Exercise 1.

Computation of mean error (each observation has the same random error – unit weight):

Obs. i	Observation l_i [m]	correction v_i [mm]	$v_i\, v_i$ [mm^2]
1	326,008	- 7,8	60,8
2	325,992	+ 8,2	67,2
3	326,009	- 8,8	77,4
4	325,988	+ 12,2	148,8
5	326,004	- 3,8	14,4
Σ	1630,001	0,0	368,6

Results:

1) Mean value: $M(l_i)=$ 326,0002 m

$$M(l_i) = \frac{\Sigma l_i}{n}$$

2) The mean error of unit weight: $\mu = \pm$ 9,6 mm

$$\mu(l_i) = \sqrt{\frac{\Sigma v_i^2}{n-1}}$$

Exercise 2.

Compute the mean error of the mean value of the observation in Exercise 1.

Note: The mean error of each observation is ± 9,6 mm, and the mean value has been computed by:

$$M(l_i) = \frac{\Sigma l_i}{n}$$

Using the error propagation law, we can compute the mean error of the mean value:

$$m_0 = \sqrt{\Sigma\, g_i^2 m_i^2}\,, where \;\; g_i = \frac{1}{n}$$

Thus:

$$m_0 = \sqrt{n\left(\frac{1}{n}\right)^2 \mu^2} = \frac{\mu}{\sqrt{n}} = \frac{9{,}6}{\sqrt{5}} = \pm 4{,}3mm$$

Exercise 3.

Compute the mean value and its mean error of the following set of observations:

l_1 = 92-30-58 l_2 = 92-30-57 l_3 = 92-31-04 l_4 = 92-31-00
l_5 = 92-30-56 l_6 = 92-30-50 l_7 = 92-30-59 l_8 = 92-31-01

The observations have been taken with the same procedure and the same instrumentation.

Obs. i	Full Observation l_i [° ' '']	Observation l_i [''']	correction v_i ['']	$v_i\, v_i$ [''^{2}]
1	92-30-58	58	0,125	0.015625
2	92-30-57	57	1,125	1.265625
3	92-31-04	64	-5,875	34.515625
4	92-31-00	60	-1,875	3.515625
5	92-30-56	56	2,125	4.515625
6	92-30-50	50	8,125	66.015625
7	92-30-59	59	-0,875	0.765625
8	92-31-01	61	-2,875	8.265625
Σ		465	0,000	118,875

1) Mean value: 92° 30' 58,1''
2) Mean error of unit weight: ±4,1''
3) Mean error of mean value: ±1,6''

Working exercises:

The observation of the following (1-4) exercises are always taken with the same instrumentation/procedure. Compute the mean values of the observations, and their mean error as well!

1) l_1 = 559,34 m l_2 = 559,36 m l_3 = 559,17 m l_4 = 559,24 m
   l_5 = 559,31 m l_6 = 559,26 m

2) l_1 = 256,010 m l_2 = 255,982 m l_3 = 256,003 m l_4 = 255,990 m

$l_5 = 256{,}011$ m $l_6 = 255{,}982$ m

3) $l_1 = 100{,}48$ m $l_2 = 100{,}47$ m $l_3 = 100{,}54$ m $l_4 = 100{,}49$ m
$l_5 = 100{,}50$ m $l_6 = 100{,}46$ m $l_7 = 100{,}40$ m $l_8 = 100{,}51$ m

4) $l_1 = 103{,}16$ m $l_2 = 103{,}10$ m $l_3 = 103{,}18$ m $l_4 = 103{,}15$ m
$l_5 = 103{,}04$ m $l_6 = 103{,}09$ m $l_7 = 103{,}18$ m $l_8 = 103{,}19$ m
$l_9 = 103{,}02$ m $l_{10} = 103{,}20$ m $l_{11} = 103{,}09$ m

Solutions:

1) M(l) = 559,28 m $\mu = \pm 71$ mm $m_0 = \pm 29$ mm

2) M(l) = 256,996 m $\mu = \pm 13{,}4$ $m_0 = \pm 5{,}5$ mm

3) M(l) = 100,481 m $\mu = \pm 41$ $m_0 = \pm 15$ mm

4) M(l) = 103,127 m $\mu = \pm 62$ $m_0 = \pm 19$ mm

Exercise 4.

Compute the mean value of the distance observations. Each observation has been made with different instrumentation or observation procedure, therefore they have different mean error. Compute the appropriate weighting and the mean error of the adjusted mean value as well.

Obs. I	Observation. l_i [mm] 325,980m +	Mean error	Variance	weights w_i [mm^{-2}]	$w_i l_i$ [mm^{-1}]	residual v_i [mm]	$w_i v_i$ [mm^{-1}]	$w_i v_i v_i$
1	28	± 8,1 mm	65,61	3	84	- 4,8	- 14,4	69,1
2	12	**± 14,1 mm**	**198,81**	1	12	+ 11,2	+ 11,2	125,4
3	29	± 7,0 mm	49	4	116	- 5,8	- 23,2	134,6
4	8	± 10,0 mm	100	2	16	+ 15,2	+ 30,4	462,1
5	24	± 6,3 mm	39,69	5	120	- 0,8	- 4,0	3,2
Σ	101			15	348		0,0	794,4

1) Compute the variance from the mean error:

$$\sigma = \mu^2$$

2) Compute the weights:
   - choose the highest variance value (this would correspond to the unit weight)
   - compute the weights of the other observations by:

$$w_i = w_0 \frac{\sigma_0}{\sigma_i}$$

3) Compute the weighted mean:

$$M(w_i l_i) = \frac{\sum w_i l_i}{\sum w_i}$$

The weighted mean is: 326,0032 mm

4) Compute the residuals (weighted mean – observation):

$$v_i = M(w_i l_i) - l_i$$

5) Compute the sum of the weighted residuals to check the previous computations:

$$\sum w_i v_i = 0$$

6) Compute the mean error of the unit weight:

$$\mu = \sqrt{\frac{\sum w_i v_i^2}{n-1}}$$

The mean error of unit weight is: ± 14,1 mm

7) Compute the mean error of the adjusted value:

$$m_0 = \frac{\mu}{\sqrt{\sum w_i}} = \frac{\pm 14{,}1}{\sqrt{15}} = \pm 3{,}64\ mm.$$

Working exercises

Compute the mean value of the observations. Each observation has been made with different instrumentation or observation procedure, therefore they have different mean error. Compute the appropriate weighting and the mean error of the adjusted mean value as well.

1) $l_1 =$ 211-05-32 $m_1 = \pm$ 8" $l_2 =$ 211-05-24 $m_2 = \pm$ 2"
$l_3 =$ 211-05-35 $m_3 = \pm$ 4" $l_4 =$ 211-05-28 $m_4 = \pm$ 2"

2) $l_1 =$ 70,710 m $m_1 = \pm$ 4 mm $l_2 =$ 70,718 m $m_2 = \pm$ 12 mm
$l_3 =$ 70,717 m $m_3 = \pm$ 6 mm

Solutions:

1) l_0 = 211-05-27,1 $w_1 =$ 1 $''^{-2}$ $w_2 =$ 16 $''^{-2}$
$w_3 =$ 4 $''^{-2}$ $w_4 =$ 16 $''^{-2}$ $w_o =$ 37 $''^{-2}$
$\mu = \pm$ 12,1 $m_0 = \pm$ 2,0"

2) l_0 = 70,7126 m $w_1 =$ 9 mm^{-2} $w_2 =$ 1 mm^{-2} $w_o =$ 14 mm^{-2}
$w_3 =$ 4 mm^{-2} $\mu = \pm$ 9,2 $m_0 = \pm$ 2,4 mm