & & \\ -\mathbf{T}_{1,2} & & & \mathbf{T}_{2,3} & \mathbf{T}_{2,4} & \\ & -\mathbf{T}_{1,3} & & -\mathbf{T}_{2,3} & & \mathbf{T}_{3,4} \\ & & -\mathbf{T}_{1,4} & & -\mathbf{T}_{2,4} & -\mathbf{T}_{3,4} \end{bmatrix} \begin{bmatrix} S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix} + \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \\ \mathbf{q}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

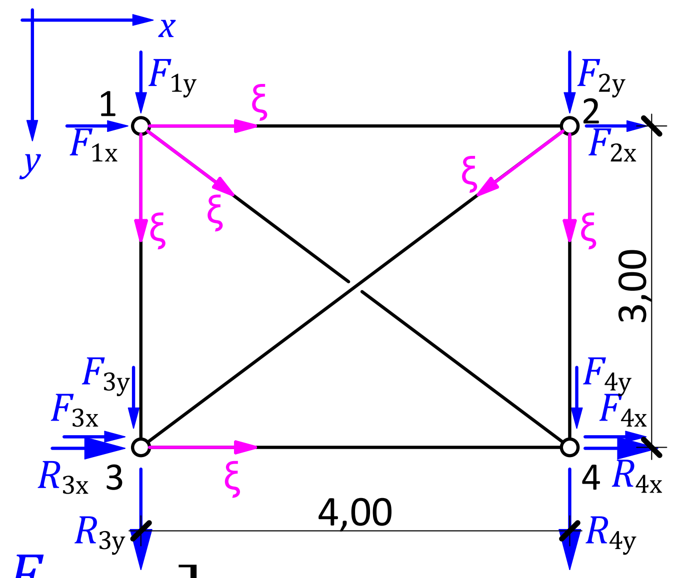


$$\begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 1\text{-}2 & 1\text{-}3 & 1\text{-}4 & 2\text{-}3 & 2\text{-}4 & 3\text{-}4 \end{array} \\ \left[\begin{array}{cccccc} 1 & 0 & 0{,}8 & & & \\ 0 & 1 & 0{,}6 & & & \\ \hdashline -1 & & & -0{,}8 & 0 & \\ 0 & & & 0{,}6 & 1 & \\ \hdashline & 0 & & 0{,}8 & & 1 \\ & -1 & & -0{,}6 & & 0 \\ \hdashline & & -0{,}8 & & 0 & -1 \\ & & -0{,}6 & & -1 & 0 \end{array}\right] \end{array} \begin{bmatrix} S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix} + \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} + R_{3x} \\ F_{3y} + R_{3y} \\ F_{4x} + R_{4x} \\ F_{4y} + R_{4y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Egyensúlyi egyenlet

$$\mathbf{G}\mathbf{s} + \mathbf{q} = \mathbf{0}$$

sk: 8x6 6 8 8

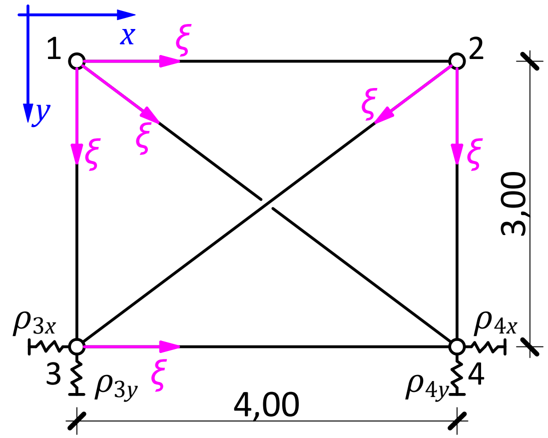


n: csomópontok száma
r: rudak száma
f: külső kényszerek (támaszerők (*R*)) fokszáma
egyenletek száma: e=2n=2x4=8
ismeretlenek száma: i=r+f=6+4=10
Statikai határozatlanság fokszáma: i-e=10-8=2

$$\begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccccc} \text{1-2} & \text{1-3} & \text{1-4} & \text{2-3} & \text{2-4} & \text{3-4} \end{array} \\
\left[\begin{array}{cccccc}
1 & 0 & 0{,}8 & & & \\
0 & 1 & 0{,}6 & & & \\ \hdashline
-1 & & & -0{,}8 & 0 & \\
0 & & & 0{,}6 & 1 & \\ \hdashline
 & 0 & & 0{,}8 & & 1 \\
 & -1 & & -0{,}6 & & 0 \\ \hdashline
 & & -0{,}8 & & 0 & -1 \\
 & & -0{,}6 & & -1 & 0
\end{array}\right]
\end{array}
\begin{bmatrix} S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix}
+
\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} + R_{3x} \\ F_{3y} + R_{3y} \\ F_{4x} + R_{4x} \\ F_{4y} + R_{4y} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Teljes szerkezet állapotegyenlete

$$\begin{bmatrix} \mathbf{C} & \mathbf{G} \\ \mathbf{G}^T & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{q} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$



C: a támaszrugókat tartalmazza negatív előjellel, $(2n)$x$(2n)$

G: egyensúlyi (geometriai) mátrix, $(2n)$x(r)

F: hajlékonysági mátrix, (r)x(r)

v: csomóponti elmozdulások vektora, $(2n)$

s: rúderők vektora, (r)

q: csomóponti terhek vektora, $(2n)$

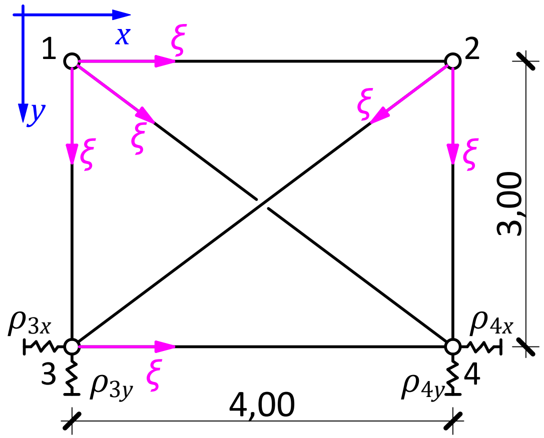
t: rudak kinematikai tehervektora, (r)

n: csomópontok száma

r: rudak száma

Teljes szerkezet állapotegyenlete

$$\begin{bmatrix} \mathbf{C} & \mathbf{G} \\ \mathbf{G}^\mathrm{T} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{q} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$



$$\left[\begin{array}{cccc|cccccc}
\mathbf{C}_1 & & & & & & & & & \\
& \mathbf{C}_2 & & & & & \mathbf{G} & & & \\
& & \mathbf{C}_3 & & & & & & & \\
& & & \mathbf{C}_4 & & & & & & \\
\hline
& & & & \frac{l_{1,2}}{EA_{1,2}} & & & & & \\
& & & & & \frac{l_{1,3}}{EA_{1,3}} & & & & \\
& & & & & & \frac{l_{1,4}}{EA_{1,4}} & & & \\
& \mathbf{G}^\mathrm{T} & & & & & & \frac{l_{2,3}}{EA_{2,3}} & & \\
& & & & & & & & \frac{l_{2,4}}{EA_{2,4}} & \\
& & & & & & & & & \frac{l_{3,4}}{EA_{3,4}}
\end{array}\right]
\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \\ S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix}
+ \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \\ \mathbf{q}_4 \\ t_{1,2} \\ t_{1,3} \\ t_{1,4} \\ t_{2,3} \\ t_{2,4} \\ t_{3,4} \end{bmatrix}
= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Teljes szerkezet állapotegyenlete

$$\begin{bmatrix} \mathbf{C} & \mathbf{G} \\ \mathbf{G}^{T} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{q} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

e=2n+r=2x4+6=14

i= 2n+r=2x4+6=14

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} &&&&&&&& \text{1-2} & \text{1-3} & \text{1-4} & \text{2-3} & \text{2-4} & \text{3-4} \end{array} \\
\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \text{1-2} \\ \text{1-3} \\ \text{1-4} \\ \text{2-3} \\ \text{2-4} \\ \text{3-4} \end{array} &
\left[\begin{array}{cccccccc:cccccc}
0 &&&&&&&& 1 & 0 & 0{,}8 &&& \\
& 0 &&&&&&& 0 & 1 & 0{,}6 &&& \\
\hdashline
&& 0 &&&&&& -1 &&& -0{,}8 & 0 & \\
&&& 0 &&&&& 0 &&& 0{,}6 & 1 & \\
\hdashline
&&&& -\rho_{3x} &&&&& 0 && 0{,}8 && 1 \\
&&&&& -\rho_{3y} &&&& -1 && -0{,}6 && 0 \\
\hdashline
&&&&&& -\rho_{4x} &&&& -0{,}8 && 0 & -1 \\
&&&&&&& -\rho_{4y} &&& -0{,}6 && -1 & 0 \\
\hdashline
1 & 0 & -1 & 0 &&&&& \frac{l_{1,2}}{EA_{1,2}} &&&&& \\
0 & 1 &&& 0 & -1 &&&& \frac{l_{1,3}}{EA_{1,3}} &&&& \\
0{,}8 & 0{,}6 &&&&& -0{,}8 & -0{,}6 &&& \frac{l_{1,4}}{EA_{1,4}} &&& \\
&& -0{,}8 & 0{,}6 & 0{,}8 & -0{,}6 &&&&&& \frac{l_{2,3}}{EA_{2,3}} && \\
&& 0 & 1 &&& 0 & -1 &&&&& \frac{l_{2,4}}{EA_{2,4}} & \\
&&&& 1 & 0 & -1 & 0 &&&&&& \frac{l_{3,4}}{EA_{3,4}}
\end{array}\right]
\end{array}
\begin{bmatrix} u_{1x} \\ v_{1y} \\ u_{2x} \\ v_{2y} \\ u_{3x} \\ v_{3y} \\ u_{4x} \\ v_{4y} \\ S_{1,2} \\ S_{1,3} \\ S_{1,4} \\ S_{2,3} \\ S_{2,4} \\ S_{3,4} \end{bmatrix} + \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \\ \Delta\ell_{1,2} \\ \Delta\ell_{1,3} \\ \Delta\ell_{1,4} \\ \Delta\ell_{2,3} \\ \Delta\ell_{2,4} \\ \Delta\ell_{3,4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Elmozdulás módszer

$$\mathbf{K_e} = \begin{bmatrix} \mathbf{T}\mathbf{B} \\ \mathbf{-T} \end{bmatrix} \mathbf{F}^{-1} \begin{bmatrix} \mathbf{B}^{\mathrm{T}}\mathbf{T}^{T} & \mathbf{-T}^{T} \end{bmatrix} = \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{B} \\ \mathbf{-E} \end{bmatrix} \mathbf{F}^{-1} \begin{bmatrix} \mathbf{B}^{T} & \mathbf{-E} \end{bmatrix} \begin{bmatrix} \mathbf{T}^{T} & \\ & \mathbf{T}^{T} \end{bmatrix} =$$

$$\mathbf{K_e} = \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \mathbf{K_e} \begin{bmatrix} \mathbf{T}^{T} & \\ & \mathbf{T}^{T} \end{bmatrix} \qquad \mathbf{K_e} = \begin{bmatrix} \mathbf{B} \\ \mathbf{-E} \end{bmatrix} \mathbf{F}^{-1} \begin{bmatrix} \mathbf{B}^{T} & \mathbf{-E} \end{bmatrix}$$

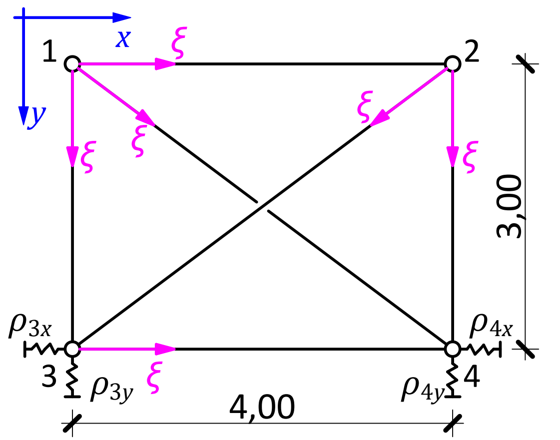
Elemi merevségi mátrix a lokális koordináta rendszerben:

$$\mathbf{K_e} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{EA}{\ell} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{EA}{\ell} & -\frac{EA}{\ell} \\ -\frac{EA}{\ell} & \frac{EA}{\ell} \end{bmatrix}$$

Elmozdulás módszer

Elemi merevségi mátrixok a globális koordináta rendszerben:

$$\mathbf{K}_{\mathrm{e}} = \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \mathbf{K}_{\mathrm{e}} \begin{bmatrix} \mathbf{T}^T & \\ & \mathbf{T}^T \end{bmatrix}$$



$$\mathbf{K}_{1,2} = \mathbf{K}_{3,4} = \begin{bmatrix} 1 & \\ 0 & \\ & 1 \\ & 0 \end{bmatrix} \begin{bmatrix} \frac{EA}{4} & -\frac{EA}{4} \\ -\frac{EA}{4} & \frac{EA}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & & \\ & & 1 & 0 \end{bmatrix} = \left[\begin{array}{cc:cc} \frac{EA}{4} & 0 & -\frac{EA}{4} & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline -\frac{EA}{4} & 0 & \frac{EA}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\mathbf{K}_{1,3} = \mathbf{K}_{2,4} = \begin{bmatrix} 0 & \\ 1 & \\ & 0 \\ & 1 \end{bmatrix} \begin{bmatrix} \frac{EA}{3} & -\frac{EA}{3} \\ -\frac{EA}{3} & \frac{EA}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & & \\ & & 0 & 1 \end{bmatrix} = \left[\begin{array}{cc:cc} 0 & 0 & 0 & 0 \\ 0 & \frac{EA}{3} & 0 & -\frac{EA}{3} \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & -\frac{EA}{3} & 0 & \frac{EA}{3} \end{array}\right]$$

Elmozdulás módszer

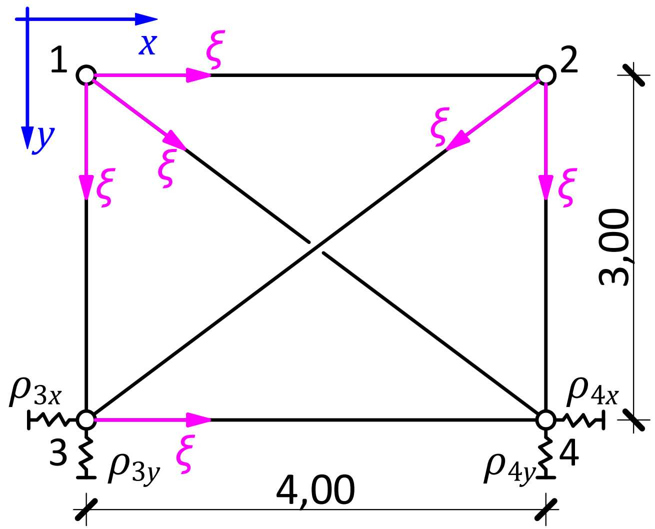
Elemi merevségi mátrixok a globális koordináta rendszerben:

$$\mathbf{K}_{1,4}=\begin{bmatrix}0{,}8 & \\ 0{,}6 & \\ & 0{,}8 \\ & 0{,}6\end{bmatrix}\begin{bmatrix}\frac{EA}{5} & -\frac{EA}{5} \\ -\frac{EA}{5} & \frac{EA}{5}\end{bmatrix}\begin{bmatrix}0{,}8 & 0{,}6 & & \\ & & 0{,}8 & 0{,}6\end{bmatrix}$$

$$=\frac{EA}{5}\left[\begin{array}{cc:cc} 0{,}8^2 & 0{,}8\cdot 0{,}6 & -0{,}8^2 & -0{,}8\cdot 0{,}6 \\ 0{,}8\cdot 0{,}6 & 0{,}6^2 & -0{,}8\cdot 0{,}6 & -0{,}6^2 \\ \hdashline -0{,}8^2 & -0{,}8\cdot 0{,}6 & 0{,}8^2 & 0{,}8\cdot 0{,}6 \\ -0{,}8\cdot 0{,}6 & -0{,}6^2 & 0{,}8\cdot 0{,}6 & 0{,}6^2 \end{array}\right]$$

$$\mathbf{K}_{2,3}=\begin{bmatrix}-0{,}8 & \\ 0{,}6 & \\ & -0{,}8 \\ & 0{,}6\end{bmatrix}\begin{bmatrix}\frac{EA}{5} & -\frac{EA}{5} \\ -\frac{EA}{5} & \frac{EA}{5}\end{bmatrix}\begin{bmatrix}-0{,}8 & 0{,}6 & & \\ & & -0{,}8 & 0{,}6\end{bmatrix}$$

$$=\frac{EA}{5}\left[\begin{array}{cc:cc} 0{,}8^2 & -0{,}8\cdot 0{,}6 & -0{,}8^2 & 0{,}8\cdot 0{,}6 \\ -0{,}8\cdot 0{,}6 & 0{,}6^2 & 0{,}8\cdot 0{,}6 & -0{,}6^2 \\ \hdashline -0{,}8^2 & 0{,}8\cdot 0{,}6 & 0{,}8^2 & -0{,}8\cdot 0{,}6 \\ 0{,}8\cdot 0{,}6 & -0{,}6^2 & -0{,}8\cdot 0{,}6 & 0{,}6^2 \end{array}\right]$$



Elmozdulás módszer

A szerkezet merevségi mátrixa:

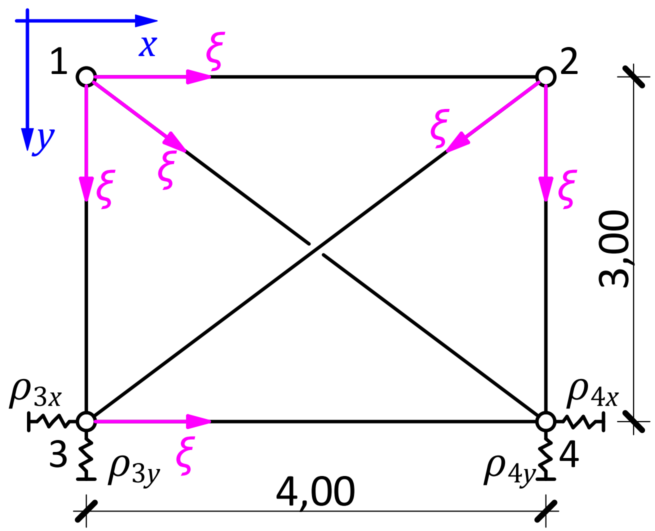
$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{1-2}^{1,1} + \mathbf{K}_{1-3}^{1,1} + \mathbf{K}_{1-4}^{1,1} & \mathbf{K}_{1-2}^{1,2} & \mathbf{K}_{1-3}^{1,3} & \mathbf{K}_{1-4}^{1,4} \\ \mathbf{K}_{1-2}^{2,1} & \mathbf{K}_{1-2}^{2,2} + \mathbf{K}_{2-3}^{2,2} + \mathbf{K}_{2-4}^{2,2} & \mathbf{K}_{2-3}^{2,3} & \mathbf{K}_{2-4}^{2,4} \\ \mathbf{K}_{1-3}^{3,1} & \mathbf{K}_{2-3}^{3,2} & \mathbf{K}_{1-3}^{3,3} + \mathbf{K}_{2-3}^{3,3} + \mathbf{K}_{3-4}^{3,3} + \mathbf{C}_3 & \mathbf{K}_{3-4}^{3,4} \\ \mathbf{K}_{1-4}^{4,1} & \mathbf{K}_{2-4}^{4,2} & \mathbf{K}_{3-4}^{4,3} & \mathbf{K}_{1-4}^{4,4} + \mathbf{K}_{2-4}^{4,4} + \mathbf{K}_{3-4}^{4,4} + \mathbf{C}_4 \end{bmatrix}$$

$$\mathbf{K}_{1,2} = \begin{bmatrix} \mathbf{K}_{1-2}^{1,1} & \mathbf{K}_{1-2}^{1,2} \\ \mathbf{K}_{1-2}^{2,1} & \mathbf{K}_{1-2}^{2,2} \end{bmatrix} \quad \mathbf{K}_{1,3} = \begin{bmatrix} \mathbf{K}_{1-3}^{1,1} & \mathbf{K}_{1-3}^{1,3} \\ \mathbf{K}_{1-3}^{3,1} & \mathbf{K}_{1-3}^{3,3} \end{bmatrix} \quad \mathbf{K}_{1,4} = \begin{bmatrix} \mathbf{K}_{1-4}^{1,1} & \mathbf{K}_{1-4}^{1,4} \\ \mathbf{K}_{1-4}^{4,1} & \mathbf{K}_{1-4}^{4,4} \end{bmatrix}$$

$$\mathbf{K}_{2,3} = \begin{bmatrix} \mathbf{K}_{2-3}^{2,2} & \mathbf{K}_{2-3}^{2,3} \\ \mathbf{K}_{2-3}^{3,2} & \mathbf{K}_{2-3}^{3,3} \end{bmatrix} \quad \mathbf{K}_{2,4} = \begin{bmatrix} \mathbf{K}_{2-4}^{2,2} & \mathbf{K}_{2-4}^{2,4} \\ \mathbf{K}_{2-4}^{4,2} & \mathbf{K}_{2-4}^{4,4} \end{bmatrix} \quad \mathbf{K}_{3,4} = \begin{bmatrix} \mathbf{K}_{3-4}^{3,3} & \mathbf{K}_{3-4}^{3,4} \\ \mathbf{K}_{3-4}^{4,3} & \mathbf{K}_{3-4}^{4,4} \end{bmatrix}$$

$$\mathbf{C}_3 = \begin{bmatrix} \rho_{3x} & \\ & \rho_{3y} \end{bmatrix} \quad \mathbf{C}_4 = \begin{bmatrix} \rho_{4x} & \\ & \rho_{4y} \end{bmatrix}$$

A **C** mátrixban már + előjellel szerepelnek a rugómerevségek! [kN/m]



Elmozdulás módszer

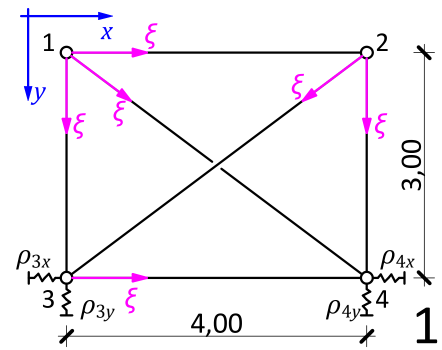
A szerkezet merevségi mátrixa:

$$\mathbf{K}=\begin{bmatrix}\mathbf{K}_{1-2}^{1,1}+\mathbf{K}_{1-3}^{1,1}+\mathbf{K}_{1-4}^{1,1} & \mathbf{K}_{1-2}^{1,2} & \mathbf{K}_{1-3}^{1,3} & \mathbf{K}_{1-4}^{1,4}\\ \mathbf{K}_{1-2}^{2,1} & \mathbf{K}_{1-2}^{2,2}+\mathbf{K}_{2-3}^{2,2}+\mathbf{K}_{2-4}^{2,2} & \mathbf{K}_{2-3}^{2,3} & \mathbf{K}_{2-4}^{2,4}\\ \mathbf{K}_{1-3}^{3,1} & \mathbf{K}_{2-3}^{3,2} & \mathbf{K}_{1-3}^{3,3}+\mathbf{K}_{2-3}^{3,3}+\mathbf{K}_{3-4}^{3,3}+\mathbf{C}_3 & \mathbf{K}_{3-4}^{3,4}\\ \mathbf{K}_{1-4}^{4,1} & \mathbf{K}_{2-4}^{4,2} & \mathbf{K}_{3-4}^{4,3} & \mathbf{K}_{1-4}^{4,4}+\mathbf{K}_{2-4}^{4,4}+\mathbf{K}_{3-4}^{4,4}+\mathbf{C}_4\end{bmatrix}$$

$$=EA\begin{bmatrix}0{,}378 & 0{,}096 & -0{,}25 & 0 & 0 & 0 & -0{,}128 & -0{,}096\\ 0{,}096 & 0{,}4053 & 0 & 0 & 0 & -0{,}3333 & -0{,}096 & -0{,}072\\ -0{,}25 & 0 & 0{,}378 & -0{,}096 & -0{,}128 & 0{,}096 & 0 & 0\\ 0 & 0 & -0{,}096 & 0{,}4053 & 0{,}096 & -0{,}072 & 0 & -0{,}3333\\ 0 & 0 & -0{,}128 & 0{,}096 & 1000{,}378 & -0{,}096 & -0{,}25 & 0\\ 0 & -0{,}3333 & 0{,}096 & -0{,}072 & -0{,}096 & 1000{,}4053 & 0 & 0\\ -0{,}128 & -0{,}096 & 0 & 0 & -0{,}25 & 0 & 1000{,}378 & 0{,}096\\ -0{,}096 & -0{,}072 & 0 & -0{,}3333 & 0 & 0 & 0{,}096 & 1000{,}4053\end{bmatrix}$$

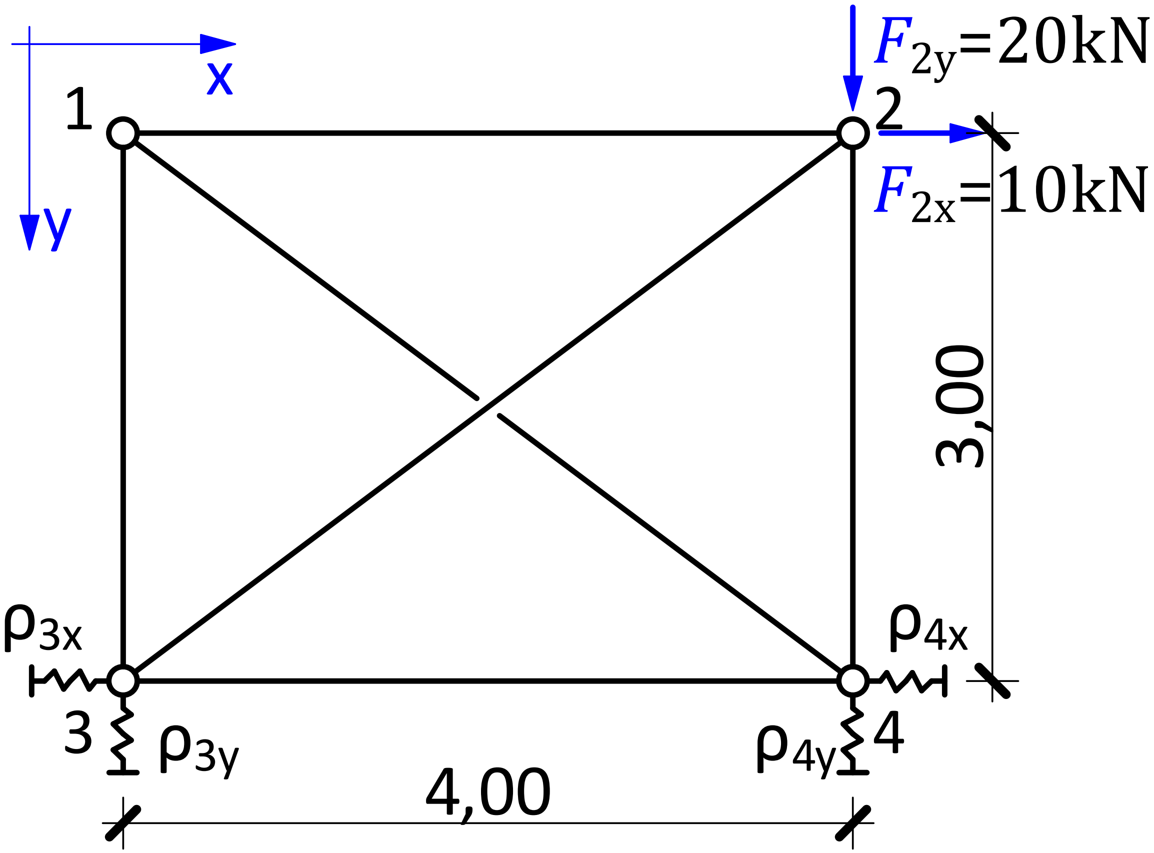
$$\mathbf{K}_{1,2}=\mathbf{K}_{3,4}=EA\begin{bmatrix}0{,}25 & 0 & -0{,}25 & 0\\ 0 & 0 & 0 & 0\\ -0{,}25 & 0 & 0{,}25 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\qquad \mathbf{K}_{1,4}=EA\begin{bmatrix}0{,}128 & 0{,}096 & -0{,}128 & -0{,}096\\ 0{,}096 & 0{,}072 & -0{,}096 & -0{,}072\\ -0{,}128 & -0{,}096 & 0{,}128 & 0{,}096\\ -0{,}096 & -0{,}072 & 0{,}096 & 0{,}072\end{bmatrix}\qquad \mathbf{C}_3=EA\begin{bmatrix}1000 & \\ & 1000\end{bmatrix}$$

$$\mathbf{K}_{1,3}=\mathbf{K}_{2,4}=EA\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0{,}3333 & 0 & -0{,}3333\\ 0 & 0 & 0 & 0\\ 0 & -0{,}3333 & 0 & 0{,}3333\end{bmatrix}\qquad \mathbf{K}_{2,3}=EA\begin{bmatrix}0{,}128 & -0{,}096 & -0{,}128 & 0{,}096\\ -0{,}096 & 0{,}072 & 0{,}096 & -0{,}072\\ -0{,}128 & 0{,}096 & 0{,}128 & -0{,}096\\ 0{,}096 & -0{,}072 & -0{,}096 & 0{,}072\end{bmatrix}\qquad \mathbf{C}_4=EA\begin{bmatrix}1000 & \\ & 1000\end{bmatrix}$$



Elmozdulás módszer

Tehervektor



$$\mathbf{q} = \begin{bmatrix} \\ \hdashline 10 \\ 20 \\ \hdashline \\ \\ \hdashline \\ \end{bmatrix}$$

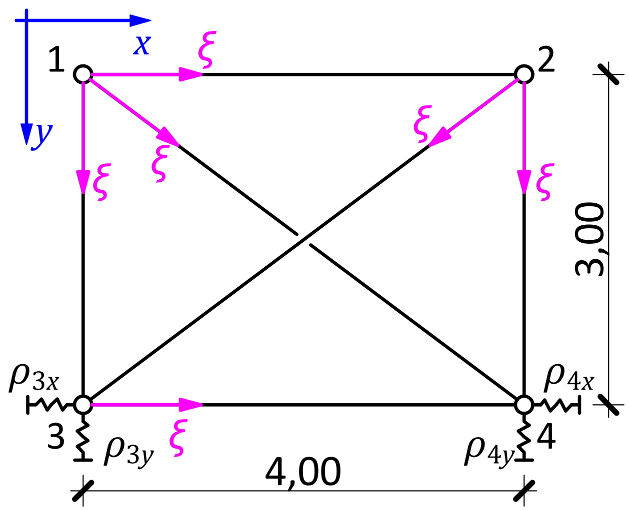
Elmozdulás módszer

Egyenletrendszer:

$\mathbf{K}\,\mathbf{v} = \mathbf{q}$

$$EA\begin{bmatrix} 0{,}378 & 0{,}096 & -0{,}25 & 0 & 0 & 0 & -0{,}128 & -0{,}096 \\ 0{,}096 & 0{,}4053 & 0 & 0 & 0 & -0{,}3333 & -0{,}096 & -0{,}072 \\ -0{,}25 & 0 & 0{,}378 & -0{,}096 & -0{,}128 & 0{,}096 & 0 & 0 \\ 0 & 0 & -0{,}096 & 0{,}4053 & 0{,}096 & -0{,}072 & 0 & -0{,}3333 \\ 0 & 0 & -0{,}128 & 0{,}096 & 1000{,}378 & -0{,}096 & -0{,}25 & 0 \\ 0 & -0{,}3333 & 0{,}096 & -0{,}072 & -0{,}096 & 1000{,}4053 & 0 & 0 \\ -0{,}128 & -0{,}096 & 0 & 0 & -0{,}25 & 0 & 1000{,}378 & 0{,}096 \\ -0{,}096 & -0{,}072 & 0 & -0{,}3333 & 0 & 0 & 0{,}096 & 1000{,}4053 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 20 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 57{,}86 \\ -13{,}70 \\ 82{,}21 \\ 68{,}84 \\ 0{,}0039 \\ -0{,}0075 \\ 0{,}0061 \\ 0{,}0275 \end{bmatrix}$$



Elmozdulás módszer

Rúderő meghatározása:

$$\mathbf{s}_{ij} = -\mathbf{q}_{ij}^{\,j} + \begin{bmatrix} \mathbf{K}_{ij}^{\,ji} & \mathbf{K}_{ij}^{\,jj} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{ij}^{\,T} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{ij}^{\,T} \end{bmatrix} \begin{bmatrix} \mathbf{v}_i \\ \mathbf{v}_j \end{bmatrix}$$

$$\mathbf{K}_e = \begin{bmatrix} \frac{EA}{\ell} & -\frac{EA}{\ell} \\ \boxed{-\frac{EA}{\ell} \quad \frac{EA}{\ell}} \end{bmatrix}$$

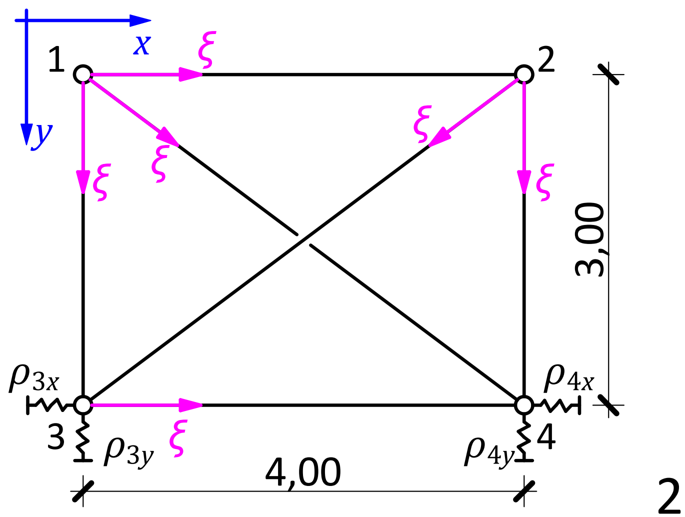
$$\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 57{,}86 \\ -13{,}70 \\ 82{,}21 \\ 68{,}84 \\ 0{,}0039 \\ -0{,}0075 \\ 0{,}0061 \\ 0{,}0275 \end{bmatrix}$$

$$S_{1-2} = EA \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & & \\ & & 1 & 0 \end{bmatrix} \frac{1}{EA} \begin{bmatrix} 57{,}86 \\ -13{,}70 \\ 82{,}21 \\ 68{,}84 \end{bmatrix} = +6{,}087\ kN$$

$$S_{2-3} = EA \begin{bmatrix} -\frac{1}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -0{,}8 & 0{,}6 & & \\ & & -0{,}8 & 0{,}6 \end{bmatrix} \frac{1}{EA} \begin{bmatrix} 82{,}21 \\ 68{,}84 \\ 0{,}0039 \\ -0{,}0075 \end{bmatrix} = +4{,}891\ kN$$

$$S_{2-4} = EA \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & & \\ & & 0 & 1 \end{bmatrix} \frac{1}{EA} \begin{bmatrix} 82{,}21 \\ 68{,}84 \\ 0{,}0061 \\ 0{,}0275 \end{bmatrix} = -22{,}937\ kN$$

$$S_{3-4} = EA \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & & \\ & & 1 & 0 \end{bmatrix} \frac{1}{EA} \begin{bmatrix} 0{,}0039 \\ -0{,}0075 \\ 0{,}0061 \\ 0{,}0275 \end{bmatrix} = +0{,}0006\ kN$$



VÉGE

Köszönöm a figyelmet!

Összeállította: Dr. Hortobágyi Zsolt
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